Firm Heterogeneity and the Melitz Model

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International Trade I at ITAM

December 18, 2017

1 Explaining firm level heterogeneity

Melitz (2003) provides a theory of international trade that accounts for many facts documented recently at the firm level as, for example, in Bernard, Jensen, and Lawrence (1995) and Bernard, Jensen, Redding, and Schott (2007), namely:

- within each industry there is multidimensional firm-level heterogeneity;
- exporting is a relatively rare within an industry;
- fixed costs seem to play an important role; more productive firms are more likely to export;
- trade liberalization leads to reallocation at the sectoral level.

Two main modeling features are included: monopolistic competition with increasing returns to scale as in Krugman (1980); and firm entry and exiting as in Hopenhayn (1992).

The Melitz's model extends Krugman's by allowing firms to be heterogenous. Also, because there will be an additional fixed cost of exporting, only the most productive firms will select into exports once free trade is allowed. This allows for an additional source of gains from trade if trade induces reallocation of labor from the least to the most productive firms.

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1.1 The Melitz's model

As stated before, the model builds from Krugman's and adds heterogeneity plus an additional equilibrium condition of entry and exiting. We start by characterizing the model implications under autarky and then open up the economy to free trade.

Household We'll follow the Krugman's model and assume the economy has a fixed supply of L workers, each supplying a single unit of labor. Each worker enjoys consumption of N varieties of the same good following utility function:

$$C = \left(\sum_{i=1}^{N} c_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{1}$$

We assume that$\sigma > 1$ is the elasticity of substitution and c_i is the level of consumption of variety i. Note that in this framework, both c_i and N will be endogenous variables of the model (more about them later).

As usual, the problem for each worker consists in minimizing her expenditure subject to a particular utility level:

$$\min_{\{c_i\}_{i=1}^{N}} \left\{ \sum_{i=1}^{N} p_i c_i \quad st \quad \left(\sum_{i=1}^{N} c_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \geq \bar{C} \right\}$$

First order conditions imply the following demand functions for each $i = 1, ..., N$:[1]

$$c_i = \left(\frac{p_i}{P} \right)^{-\sigma} C \tag{2}$$

where

$$P = \left(\sum_{i=1}^{N} p_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{3}$$

$$C = \left(\sum_{i=1}^{N} c_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{4}$$

For later purposes, it is useful to derive the demand equation (2) in terms of expenditure

[1]For the derivations of the demand functions, refer to the Armington model handout.

level per variety consumed i. That is, multiplying p_i in both sides of (2) yields:

$$\begin{aligned}
c_i &= \left(\frac{p_i}{P}\right)^{-\sigma} C \\
\Rightarrow \quad p_i c_i &= p_i \left(\frac{p_i}{P}\right)^{-\sigma} \frac{P}{P} C \\
\Rightarrow L p_i c_i &= \left(\frac{p_i}{P}\right)^{1-\sigma} LPC \\
\Rightarrow \quad r_i &= \left(\frac{p_i}{P}\right)^{1-\sigma} R
\end{aligned} \tag{5}$$

where r_i is the expenditure of all consumers on variety i and R is her the level of aggregate expenditure as, with a CES utility function, $LPC = \sum_{i=1}^{N} L p_i c_i \equiv R$.

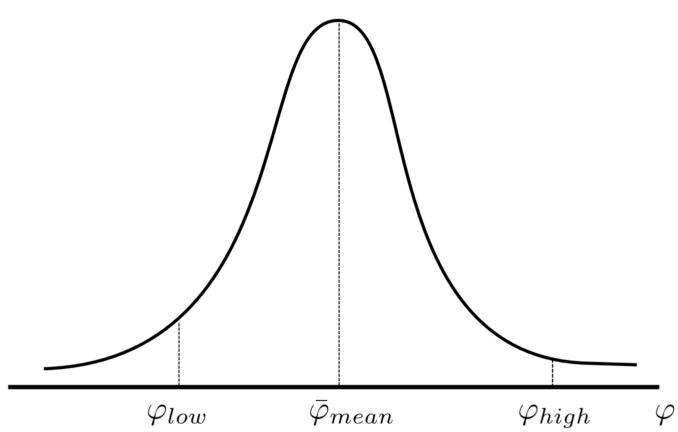
Finally, each worker is constraint to her budget constraint, meaning that, individually, expenditure must equal income:

$$\sum_{i=1}^{N} p_i c_i = w + \ddot{\pi} \tag{6}$$

To simplify our later algebra, we will normalize $w = 1$. That implies that all prices that we will further derive below should be interpreted as a the price of the good relative to labor. The $\ddot{\pi}$ is a measure of residual profits, which can be potentially positive, but we'll see that in equilibrium will equate zero.

Firms Contrary to what we assumed in the Krugman's model, we allow now explicitly for firm heterogeneity. In particular, we will assume that there is a non degenerate distribution of productivities φ across the economy as depicted in figure 1.

Figure 1: Example of a distribution of firms productivities



This implies that we'll now index each firms technology by φ. It follows that for a firm with technology φ, wishing to produce variety i, its labor requirements amount to:

$$l_i(\varphi) = f + q_i/\varphi \tag{7}$$

Profit maximization implies that firms choose the optimal price that maximizes the following function:

$$\begin{aligned}
\pi_i(\varphi) &= p_i(\varphi)\, q_i - l_i(\varphi) \\
&= p_i(\varphi)\, q_i - (f + q_i/\varphi) \\
&= p_i(\varphi)\, c_i L - (f + c_i L/\varphi) \\
&= L\left(\frac{p_i(\varphi)}{P}\right)^{-\sigma} C p_i - \left(f + L\left(\frac{p_i(\varphi)}{P}\right)^{-\sigma} C/\varphi\right)
\end{aligned}$$

Maximization of the above expression with respect to $p_i(\varphi)$ yields the usual formula[2]:

$$\begin{aligned}
p_i(\varphi) &= \frac{\sigma}{\sigma - 1} \cdot \frac{1}{\varphi} \\
&= \frac{1}{\rho\varphi}
\end{aligned} \tag{8}$$

here $\rho = (\sigma - 1)/\sigma$ is just a variable change to simplify our further notation. Note from the right hand side of equation (8) that the optimal price depends only on φ and not in i. That means that a firm producing any variety $i = 1, ..., N$ decides its optimal price just based on its productivity. For that reason, we can drop the subscripts on the optimal prices:

$$p_i(\varphi) = p(\varphi)$$

Note that the individual expenditure represented in (5) equals to the individual revenue that a firms generate from selling one variety. Then, if we substitute the optimal price (8)

[2]For details of the derivation please have a look at the Krugman's model.

on (5), we get the optimal revenue function:

$$
\begin{aligned}
r(\varphi) &= \left(\frac{p(\varphi)}{P}\right)^{1-\sigma} R \\
\Rightarrow r(\varphi) &= \left(\frac{1}{P\varphi\rho}\right)^{1-\sigma} R \\
\Rightarrow r(\varphi) &= (P\varphi\rho)^{\sigma-1} R
\end{aligned} \tag{9}
$$

Note also that, by definition, $r(\varphi) = Lp(\varphi) \cdot c(\varphi)$. On its turn, (9) implies that the optimal profit function is linear in the revenue:

$$
\begin{aligned}
\pi(\varphi) =& p(\varphi) q(\varphi) - l(\varphi) \\
=& r(\varphi) - f - Lc(\varphi)/\varphi \\
=& r(\varphi) - f - \frac{r(\varphi)}{\varphi \cdot p(\varphi)} \\
=& r(\varphi) - f - \frac{r(\varphi)}{\varphi} \cdot \varphi\rho \\
=& (1-\rho) r(\varphi) - f \\
=& r(\varphi)/\sigma - f
\end{aligned} \tag{10}
$$

and, in the last equality, we made use of $\rho = (\sigma - 1)/\sigma \Rightarrow 1/\sigma = 1 - \rho$. Note that (10) implies that the optimal profit is a linear function of the revenues. From equations (2), (9), and (10), three comments are in order:

1. Firms with higher productivity produce more: let $\varphi_2 > \varphi_1$, then:

$$
\begin{aligned}
\frac{q(\varphi_2)}{q(\varphi_1)} &= \frac{Lc(\varphi_2)}{Lc(\varphi_1)} \\
&= \frac{r(\varphi_2)/p(\varphi_2)}{r(\varphi_1)/p(\varphi_1)} \\
&= \frac{(P\varphi_2\rho)^{\sigma-1} \varphi_2\rho}{(P\varphi_1\rho)^{\sigma-1} \varphi_1\rho} \\
&= \left(\frac{\varphi_2}{\varphi_1}\right)^{\sigma} > 1
\end{aligned}
$$

2. Firms with higher productivity generate larger revenues: let $\varphi_2 > \varphi_1$, then:

$$\begin{aligned}\frac{r(\varphi_2)}{r(\varphi_1)} &= \frac{(P\varphi_2\rho)^{\sigma-1}}{(P\varphi_1\rho)^{\sigma-1}} \\ &= \left(\frac{\varphi_2}{\varphi_1}\right)^{\sigma-1} > 1 \end{aligned} \tag{11}$$

3. Firms with higher productivity have larger profits: this is immediate from the linearity of $\pi(\varphi)$ in $r(\varphi)$ reflected in (10).

Aggregation Given our optimal formulas for demand and supply, we are in order to derive some expressions for the aggregate variables in prices P, consumption C, revenues R, and profits Π. This exercise will allow us to better understand the differences and similarities that emerge between the Melitz's and the Krugman's model.

