

# International supply chains

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## 1 Supply chains

Supply chains emerge when firms can allocate part of production to outside of their own boundaries. This happens, for example, when a firm buys materials/ intermediate inputs from other firms required to produce a final output. Whenever this type of production fragmentation occurs across national borders, we then refer to international supply chains. Vertical specialization is sometimes also used a synonymous term of supply chain.

Given the recent growing importance of vertical specialization at accounting for international trade flows (for example [Yi, 2003](#), or [Costinot et al., 2012](#)), it becomes important to have a theoretical framework that allows us to understand how to interpret these trends, and how disruptive are trade barriers in environments with vertical specialization. In the next section, we present a Ricardian model that allows for vertical specialization occurring at an international level. This simple model help on generating a clear view on:

- why has vertical specialization has become more important in recent years at accounting international trade
- why international trade responds non-linearly to change in tariffs and other trade barriers
- how small variations in trade barriers can have large effects on vertical specialization

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## 1.1 A Ricardian model of international trade with vertical specialization

This Ricardian model is a generalization of another version of this model that we've used to study trade between two countries with multiple goods (Dornbusch et al., 1977). The main difference consists in assuming that production of one final good can be decomposed into three sequential stages of production:

1. the production of simple intermediate goods, potentially traded, using labor as an input of production
2. the production of more complex intermediate goods, also traded, using labor and the stage 1 intermediate as inputs
3. the production of a single final consumption good, considered to be non-traded, using all intermediate goods produced in stage 2 as inputs

This model structure, proposed by Yi (2003), allows for international vertical specialization when there are two trading partners (say, home and foreign), if the production of the intermediate good 2 occurs at home and the intermediate 1 is imported, or if the production of intermediate good 1 occurs at home while good two is imported.

Formally, allow for two countries  $H, F$  and a continuous of intermediate goods  $z \in [0, 1]$  that required two sequential stages of production characterized by the following production functions:

$$\begin{aligned} y_1^i(z) &= A_1^i(z) l_1^i(z) \\ y_2^i(z) &= [y_1^i(z)]^\theta [A_2^i(z) l_2^i(z)]^{1-\theta} \end{aligned}$$

for  $i = H, F$ ,  $A_j^i(z)$  is a productivity parameter associated with the production of intermediate good  $z$  during stage  $j$  in country  $i$ ,  $l_j^i(z)$  is the corresponding labor demand, and  $\theta$  is an elasticity associated with a Cobb-Douglas production function. To impose some structure on the comparative advantage across the two economies, assume that:

$$\frac{A_j^H(z)}{A_j^F(z)} \equiv A_j(z) \tag{1}$$

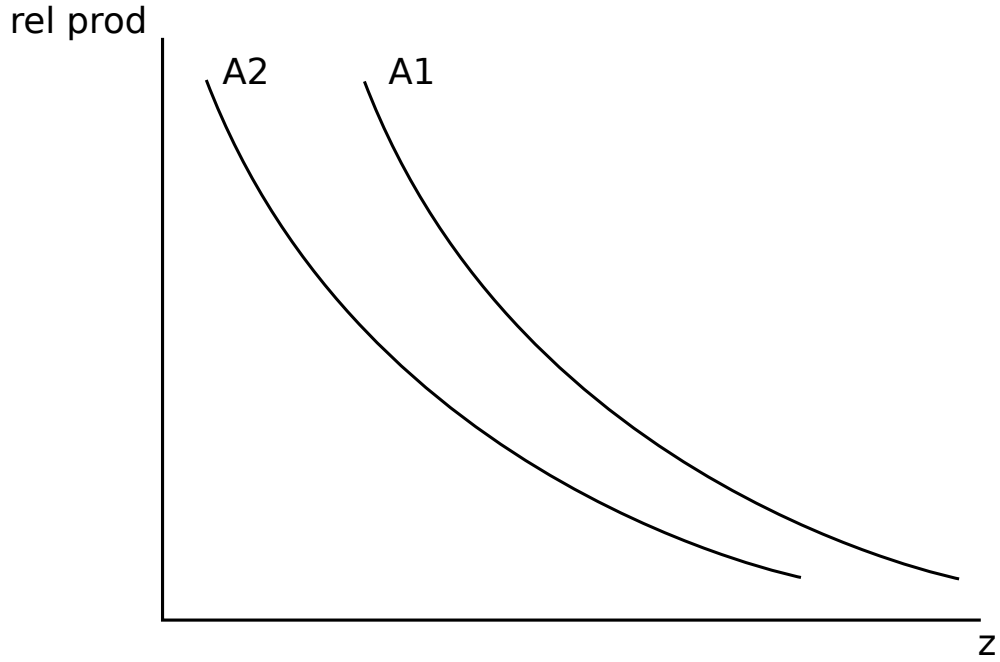
is a decreasing function of  $z$ , that is, the home economy has stronger comparative advantage

