SEARCH AND MATCHING MODELS OF THE LABOR MARKET

Traditional models see unemployment as a disequilibrium phenomenon in the labor market

- As such, is a temporary situation of excess supply of workers

- Role of wage rigidities

Models of search and matching emphasize that unemployment in an *activity*: search for jobs

- Labor market frictions imply that job search can take time

- We can have unemployment in equilibrium even in the long run

Mortensen-Pissarides (1994): prototype model in the literature

Moreover, this class of models allow for a richer description of the labor market

`Introduction: Looking at Labor Market Data`

We can divided the working age population (POP) in three basic categories:

- Employed (E)

- Unemployed (U)

- Inactive (I)

Inactive workers do not participate in the labor force (students, retirees, at home); unemployed do participate searching actively for a job

The sum of employed and unemployed is equal to the *labor force*

With these three occupational categories, we can define the following labor market rates:

- Employment rate: E / POP
- Unemployment rate: U / (E+U)
- Participation rate: (E+U) / POP

where POP = E+U+I

Each of these indicators captures a different dimension of the labor market

For instance, comparing México and the U.S. (2019)

	Mexico	U.S.
Employment rate (%)	57.1	61.0
Unemployment rate (%)	3.6	3.5
Participation rate (%)	60.3	63.6

Moreover, in Mexico the informality rate (fraction of total employment under informality conditions) is about 55%

Labor surveys have a panel structure that follows the same individuals over time between consecutive periods (quarters)

This allows to compute nine different gross flows across occupational categories; for instance

- Employed workers moving to inactivity (E$\rightarrow$I)

- Unemployed workers finding a job (U$\rightarrow$E)

- Inactive individuals remaining out of the labor force (I$\rightarrow$I)

We can summarize these flows using the following rates:

- Job creation rate: JC = [(U→E) + (I→E)] / E
- Job destruction rate: JD = [(E→U) + (E→I)] / E
- Job finding probability: (U→E) / U

Subtracting the first two we obtain the *net* job creation

$$E_{t+1} = \left(1 + \underbrace{JC_t - JD_t}_{net\ creation}\right) E_t$$

... while adding them (JC+JD) we get an indicator of labor market *turnover*

As an example, in the U.S. (average 1978-2012)

from\to	E	U	I
E	0.96	0.01	0.03
U	0.25	0.54	0.21
I	0.05	0.03	0.93

... implying job creation/destruction rates of around 4-5%

... and a job finding probability around 25%

Finally, some labor surveys allow to distinguish between the *intensive* margin (hours worked per person) and the *extensive* margin (employed individuals) of employment

Most models do not allow to distinguish between these two margins

In the data, the extensive margin seems to be more important to understand employment fluctuations

Cyclical Behavior

For each labor market indicator, using a sufficiently long time series, we can compute several business cycle statistics, as:

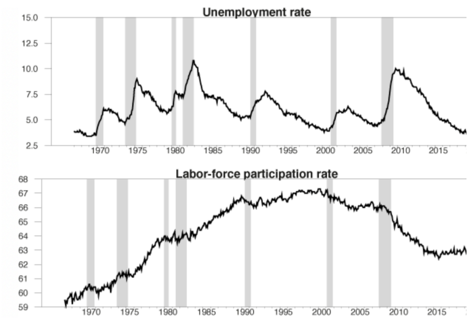
- Volatility (standard deviation), absolute or relative to output

- Correlation with output

In most economies:

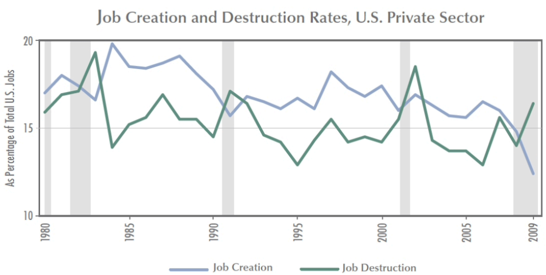
- Employment and participation rates are procyclical and less volatile than output; the unemployment rate is counter-cyclical and more volatile

For instance, in the U.S.



