Implications and Extensions of the Ricardian Model

Tiago Tavares*
International Trade I at ITAM

August 28, 2017

1 Implications and extensions of the Ricardian model

Following the notes from handout 2, we assume that, under the 2-countries/2-goods Ricardian model, the domestic economy has a comparative advantage in the production of good 1, that is:

$$\frac{z_1}{z_2} \geq \frac{z_1^*}{z_2^*}$$

where z_i^j is the labor productivity of good i in country j.

1.1 Absolute advantage and relative wages

From the simple 2-countries/2-goods Ricardian model one could conclude that it is the notion of comparative advantage and not of absolute advantage that determines the patterns of trade across countries. This, however, doesn't mean that absolute advantage is unimportant. The reason is that productivity differences, that determine absolute advantages, play an important role in determining **relative wages**, *i.e*, the wages of the domestic economy relative to the foreign economy. On it's turn, relative wages are indicative of the relative living of standards across countries.

So see this point, note that under full specialization (domestic producing good 1, while foreign producing good 2) the following wage conditions need to hold:

$$\begin{aligned} w &= p_1^T \cdot z_1 \\ w^* &= p_2^T \cdot z_2^* \end{aligned}$$

*Email: Please contact me to tgstavares@gmail.com if you find any errors in the text.

where p_1^T, p_2^T are the prices that prevail under free trade. The correspondent relative wage becomes:

$$\frac{w}{w^*} = \frac{p_1^T}{p_2^T} \cdot \frac{z_1}{z_2^*} \tag{1}$$

Note also that, under free trade, the relative price of good 1 is between the relative autarky prices that would prevail in both the domestic and foreign economy:

$$\frac{p_1}{p_2} = \frac{1/z_1}{1/z_2} \leq \frac{p_1^T}{p_2^T} \leq \frac{p_1^*}{p_2^*} = \frac{1/z_1^*}{1/z_2^*} \tag{2}$$

Putting together (1) and (2) and rearranging yields the following relationship:

$$\frac{z_2}{z_2^*} \leq \frac{w}{w^*} \leq \frac{z_1}{z_1^*}$$

That is, the relative wage across countries lies between the ratio of the productivities in each sector. Under free trade, even if the domestic economy is not producing good 2 (without comparative advantage), a very large productivity in that sector would imply a large relative wage. This shows that productivity differentials are important even in those sectors where an economy is not specializing as these determine the relative wages under trade.

1.2 Gains from trade

To see the welfare gain from free trade under the Ricardian model, it is sufficient to look at the real wages. This is the case since labor income is the only revenue available to households' consumption.

Note that under autarky, the domestic country has to produce on both sectors thus, from a zero profit condition, wages under autarky w^a must observe that:

$$z_2 p_2^a - w^a = 0$$

Similarly, because under free trade the domestic economy specializes on good 1, it must be that

$$z_2 p_2^T - w^T \leq 0$$

(note that if specialization is full, the inequality is strict). Substituting z_2 from the first equation into the inequality yields:

$$\frac{w^T}{p_2^T} \geq \frac{w^a}{p_2^a}$$

But this implies that, under free trade, real wages are larger than under autarky. Similar arguments allow us to conclude the same for the foreign economy.

The idea is that, by allowing for trade, labor is reallocated into those sectors where revenue at world market prices is higher thus pushing wages up.

1.3 Ricardian model extension: multiple goods

In the previous handout we saw that in the two-good model, comparative advantage on the production of sector 1 implies:

$$\begin{aligned} & \frac{1/z_1}{1/z_2} \leq \frac{1/z_1^*}{1/z_2^*} \\ \Rightarrow \quad & \frac{1/z_1}{1/z_1^*} \leq \frac{1/z_2}{1/z_2^*} \end{aligned}$$

Such two-good type of economies can be easily generalized to multiple goods. For example, we can let be N goods and rebrand those goods such that comparative advantage is highest for sectors with a low index, that is:

$$\frac{1/z_1}{1/z_1^*} \leq \frac{1/z_2}{1/z_2^*} \leq \cdots \leq \frac{1/z_i}{1/z_i^*} \leq \cdots \leq \frac{1/z_N}{1/z_N^*}$$