for those goods with lower index  $z$ . Also, assume that:

$$A_1(z) > A_2(z) \quad \forall z \in [0, 1] \quad (2)$$

implying that the home economy has a relatively higher comparative advantage in the intermediate good 1 than on 2. These relative productivity functions can be represented in a diagram as in figure 1.

Figure 1: Description of technology for the model



The final consumption good, uses a Cobb-Douglas production function that aggregates all goods produced in stage 2:

$$y^i = \int_{z \in [0,1]} [y_2^i(z)]^b dz$$

where  $b$  is such that

$$\int_{z \in [0,1]} b dz = 1$$

**Patterns of specialization** Note that the economy represented above implies different patterns of specialization on the production of an intermediate good : 1 and 2 are produced

at home  $(H, H)$ ; 1 and 2 are produced abroad  $(F, F)$ ; 1 is produced at home while 2 is produced abroad  $(H, F)$ ; 1 is produced abroad while 2 is produced at home  $(F, H)$ . Naturally, vertical specialization only includes  $(H, F)$  and  $(F, H)$ .

Note that profits in the first stage production of the intermediate good is given by  $(i = H, F)$ :

$$\pi_1^i(z) = p_1(z) y_1^i(z) - w^i l_1^i(z)$$

while that of the second stage we use:

$$\pi_2^i(z) = p_2(z) y_2^i(z) - w^i l_1^i(z) - p_1(z) y_1^i(z)$$

if intermediate good 1 is produced domestically, or, if it's imported:

$$\pi_2^i(z) = p_2(z) y_2^i(z) - w^i l_1^i(z) - \tau p_1(z) y_1^i(z)$$

where  $\tau \geq 1$  represents an iceberg cost capturing tariffs and other trade costs. Finally, the production of the final consumption good implies the following profit:

$$\pi^i = p^i y^i - \int_{z \in (H, H), (F, H)} p_2^i(z) \cdot y_2^i(z) dz - \int_{z \in (F, F), (H, F)} \tau p_2^i(z) y_2^i(z) dz$$

**Cut-off sectors (without iceberg costs  $\tau = 1$ )** Given the competitive framework, zero profit conditions in the production of intermediate good 1 implies:

$$p_1(z) A_1^i(z) - w^i \leq 0$$

Since the above equation must hold for both equations we can show that there's a cut-off sector  $z_1$  that divides production between the home and the foreign economy on that intermediate good:

$$p_1(z_1) A_1^H(z_1) - w^H = 0$$

$$p_1(z_1) A_1^F(z_1) - w^F = 0$$

implying:

$$\frac{w^H}{w^F} = \frac{A_1^H(z_1)}{A_1^F(z_1)} \equiv A_1(z_1) \quad (3)$$

Therefore for any  $z < z_1$  production is located at home, while for  $z \geq z_1$  production is located in the foreign economy.

As for the intermediate good of stage 2, cost minimization implies the following demand functions:

$$\begin{aligned} w^i l_2^i(z) &= \lambda (1 - \theta) y_2^i(z) \\ p_1(z) y_1^i(z) &= \lambda \theta y_2^i(z) \end{aligned}$$

where  $\lambda$  is a Lagrangean multiplier representing the marginal cost. Substituting these demand functions into the production function, one get

$$\begin{aligned} y_2^i(z) &= [y_1^i(z)]^\theta [A_2^i(z) l_2^i(z)]^{1-\theta} \\ \Rightarrow y_2^i(z) &= \left[ \frac{\lambda \theta y_2^i(z)}{p_1(z)} \right]^\theta \left[ \frac{A_2^i(z) \lambda (1 - \theta) y_2^i(z)}{w^i} \right]^{1-\theta} \\ \Rightarrow MC = \lambda &= \left[ \frac{p_1(z)}{\theta} \right]^\theta \left[ \frac{w^i}{(1 - \theta) A_2^i(z)} \right]^{1-\theta} \end{aligned}$$