(the shaded areas correspond to recessions)

- Job creation and the job finding probability are counter-cyclical; job destruction is procyclical



This is the kind of regularities that search and matching models try to account for

The Basic Mortensen-Pissarides Model

In each period t,

- U_t individuals search for a job (unemployed)

- Firms post V_t vacancies

The *matching function* $m(U_t, V_t)$ determines the number of hirings resulting from a random search process

This function introduces frictions to the labor market adjustment in the model

We assume that the matching function features constant returns to scale, is increasing in each argument (unemployed, vacancies) and these two inputs are complementary

We define

- Job finding probability: $p_t = \frac{m(U_t, V_t)}{U_t} = m\left(1, \frac{V_t}{U_t}\right) = p\left(\theta_t\right)$

- Vacancy filling probability: $q_t = \frac{m(U_t, V_t)}{V_t} = m\left(\frac{U_t}{V_t}, 1\right) = q\left(\theta_t\right)$

these two probabilities depend on the labor market *tightness*, defined as $\theta_t = \frac{V_t}{U_t}$

We verify that p_t is increasing and q_t decreasing in θ_t

Total employment satisfies the law of motion

$$
\begin{aligned}
L_t &= L_{t-1} + q_t V_t - s L_{t-1} \\
&= (1-s) L_{t-1} + q_t V_t
\end{aligned}
$$

- $q_t V_t$: hirings (job creation)

- $s L_{t-1}$: separations (job destruction)

In the most basic version of the model, the separation rate s is exogenous

On the other hand, in each period,

$$
L_t + U_t = \bar{L}
$$

again in the basic model there is no participation decision

Household's Problem

In its simplest version, using per-capita variables (divided by $\bar{L}$)

$$\begin{aligned} \max \quad & \sum_{t=0}^{\infty} \beta^t c_t \\ s.t. \quad c_t &= w_t l_t \\ l_t &= (1-s)\, l_{t-1} + p_t u_t \\ l_t + u_t &= 1 \end{aligned}$$

There is no capital nor savings, although these could be added

Linear utility function (also can be generalized)

Lagrangian:

$$L = \sum_{t=0}^{\infty} \left\{ \beta^t c_t - \lambda_{1t} \left[c_t - w_t l_t \right] - \lambda_{2t} \left[l_t - (1-s)\, l_{t-1} - p_t \,(1-l_t) \right] \right\}$$

and first order conditions:

$$\frac{\partial L}{\partial c_t} = \beta^t - \lambda_{1t} = 0$$
$$\frac{\partial L}{\partial l_t} = \lambda_{1t} w_t - \lambda_{2t}\,(1+p_t) + \lambda_{2t+1}\,(1-s) = 0$$

from which, defining $W_t \equiv \frac{\lambda_{2t}}{\lambda_{1t}}$,

$$W_t = \left[w_t - p_t W_t \right] + \beta\,(1-s)\, W_{t+1}$$

The multiplier W_t represents recursively the net value for the household of having one worker employed

Wage Bargaining and the Decision to Post Vacancies

In each period, a matched workers produces A units of output, hence in the aggregate

$$y_t = Al_t$$

(we could add capital and/or productivity shocks)

In competitive markets, workers would be payed their marginal product $w_t = A$

However, search frictions implied that, once matched, a worker and a firm enter a bilateral bargaining process

Firms pay a cost η for each vacancy posted, and recover this cost extracting a surplus from the match over time

The value of a worker for a firm (J_t) can be written recursively as:

$$J_t = [A - w_t] + \beta (1 - s) J_{t+1}$$

The zero profit condition implies in this model that

$$q_t J_t = \kappa$$

this is, the vacancy cost needs to be exactly compensated by the value of having a new worker, times the vacancy filling probability

Once matched, we assume that the worker and the firm bargain each period the wage rate according to the *Nash protocol*

