Understanding What Instrumental Variables Estimate in Models with Essential Heterogeneity

Advanced Microeconometrics

Instructor: Cristián Sánchez

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This Lecture

- Heckman, Urzua, and Vytlacil (2006) (Essential Heterogeneity)
- Heckman and Vytlacil (2007) (Treatment Parameters)

Main Ideas of this Talk

- To present and discuss potential limitations of the *IV* method.
- We relate the standard approach to economic choice models. This allows us to show that, in general, the *IV* approach does not provide answers to well defined questions.
- We illustrate these points using theoretical and empirical examples.

The Instrumental Variable Approach

- Suppose the availability of a cross-section.
- In the sample we have information on:

  Outcomes Y

  Choices/Treatment D

  Instruments Z

- Examples:
  - Y educational outcomes (e.g. Achievement Test Scores), $D = 1$ if enrolled in college, Z tuition or family background.
  - Y firm performance, $D = 1$ if access to financial sector, Z distance to the bank.

- The researcher postulates the empirical model

$$Y = \alpha + \beta D + \varepsilon$$

where, due to the endogeneity of D (selection bias), he assumes

$$D \not\perp\!\!\!\perp \varepsilon$$

- OLS in this model produces a biased estimate of β.

- The availability of an instrument Z allows to obtain β ("the effect" of D on Y)

$$\frac{Cov(Y, Z)}{Cov(D, Z)} = \beta$$

if $Z \perp\!\!\!\perp \varepsilon$ and $Z \not\perp\!\!\!\perp D$.

A More General Approach: A Model of Essential Heterogeneity

- Two potential outcomes

$$
\begin{aligned}
Y_1 &= \mu_1(X) + U_1 = \alpha + \varphi + U_1 \\
Y_0 &= \mu_0(X) + U_0 = \alpha + U_0
\end{aligned}
$$

- The choice model

$$
D = \begin{cases} 1 & \text{if } D^* = \mu_D(Z) - V \geq 0 \\ 0 & \text{otherwise} \end{cases}
$$

and

$$
U_1 \not\perp\!\!\!\perp U_0 \not\perp\!\!\!\perp V.
$$

- And, as in the previous case, we observe D, Y, Z, but notice that

$$
Y = Y_1 D + Y_0(1 - D)
$$

- Interestingly,

$$\begin{aligned} Y &= Y_1 D + Y_0(1 - D) \\ &= (\alpha + \varphi + U_1)D + (\alpha + U_0)(1 - D) \\ &= \alpha + (\varphi + U_1 - U_0)D + U_0 \\ &= \alpha + \varphi D + U_0 + D(U_1 - U_0) \\ &= \alpha + \varphi D + \varepsilon \end{aligned}$$

  where $D \not\perp \varepsilon$.

- In principle, we could apply the same logic as in the *IV* approach. Thus, using Z we could compute

$$\frac{Cov(Y, Z)}{Cov(D, Z)}$$

  to obtain "the effect" of D on Y.

- But, by assumption
$$Z \not\perp\!\!\!\perp \varepsilon \Leftrightarrow Z \not\perp\!\!\!\perp U_0 + D(U_1 - U_0)$$
even if $Z \perp\!\!\!\perp (U_1, U_0)$.

- The main consequence of this model is that we cannot define "the effect" of D on Y.

- In general, there is an entire distribution of gains $(Y_1 - Y_0)$
$$\begin{aligned} Y &= Y_1 D + Y_0(1 - D) \\ &= Y_0 + (Y_1 - Y_0)D \\ &= \alpha + \beta D + U_0 \end{aligned}$$
where $\beta = Y_1 - Y_0$.