Therefore:

$$p_2(z) \leq \left[ \frac{p_1(z)}{\theta} \right]^\theta \left[ \frac{w^i}{(1 - \theta) A_2^i(z)} \right]^{1-\theta}$$

But since the above equation holds simultaneous for  $i = H, F$ , we must have a cut-off productivity  $z_2$ , such that:

$$\begin{aligned} p_2(z_2) &= \left[ \frac{p_1(z_2)}{\theta} \right]^\theta \left[ \frac{w^H}{(1 - \theta) A_2^H(z_2)} \right]^{1-\theta} \\ p_2(z_2) &= \left[ \frac{p_1(z_2)}{\theta} \right]^\theta \left[ \frac{w^F}{(1 - \theta) A_2^F(z_2)} \right]^{1-\theta} \end{aligned}$$

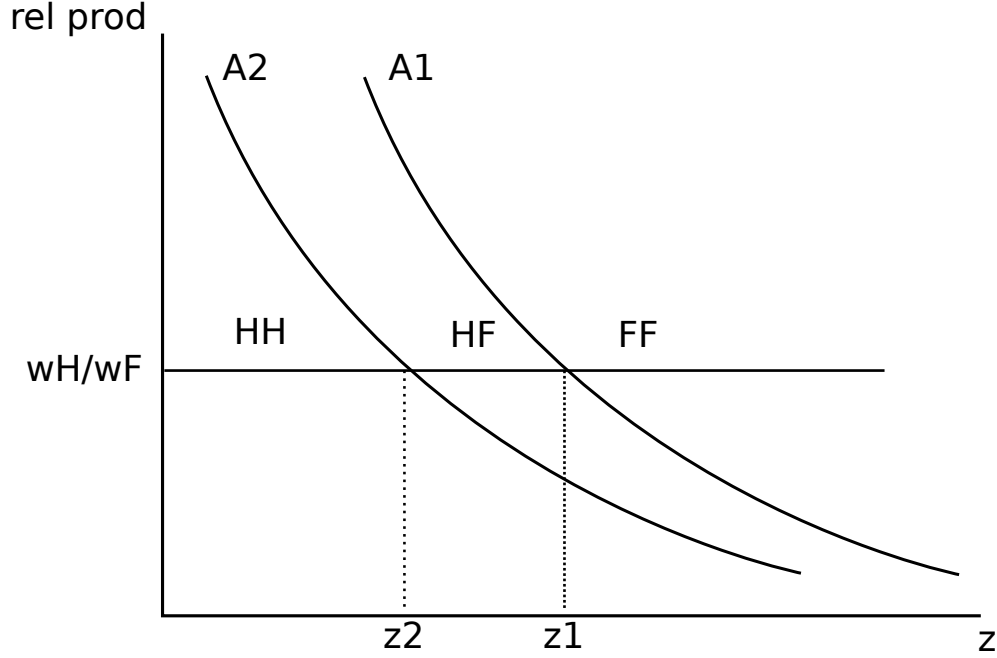
implying:

$$\frac{w^H}{w^F} = \frac{A_2^H(z_2)}{A_2^F(z_2)} \equiv A_2(z_2) \quad (4)$$

where for any  $z < z_2$  production of intermediate 2 is located at home, while for  $z \geq z_2$  production is located in the foreign economy.

Note that given the structure that we imposed on relative productivities in (1) and (2), together with (3) and (4) imply that we can determine the pattern of specialization associated with the production of any good  $z \in [0, 1]$ . This can be represented in figure 2.

