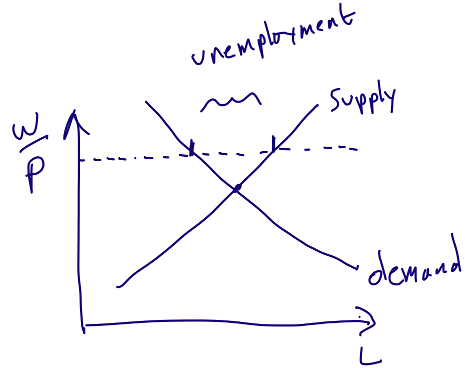
SEARCH AND MATCHING MODELS OF THE LABOR MARKET

Traditional models see unemployment as a disequilibrium phenomenon in the labor market

- As such, is a temporary situation of excess supply of workers

- Role of wage rigidities



Models of search and matching emphasize that unemployment in an *activity*: search for jobs

- Labor market frictions imply that job search can take time

- We can have unemployment in equilibrium even in the long run

Mortensen-Pissarides (1994): prototype model in the literature

Moreover, this class of models allow for a richer description of the labor market

Introduction: Looking at Labor Market Data

We can divided the working age population (POP) in three basic categories:

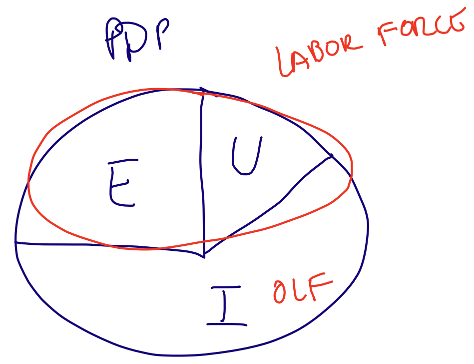
- Employed (E)

- Unemployed (U)

- Inactive (I)

Inactive workers do not participate in the labor force (students, retirees, at home); unemployed do participate searching actively for a job

The sum of employed and unemployed is equal to the *labor force*



With these three occupational categories, we can define the following labor market rates:

- Employment rate: E / POP 57% → L input
- Unemployment rate: U / (E+U) 3-4%
- Participation rate: (E+U) / POP 60%

where POP = E+U+I

Each of these indicators capture a different dimension of the labor market

For instance, comparing México and the U.S. (2019)

	Mexico	U.S.
Employment rate (%)	57.1	61.0
Unemployment rate (%)	3.6	3.5
Participation rate (%)	60.3	63.6

Moreover, in Mexico the informality rate (fraction of total employment under informality conditions) is about 55%

Labor surveys have a panel structure that follows the same individuals over time, for instance between consecutive quarters

This allows to compute nine different gross flows across occupational categories; for instance

- Employed workers moving to inactivity (E→I)
- Unemployed workers finding a job (U→E)
- Inactive individuals remaining out of the labor force (I→I)

We can summarize these flows using the following rates:

- Job creation rate: JC = [(U→E) + (I→E)] / E
- Job destruction rate: JD = [(E→U) + (E→I)] / E
- Job finding probability: (U→E) / U

JC − JD

JC + JD

Subtracting the first two we obtain the *net* job creation

$$E_{t+1} = \left(1 + \underbrace{JC_t - JD_t}_{net\ creation}\right) E_t$$

... while adding them (JC+JD) we get an indicator of labor market *turnover*

$$JD = 0.01 + 0.03$$

$$JC = 0.25 \times \frac{U}{E} + 0.05 \frac{I}{E}$$

As an example, in the U.S. (average 1978-2012)

from\to	E	U	I
E	0.96	0.01	0.03
U	0.25	0.54	0.21
I	0.05	0.03	0.93

... implying job creation/destruction rates of around 4-5%

turnover: 8-10 %

... and a job finding probability around 25%

Finally, some labor surveys allow to distinguish between the *intensive* margin (hours worked per person) and the *extensive* margin (employed individuals) of employment

Most models do not allow to distinguish between these two margins

In the data, the extensive margin seems to be more important to understand employment fluctuations

Cyclical Behavior

For each labor market indicator, using a sufficiently long time series, we can compute several business cycle statistics, as:

- Volatility (standard deviation), absolute or relative to output
- Correlation with output

In most economies:

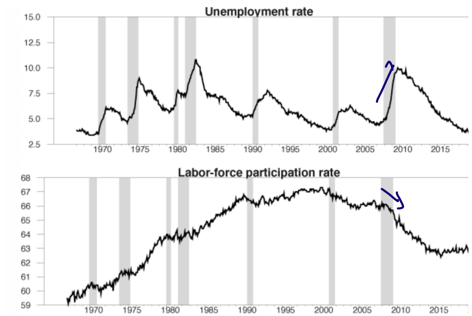
- Employment and participation rates are procyclical and less volatile than output; the unemployment rate is counter-cyclical and more volatile

recession $\rightarrow$ $E \downarrow$

recession $\rightarrow$ $U \uparrow$

recession $\rightarrow$ $I \uparrow$

For instance, in the U.S.

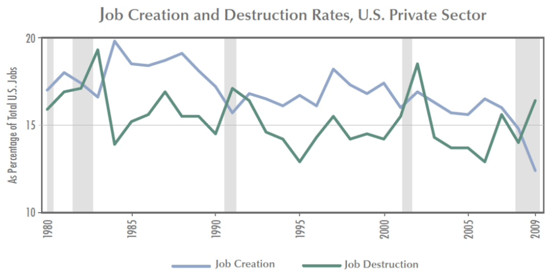


unemployment rate
procyclical

participation rate
countercyclical

(the shaded areas correspond to recessions)

- Job creation and the job finding probability are counter-cyclical; job destruction is procyclical



This is the kind of regularities that search and matching models try to account for

`The Basic Mortensen-Pissarides Model`

In each period t,

- U_t individuals search for a job (unemployed)
- Firms post V_t vacancies

The *matching function* $m(U_t, V_t)$ determines the number of hirings resulting from a random search process

This function introduces frictions to the labor market adjustment in the model

U_t

V_t

$m(U_t, V_t)$ hirings

example

$m(U,V) = U^{\phi} V^{1-\phi}$

We assume that the matching function features constant returns to scale, is increasing in each argument (unemployed, vacancies) and these two inputs are complementary

We define

CRS

labor market tightness

- Job finding probability: $p_t = \frac{m(U_t,V_t)}{U_t} = m\left(1, \boxed{\frac{V_t}{U_t}}\right) = p\left(\theta_t\right)$
- Vacancy filling probability: $q_t = \frac{m(U_t,V_t)}{V_t} = m\left(\frac{U_t}{V_t}, 1\right) = q\left(\theta_t\right)$

$\theta_t \uparrow$ $p_t \uparrow$ $q_t \downarrow$

these two probabilities depend on the labor market *tightness*, defined as $\theta_t = \frac{V_t}{U_t}$

We verify that p_t is increasing and q_t decreasing in θ_t

# of matches (JC) = m(U_t, V_t)

= p_t U_t

J D

Total employment satisfies the law of motion

$$
\begin{aligned}
L_t &= L_{t-1} + q_t V_t - s L_{t-1} \\
&= (1-s) L_{t-1} + q_t V_t
\end{aligned}
$$

- $q_t V_t$: hirings (job creation)
- $s L_{t-1}$: separations (job destruction)

In the most basic version of the model, the separation rate s is exogenous

On the other hand, in each period,

$$L_t + U_t = \bar{L} \quad \text{pop}$$

Inactivity = 0

again in the basic model there is no participation decision

$$l_t \equiv \frac{L_t}{\bar{L}}$$

Household's Problem

In its simplest version, using per-capita variables (divided by $\bar{L}$)

$$\max \quad \sum_{t=0}^{\infty} \beta^t c_t \quad \longleftarrow \quad \text{no-leisure}$$

$$\begin{aligned} s.t. \qquad c_t &= w_t l_t \\ l_t &= (1-s) l_{t-1} + p_t u_t \\ l_t + u_t &= 1 \end{aligned}$$