We can summarize this distribution of gains in many ways:

- The average treatment effect

$$\Delta^{ATE} = E(Y_1 - Y_0)$$

- Mean causal effect for those for whom $D = 1$

$$\Delta^{TT} = E(Y_1 - Y_0 \mid D = 1)$$

  or whom $D = 0$

$$\Delta^{TUT} = E(Y_1 - Y_0 \mid D = 0)$$

- The marginal treatment effect (MTE)

$$E(Y_1 - Y_0 \mid V = v, D^* = 0)$$

  is the mean gain in terms of $Y_1 - Y_0$ for persons who would be indifferent between treatment or not when $V = v$.

Numerical Example

The Model

Outcomes	Choice Model
$Y_1 = \alpha + \varphi + U_1$	$D = \begin{cases} 1 \text{ if } D^* > 0 \\ 0 \text{ if } D^* \leq 0 \end{cases}$
$Y_0 = \alpha + U_0$	$D^* = Y_1 - Y_0 - C$

General Case

$$(U_1 - U_0) \not\perp D$$

$$ATE \neq TT \neq TUT$$

The Researcher Observes (Y, D, C)

$$Y = \alpha + \beta D + U_0 \text{ where } \beta = Y_1 - Y_0$$

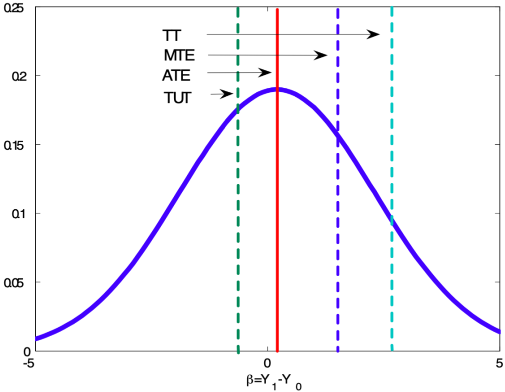
Parameterization

$$\alpha = 0.67 \qquad (U_1, U_0) \sim N(0, \Sigma) \qquad D^* = Y_1 - Y_0 - C$$

$$\varphi = 0.2 \qquad \Sigma = \begin{bmatrix} 1 & -0.9 \\ -0.9 & 1 \end{bmatrix} \qquad C = 1.5$$

Figure 1. Distribution of Gains under Different Assumptions
The Roy Economy

$$U_1 - U_0 \not\perp D$$



$$TT = 2.66,\ TUT = -0.63$$

$$MTE = C = 1.5$$

$$ATE = \mu_1 - \mu_0 = 0.2$$

What is the role of the instrument in this framework?

- Suppose
  - $Z \perp\!\!\!\perp (U_1, U_0)$ **(Independence)**
  - $\Pr(D = 1 \mid Z)$ depends on Z **(Rank)**

- We can understand what IV basically does by noting:

$$
\begin{aligned}
E[Y \mid Z = z] - E[Y \mid Z = z'] = \\
& E[Y_0 + D(Y_1 - Y_0) \mid Z = z] - E[Y_0 + D(Y_1 - Y_0) \mid Z = z'] \\
& = E[D(Y_1 - Y_0) \mid Z = z] - E[D(Y_1 - Y_0) \mid Z = z'].
\end{aligned}
$$

From the independence assumption and the definition of $D(z)$ and $D(z')$, we may write this expression as $E[(Y_1 - Y_0)(D(z) - D(z'))]$. Using the law of iterated expectations,

$$
\begin{aligned}
E[Y \mid Z = z] - E[Y \mid Z = z'] = \\
& E[Y_1 - Y_0 \mid D(z) - D(z') = 1] \Pr\big[D(z) - D(z') = 1\big] \\
& + E[Y_1 - Y_0 \mid D(z) - D(z') = -1] \Pr\big[D(z) - D(z') = -1\big].
\end{aligned}
$$

- Additional assumption:
  Letting $D_i(z)$ be the indicator (= 1 if adopted; = 0 if not) for choice or treatment if $Z = z$ for person i. For any distinct values z and z', $D_i(z) \geq D_i(z')$ or $D_i(z) \leq D_i(z'), i = 1, ..., I$ **(Uniformity)**