Given this multitude of goods the question becomes: what goods are produced domestically and what are produced abroad. Grossly speaking, international competition determines that, given equilibrium wages, goods are produced where they are less expensive. For example, good i would be produced in the domestic economy if:

$$w \cdot 1/z_i \leq w^* \cdot 1/z_i^* \tag{3}$$

where w is the wage rate and $1/z_i$ is the labor requirement to produce one unit of good i. Rearranging the above equation gives us a criterium about the goods that are produced at home. Essentially, a good i is produced at home if

$$\frac{w}{w^*} \leq \frac{z_i}{z_i^*}$$

The following assumptions will help us to determine the **equilibrium** $\hat{i}$ such that bellow $\hat{i}$ production occurs at home and the relative wage w/w^*.

Following the classical work of Dornbusch, Fischer, and Samuelson (1977), let $N \to \infty$ and define a continuum of goods indexed by $i \in [0, 1]$. This implies that, for example, home labor productivity of good i is $z(i)$. From the previous discussion, goods with a low i index have a higher comparative advantage. That is for $j > i$

$$\frac{1/z(i)}{1/z^*(i)} \leq \frac{1/z(j)}{1/z^*(j)}$$

Note that for any i, profit maximization implies that

$$p(i)z(i) - w \leq 0 \qquad \text{(equality if i is produced at home)}$$
$$p(i)z^*(i) - w^* \leq 0 \qquad \text{(equality if i is produced at foreign)}$$

Given this structure it can be shown that there is a $\hat{i}$ such that production at home occurs with $i < \hat{i}$ while for $i > \hat{i}$ it occurs abroad. But this implies that at $\hat{i}$, both countries are indifferent between producing or not:

$$p(\hat{i})z(\hat{i}) - w = 0 \tag{4}$$
$$p(\hat{i})z^*(\hat{i}) - w^* = 0 \tag{5}$$

Dividing one equation into the other implies:

$$\frac{w}{w^*} = \frac{z(\hat{i})}{z^*(\hat{i})} \equiv 1/A(\hat{i}) \tag{6}$$

where $A(i) \equiv \frac{1/z(i)}{1/z^*(i)}$ is, by definition, an increasing function (reflecting decreasing comparative advantage for goods with large i index).

From the demand side of the economy we assume that the consumer utility function is characterized by a Cobb-Douglas function with constant expenditure share $b(i)$ such that[1]:

$$\int_0^1 b(i)di = 1$$

[1]For example:

$$U = \int_0^1 b(i) \log x(i) di$$

Given this utility function, demand of good i is characterized by being a constant share of real income:

$$p(i)x(i) = b(i) \cdot wL$$

Assuming that preferences are the same between the foreign and the domestic economy, it follows that for each country **the expenditure share** on goods produced at home equal to:

$$\theta(\hat{i}) = \int_0^{\hat{i}} b(i)di$$

But then **trade balance** for the home economy implies that exports must equate imports:

$$\theta(\hat{i})w^*L^* = \left[1 - \theta(\hat{i})\right] wL \tag{7}$$

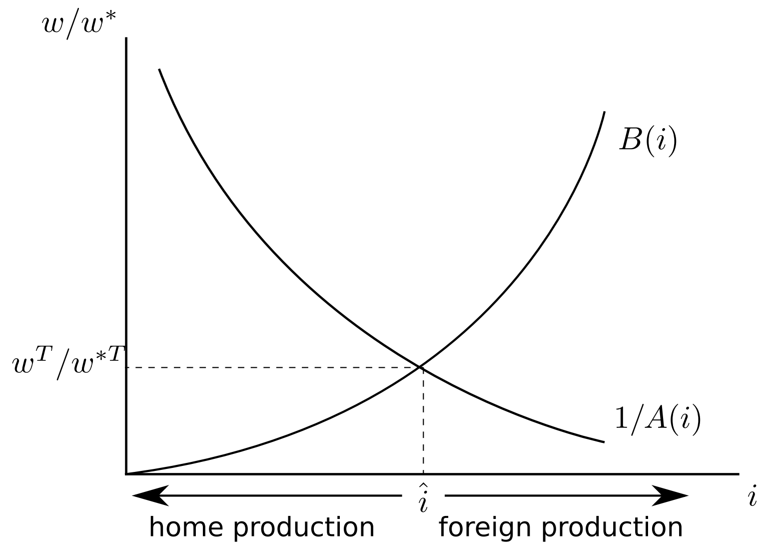
where the LHS of the equation are the exports of the home economy and the RHS the imports. Rearranging equation (7) yields:

$$\frac{w}{w^*} = \frac{\theta(\hat{i})}{1 - \theta(\hat{i})} \frac{L^*}{L} \equiv B(i) \tag{8}$$

And $B(i)$ is an increasing function (an increase in $\hat{i}$ would imply a trade surplus of the home economy, requiring a higher wage at the domestic economy to equilibrate the market).