This protocol is equivalent to maximize a weighted average of the surplus of each side

$$\max_{w_t} \left(W_t\left(w_t\right)\right)^{\gamma} \left(J_t\left(w_t\right)\right)^{1-\gamma}$$

where the parameter γ measures the worker's bargaining power

Nash bargaining generates endogenously the *sharing rule*

$$\left(1-\gamma\right) W_t = \gamma J_t$$

$\theta_t = \frac{v_t}{u_t}$

→ for the household

→ for the firm

`Equilibrium`

A *Nash-bargaining equilibrium* for this economy is a set of sequences for quantities c_t, l_t, u_t, y_t , v_t, θ_t, probabilities p_t, q_t, values W_t, J_t and wages w_t such that:

i) Given l_{-1}, p_t, w_t, the sequences c_t, u_t, l_t solve the household's problem:

$$\max \sum_{t=0}^{\infty} \beta^t c_t$$

exogenous separation

$$\begin{aligned} s.t. \quad c_t &= w_t l_t \\ l_t &= (1-s)\, l_{t-1} + p_t u_t \\ l_t + u_t &= 1 \end{aligned}$$

→ new hires

ii) In each period t, we define the values of a worker for the household and for the firm recursively:

$$W_t = [w_t - p_t W_t] + \beta (1-s) W_{t+1}$$

$$J_t = [A - w_t] + \beta (1-s) J_{t+1}$$

matching function

v_t / u_t

$$p_t = \frac{m(u_t, v_t)}{u_t} = p(\theta_t)$$

$$q_t = \frac{m(u_t, v_t)}{v_t} = q(\theta_t)$$

iii) In each period t, the probabilities are given by: $p_t = p(\theta_t)$ y $q_t = q(\theta_t)$, with $\theta_t = \frac{v_t}{u_t}$

iv) In each period t, the sharing rule, zero-profit condition and market clearing conditions are all satisfied

$$(1-\gamma) W_t = \gamma J_t \qquad q_t J_t = \eta$$

$$y_t = A l_t = c_t + \eta v_t$$

cost of a vacancy

Nash-bargaining

$$(1-\gamma)\,W_t = {}^{(1-\gamma)}[w_t - p_t W_t] + \beta(1-s)\,{}^{(1-\gamma)}W_{t+1}$$

$$\gamma\, J_t = {}^{\gamma}[A_t - w_t] + \beta(1-s)\,{}^{\gamma}J_{t+1}$$

$$\Rightarrow (1-\gamma)[w_t - p_t W_t] = \gamma[A_t - w_t]$$

Characterizing the Equilibrium

Combining the recursive definitions of W_t and J_t with the sharing rule, we obtain

$$(1-\gamma)[w_t - p_t W_t] = \gamma[A - w_t]$$

from which,

$$w_t = \gamma A + (1-\gamma)\,p_t W_t$$

$$w_t = \gamma A_t + (1-\gamma)\,p_t W_t$$

and, coming back to the definition of W_t,

$$(1+\gamma p_t)\,W_t = \gamma A + \beta(1-s)\,W_{t+1} \qquad (I)$$

$$W_t = [\gamma A + (1-\gamma)p_t W_t - p_t W_t] + \beta(1-s)W_{t+1}$$

$$(1+\gamma p_t)\,W_t = \gamma A + \beta(1-s)\,W_{t+1}$$

$$\gamma J_t = (1-\gamma) W_t \longrightarrow J_t = \left(\frac{1-\gamma}{\gamma}\right) W_t$$

Using the zero profit condition and, again, the sharing rule,

$$J_t = \frac{\eta}{q_t} = \left(\frac{1-\gamma}{\gamma}\right) W_t$$

from which,

$$\left(\frac{1-\gamma}{\gamma}\right) q_t W_t = \eta \qquad (II)$$

and, coming back to the result for the equilibrium wage,

$$w_t = \gamma \left[A + \eta \frac{p_t}{q_t}\right] = \gamma \left[A + \eta \theta_t\right]$$