Starting with prices, by the definition of our perfect price level in (3), we have that:

$$\begin{aligned}
&P = \left(\sum_{i=1}^{N} p_i^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \\
&\Rightarrow P = N^{\frac{1}{1-\sigma}}\left(\frac{1}{N}\sum_{i=1}^{N} p_i^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \\
&\Rightarrow P = N^{\frac{1}{1-\sigma}}\left(\frac{1}{N}\sum_{i=1}^{N}\left(\frac{1}{\rho\varphi_i}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \\
&\Rightarrow P = N^{\frac{1}{1-\sigma}}\frac{1}{\rho}\left(\frac{1}{N}\sum_{i=1}^{N}(\varphi_i)^{\sigma-1}\right)^{\frac{1}{1-\sigma}} \\
&\Rightarrow P = N^{\frac{1}{1-\sigma}}\frac{1}{\rho}\cdot\frac{1}{\left(\frac{1}{N}\sum_{i=1}^{N}(\varphi_i)^{\sigma-1}\right)^{\frac{1}{\sigma-1}}} \\
&\Rightarrow P = N^{\frac{1}{1-\sigma}}\frac{1}{\rho\tilde{\varphi}} = N^{\frac{1}{1-\sigma}}p(\tilde{\varphi})
\end{aligned} \tag{12}$$

where we defined $\tilde{\varphi} = \left(\frac{1}{N}\sum_{i=1}^{N}(\varphi_i)^{\sigma-1}\right)^{\frac{1}{\sigma-1}}$. Note that $\tilde{\varphi}$ can be interpreted as the average productivity of the firms that operate in the market (an weighted average in accordance with the CES utility function). Note that (12) is exactly the same expression that we have for the P in the Krugman model if we imposed a level of productivity $\tilde{\varphi}$. This implies that the Melitz model collapses into the Krugman model for the "average" firm operating in the market with productivity $\tilde{\varphi}$. Of course, we still need to determine $\tilde{\varphi}$ and N which are endogenous variables in our model (more about this in later sections).

Similarly to what we did for P, other aggregation formulas can be derived, for example, for the aggregate consumption C per worker:

$$\begin{aligned} c(\tilde{\varphi}) &= \left(\frac{p(\tilde{\varphi})}{P}\right)^{-\sigma} C \\ \Rightarrow c(\tilde{\varphi}) &= (P\tilde{\varphi}\rho)^{\sigma} C \\ \Rightarrow c(\tilde{\varphi}) &= N^{\frac{\sigma}{1-\sigma}} C \\ \Rightarrow \quad C &= N^{\frac{\sigma}{\sigma-1}} c(\tilde{\varphi}) \end{aligned} \tag{13}$$

or for the aggregate revenue R:

$$\begin{aligned} R &= LPC \\ \Rightarrow R &= N^{\frac{\sigma}{\sigma-1}} Lc(\tilde{\varphi}) N^{\frac{1}{1-\sigma}} p(\tilde{\varphi}) \\ \Rightarrow R &= Nr(\tilde{\varphi}) \end{aligned} \tag{14}$$

or even for aggregate profits Π:

$$\begin{aligned} \pi(\tilde{\varphi}) &= r(\tilde{\varphi})/\sigma - f \\ \Rightarrow \quad \pi(\tilde{\varphi}) &= R/N/\sigma - f \\ \Rightarrow N\pi(\tilde{\varphi}) &= R/\sigma - Nf \\ \Rightarrow \quad \Pi &= N\pi(\tilde{\varphi}) \end{aligned} \tag{15}$$

Again, note that formulas (13), (14), and (15) are all consistent with the Krugman model when productivity equals $\tilde{\varphi}$. In order to determine $\tilde{\varphi}$ and N, we need to define how entry and exit works in this market. This is done in the next section.

Firm entry and exit As stated before $\tilde{\varphi}$ and N are equilibrium objects in the Melitz's model. In order to determine these, we need to specify how firms decide whether to operate or not in this economy. The modeling approach used is similar to the one introduced by Hopenhayn (1992), namely:

1. There is a potential large number of entrants

2. Each entrant decides whether or not to pay a fixed cost f_e to sample a productivity

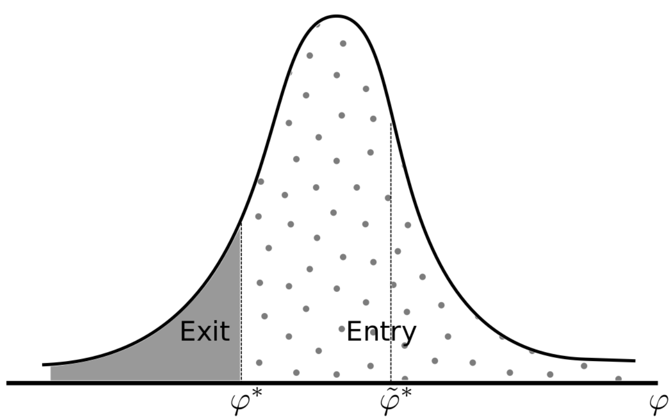
level from a known distribution discrete distribution $G(\varphi)$ with $p_j = Prob\,(\varphi = \dot{\varphi}_j)$ for $j = 1, ..., M$ and $\dot{\varphi}_j > \dot{\varphi}_i$ for $j > i$. That is, before they pay f_e, entrepreneurs don't know about their exact productivity.

3. Knowing their productivity φ, they decide whether or not to produce

4. Active firms die with an exogenous probability δ

To understand how does this mechanism of entry works, note that if a firms produces, it has to pay a fixed cost equal to f. Because profits are an increasing function of productivity, entrants that turn out to very unproductive decide not to enter the market as they would have generated negative profits. All the remaining firms are the ones that decide to stay and operate in the market (as in figure 2).

Figure 2: Entry in the Melitz's model



Note that the value of staying in the market for a firm that already payed f_e to sample its productivity amounts to:

$$
\begin{aligned}
V(\varphi) &= \max\left\{0, \sum_{t=1}^{\infty} (1-\delta)^t \pi(\varphi)\right\} \\
&= \max\left\{0, \pi(\varphi)/\delta\right\}
\end{aligned}
$$

This last equation essentially means that firms only operate in the market if they can generate positive profits (as exiting the market is always possible which would generate zero profits), and, given the exogenous exiting probability δ, their expected lifetime at entry is $1/\delta$. For this reason, the condition for stay in the market is:

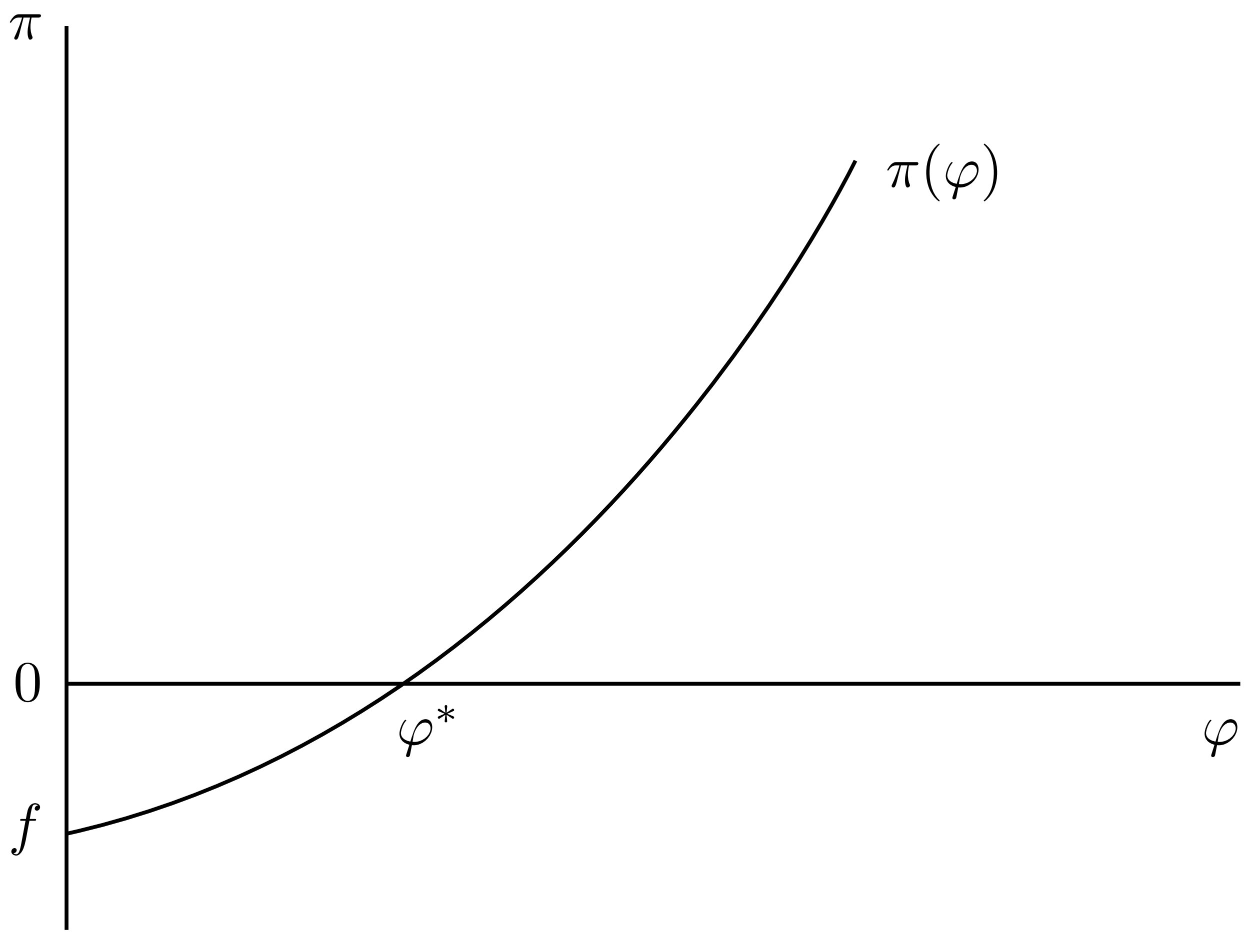
$$V(\varphi) > 0$$

but because $\pi(\varphi)$ is an increasing function in φ, there must be a cut-off productivity φ^* such that firms are indifferent between exiting and staying at the market:

$$\exists\varphi^* : \quad \pi(\varphi^*) = 0 \Rightarrow V(\varphi^*) = 0$$

This property of the profits function can be seen in the diagram of figure 3.

Figure 3: Profit function and the cut-off productivity φ^*



It is easy to see that for any $\varphi > \varphi^*$, firms decide to enter the market, while for $\varphi < \varphi^*$, firms decide to exit.

In order to determine the expected benefit of paying the fixed cost of survey the productivity f_e in advance, we will need to specify a particular distribution of productivities. With that distribution at hand, we'll be able to compute the probability $Prob(\varphi > \varphi^*)$, a fundamental object for the characterization of the final equilibrium.