Because equation (6) is decreasing while (8) is increasing, the equilibrium variables w/w^* and $\hat{i}$ can be determined. This can be easily seen in the following figure.

Figure 1: Equilibrium in a Ricardian model with multiple goods



In this example, specialization is complete with the domestic economy producing all goods at the left of $\hat{i}$ while the foreign economy produces all the goods at the right of $\hat{i}$.

Comparative statics analysis is particular simple in this model. Using equations (6) and (8) it is easy to see that, for example, an increase in the foreign labor supply L^* implies an increase in w/w^* and a decrease in $\hat{i}$; or a shift up of foreign productivity in all sectors $z^*(i)\forall i$ implies a fall in w/w^* and $\hat{i}$.

1.4 Trading costs in the Ricardian Model

Suppose now that for each unit that is exported to the foreign country, only a fraction $1/\gamma$ arrives to the destination where $\gamma > 1$. These type of costs, colloquially named iceberg costs, can represent depreciation of goods, transportation costs, or many other barriers to commerce. To understand the role of these iceberg costs consider the price that an economy j (foreign) faces that wishes to import goods produced from another economy i (home):

$$p_{ij}\frac{z_i}{\gamma} - w = 0$$

where we divide the quantity of goods z_i produced by one unit of labor to take into account the fact that transportation implies an iceberg cost, and impose an equality to impose zero profits. That is, a zero-profit exporting firm that hires 1 unit of labor, produces z_i units of final good, where only z_i/γ units arrive at the foreign country, and charges a price of $p_{ij} = w\gamma/z_i$. Note however that goods produced at i to be consumed in i don't carry any transportation costs, that is, $p_{ii}z_i = w \Rightarrow p_{ii} = w/z_i$. But then, the above equation can be written as:

$$p_{ij} = p_{ii}\gamma$$

In words, the price of a non-domestic good at the destination has a wedge over the price of that same good at the origin to take into account for the transportation cost γ. Note that a different interpretation of γ is to associate that to an import *add valorem* tariff.

Using this same analysis on equations (4) and (5) implies that the decision of producing goods at home versus importing them from abroad verifies:

$$p\left(\hat{i}\right) z(\hat{i}) - w = 0$$
$$p(\hat{i})z^*(\hat{i})/\gamma - w^* = 0$$

But then, equation (6) is shift up by the iceberg cost factor:

$$\frac{w}{w^*} = \frac{1/z^*(\hat{i})}{1/z(\hat{i})} \cdot \gamma \equiv \frac{1}{A\left(\hat{i}\right)} \cdot \gamma \equiv 1/A^{dom}(\hat{i})$$

and because $A(i) = z^*(i)/z(i)$ and $A^{dom}(i) = (1/\gamma) \cdot z(i)^*/z(i)$, $A^{dom}(i) < A(i)$.

Equivalently for the goods that are produced abroad $A^{for}(i) > A(i)$, implying:

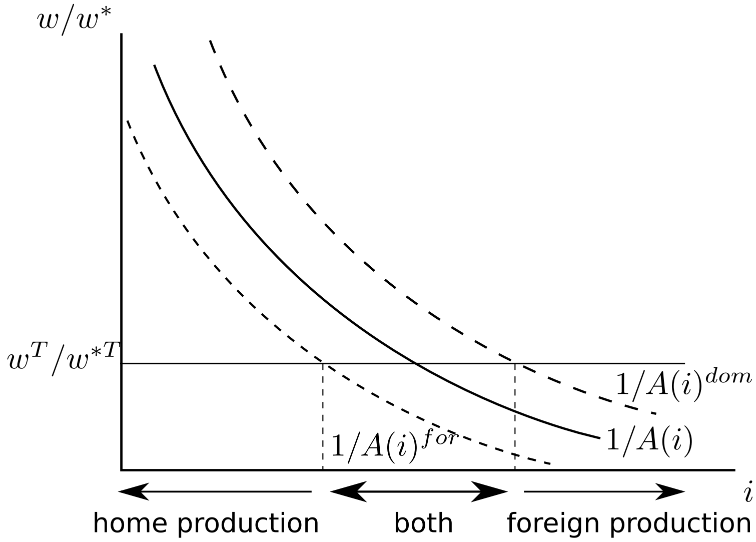
$$\frac{w}{w^*} = \frac{1/z^*(\hat{i})}{1/z(\hat{i})} \cdot \frac{1}{\gamma} \equiv 1/A^{for}(\hat{i})$$

