Game Theory

Introduction

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Game theory is the study of strategic interaction among multiple agents.

1 Leading Example: Braess's (1968) Paradox

There are 10 people who have to travel from S (Start) to G (Goal) in Figure 1. Each wants to minimize his/her travel time. There are two routes: route SAG passes through A and route SBG that passes through B. Route SABG, indicated by the dashed arrow, is unavailable. Roads SA and BG each are "narrow" and it takes x minutes if x people use them (due to congestion). Roads AG and SB each are "wide" and it takes 11 minutes regardless of the number of people using the roads.

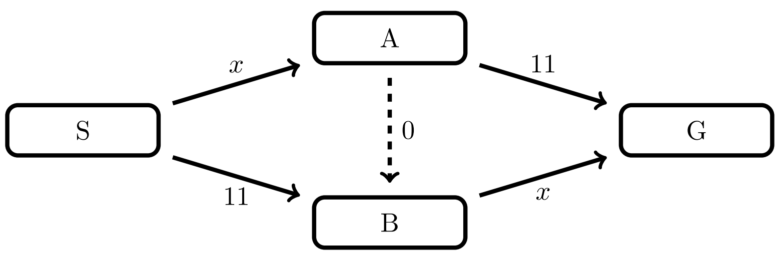

Figure 1: Braess's paradox

When road AG is unavailable, it is natural to expect that 5 use route SAG and 5 use route SBG.[1] The total travel time per person is $5 + 11 = 16$ minutes.

Suppose that road AB is now available, with travel time of 0 minute (for simplicity). Then, how would you travel? Let a and b be the number of all people (other than you) who use the roads SA and BG, respectively ($a, b \leq 9$). To arrive at G, each goes through either A or B (or both). Suppose that you are now at A. If you go straight to G, then it takes 11 minutes, but if you go to G via B then it takes $b + 1$ minutes. Since $b \leq 9$, you strictly prefer route ABG. Next, suppose that you are at S. Which should you go to, A or B? If you go to A, then since you will choose route ABG next, it takes a total of $(a+1)+0+(b+1)$ minutes, but if you go to B then it takes a total of $11 + (b + 1)$ minutes. Since $a \leq 9$, you strictly prefer to go to A. In the end, you will choose route SABG. This argument works for all people, not only for you. Hence, all will choose route SABG.[2] As a result, it takes 20 minutes, longer than 16 minutes! More options get available, all get worse-off!

[1] This kind of argument will be formalized as a Nash equilibrium.
[2] This kind of argument will be formalized as a strictly dominant strategy.

2 Game Theory as Interactive Decision Theory

This paradoxical phenomenon is unique to multi-agent decision-making. If you were the only one person who is privileged to use road AB, then since everyone else still has to use route SAG or SBG, you would choose route SABG, with travel time of 11 minutes at most. In single-agent decision-making, more options get available, you must be (weakly) better-off.

Braess's paradox illustrates that this is no longer the case if multiple agents "strategically interact" with each other. That is, your opponents, who strategically think that you will change your behavior, will also change their behavior. This change in action of your opponents may be harmful for you.

Game theory is a paradigm to analyze such strategic interaction among multiple agents. It thus could be more appropriately called interactive decision theory.

References

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