For that end, let's assume that we have a distribution of M different productivities, characterized by the following sequence: $\varphi_1 = \gamma > 1$, $\varphi_2 = \gamma^2$, $\varphi_3 = \gamma^3$,..., $\varphi_M = \gamma^M$. That is:

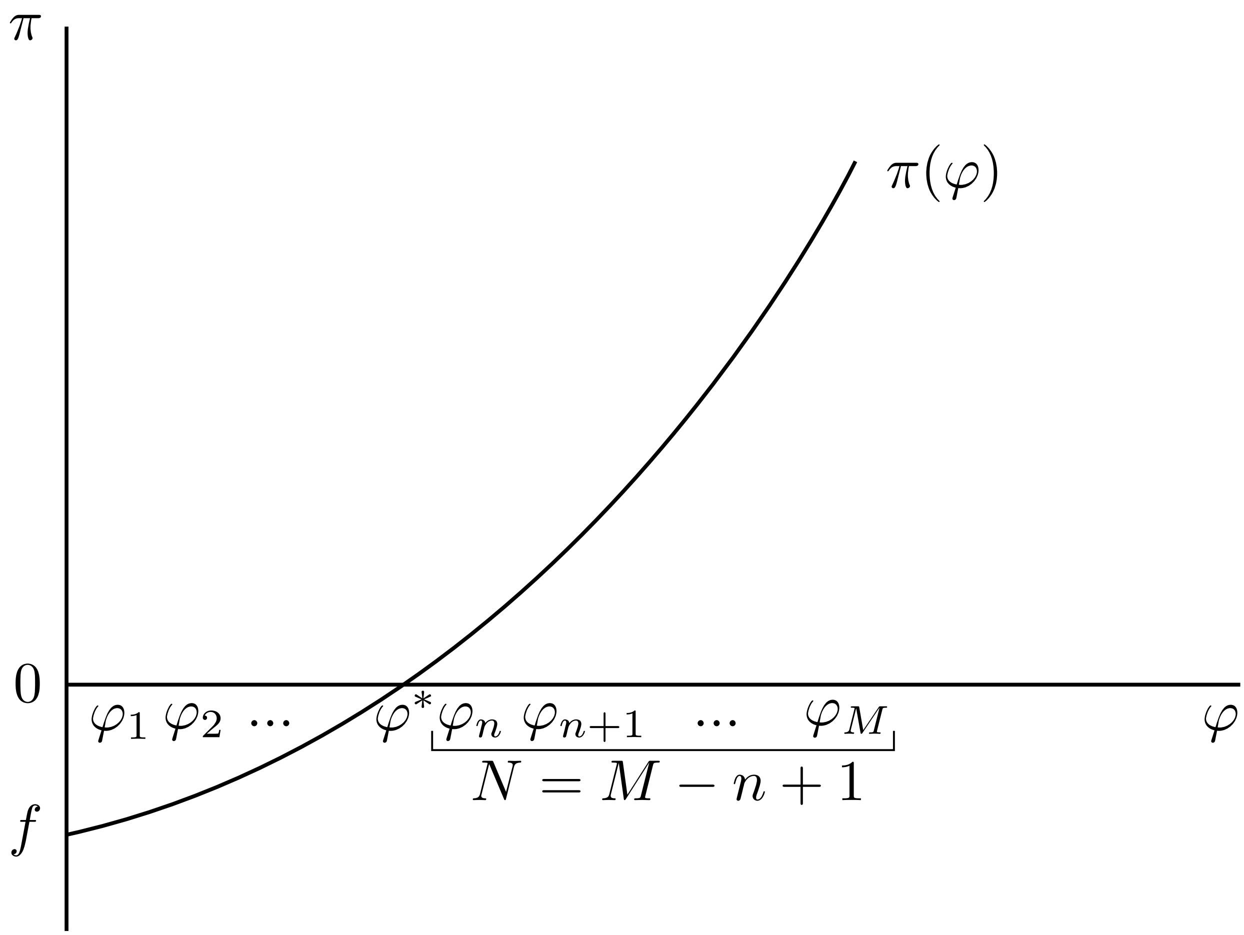
$$\begin{aligned} \varphi_1 &= \gamma > 1 \\ \varphi_j &= \gamma \varphi_{j-1} \quad for \quad j = 2, ..., M \end{aligned}$$

In terms of probabilities, we assume a uniform distribution:

$$Prob\,(\varphi_i) = 1/M \quad \forall i = 1, ..., M$$

Note from figure 4 that this distribution implies that only the $N = M - n + 1$ elements at the right of φ^* are acceptable for potential entrants.

Figure 4: Profit function and the cut-off productivity φ^*



It follows that

$$Prob(\varphi > \varphi^*) \equiv P^N(\varphi^*) = \frac{N(\varphi^*)}{M}$$

where, it is clear that N is decreasing with φ^* - such probability corresponds to the dot area of figure 2.

We are now able to specify the free-entry condition. Essentially, potential entrants are willing to pay the fix cost f_e if their expect value of sample their productivity is larger than zero:

$$EV = \underbrace{0 \cdot \left[1 - P^N(\varphi^*)\right] + P^N(\varphi^*)\,\tilde{\pi}/\delta}_{expected\ benefit} - \underbrace{f_e}_{cost}$$

Equating $EV = 0$ yields:

$$\begin{aligned} EV &= 0 \\ \Rightarrow \tilde{\pi} &= \frac{\delta f_e}{P^N\left(\varphi^*\right)} \end{aligned} \tag{FE}$$

Equation (FE) is known as the free-entry condition and relates the average profit for firms operating in the market $\tilde{\pi}$ with the cutoff productivity φ^*. This is an increasing function as entrants need to be remunerated with larger profits whenever the conditions to enter the market become harsher (all else equal).

We should suspect that we need an additional equation to be able to determine both φ^* and $\tilde{\pi}$. Such equation can bee derived from the profit maximization condition, summarized in (10):

$$\begin{aligned} \tilde{\pi} \equiv \pi\left(\tilde{\varphi}\right) &= r\left(\tilde{\varphi}\right)/\sigma - f \\ &= f\left[\frac{r\left(\tilde{\varphi}\right)}{f\sigma} - 1\right] \end{aligned} \tag{16}$$

Note also that at φ^*:

$$\pi\left(\varphi^*\right) = r\left(\varphi^*\right)/\sigma - f = 0 \Rightarrow r\left(\varphi^*\right) = f\sigma \tag{17}$$

Meaning that we can substitute $f\sigma$ into (16) to get:

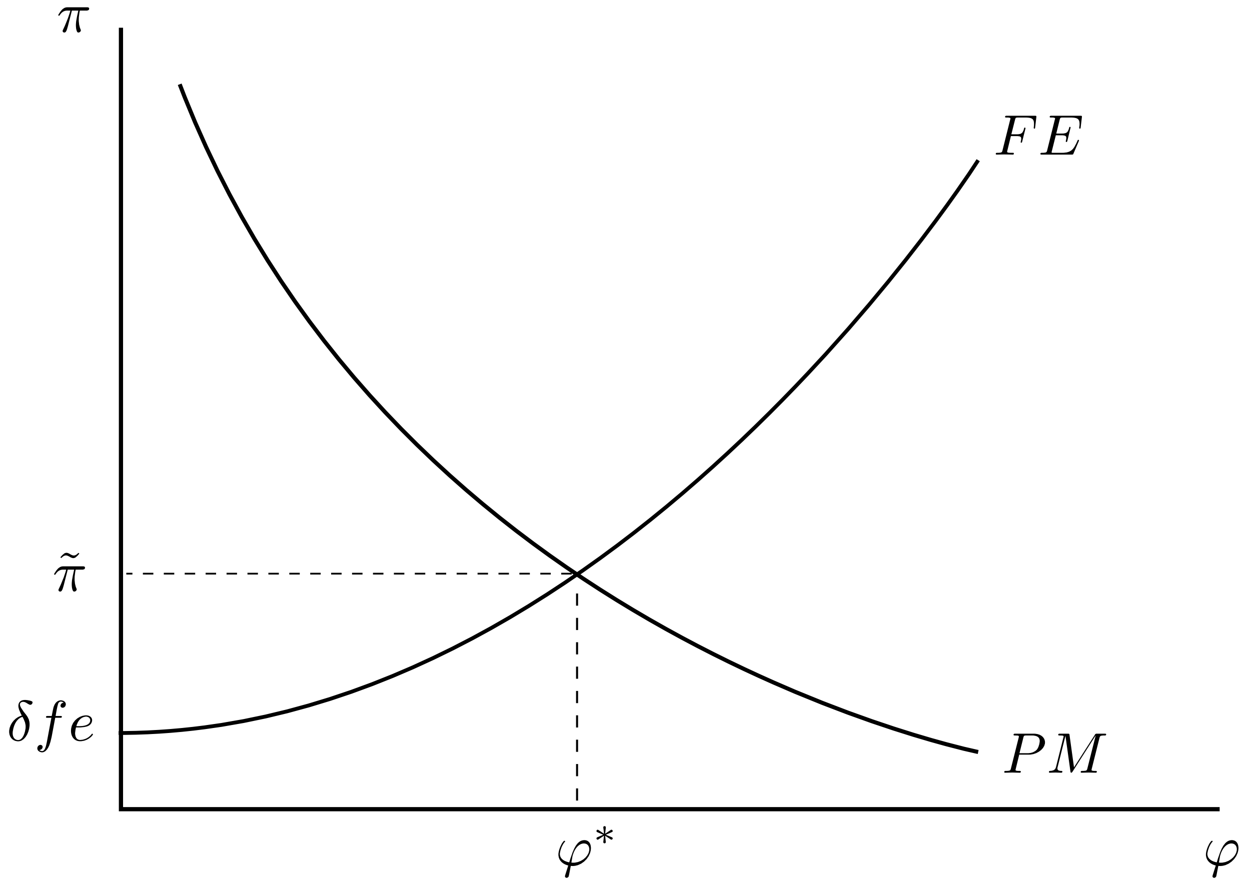
$$\begin{aligned} \tilde{\pi} &= f\left[\frac{r\left(\tilde{\varphi}\right)}{f\sigma} - 1\right] \\ \Rightarrow \tilde{\pi} &= f\left[\frac{r\left(\tilde{\varphi}\right)}{r\left(\varphi^*\right)} - 1\right] \\ \Rightarrow \tilde{\pi} &= f\left[\left(\frac{\tilde{\varphi}\left(\varphi^*\right)}{\varphi^*}\right)^{\sigma-1} - 1\right] \end{aligned} \tag{PM}$$

It's easy to show that (PM) is decreasing in φ^*[3]. Intuitively, after being in the market, harder market conditions reduces profits for all the operating firms.