- Under this assumption, w.l.o.g. suppose $\Pr[D(z) - D(z') = -1] = 0$, then

$$E[Y \mid Z = z] - E[Y \mid Z = z'] =$$
$$E[Y_1 - Y_0 \mid D(z) - D(z') = 1] \Pr\big[D(z) - D(z') = 1\big].$$

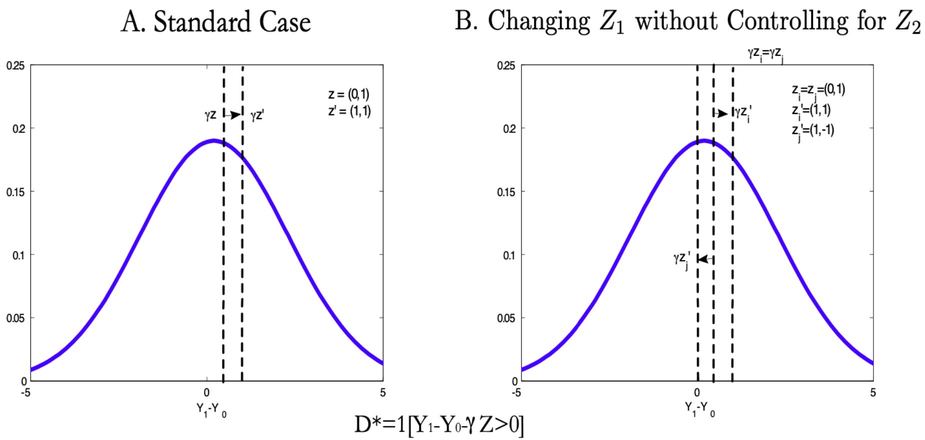
- Dividing by $\Pr[D(z) - D(z') = 1]$, we obtain the local average treatment effect

$$LATE = \frac{E[Y \mid Z = z] - E[Y \mid Z = z']}{\Pr[D(z) - D(z') = 1]}$$

- This is the mean gain to those induced to switch from "0" to "1" by a change in Z from z' to z.

- Notice the importance of uniformity. If uniformity is violated, IV estimates an average response of those induced to switch into the program and those induced to switch out of the program by the change in the instrument.

Figure 2. Uniformity
The Extended Roy Economy



- Thus, IV in general, is not the same as any of the treatment parameters presented above.

- If uniformity is violated, IV estimates an average response of those induced to switch into the program and those induced to switch out of the program by the change in the instrument.

- If the analyst is interested in knowing the average response, the effect of the policy on the outcomes of individuals that are treated, there is no guarantee that the IV estimator comes any closer to the desired target than the OLS estimator and indeed it may be more biased than OLS.

- The question to ask in this more general model is "What parameter is being identified by the instrument?".
- Two economists using the same valid instrument and the same outcome equations but maintaining different models of economic choice will interpret the same point estimate differently.
- The agnostic and robust features of IV in its classical setting disappear in a model with essential heterogeneity.

The Marginal Treatment Effect

- The choice model $D = \mathbf{1}[\mu_D(Z) - V > 0]$.

- Thus, without loss of generality,

$$D = \mathbf{1}[F_V(\mu_D(Z)) > F_V(V)] = \mathbf{1}[P(Z) > U_D]$$

where $U_D = F_V(V)$ and $P(Z) = F_V(\mu_D(Z)) = \Pr[D = 1 \mid Z]$, the propensity score.

- Thus, the marginal treatment effect (MTE) is

$$\Delta^{MTE}(u_D) = E[Y_1 - Y_0 \mid U_D = u_D, U_D = P(z)].$$

- The MTE can be identified by taking derivatives of $E[Y \mid Z]$ with respect to $P(z) = p$. This derivative is called the local instrumental variable (LIV).