We can show that the equilibrium wage increases with: (i) labor productivity; (ii) worker's bargaining power; (iii) labor market tightness and (iv) the vacancy cost

$$w_t = \gamma A + (1-\gamma) p_t W_t$$

but $W_t = \left(\frac{\gamma}{1-\gamma}\right)\left(\frac{\eta}{q_t}\right)$

$$w_t = \gamma A + \gamma \left(\frac{p_t}{q_t}\right) \eta$$

$$= \gamma \left[A + \eta \frac{p_t}{q_t}\right]$$

$$p_t \equiv \frac{m(u_t, v_t)}{u_t}$$

$$q_t \equiv \frac{m(u_t, v_t)}{v_t}$$

$$\frac{p_t}{q_t} = \frac{v_t}{u_t} = \theta_t$$

$$A l_t = c_t + \eta v_t$$

$$= w_t l_t + \eta v_t$$

$$[A - w_t] \, l_t = \eta v_t$$

$$[A - w_t](1 - u_t) = \eta v_t$$

$$[A - \gamma(A + \eta\theta_t)](1 - u_t) = \eta v_t$$

$$[(1-\gamma)A - \gamma\eta\theta_t](1 - u_t) = \eta v_t$$

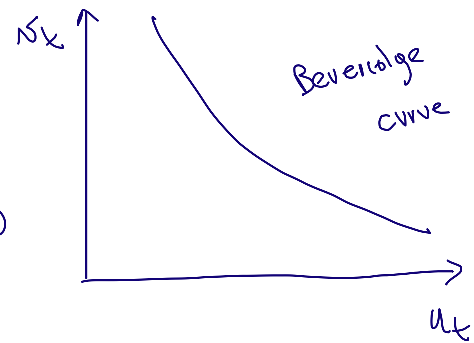
Now, combining the household's budget constraint with the market clearing condition:

$$(A - w_t)\, l_t = \eta v_t$$

from where, using again the equilibrium wage,

$$[(1-\gamma)\, A - \gamma\eta\theta_t]\,(1 - u_t) = \eta v_t$$

we obtain a negative relation between unemployment and vacancies, named in the literature the *Beveridge curve*



chooses c_t, l_t, u_t, v_t

`Social Planner's Problem`

The efficient solution of the model solves the problem

$$\max \quad \sum_{t=0}^{\infty} \beta^t c_t$$

feasibility constraint

law of motion labor

$$\text{s.t.} \quad \begin{aligned} c_t + \eta v_t &= A l_t \\ l_t &= (1-s)\, l_{t-1} + m(u_t, v_t) \\ l_t + u_t &= 1 \end{aligned}$$

time use constraint

For the special case of a Cobb-Douglas matching function: $m(u,v) = u^{\phi} v^{1-\phi}$

$m(u,v) = u^{\phi} v^{1-\phi}$

$p_t = \frac{m}{u} = \left(\frac{v}{u}\right)^{1-\phi}$

$q_t = \frac{m}{v} = \left(\frac{v}{u}\right)^{-\phi}$

... the *Hosios condition* states that the Nash-bargaining equilibrium is efficient if and only if $\gamma = \phi$

weight of unemployment in matching function

worker's bargaining power

$$m(u_t, v_t) = m(1-l_t, v_t) = (1-l_t)^\phi v_t^{1-\phi}$$

Lagrangian:

$$\begin{aligned} L &= \sum_{t=0}^{\infty} \Big\{ \beta^t c_t - \lambda_{1t}\left[c_t + \eta v_t - A l_t\right] \\ &\quad - \lambda_{2t}\left[l_t - (1-s)\, l_{t-1} - (1 - l_t)^\phi v_t^{1-\phi}\right] \Big\} \end{aligned}$$

and first order conditions:

$$\begin{aligned} \frac{\partial L}{\partial c_t} &= \beta^t - \lambda_{1t} = 0 \\ \frac{\partial L}{\partial v_t} &= -\lambda_{1t}\eta + \lambda_{2t}(1-\phi)\left(\frac{1-l_t}{v_t}\right)^\phi = 0 \\ \frac{\partial L}{\partial l_t} &= \lambda_{1t} A - \lambda_{2t}\left(1 + \phi\left(\frac{1-l_t}{v_t}\right)^{\phi-1}\right) + (1-s)\,\lambda_{2t+1} = 0 \end{aligned}$$

$$\lambda_{1t} = \beta^t$$

$$\frac{\lambda_{2t}}{\lambda_{1t}} = \frac{\eta}{(1-\phi)\underbrace{\left(\frac{1-l_t}{v_t}\right)^\phi}_{q_t}}$$