But then, since we have a (FE) which is increasing in φ^* and a (PM) that is decreasing in φ^*, equilibrium of the model is given by the intersection of the two curves as show in figure 5.

[3] To see that, note that the sign of $\tilde{\pi}$ with respect to φ^* depends only on $\tilde{\varphi}\left(\varphi^*\right)/\varphi^*$. From the definition

Figure 5: Market equilibrium in the Melitz's model



Comparative static exercises are done in the usual way. For example, let's say that we

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of $\tilde{\varphi}(\varphi^*)/\varphi^*$:

$$
\begin{aligned}
\tilde{\varphi}(\varphi^*)/\varphi^* &= \frac{1}{\varphi^*}\left(\sum_{i=1}^{N}\frac{1}{N}\varphi_i^{\sigma-1}\right)^{\frac{1}{\sigma-1}} \\
&= \frac{1}{\varphi^*}\left(\frac{\varphi_n^{\sigma-1}+\varphi_n^{\sigma-1}\gamma^{\sigma-1}+\varphi_n^{\sigma-1}\gamma^{2(\sigma-1)}+...+\varphi^{\sigma-1}\gamma^{(N-1)(\sigma-1)}}{N}\right)^{\frac{1}{\sigma-1}} \\
&= \frac{\varphi_n}{\varphi^*}\left(\frac{1+\gamma^{\sigma-1}+\gamma^{2(\sigma-1)}+...++\gamma^{N(\sigma-1)}}{N}\right)^{\frac{1}{\sigma-1}} \\
&\approx \left(\frac{1-\gamma^{N(\sigma-1)}}{1-\gamma^{\sigma-1}}\cdot\frac{1}{N}\right)^{\frac{1}{\sigma-1}}
\end{aligned}
$$

And note that $\tilde{\varphi}(\varphi^*)/\varphi^*$ is increasing in N, but N is decreasing in φ^*, which implies that $\tilde{\varphi}(\varphi^*)/\varphi^*$ is decreasing in φ^*. One can show that this property in not exclusively of the particular distribution that we are using, but it also includes many know continuous distributions (see Melitz, 2003).

have an increase in f_e, then the curve (FE) shifts up, leading to a higher $\tilde{\pi}$ and a lower φ^*. Intuitively, less firms survey the market, implying lower competition, but then we have more profits. As another example, we can think of an increase in f. This actually implies that the (PM) increases, leading to higher profits and a higher φ^*: more firms exit the market, which implies that only the most productive firms stay in the market, leading to an increase in profits.

Market clearing conditions To conclude the characterization of the equilibrium, we still need to impose some market clearing conditions. Because we have two main markets - market for varieties and market for labor - we need the following set of expressions that essentially equate supply and demand:

$$q_i = Lc_i \tag{varieties markets}$$

$$L = \sum_{i=1}^{N} l_i = Nf + \sum_{i=1}^{N} q_i/\varphi_i + N\frac{\delta f_e}{P^N} \tag{labor market}$$

where the last term in (labor market) is the labor that is required to pay for the fixed cost of surveying the market (because in the equilibrium we have N firms operating in the market, it must be that $N\delta$ of those die in every period, and therefore $N\delta/P^N$ new firms have to survey the market by paying f_e).

Equilibrium number of varieties To determine the number of varieties, we make use of the (labor market) condition and substitute in the equilibrium conditions:

$$\begin{aligned}
L =& Nf + \sum_{i=1}^{N} q_i/\varphi_i + N\frac{\delta f_e}{P^N} \\
\Rightarrow L =& Nf + \sum_{i=1}^{N} Lc(\varphi_i)/\varphi_i + N\tilde{\pi} \\
\Rightarrow L =& Nf + \rho\sum_{i=1}^{N} r(\varphi_i)/p(\varphi_i)/(\rho\varphi_i) + N\tilde{\pi} \\
\Rightarrow L =& Nf + \rho\sum_{i=1}^{N} r(\varphi_i)/p(\varphi_i)/(\rho\varphi_i) + N\tilde{\pi} \\
\Rightarrow L =& Nf + \rho\sum_{i=1}^{N} r(\varphi_i) + N\tilde{\pi} \\
\Rightarrow L =& Nf + \rho R + N\tilde{\pi} \\
\Rightarrow L =& Nf + (1-1/\sigma) N(\sigma\tilde{\pi} + \sigma f) + N\tilde{\pi} \\
\Rightarrow N =& \frac{L}{\sigma(\tilde{\pi} + f)}
\end{aligned}$$

Note that in the Krugman model $N = L/(\sigma f)$. The difference compared with this model is that, since operating firms make positive profits in equilibrium, the number of varieties won't be as large.

A different way to derive the same result is to note that in equilibrium, expenditure equals income, that is:

$$\begin{aligned}
R =& L \\
\Rightarrow \quad N r(\tilde{\varphi}) =& L \\
\Rightarrow N\sigma(\pi(\tilde{\varphi}) + f) =& L \\
\Rightarrow \quad N =& \frac{L}{\sigma(\tilde{\pi} + f)}
\end{aligned}$$

Price index and utility Given the CES utility structuring in (1), we should be able to analyze the welfare shocks in this economy by just evaluating the price index as (when $w = 1$):

$$C|_{E=1} = 1/P$$

Recall from our aggregation results (12) that:

$$\begin{aligned} P &= N^{\frac{1}{1-\sigma}} p(\tilde{\varphi}) \\ &= N^{\frac{1}{1-\sigma}} p(\tilde{\varphi}) \\ &= N^{\frac{1}{1-\sigma}} p\left[(\tilde{\pi}/f+1)^{\frac{1}{\sigma-1}} \varphi^*\right] \end{aligned} \tag{18}$$

This is, in general a more difficult equation to analyze that the equivalent in the Krugman model. However, we can see, for example, that an increase in f_e, which implies a higher $\tilde{\pi}$ and a lower φ^*, should have a negative impact in an lower number of varieties, and an ambiguous impact on the average price of the the final good. We can actually show that higher f_e leads to a higher price index, that is, lower welfare. To see that note that under autarky

$$\left(\frac{\tilde{\varphi}}{\varphi^*}\right)^{\sigma-1} = \frac{r(\tilde{\varphi})}{r(\varphi^*)} = \frac{R/N}{f\sigma} = \frac{L/N}{f\sigma}$$

here we made use of (11); $R = r(\tilde{\varphi})N$ in (14); that at φ^*, $r(\varphi^*) = f\sigma$ in (17), and finally that $L = R$ (this last from the fact that income wL must equal sales revenues R at the aggregate). Substituting this in the price index:

$$\begin{aligned} P &= N^{-\frac{1}{\sigma-1}} \frac{1}{\rho\tilde{\varphi}} \\ &= N^{-\frac{1}{\sigma-1}} \cdot \frac{1}{\rho} \cdot \left(\frac{L/N}{f\sigma}\right)^{-\frac{1}{\sigma-1}} \cdot \frac{1}{\varphi^*} \\ &= \left(\frac{L}{f\sigma}\right)^{-\frac{1}{\sigma-1}} \cdot \frac{1}{\rho\varphi^*} \end{aligned}$$

But then, since φ^*, this implies that a lower f_e implies a larger consumer welfare.

1.2 The Melitz's model with free trade between two identical economies

To think about free trade in this model, two additional elements are included in the previous model: iceberg costs of exporting $\tau > 1$; and a fixed exporting cost f_{ex}. Moreover we will consider that the trading partner is symmetric to the home economy. Most of the features of the previous set up apply easily to free-trade.

Prices, Revenues, Profits Prices are derived as before, but now we need to take into account that goods that are exported will have an increment in the price equal to τ, the iceberg cost:

$$p_d(\varphi) = \frac{1}{\rho\varphi}$$
$$p_x(\varphi) = \frac{\tau}{\rho\varphi}$$

where p_d are prices of domestically produced goods, p_x are prices for exports.

Similarly revenues become:

$$r_d(\varphi) = (P\rho\varphi)^{\sigma-1} R$$
$$r_x(\varphi) = (P\rho\varphi)^{\sigma-1} \tau^{1-\sigma} R$$

and profits:

$$\pi_d(\varphi) = r_d(\varphi)/\sigma - f \tag{19}$$
$$\pi_x(\varphi) = r_x(\varphi)/\sigma - f_x \tag{20}$$

Aggregation As usual, the price index is given by a weighted sum of all variety prices in this economy (let $N = N^d + N^x$ be the total number of varieties with N^d, the domestically produced varieties while N^x the foreign produced varieties; and $a = N^d/N$ and $1 - a =$

