Examples: Joint Densities and Joint Mass Functions

Example 1: X and Y are jointly continuous with joint pdf

$$f(x,y) = \begin{cases} cx^2 + \frac{xy}{3} & \text{if } 0 \le x \le 1,\ 0 \le y \le 2 \\ 0, & \text{otherwise.} \end{cases}$$

(a). Find c. (b). Find $P(X+Y \ge 1)$. (c). Find marginal pdf's of X and of Y. (d). Are X and Y independent (justify!). (e). Find $E(e^X \cos Y)$. (f). Find $cov(X,Y)$.

Example 2: X and Y are jointly continuous with joint pdf

$$f(x,y) = \begin{cases} cxy & \text{if } 0 \le x,\ 0 \le y,\ x+y \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

(a). Find c. (b). Find $P(Y \ge X)$. (c). Find marginal pdf's of X and of Y. (d). Are X and Y independent (justify!).

Example 3: X and Y are jointly continuous with joint pdf

$$f(x,y) = \begin{cases} cxy & \text{if } 0 \le x \le 1,\ 0 \le y \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

(a). Find c. (b). Find $P(|Y - 2X| \le 0.1)$. (c). Find marginal pdf's of X and of Y. (d). Are X and Y independent (justify!).

Example 4: X and Y are independent continuous random variables, each with pdf

$$g(w) = \begin{cases} 2w & \text{if } 0 \le w \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

(a). Find $P(X+Y \le 1)$. (b). Find the cdf and pdf of $Z = X+Y$.

Example 5: X and Y are jointly continuous with joint pdf

$$f(x,y) = \begin{cases} e^{-(x+y)} & \text{if } 0 \le x,\ 0 \le y \\ 0, & \text{otherwise.} \end{cases}$$

Let $Z = X/Y$. Find the pdf of Z.

Example 6: X and Y are independent, each with an exponential(λ) distribution. Find the density of $Z = X+Y$ and of $W = Y - X^2$.

Example 7: X and Y are jointly continuous with (X,Y) uniformly distributed over the union of the two squares $\{(x,y) : 0 \le x \le 1, 0 \le y \le 1\}$ and $\{(x,y) : 0 \le x \le 1, 3 \le y \le 4\}$.
(a). Find $E(Y)$. (b). Find the marginal densities of X and Y. (c). Are X and Y independent? (d). Find the pdf of $Z = X+Y$.

Example 8: X and Y have joint density

$$f(x,y) = \begin{cases} x+y & \text{if } 0 \le x,y \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

Find the joint cdf, $F_{X,Y}(x,y)$, for all x and y. Compute the covariance and correlation of X and Y.

Example 9: Suppose that X and Y have joint mass function as shown in the table below. (Here, X takes on possible values in the set $\{-2, 2\}$, Y takes on values in the set $\{-2, 0, 2, 3.1\}$.)

	-2	0	2	3.1
-2	.04	.08	.12	.16
2	.06	.12	.18	.24

(a). (6 points) Compute $P(|X + Y^2| < 1)$. (b). (6 points) Find the marginal mass function of Y and plot it. (be very explicit!) (c). (6 points) Compute $var(X^2 - Y)$ and $cov(X, Y)$. (d). (2 points) Are X and Y independent? (Why or why not?)

Example 10: Two fair dice are rolled. Let X be the larger of the two values shown on the dice, and let Y be the absolute value of the difference of the two values shown. Give the joint pmf of X and Y. Compute $cov(X, Y)$, $E(X)$, $E(Y^X)$, $P(X > 2Y)$.

Example 11: Alice and Bob plan to meet at a cafe to do AMS311 homework together. Alice arrives at the cafe at a random time (uniform) between noon and 1:00pm today; Bob independently arrives at a random time (uniform) between noon and 2:00pm today. (a). What is the expected amount of time that somebody waits for the other? (b). What is the probability that Bob has to wait for Alice?

Example 12: Suppose X and Y are independent and that X is exponential with mean 0.5 and Y has density

$$f_Y(y) = \begin{cases} 3e^{-3y} & \text{if } y > 0 \\ 0 & \text{otherwise} \end{cases}$$

Find the density of the random variable $W = \min\{X, Y\}$ and the random variable $Z = X + Y$.