$$\frac{\lambda_{2t}}{\lambda_{1t}} = \frac{\eta}{(1-\phi)\, q_t}$$

$$\frac{\lambda_{2t}}{\lambda_{1t}}\left(1 + \phi\, P_t\right) = A + \left(\frac{\lambda_{2t+1}}{\lambda_{1t+1}}\right)\underbrace{\left(\frac{\lambda_{1t+1}}{\lambda_{1t}}\right)}_{\beta}(1-s)$$

$$(1 + \phi\, P_t)\,\frac{\lambda_{2t}}{\lambda_{1t}} = A + \beta(1-s)\,\frac{\lambda_{2t+1}}{\lambda_{1t+1}}$$

define $W_t^p \equiv \phi \frac{\lambda_{2t}}{\lambda_{1t}}$

$$\begin{cases} (1+\phi p_t) W_t^p = \phi A + \beta(1-s) W_{t+1}^p & \checkmark \\ W_t^p = \left(\frac{\phi}{1-\phi}\right) \frac{\eta}{q_t} & \checkmark \end{cases}$$

Defining $W_t^p \equiv \phi \frac{\lambda_{2t}}{\lambda_{1t}}$ and, as before, $p_t \equiv \left(\frac{1-l_t}{v_t}\right)^{\phi-1}$ and $q_t \equiv \left(\frac{1-l_t}{v_t}\right)^{\phi}$,

we can rewrite these first order conditions as:

$$\begin{cases} (1+\phi p_t)\, W_t^p = \phi A + \beta\,(1-s)\, W_{t+1}^p \\ q_t \left(\frac{1-\phi}{\phi}\right) W_t^p = \eta \end{cases}$$

$\mu = \phi$

Using the Hosios condition, we recover the same equations (I) and (II) from the Nash bargaining equilibrium

Steady state $W_t^p = W_{t+1}^p = W^*$

(I) $(1+\phi p^*) W^* = \phi A + \beta(1-s) W^*$

$$W^* = \frac{\phi A}{1+\phi p^* - \beta(1-s)}$$

(2) $\quad q^* \left(\frac{1-\phi}{\phi}\right) W^* = \eta$

combining (1) and (2)

$$q^* \left(\frac{1-\phi}{\phi}\right) \frac{\phi A}{1+\phi p^* - \beta(1-s)} = \eta$$

Steady State

In a stationary equilibrium

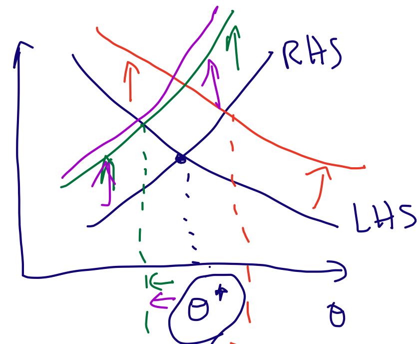
$$W^* = \frac{\phi A}{1+\phi p^* - \beta\,(1-s)} \qquad q^* \left(\frac{1-\phi}{\phi}\right) W^* = \eta$$

$p^* = p(\theta^*) \qquad q^* = q(\theta^*)$

so that

$$\underbrace{q\,(\theta^*)\,(1-\phi)\,A}_{LHS} = \underbrace{\eta\,[1+\phi p\,(\theta^*) - \beta\,(1-s)]}_{RHS}$$

This equation determines the labor market tightness θ^* in steady state $\Rightarrow$



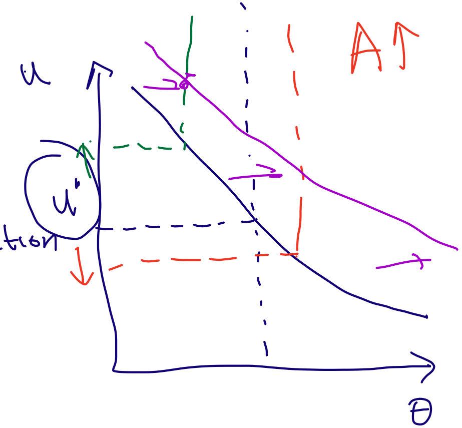
On the other hand, the stationary condition