N^x/N). Using the typical definition we have that:

$$
\begin{aligned}
P =& \left(\sum_{i=1}^{N} p_i^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \\
=& N^{\frac{1}{1-\sigma}} \left(\frac{1}{N}\sum_{i=1}^{N} p_i^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \\
=& N^{\frac{1}{1-\sigma}} \left(\frac{1}{N}\sum_{i=1}^{N^d} p_i^{1-\sigma} + \frac{1}{N}\sum_{i=1}^{N^x} p_i^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \\
=& N^{\frac{1}{1-\sigma}} \frac{1}{\rho} \left(\frac{N^d}{N} \cdot \frac{1}{N^d}\sum_{i=1}^{N^d} \varphi_i^{\sigma-1} + \frac{N^x}{N} \cdot \frac{1}{N^x}\sum_{i=1}^{N^x} (\varphi_i/\tau)^{\sigma-1}\right)^{\frac{1}{1-\sigma}} \\
=& N^{\frac{1}{1-\sigma}} \frac{1}{\rho} \left(a \cdot \frac{1}{N^d}\sum_{i=1}^{N^d} \varphi_i^{\sigma-1} + (1-a) \cdot \frac{1}{N^x}\sum_{i=1}^{N^x} (\varphi_i/\tau)^{\sigma-1}\right)^{\frac{1}{1-\sigma}} \\
=& N^{\frac{1}{1-\sigma}} \frac{1}{\rho} \left(a \cdot \left(\frac{1}{N^d}\sum_{i=1}^{N^d} \varphi_i^{\sigma-1}\right)^{\frac{\sigma-1}{\sigma-1}} + (1-a) \cdot \left(\frac{1}{N^x}\sum_{i=1}^{N^x} (\varphi_i/\tau)^{\sigma-1}\right)^{\frac{\sigma-1}{\sigma-1}}\right)^{\frac{1}{1-\sigma}} \\
=& N^{\frac{1}{1-\sigma}} \frac{1}{\rho} \left(a \cdot \tilde{\varphi}^{\sigma-1} + (1-a) \cdot \tilde{\varphi}_x^{\sigma-1}\right)^{\frac{1}{1-\sigma}} \\
=& N^{\frac{1}{1-\sigma}} \frac{1}{\rho} \frac{1}{\left(a \cdot \tilde{\varphi}^{\sigma-1} + (1-a) \cdot \tilde{\varphi}_x^{\sigma-1}\right)^{\frac{1}{\sigma-1}}} \\
\Rightarrow P =& N^{\frac{1}{1-\sigma}} \frac{1}{\rho \tilde{\varphi}_t}
\end{aligned}
\tag{21}
$$

where the average productivity of domestic firms is given by:

$$
\tilde{\varphi} = \left(\frac{1}{N^d}\sum_{i=1}^{N^d} \varphi_i^{\sigma-1}\right)^{\frac{1}{\sigma-1}}
$$

while the average productivity for exporting firms is:

$$
\tilde{\varphi}_x = \left(\frac{1}{N^x}\sum_{i=1}^{N^x} (\varphi_i/\tau)^{\sigma-1}\right)^{\frac{1}{\sigma-1}}
$$

and finally, the total productivity associated with all varieties consumed is given by:

$$\tilde{\varphi}_t = \left(a \cdot \tilde{\varphi}^{\sigma-1} + (1-a) \cdot \tilde{\varphi}_x^{\sigma-1}\right)^{\frac{1}{\sigma-1}}$$

Proceeding as before, once we know $\tilde{\varphi}_t$, we can compute all the relevant aggregate variables:

$$\begin{aligned}
P =& N^{-\frac{1}{\sigma-1}} \cdot \frac{1}{\rho\tilde{\varphi}_t} = N^{-\frac{1}{\sigma-1}} p\left(\tilde{\varphi}_t\right) \\
C =& N^{\frac{\sigma}{\sigma-1}} c(\tilde{\varphi}_t) \\
R =& N r(\tilde{\varphi}_t) \\
\Pi =& N \pi(\tilde{\varphi}_t)
\end{aligned}$$

We should note that N is the number of varieties produced by both domestic firms and foreign firms. That implies that R/N is not the average revenue in the domestic economy. Instead that average should be given by R/N^d as N^d represents the number of varieties (and firms) that are produced in the home economy. The relevant productivity average for these firms is $\tilde{\varphi}$. Therefore the average domestic revenues are given by:

$$\begin{aligned}
\frac{R}{N^d} &= r(\tilde{\varphi}) \\
&= r_d(\tilde{\varphi}) + P^{X|N} r_x(\tilde{\varphi}_x) \qquad (22)
\end{aligned}$$

where $P^{X|N}$ is the probability of exporting conditional on being in the market (this will be determined later). This essentially means that domestic firms have two different sources of revenues: from producing for domestic markets and from producing to foreign markets (if they are productive enough). Similarly, average domestic profits can be defined as:

$$\begin{aligned}
\frac{\Pi}{N^d} &= \pi(\tilde{\varphi}) \\
&= \pi_d(\tilde{\varphi}) + P^{X|N} \pi_x(\tilde{\varphi}_x)
\end{aligned}$$

Free entry After knowing its productivity, a firm's value equals:

$$V\left(\varphi\right) = \max\left\{0, \sum_{t=0}^{\infty}\left(1-\delta\right)^t \pi\left(\varphi\right)\right\} = \max\left\{0, \pi\left(\varphi\right)/\delta\right\}$$

where now profits can be derived from selling goods in domestic or export markets:

$$\pi(\varphi) = \pi_d(\varphi) + \max\{0, \pi_x(\varphi)\}$$

that is, the firm only exports if it can get positive profits by doing so. From the fact that profit functions are increasing with the productivity level, there must be two productivity thresholds φ^* and φ_x^* such that:

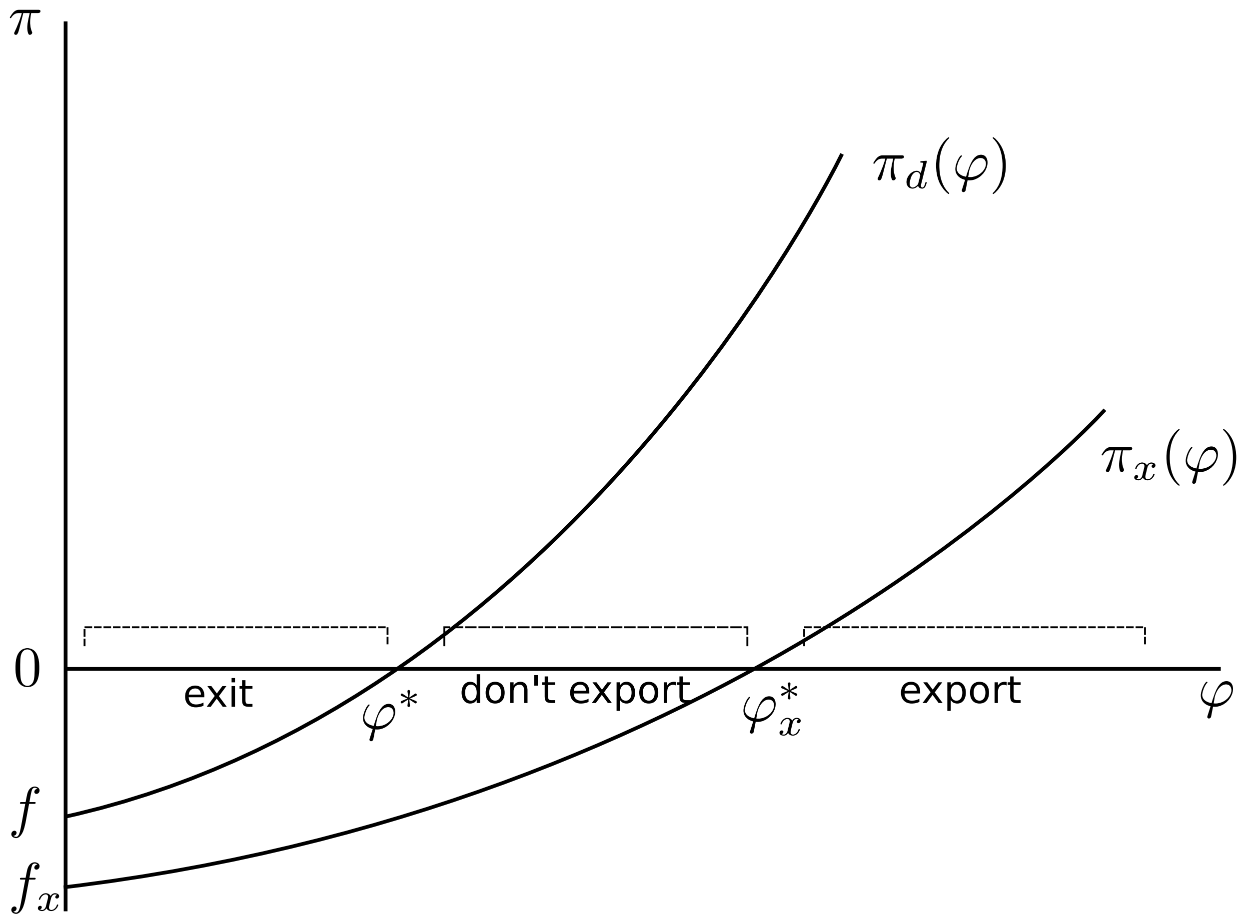
$$\begin{aligned}\pi(\varphi^*) &= 0 \\ \pi_x(\varphi_x^*) &= 0\end{aligned}$$

Note that as long as f_x is sufficiently larger (how much?) than f, $\varphi^* < \varphi_x^*$ which implies also that $\pi_d(\varphi^*) = 0$. But then:

$$\begin{aligned}\pi_d(\varphi) =& r_d(\varphi)/\sigma - f = (P\rho\varphi)^{\sigma-1} R/\sigma - f \\ \pi_x(\varphi) =& r_x(\varphi)/\sigma - f_x = (P\rho\varphi)^{\sigma-1} R\tau^{1-\sigma} - f_x\end{aligned}$$

To understand how does the selection play a role in this model, the next figure plots both these profit functions.