Figure 2: Patterns of specialization in the model



### 1.1.1 Adding an iceberg cost

Suppose now that we allow for a tariff  $\tau > 1$ . With respect to the production of the intermediate good 2, the home economy has to decide now whether to produce that good domestically or to import that good from the foreign economy (and face a transportation cost). The new cut-off productive consistent with indifference,  $z_2^\tau$ , implies:

$$p_2(z_2^\tau) = \left[ \frac{p_1(z_2^\tau)}{\theta} \right]^\theta \left[ \frac{w^H}{(1-\theta) A_2^H(z_2^\tau)} \right]^{1-\theta}$$

if the good 2 is produced at home, or, if it's imported:

$$p_2(z_2) = \tau \left[ \frac{\tau p_1(z_2^\tau)}{\theta} \right]^\theta \left[ \frac{w^F}{(1-\theta) A_2^F(z_2^\tau)} \right]^{1-\theta}$$

where in the last equation we add the transportation cost  $\tau$  to the origin price of producing a good in the foreign economy (that implies imports of the intermediate 1). Dividing the two equations we get:

$$\frac{w^H}{w^F} = \frac{A_2^H(z_2^\tau)}{A_2^F(z_2^\tau)} \cdot \tau^{\frac{1+\theta}{1-\theta}} \equiv A_2(z_2^\tau) \cdot \tau^{\frac{1+\theta}{1-\theta}}$$

but since  $\tau > 1$  and  $\frac{1+\theta}{1-\theta} > 1$ ,  $z_2^\tau > z_2$  which implies a shift to the right of the  $A_2$  curve in figure 2.

Similarly, the foreign economy has to consider whether to produce good 1 or import it from abroad. This should imply two indifference prices at  $z_1^\tau$ :

$$p_1(z_1^\tau) = \frac{w^F}{A_1^F(z_1^\tau)}$$

$$p_1(z_1^\tau) = \tau \frac{w^H}{A_1^H(z_1^\tau)}$$

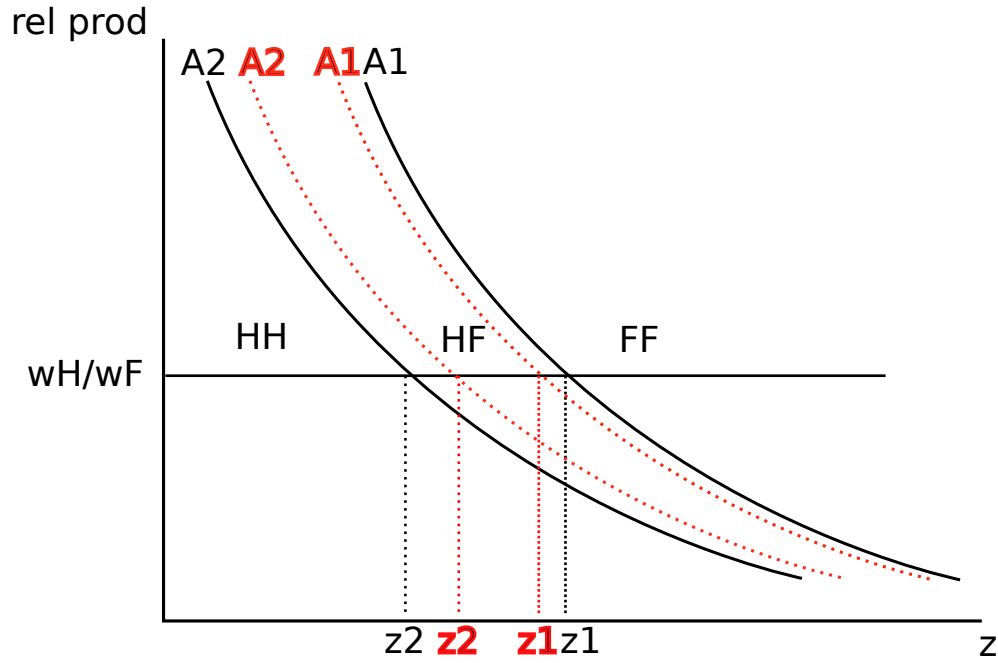
or:

$$\frac{w^H}{w^F} = \frac{A_1^H(z_1^\tau)}{A_1^F(z_1^\tau)} \tau^{-1} \equiv A_1(z_1^\tau) \tau^{-1}$$

this, instead, implies a shift to the left of the  $A_1$  curve in figure 2.

It is easy to see that the combined effect of the two curve shifts, implies a smaller region where countries can engage in vertical specialized trade, as depicted in figure 3.