- Note that $E(Y \mid Z = z) = E(Y \mid P(Z) = p)$, and thus

$$\begin{aligned} E[Y \mid P(Z) = p] &= E[DY_1 + (1-D)Y_0 \mid P(Z) = p] \\ &= E[Y_0] + E[D(Y_1 - Y_0) \mid P(Z) = p] \\ &= E[Y_0] + E[(Y_1 - Y_0) \mid D = 1]\, p \\ &= E[Y_0] + \int_0^p E[(Y_1 - Y_0) \mid U_D = u_D]\, du_D \end{aligned}$$

- As a consequence,

$$\frac{\partial}{\partial p} E[Y \mid P(Z) = p]\bigg|_{P(z)=p} = E[Y_1 - Y_0 \mid U_D = p].$$

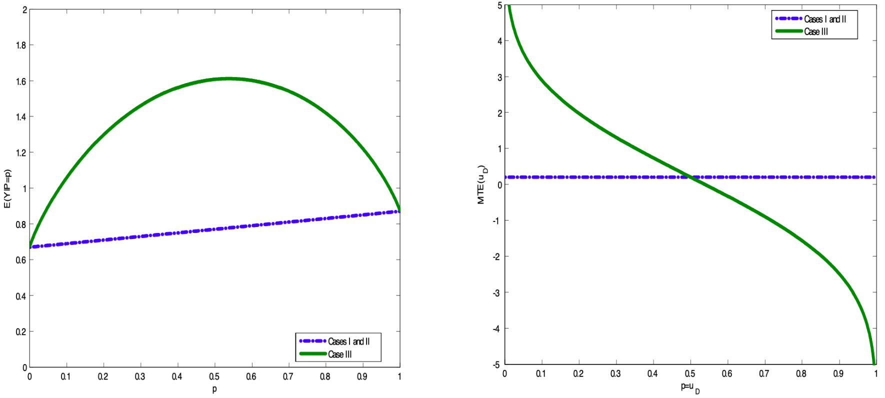
- At the point of evaluation $u_D = P(z)$ $(= p)$, MTE is the gross gain of going from "0" to "1" for agents who are indifferent between the sectors when their mean utility is $\mu_D(z) = v \Leftrightarrow P(z) = u_D$.

- When $\Delta^{MTE}(u_D)$ does not depend on u_D, all of the treatment effects are the same, and IV estimates all of them. In this case, we are back to the conventional model of homogeneous responses.

Figure 3. Conditional Expectation of Y on $P(Z)$ and the Marginal Treatment Effect (MTE)
The Extended Roy Economy

A. $E[Y \mid P(Z) = p]$

B. $MTE(u_D)$



Case I	Case II	Case III
$U_1 = U_0$ $\beta = ATE = TT = TUT = IV$	$U_1 - U_0 \perp\!\!\!\perp D$ $\beta = ATE = TT = TUT = IV$	$U_1 - U_0 \not\perp\!\!\!\perp D$ $\beta \neq ATE \neq TT \neq TUT \neq IV$

Table 1A
Treatment Effects and Estimands as Weighted Averages of the Marginal Treatment Effect

$$ATE = E(Y_1 - Y_0) = \int_0^1 \Delta^{MTE}(u_D)\, du_D$$

$$TT = E(Y_1 - Y_0 \mid D = 1) = \int_0^1 \Delta^{MTE}(u_D) h_{TT}(u_D)\, du_D$$

$$TUT = E(Y_1 - Y_0 \mid D = 0) = \int_0^1 \Delta^{MTE}(u_D) h_{TUT}(u_D)\, du_D$$

Table 1B
Weights

$$h_{ATE}(u_D) = 1$$

$$h_{TT}(u_D) = \left[\int_{u_D}^1 f(p)\, dp\right] \frac{1}{E(P)}$$

$$h_{TUT}(x, u_D) = \left[\int_0^{u_D} f(p)\, dp\right] \frac{1}{E(1 - P)}$$

Source: Heckman and Vytlacil (2005)