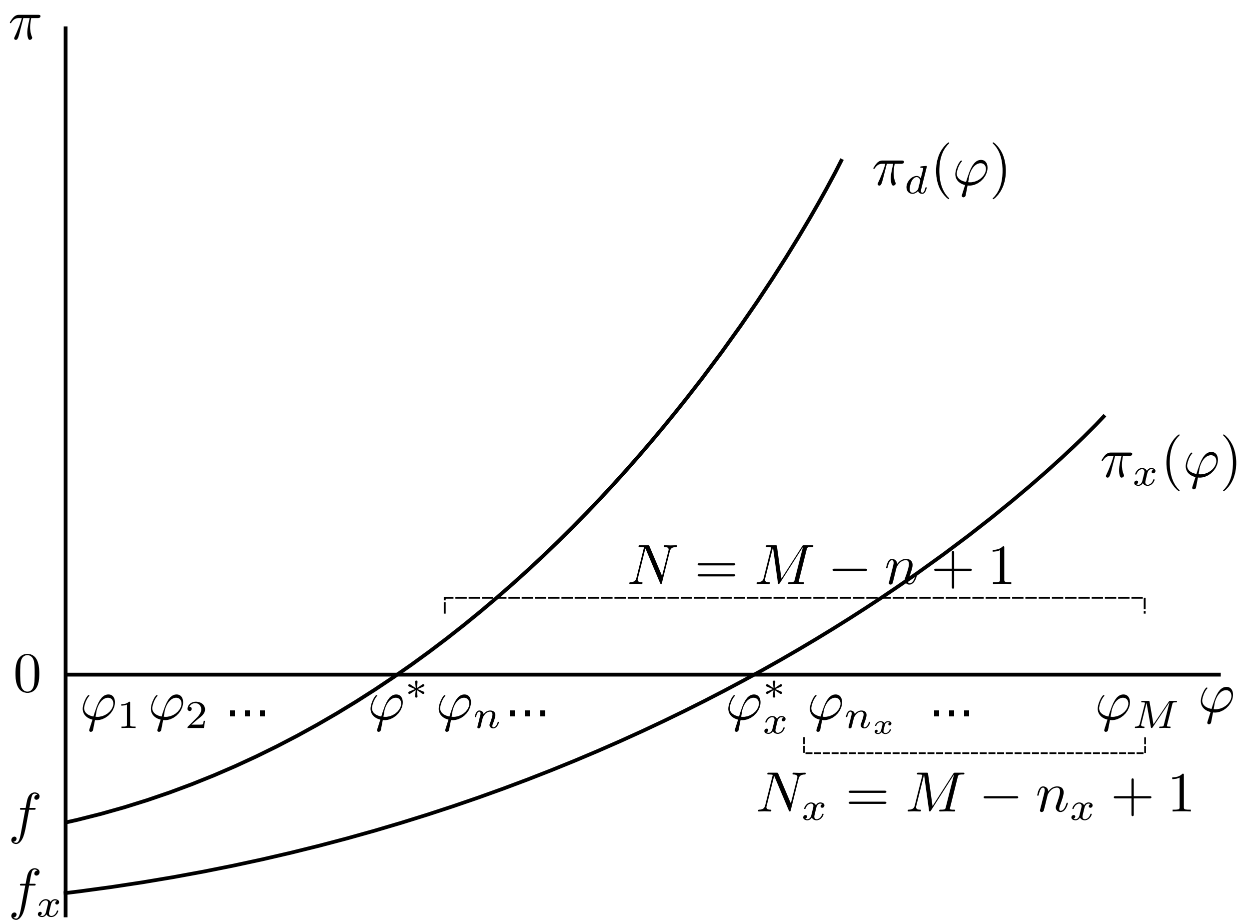
Figure 6: Profits in the Melitz model with free-trade



The figure shows that only more productive firms select into export markets as they are the only ones that make positive profits in those markets. At the same time medium productivity firms produce only for domestic markets, while low productivity firms exit the market. This directly implies that large firms are the ones that select into export markets.

Equilibrium As before, our equilibrium characterization will rely in a free entry condition and a profit maximization condition. Before going to the specific equations, note on the probabilities that figure 7 implies.

Figure 7: Profits in the Melitz model with free-trade



That is:

$$\begin{aligned}
Prob(\varphi > \varphi^*) \equiv \quad & P^N(\varphi^*) = \quad \frac{N(\varphi^*)}{M} \\
Prob(\varphi > \varphi_x^*) \equiv \quad & P^X(\varphi_x^*) = \quad \frac{N_x(\varphi_x^*)}{M} \\
Prob(\varphi > \varphi_x^* | \varphi > \varphi^*) \equiv \quad & P^{X|N}(\varphi^*) = \quad \frac{N_x(\varphi_x^*)}{N(\varphi^*)}
\end{aligned}$$

For the free entry condition, we have the exactly the same equation as before:

$$EV = \left[1 - P^N(\varphi^*)\right] \cdot 0 + P^N(\varphi^*) \cdot \frac{\tilde{\pi}}{\delta} - f_e = 0$$

$$\Rightarrow \tilde{\pi} = \frac{\delta fe}{P^N(\varphi^*)} \tag{FE-FT}$$

with the difference that now:

$$\tilde{\pi} \equiv \pi\left(\tilde{\varphi}_t\right) = \pi_d\left(\tilde{\varphi}\right) + P^{X|N}\left(\varphi^*\right)\pi_x\left(\tilde{\varphi}_x\right)$$

where $P^{X|N}\left(\varphi^*\right)$ is the probability of exporting conditional on a successful entry.

As for the profit maximization, note that:

$$\begin{aligned}
\pi_d\left(\varphi^*\right) = 0 \quad &\Leftrightarrow \quad r_d\left(\varphi^*\right) = \sigma f \\
\pi_x\left(\varphi_x^*\right) = 0 \quad &\Leftrightarrow \quad r_x\left(\varphi_x^*\right) = \sigma f_x
\end{aligned}$$

implying

$$\begin{aligned}
\frac{r_x\left(\varphi_x^*\right)}{r_d\left(\varphi^*\right)} &= \frac{f_x}{f} \\
\Leftrightarrow \frac{R\left(\varphi_x^*\rho P\right)^{\sigma-1}\tau^{1-\sigma}}{R\left(\varphi^*\rho P\right)^{\sigma-1}} &= \frac{f_x}{f} \\
\Leftrightarrow \qquad \varphi_x^* &= \tau\left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}}\varphi^*
\end{aligned} \tag{23}$$

But then:

$$\begin{aligned}
\tilde{\pi} &= \pi_d\left(\tilde{\varphi}\right) + P^{X|N}\left(\varphi^*\right)\pi_x\left(\tilde{\varphi}_x\right) \\
\Rightarrow \tilde{\pi} &= f\left(\frac{r_d\left(\tilde{\varphi}\right)}{\sigma f} - 1\right) + P^{X|N}\left(\varphi^*\right)\cdot f_x\left(\frac{r_x\left(\tilde{\varphi}_x\right)}{\sigma f_x} - 1\right) \\
\Rightarrow \tilde{\pi} &= f\left(\frac{r_d\left(\tilde{\varphi}\right)}{r_d\left(\varphi^*\right)} - 1\right) + P^{X|N}\left(\varphi^*\right)\cdot f_x\left(\frac{r_x\left(\tilde{\varphi}_x\right)}{r_x\left(\varphi_x^*\right)} - 1\right) \\
\Rightarrow \tilde{\pi} &= f\left(\left(\frac{\tilde{\varphi}\left(\varphi^*\right)}{\varphi^*}\right)^{\sigma-1} - 1\right) + P^{X|N}\left(\varphi^*\right)\cdot f_x\left(\left(\frac{\tilde{\varphi}_x\left(\varphi_x^*\left(\varphi^*\right)\right)}{\varphi_x^*\left(\varphi^*\right)}\right)^{\sigma-1} - 1\right)
\end{aligned} \tag{PM-FT}$$

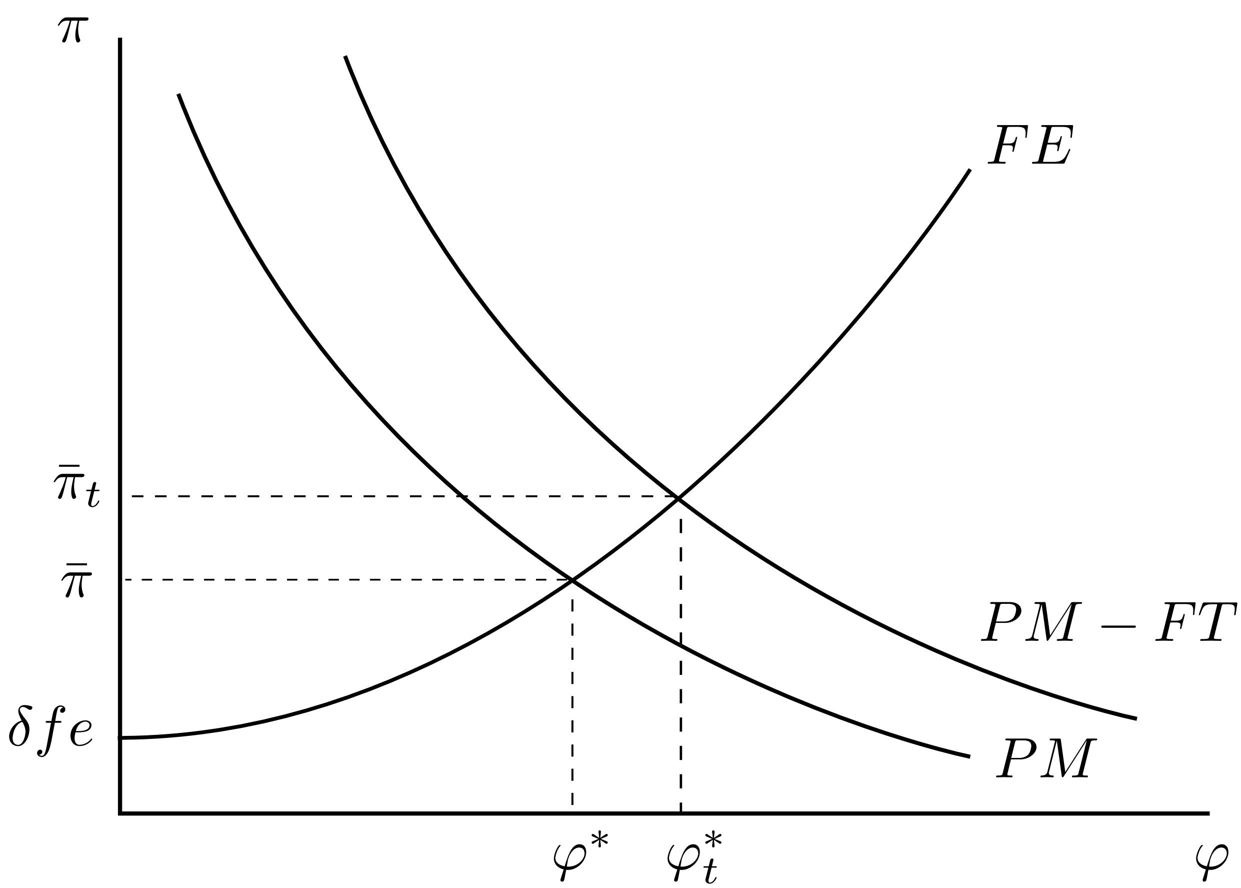
Again the relationship between $\tilde{\pi}$ and φ^* is negative for most distributions, which includes the distribution we assumed for φ[4]. The difference, compared with the autarky case, is

[4] To see that note that from (23), $\varphi_x^*\left(\varphi^*\right)$ is an increasing function, which implies that both $\pi_d\left(\tilde{\varphi}\right)$ and $\pi_x\left(\tilde{\varphi}_x\right)$ are both decreasing in φ^* using an argument similar to the one used in footnote 3. It remains to be seen that $P^{X|N}\left(\varphi^*\right)$ is also decreasing in φ^*. Note from the definition that:

$$P^{X|N}\left(\varphi^*\right) = \frac{N_x\left(\varphi_x^*\right)}{N\left(\varphi^*\right)}$$

we already showed that both N_x and N are both decreasing in φ_x^* and φ^*, respectively, for exactly the same that the (PM-FT) now shifts to the right side as shown in figure 8, as the first term of the right hand side is the same as in the autarky case, while the second term is always non-negative. Note how both profits and the cut-off productivity increase in the free-trade case. Intuitively, for exporters, profits increase due to higher exporting opportunities, while for non-exporters entry of foreign firms in the market makes them exit the market.