Figure 3: Patterns of specialization in the model with an increase of  $\tau$



### 1.1.2 Trade without vertical specialization

Suppose we were to increase  $\tau$  such that the region  $(HF)$  in figure 3 shrinks completely to 0. Then, we would no longer generate any trade of the intermediate good 1 in this world economy, that is to say, both economies had to integrate their own production of 1 when producing 2. In practical terms, that implies that the effective production function of intermediate good 2 in each country aggregates the production function of good 1:

$$y_2^i(z) = (A_1^i(z) l_1^i(z))^\theta (A_2^i(z) l_2^i(z))$$

proceeding with analogy, that implies an optimal price of:

$$p_2(z) = \left[ \frac{w^i}{\theta A_1^i(z)} \right]^\theta \left[ \frac{w^i}{(1-\theta) A_2^i(z)} \right]^{1-\theta}$$

When the home economy considers whether to produce or import an intermediate good 2, we should generate an indifference sector  $z^{\tau\tau}$  characterized by:

$$p_2(z^{\tau\tau}) = \left[ \frac{w^H}{\theta A_1^H(z^{\tau\tau})} \right]^\theta \left[ \frac{w^H}{(1-\theta) A_2^H(z^{\tau\tau})} \right]^{1-\theta}$$

if production is located at home, while

$$p_2(z^{\tau\tau}) = \tau \left[ \frac{w^F}{\theta A_1^F(z^{\tau\tau})} \right]^\theta \left[ \frac{w^F}{(1-\theta) A_2^F(z^{\tau\tau})} \right]^{1-\theta}$$

if that good is instead imported. The above two equations also imply:

$$\begin{aligned} \frac{w^H}{w^F} &= \left( \frac{A_1^H(z^{\tau\tau})}{A_1^F(z^{\tau\tau})} \right)^\theta \left( \frac{A_2^H(z^{\tau\tau})}{A_2^F(z^{\tau\tau})} \right)^{1-\theta} \cdot \tau \\ &= \gamma \left( \frac{A_2^H(z^{\tau\tau})}{A_2^F(z^{\tau\tau})} \right) \cdot \tau \\ \Rightarrow \frac{w^H}{w^F} &= \gamma A_2(z^{\tau\tau}) \cdot \tau \end{aligned} \tag{5}$$

where in the second equality we make an assumption that  $\frac{A_1^H(z)}{A_1^F(z)} = \gamma \frac{A_2^H(z)}{A_2^F(z)}$ .

Note that the effect of a tariff on trade is very different when we have vertical specialization (4) versus when we don't (5). For fixed wages, a 1% increase in tariff implies a 1% shift to the right of the  $A_2(z)$  curve whenever this home economy is not engaged in vertical



specialization (as seen in equation 5). If a country does however vertical specialization, a 1% increase in the tariff, implies a  $\frac{1+\theta}{1-\theta}\%$  shift of the  $A_2(z)$  curve. This implies that a tariff impact is much larger with vertical specialization when compared with no vertical specialization. To be a little bit more specific, suppose that imports move 1-to-1 with the shift of the curve  $A_2(z)$  and that  $\theta = 2/3$ . The elasticity of imports with respect to the tariff, without vertical specialization is only 1, while it's 5 when we allow for vertical specialization. That is, the elasticity of imports with respect to the tariff becomes much larger once a country starts specializing vertically on its chain of production

## 1.2 Intuition and relevance

What is the intuition for the much larger impact of the tariff when a country engages in vertical specialization? The reason related with the fact that this tariff has to be paid every time a good is exported. For that reason, fragmenting production into two countries may imply a tariff payment that occurs two times or more. Think, for example, of the case where the home economy exports an unfinished good to a foreign economy, and therefore a tariff is paid, and when that good is transformed in the foreign economy and re-exported to the home economy, there's another tariff payment. This type of double tariff payment is inexistent when trade occurs only at the level of end-of-line production or final consumption goods where tariffs are incurred only once.

This intuition should allow us to interpret better the non-linearity of trade with respect to tariffs that we observed in the data (Yi, 2003). During the 70's trade was responding very slowly to the reduction in tariffs because there was not much vertical specialization across countries. More recently, during the 90's, vertical specialization started to become more prevalent and that's also the period where trade responds more strongly to changes in tariffs.

The inclusion of vertical specialization in our analysis should also reveal the potential risks of scrapping some trade agreements. When trade agreements push tariffs to close to zero, it is natural that economies become more entangled in the sense that more vertical specialization and international supply chains become more prevalent. When that happens, then an exit of a trade agreement, even if it implies a moderate increase in tariffs may have a huge effect of trade reduction since these supply chains will be disrupted.

## References

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