What does linear IV estimate under Essential Heterogeneity

- Let $J(Z)$ be a scalar-valued instrument

$$\beta_{IV}(J) \equiv \frac{Cov(J(Z), Y)}{Cov(J(Z), D)} = \int_0^1 \Delta^{MTE}(u_D) h_{IV}(u_D \mid J)\, du_D$$

$$h_{IV}(u_D \mid J) = \frac{E[J(Z) - E(J(Z)) \mid P(Z) \geq u_D] \Pr(P(Z) \geq u_D)}{Cov(J(Z), P(Z))}$$

with $\int_0^1 h_{IV}(u_D \mid x; J)\, du_D = 1$

- $h_{IV}(u_D \mid J)$ is non-negative for all u_D if $E(J \mid P(Z) \geq u_D)$ is weakly monotonic in u_D (Monotonicity).

- There is no guarantee that the weights for a general $J(Z)$ will be nonnegative for all u_D, although the weights integrate to unity and thus must be positive over some range of evaluation points.

- If we redefine IV for Z_1 to be conditional on $Z_2 = z_2, \ldots, Z_K = z_K$, holding the other arguments fixed, then the weights are positive.

- Using $P(Z)$ as the instrument implies positive weights:

$$h_{IV}(u_D \mid P(Z)) = \frac{[E(P(Z) \mid P(Z) \geq u_D) - E(P(Z))] \Pr(P(Z) \geq u_D)}{Var(P(Z))}$$

- Suppose Z is discrete. Thus the support of the distribution of $P(Z)$ contains a finite number of values, $\{p_1, ..., p_K\}$ with $p_1 < p_2 < \cdots < p_K$. The support of $J(Z)$ is also discrete, taking I distinct values, where I and K may be distinct. Thus,

$$\Delta^{IV} = \sum_{\ell=1}^{K-1} \Delta^{LATE}(p_\ell, p_{\ell+1})\lambda_\ell.$$

where

$$\Delta^{LATE}(p_\ell, p_{\ell+1}) = \frac{E(Y \mid P(Z) = p_{\ell+1}) - E(Y \mid P(Z) = p_\ell)}{p_{\ell+1} - p_\ell}$$

$$\lambda_\ell = (p_{\ell+1} - p_\ell)\frac{\sum_{i=1}^{I}(j_i - E(J))\sum_{t>\ell}^{K}(f(j_i, p_t))}{Cov(J(Z), D)}$$

where $\sum_{\ell=1}^{K-1}\lambda_\ell = 1$ but the weights can be positive or negative for any ℓ.

- Our expression for the weight on *LATE* generalizes the expression presented by Imbens and Angrist (1994) who in their analysis of the case of vector Z only consider the case where $J(Z)$ and $P(Z)$ are perfectly dependent.

- Using $P(Z)$ as the instrument implies positive weights:

$$\lambda_\ell = (p_{\ell+1} - p_\ell)\frac{\sum_{i=\ell}^{K}(p_i - E(P(Z)))f_{P(Z)}(p_i)}{Var(P(Z))}$$

  which is the expression in Imbens and Angrist (1994).

Theoretical Example 1: Discrete Instruments

Outcomes

$$Y_1 = \alpha + \varphi + U_1$$

$$Y_0 = \alpha + U_0$$

Choice Model

$$D = \begin{cases} 1 \text{ if } Y_1 - Y_0 - \gamma Z > 0 \\ 0 \text{ if } Y_1 - Y_0 - \gamma Z \leq 0 \end{cases}$$

$$\text{with } \gamma Z = \gamma_1 Z_1 + \gamma_2 Z_2$$

Parametrization

$$(U_1, U_0) \sim N(0, \Sigma),\ \Sigma = \begin{bmatrix} 1 & -0.9 \\ -0.9 & 1 \end{bmatrix},\ \alpha = 0.67,\ \varphi = 0.2,\ \gamma = (0.5, 0.5)$$

$$Z_1 = \{-1, 0, 1\} \text{ and } Z_2 = \{-1, 0, 1\}$$

$$Cov(Z_1, Z_2) = -0.5468$$

Figure 4. *IV* Weight and its Components under Discrete Instruments when $P(Z)$ is the instrument
The Extended Roy Economy