Figure 8: Equilibrium in the Melitz model with free-trade

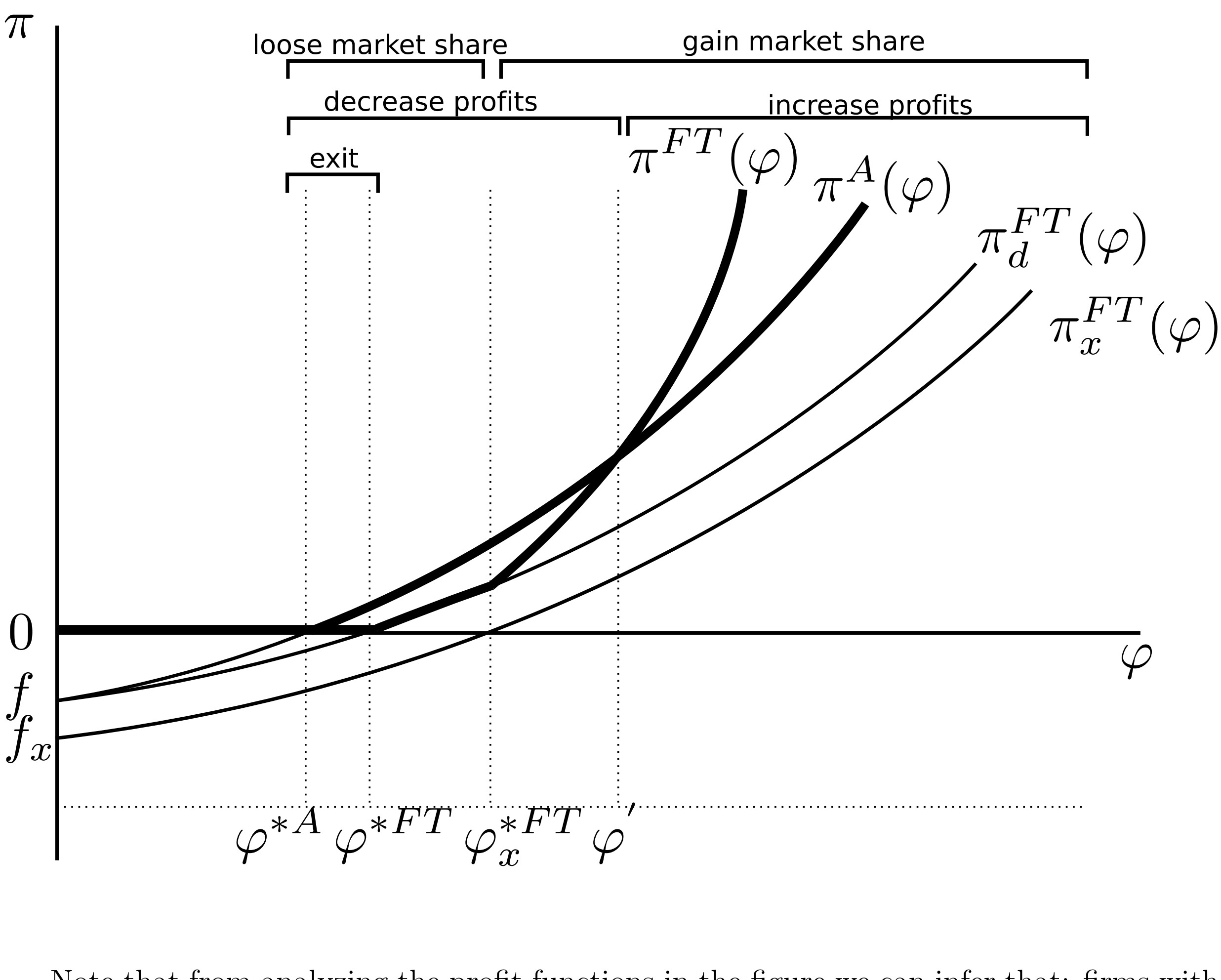


This intuition can also be seen from the drawing the profit functions in a graph. We can plot initially 3 profit function associated with the profit under autarky $\pi^A(\varphi)$ as in (10), the profit for domestic producers only under free trade $\pi_d^{FT}(\varphi)$ as in (19), and the profit of exporting $\pi_x^{FT}(\varphi)$ as in (20). Because opening up to free trade implies that φ^*

reasons. So we need to show which one falls faster. Note, from (23), that the ratio of φ_x^*/φ^* is constant. But that implies that when φ^* increases by 1%, φ_x^* also increases by 1%. But then the impact should be more less the same for N and N_x, meaning that $P^{X|N}(\varphi^*)$ is not that responsive with respect to φ^*. But then (PM-FT) must be decreasing in φ^*, which is what we wanted to show.

moves to the right (due to higher competition), $\pi_d^{FT}(\varphi)$ shifts to the right of $\pi^A(\varphi)$. Note how in the free-trade case, the profit function $\pi(\varphi)$ is the envelope of the profits earned domestically and abroad, that is, the profits are given by the sum of $\pi_d^{FT}(\varphi)$ and $\pi_x^{FT}(\varphi)$. In terms of selection note that, as we open to trade and from increased competition, the cut-off productivity moves to the right side, implying that the least productive firms drop from the market. at the same time, for those small firms that survive, they will also have an negative impact from foreign competition through lower profits; while the most productive firms, despite that impact from foreign competition, will be able to increase their profits due to the access of a new set of market opportunities. All these effects can be seen in figure 9.

Figure 9: Selection in the Melitz's model from shifting from autarky to free trade



Note that from analyzing the profit functions in the figure we can infer that: firms with productivity $\varphi \in \left[\varphi^{*A}, \varphi^{*FT}\right]$ exit the market; those with $\varphi \in \left[\varphi^{*A}, \varphi_x^{*FT}\right]$ loose market share[5]; and those with productivity lower than φ' observe a reduction in profits. These are the effects from trade liberalization that are behind the reallocation of resources within sectors that is observed in the data.

Equilibrium number of varieties As before markets need to clear which implies that:

$$q_i = 2Lc_i \qquad \text{(varieties markets exporters)}$$

$$q_i = Lc_i \qquad \text{(varieties markets only domestic)}$$

$$L = \sum_{i=1}^{N^d} l_i = N^d f + N^x f_x + \sum_{i=1}^{N^d} q_i/\varphi_i + N^d \frac{\delta f_e}{P^N} \qquad \text{(labor market)}$$

[5] To see this effect on market share note that we can bound the revenue functions in the following sense:

$$r_d(\varphi) < r_a(\varphi) < r_d(\varphi) + r_x(\varphi) \quad \forall \varphi \geq \varphi^*$$

This is because: $r(\varphi) = (\varphi/\varphi^*)^{\sigma-1}\sigma f$, and since φ^* increases under free-trade, $r_d(\varphi) < r_a(\varphi)$. We can also show that $r_d(\varphi) + r_x(\varphi)$ is decreasing with τ, but since $r_a = r_d$ as $\tau \to \infty$, the result follows.

The first part of the inequality indicates that all firms incur a loss in domestic sales in the open economy. A firm who does not export then also incurs a total revenue loss. The second part of the inequality indicates that a firm who exports more than makes up for its loss of domestic sales with export sales and increases its total revenues. Thus, a firm who exports increases its share of industry revenues while a firm who does not export loses market share. The market share of the least productive firms in the autarky equilibrium - with productivity between φ_a^* and φ^* - drops to zero as these firms exit.

Substituting in optimality conditions in (labor market):

$$
\begin{aligned}
L \quad &= N^d f + N^x f_x + \sum_{i=1}^{N^d} q_i/\varphi_i + N^d \frac{\delta f_e}{P^N} \\
\Rightarrow L \quad &= N^d f + N^x f_x + \sum_{i=1}^{N_d - N_x} q_i/\varphi_i + \sum_{i=1}^{N_x} q_i/\varphi_i + N^d \frac{\delta f_e}{P^N} \\
\Rightarrow L \quad &= N^d f + N^x f_x + \rho L \sum_{i=1}^{N^d - N_x} \frac{c(\varphi_i)}{\rho \varphi_i} + \rho 2L \sum_{i=1}^{N_x} \frac{c(\varphi_i)}{\rho \varphi_i} + N^d \tilde{\pi} \\
\Rightarrow L \quad &= N^d f + N^x f_x + \rho \sum_{i=1}^{N^d - N_x} r_d(\varphi_i) + \rho \sum_{i=1}^{N_x} r_x(\varphi_i) + N^d \tilde{\pi} \\
\Rightarrow L \quad &= N^d f + \rho (N_d - N_x) r_d(\tilde{\varphi}_d) + \rho N_x r(\tilde{\varphi}_x) + N^d \tilde{\pi} \\
\Rightarrow L \quad &= N^d f + \rho N_d \left[\frac{(N_d - N_x)}{N_d} r_d(\tilde{\varphi}_d) + \frac{N_x}{N_d} r_x(\tilde{\varphi}_x) \right] + N^d \tilde{\pi} \\
\Rightarrow L \quad &= N^d f + \rho N_d r(\tilde{\varphi}) + N^d \tilde{\pi} \\
\Rightarrow L \quad &= N^d f + N^x f_x + (1 - 1/\sigma) N^d \sigma \left[\pi(\tilde{\varphi}) + f + P^{X|N} f_x \right] + N^d \tilde{\pi} \\
\Rightarrow L \quad &= N^x f_x - N^d P^{X|N} f_x + N^d \sigma \left[\pi(\tilde{\varphi}) + f + P^{X|N} f_x \right] \\
& N^x f_x - N^x f_x + N^d \sigma \left[\pi(\tilde{\varphi}) + f + P^{X|N} f_x \right] \\
\Rightarrow L \quad &= N^d \sigma \left[\pi(\tilde{\varphi}) + f + P^{X|N} f_x \right] \\
\Rightarrow N^d &= \frac{L}{\sigma \left(\tilde{\pi} + f + P^{X|N} f_x \right)}
\end{aligned}
$$