Note that for any i, $A^{dom}(i) < A(i) < A^{for}(i)$. This implies that for a particular relative wage w/w^* there will be a no specialization region defined by:

$$\left\{i : 1/A^{for}(i) < w/w^* < 1/A^{dom}(i)\right\}$$

The equivalent graphical representation can be seen in the next figure where under the region 'both', both countries are producing those goods, which means that specialization is incomplete.

Figure 2: Iceberg costs in a Ricardian model with multiple goods



To understanding the meaning of the above equations recall that from our production criteria (3), output in sector i is produced abroad if it's cheaper to do so, that is:

$$w\frac{1}{z(i)} \geq w^*\frac{1}{z^*(i)}\gamma$$

where we need to include in the RHS the iceberg cost $\gamma > 1$ since the indifference should be in producing at home or importing from abroad where, to get one unit of the good at the destination, only γunits have to be produced at the origin to accommodate for transportation losses. Rearranging the previous equation yields:

$$\frac{w}{w^*} \geq \frac{z(i)}{z^*(i)}\gamma \equiv \frac{1}{A^{dom}(i)}$$

which is precisely the region to the right of the intersection of $1/A^{dom}$ with w/w^* in the graph. Further more, if we take logs of the above equation:

$$\begin{aligned} & \log w - \log w^* \geq \log z(i) - \log z^*(i) + \log\gamma \\ \Rightarrow \quad & \log z^*(i) - \log z(i) \geq \log w^* - \log w + \log\gamma \end{aligned}$$

Which has the following approximated interpretation:

$$\%diff\,productivity\,btw\,F\,and\,H \geq \%diff\,wages\,btw\,F\,and\,H + \%loss\,transportation$$

That is, in order for production to be located abroad, it must be that the percentage difference between foreign productivity with respect to home, is larger than the percentage difference in wages between the two countries to which we should add up the percentage lost due to transportation[2]. This means that for those sectors where the difference of productivity between the home and foreign economies is very small, it is unlikely that will be tradable in the sense that differences in productivity are not enough to compensate the transportation cost $\log \gamma > 0$.

Another point worth making related with this theory of trade is the one related with the empirical fact that when expressed in terms of a single currency, countries' price levels are positively related to the level of real income per capita. In other words, a dollar, when converted to local currency at the market exchange rate, generally goes much further in a poor country than in a rich one. This empirical regularity has become known as the **Balassa-Samuelson**[3] effect, and makes use of the assumption that poor countries are generally less productive on traded goods with respect to rich countries, but have more or less the same productivity in non-traded goods. Defining the real exchange rate as the ratio of the foreign price level to the domestic price level evaluated at the same exchange rate, that is, the cost of a foreign bundle of consumption in terms of domestic bundles of consumption; and assuming that the price level is positively related with the price of tradables and non-tradables; then, despite the fact that price of tradables is equalized across the world, the price of non-tradables should be lower in countries with low productivity in the production of tradables given the predicted effect of wages. This should account on why the real exchange rate looks so much more depreciated (that is, the ratio defined before is large) in poor countries than in rich countries. This also explains why, as poor countries catch up in productivity with rich countries, the real exchange rate tends to appreciate. For more details on the Balassa-Samuelson effect you can check chapter 16 of Krugman et al. (2012).

[2]Note that for x that is close enough to zero, a Taylor first order approximation implies:

$$\log(1+x) \approx \log 1 + x = x$$

[3]On the original formulation of the effect see, Samuelson (1964) and Balassa (1964). For a more recent documentation of the empirical regularity see O'Connell (1998).

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