$$p\,(\theta^*)\,u^* = s\,(1-u^*)$$

determines a negative relation between labor market tightness and the long run unemployment rate

$$l_t = (1-s)\,l_{t-1} + m(u_t, v_t)$$

in SS $\quad l_t = l^* \quad \forall t \quad \Rightarrow \quad \underbrace{m(u^*, v^*)}_{\text{Job creation}} = \underbrace{s\, l^*}_{\text{Job destruction}}$



$$p(\theta^*)\,u^* = s(1-u^*)$$

$$P(\theta^*) = \underline{s}\left(\frac{1-\alpha}{v^*}\right)$$

For example, an increase in productivity A:

- Increases labor market tightness
- Reduces long term unemployment

On the other hand, an increase in the separation rate s or in the vacancy cost η:

- Reduces labor market tightness
- Increases long term unemployment more

business cycle

The RBC Model with Labor Market Frictions

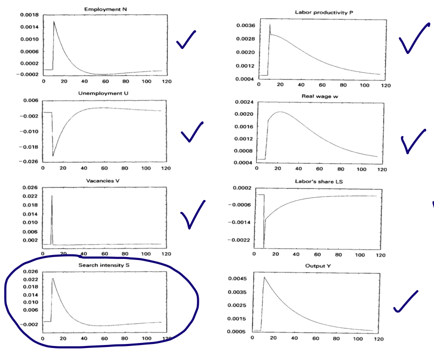
Merz (1995) and Andolfatto (1996) combine the real business cycle model with labor market frictions, à la Mortensen and Pissarides

Some important differences with the basic setup:

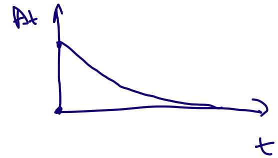
- Utility function concave in consumption and adding leisure
- Search cost for unemployed - variable search intensity merz
- Capital and investment
- Technology and matching shocks andolfatto

$B m(u,v)$
$\uparrow$

Impulse-response functions to a technology shock (Merz)



$\frac{wL}{Y}$ $\frac{w}{Y/L}$



The model captures the pro-cyclicality of employment, vacancies, labor productivity (Y/L) and the real wage, and the counter-cyclicality of the unemployment rate

Business cycle statistics (Andolfatto):

TABLE 1—CYCLICAL PROPERTIES: U.S. ECONOMY AND MODEL ECONOMIES

	U.S. economy $\sigma(y) = 1.58$			RBC economy $\sigma(y) = 1.22$			Search economy $\sigma(y) = 1.45$		
Variable (x)	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
Consumption	0.56	0.74	0	0.34	0.90	0	0.32	0.91	0
Investment	3.14	0.90	0	3.05	0.99	0	2.98	0.99	0
Total hours	0.93	0.78	+1	0.36	0.98	0	0.59	0.96	0
Employment	0.67	0.73	+1	0.00	0.00	0	0.51	0.82	+1
Hours/worker	0.34	0.66	0	0.36	0.98	0	0.22	0.66	0
Wage bill	0.97	0.76	+1	1.00	1.00	0	0.94	1.00	0
Labor's share	0.68	−0.38	−3	0.00	0.00	0	0.10	−0.62	−1
Productivity	0.64	0.43	−2	0.64	0.99	0	0.46	0.94	0
Real wage	0.44	0.04	−4	0.64	0.99	0	0.39	0.95	0

Notes: $\sigma(y)$ is the percentage standard deviation in real per-capita output. Column (1) is $\sigma(x)/\sigma(y)$. Column (2) is the correlation between x and y. Column (3) is the phase shift in x relative to y: $-j$ or $+j$ corresponds to a lead or lag of j quarters.

Search and matching frictions in the labor market improve the model predictions with respect to:

- Employment fluctuations
- Real wage fluctuations (relative to productivity)

The model also generates a negative correlation between unemployment and vacancies (Beveridge curve), as observed in the data

However, Shimer (2005) shows that the model generates too little volatility in the vacancy/unemployment ratio (labor market tightness) relative to productivity, compared to the data