This expression is derived by noting that labor is demand by firms that export or not and by potential entrants that need to pay a fixed cost of survey their productivity. A simpler way of getting the same number of equilibrium domestic variables is to realize that in equilibrium:

$$
\begin{aligned}
R \quad &= L \\
\Rightarrow \quad N^d \tilde{r} \quad &= L \\
\Rightarrow \quad N^d \quad &= \frac{L}{\tilde{r}} \\
\Rightarrow \quad N^d \quad &= \frac{L}{\sigma \left(\tilde{\pi} + f + P^{X|N} f_x \right)}
\end{aligned} \tag{24}
$$

where we made use of our result in (22). This in turn determines the number of varieties available to the consumer as:

$$\begin{aligned}
N &= N^d + N^x \\
&= N^d \left(1 + \frac{N^x}{N^d}\right) \\
&= N^d \left(1 + P^{X|N}\right)
\end{aligned} \tag{25}$$

Price index and utility As before, the price index should be a sufficient statistics for the welfare impact of shocks in these economy. From our aggregation results we have that:

$$P = N^{-\frac{1}{\sigma-1}} p\left(\tilde{\varphi}_t\right) \tag{26}$$

where (25) and (24) should help us understand the changes in utility through varieties, while (FE-FT) and (PM-FT), the changes in utility through average prices.

One welfare impact we are interested in is the consequences of opening an economy to free trade. Essentially, the comparison of P^{FT} in (26) with P^A in (18):

$$\begin{aligned}
P^{FT} &= N_{FT}{}^{-\frac{1}{\sigma-1}} p\left(\tilde{\varphi}_{FT,t}\right) \\
P^{A} &= N_{A}{}^{-\frac{1}{\sigma-1}} p\left(\tilde{\varphi}_{A}\right)
\end{aligned}$$

Note that under autarky

$$\left(\frac{\tilde{\varphi}_A}{\varphi_A^*}\right)^{\sigma-1} = \frac{r\left(\tilde{\varphi}_A\right)}{r\left(\varphi_A^*\right)} = \frac{R_A/N_A}{f\sigma} = \frac{L/N_A}{f\sigma}$$

here we made use of (11); $R = r\left(\tilde{\varphi}\right) N$ in (14); that at φ^*, $r\left(\varphi^*\right) = f\sigma$ in (17), and finally that $L = R$. Similarly, we can derive that under free trade:

$$\left(\frac{\tilde{\varphi}_{FT}}{\varphi_{FT}^*}\right)^{\sigma-1} = \frac{r\left(\tilde{\varphi}_{FT}\right)}{r\left(\varphi_{FT}^*\right)} = \frac{R_{FT}/N_{FT}}{f\sigma} = \frac{L/N_{FT}}{f\sigma}$$

Substituting these back in the price indexes we get:

$$
\begin{aligned}
P^{FT} &= N_{FT}{}^{-\frac{1}{\sigma-1}} \frac{1}{\rho \tilde{\varphi}_{FT,t}} \\
&= N_{FT}{}^{-\frac{1}{\sigma-1}} \cdot \frac{1}{\rho} \cdot \left(\frac{L/N_{FT}}{f\sigma} \right)^{-\frac{1}{\sigma-1}} \cdot \frac{1}{\varphi^*_{FT}} \\
&= \left(\frac{L}{f\sigma} \right)^{-\frac{1}{\sigma-1}} \cdot \frac{1}{\rho \cdot \varphi^*_{FT}} \\
P^{A} &= N_{A}{}^{-\frac{1}{\sigma-1}} \frac{1}{\rho \tilde{\varphi}_{A,t}} \\
&= \left(\frac{L}{f\sigma} \right)^{-\frac{1}{\sigma-1}} \cdot \frac{1}{\rho \cdot \varphi^*_{A}}
\end{aligned}
\tag{27}
$$

But then, since $\varphi^*_{FT} > \varphi^*_A$, this implies that $P^A > P^{FT}$ which means that free trade leads to higher welfare.

Note that the gains from trade accrue from 2 potential effects: an increased number of varieties; and a increase in the aggregate productivity (since $\tilde{\varphi}_{FT} > \tilde{\varphi}_A$). This is despite the fact that domestic firms reduce the number of varieties that they produce, that is, $N^d = \frac{L}{\sigma\left(\tilde{\pi}_{FT}+f+P^{X|N}f_x\right)} < N^A = \frac{L}{\sigma(\tilde{\pi}_A+f)}$. Although the number of firms in a country decreases after the transition to trade, consumers in the country still typically enjoy greater product variety ($N^{FT} = (1+P^{X|N})N^d > N^A$). That is, the decrease in the number of domestic firms following the transition to trade is typically dominated by the number of new foreign exporters. It is nevertheless possible, when the export costs are high, that these foreign firms replace a larger number of domestic firms (if the latter are sufficiently less productive). Although product variety then impacts negatively on welfare, this effect is necessarily dominated by the positive contribution of the aggregate productivity gain. Trade - even though it is costly - necessarily generates a welfare gain.

Summary of the intuition We should note that the elasticity of demand is unaffected by trade opening, so the fall in profits for domestic producers is not explained by a fall in mark-ups driven by increased foreign competition. The actual channel operates through the domestic factor market. In particular, trade translates into increased profitable opportunities for firms (recall that $\tilde{\pi}$ goes up). This translates into more entry, thereby increasing labor demand and (given the fixed supply of labor) leading to a rise in the real wage. This, in turn, brings down the profit level of the least productive firms to a level that forces them to exit.

A Appendix

A.1 Some other welfare impacts in the Melitz model

Here we study in further detail some other impacts on welfare that can be derived from the Metlitz model.

Start by noting that:

$$
\begin{aligned}
Prob(\varphi > \varphi^*) \equiv \quad P^N(\varphi^*) = \quad & \frac{N(\varphi^*)}{M} \\
Prob(\varphi > \varphi_x^*) \equiv \quad P^X(\varphi_x^*) = \quad & \frac{N_x(\varphi_x^*)}{M} \\
Prob(\varphi > \varphi_x^* | \varphi > \varphi^*) \equiv \quad P^{X|N}(\varphi^*) = \quad & \frac{N_x(\varphi_x^*)}{N(\varphi^*)}
\end{aligned}
$$

together with:

$$
\begin{aligned}
\frac{r_x(\varphi_x^*)}{r_d(\varphi^*)} =& \frac{f_x}{f} \\
\Leftrightarrow \frac{R(\varphi_x^* \rho P)^{\sigma-1} \tau^{1-\sigma}}{R(\varphi^* \rho P)^{\sigma-1}} =& \frac{f_x}{f} \\
\Leftrightarrow \qquad \varphi_x^* =& \tau \left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}} \varphi^*
\end{aligned} \tag{28}
$$

and with

$$
\tilde{\pi} = f\left(\left(\frac{\tilde{\varphi}(\varphi^*)}{\varphi^*}\right)^{\sigma-1} - 1\right) + P^{X|N}(\varphi^*) \cdot f_x \left(\left(\frac{\tilde{\varphi}_x(\varphi_x^*(\varphi^*))}{\varphi_x^*(\varphi^*)}\right)^{\sigma-1} - 1\right) \tag{PM-FT}
$$

$$
\tilde{\pi} = \frac{\delta f e}{P^N(\varphi^*)} \tag{FE-FT}
$$

Lets substitute now $P^{X|N}$ and φ_x^* in (PM-FT):

$$
\tilde{\pi} = f\left(\left(\frac{\tilde{\varphi}(\varphi^*)}{\varphi^*}\right)^{\sigma-1} - 1\right) + \frac{N_x\left(\tau\left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}} \varphi^*\right)}{N(\varphi^*)} \cdot f_x \left(\left(\frac{\tilde{\varphi}_x\left(\tau\left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}} \varphi^*\right)}{\tau\left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}} \varphi^*}\right)^{\sigma-1} - 1\right) \tag{29}
$$

- As before, f_e or δ only change (FE-FT). So the analysis should be easy.

- Consider now an increase in f. From (29) this should imply a increase in the first term, and a decrease in φ_x^* from (PM-FT). Thus the ratio $N_x\left(\tau\left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}}\varphi^*\right)/N\left(\varphi^*\right)$ should increase as well as the second term in (29). This implies that we have 2 effects: on one hand, the fact that higher fixed cost of producing to the domestic economy implies that some low productivity firms exit the market, thus increasing aggregate profits; on the other hand, larger fixed cost f, decreases the cutoff access to foreign markets as the wedge between exporting and not exporting becomes smaller, thus aggregate profits should increase even further as firms have access to larger markets now. This is a shift to the right of the (PM-FT) equation.

- Thinking about an increase in f_x. It is immediate that a larger f_x increases φ_x^*. Now, the inverse happens and both $N_x\left(\tau\left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}}\varphi^*\right)/N\left(\varphi^*\right)$ decrease as well as the second term in (29). But we still have the direct impact of f_x. As before we can see 2 effects: some firms exit the market, thus leading to larger aggregate profits; at the same time, exporting is now harder, implying that firms find it not so easier to access foreign markets thus implying lower profits. Which of these two effects is stronger depends on the exact model parameters so we cannot say for sure how (PM-FT) shifts[6]. However, note that as $f_x \to \infty$, $\varphi_x^* \to \infty$, implying that no firm would export. Thus, for that limiting case, (PM-FT) becomes equal to the one under autarky, that is, a shift to the left. This is a somehow lose argument for justifying that increases in f_x shift (PM-FT) to the left.

- Now consider an increase in τ. It is easy to see that both $N_x\left(\tau\left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}}\varphi^*\right)/N\left(\varphi^*\right)$ decrease as well as the second term in (29). This is a shift to the left side of (PM-FT).

The welfare impact of such shocks can be seen through the lenses of the price index in (27) or (26).

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[6] In fact, for a somehow general continuous distribution of probably for φ, Melitz (2003) shows that an increase in f_x implies a shift to the left of (PM-FT).

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