There is no capital nor savings, although these could be added

Linear utility function (also can be generalized)

$$l_t = (1-s) l_{t-1} + p_t (1 - l_t)$$

$$(1+p_t)\, l_t = p_t + (1-s)\, l_{t-1}$$

$$(1+p_t)\, l_t - (1-s)\, l_{t-1} - p_t$$

Lagrangian:

$$L = \sum_{t=0}^{\infty} \left\{ \beta^t c_t - \lambda_{1t} [c_t - w_t l_t] - \lambda_{2t} [l_t - (1-s)\, l_{t-1} - p_t \overbrace{(1-l_t)}^{u_t}] \right\}$$

and first order conditions:

$$\frac{\partial L}{\partial c_t} = \beta^t - \lambda_{1t} = 0 \quad \Rightarrow \lambda_{1t} = \beta^t \Rightarrow \frac{\lambda_{1t+1}}{\lambda_{1t}} = \beta$$

$$\frac{\partial L}{\partial l_t} = \lambda_{1t} w_t - \lambda_{2t} (1 + p_t) + \lambda_{2t+1} (1-s) = 0$$

from which, defining $W_t \equiv \frac{\lambda_{2t}}{\lambda_{1t}}$,

$$(1 + p_t)\, W_t = w_t + \beta\, (1-s)\, W_{t+1}$$

The multiplier W_t represents recursively the value for the household of having one worker employed

$$w_t = \frac{\lambda_{2t}}{\lambda_{1t}} (1+p_t) - \frac{\lambda_{2t+1}}{\lambda_{1t}} (1-s)$$

$$(1+p_t) \frac{\lambda_{2t}}{\lambda_{1t}} = w_t + \underbrace{\left(\frac{\lambda_{1t+1}}{\lambda_{1t}}\right)}_{\beta} \left(\frac{\lambda_{2t+1}}{\lambda_{1t+1}}\right) (1-s)$$

$$(1+p_t)\, W_t = w_t + \beta (1-s)\, W_{t+1}$$

Wage Bargaining and the Decision to Post Vacancies

In each period, a matched workers produces A units of output, hence in the aggregate

$$y_t = Al_t$$

(we could add capital and/or productivity shocks)

In competitive markets, workers would be payed their marginal product $w_t = A$ $\Rightarrow$ profits $= 0$

labor

However, search frictions implied that, once matched, a worker and a firm enter a bilateral bargaining process fixed cost → lasts for only one period

Firms pay a cost η for each vacancy posted, and recover this cost extracting a surplus from the match over time

The value of a worker for a firm (J_t) can be written recursively as:

$$J_t = [A - w_t] + \beta (1 - s) J_{t+1}$$

→ current profits

The zero profit condition implies in this model that

ex-ante

$$q_t J_t = \cancel{\kappa} \quad (\eta)$$

this is, the vacancy cost needs to be exactly compensated by the value of having a new worker, times the vacancy filling probability

Once matched, we assume that the worker and the firm bargain each period the wage rate according to the *Nash protocol*

This protocol is equivalent to maximize a weighted average of the surplus of each side

$$\max_{w_t} \left(W_t\left(w_t\right)\right)^{\gamma} \left(J_t\left(w_t\right)\right)^{1-\gamma}$$

where the parameter γ measures the worker's bargaining power

Nash bargaining generates endogenously the *sharing rule*

$$\left(1-\gamma\right) W_t = \gamma J_t$$

$\theta_t = \frac{v_t}{u_t}$

Equilibrium

A *Nash-bargaining equilibrium* for this economy is a set of sequences for quantities c_t, l_t, u_t, y_t , v_t, θ_t, probabilities p_t, q_t, values W_t, J_t and wages w_t such that:

i) Given l_{-1}, p_t, w_t, the sequences c_t, u_t, l_t solve the household's problem:

$$\max \quad \sum_{t=0}^{\infty} \beta^t c_t$$

$$\begin{aligned} s.t. \qquad c_t &= w_t l_t \\ l_t &= (1-s)\, l_{t-1} + p_t u_t \\ l_t + u_t &= 1 \end{aligned}$$

ii) In each period t, we define the values of a worker for the household and for the firm recursively:

$$(1 + p_t)\, W_t = w_t + \beta\,(1-s)\, W_{t+1}$$

$$J_t = [A - w_t] + \beta\,(1-s)\, J_{t+1}$$

iii) In each period t, the probabilities are given by: $p_t = p\,(\theta_t)$ y $q_t = q\,(\theta_t)$, with $\theta_t = \frac{v_t}{u_t}$

iv) In each period t, the sharing rule, zero-profit condition and market clearing conditions are all satisfied

$$(1-\gamma)\, W_t = \gamma J_t \qquad\qquad q_t J_t = \eta$$

$$y_t = Al_t = c_t + \eta v_t$$