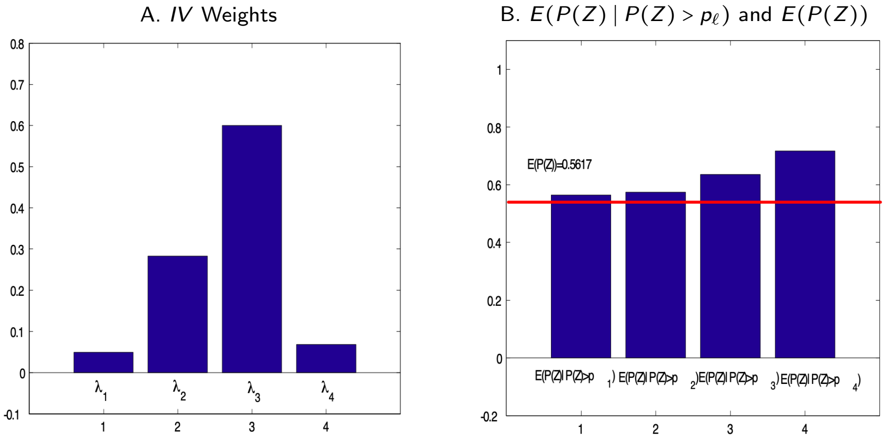
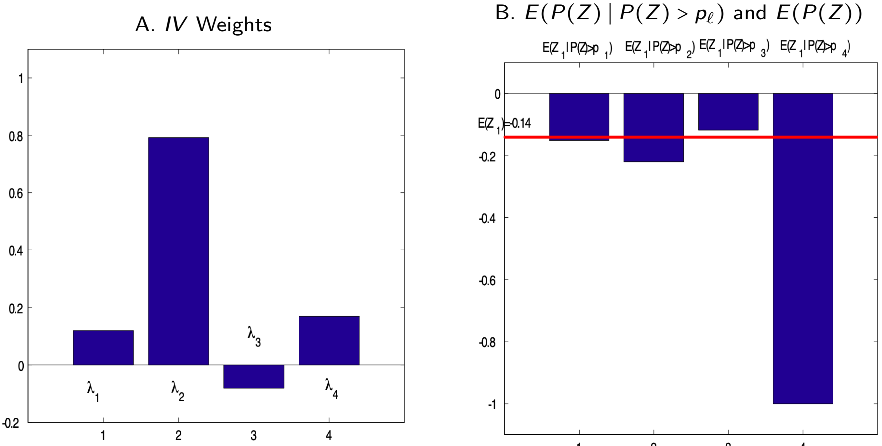


Figure 5. *IV* Weight and its Components under Discrete Instruments when Z_1 as the instrument
The Extended Roy Economy



Treatment Parameters

ATE	0.2
TT	0.5953
TUT	-0.4845
$\Delta^{LATE}(0.3516, 0.4432)$	0.7474
$\Delta^{LATE}(0.4432, 0.5379)$	0.2498
$\Delta^{LATE}(0.5379, 0.6306)$	-0.2479
$\Delta^{LATE}(0.6306, 0.7161)$	-0.7455

$$\Delta^{IV}_{P(Z)} = \sum_{l=1}^{K-1} \Delta^{LATE}(p_l, p_{l+1})\lambda_l = -0.0918$$

$$\Delta^{IV}_{Z_1} = \sum_{l=1}^{K-1} \Delta^{LATE}(p_l, p_{l+1})\lambda_l = 0.1815$$

$$Cov(Z_1, Z_2) = -0.5468$$

Table 2. The Conditional Instrumental Variable Estimator $\left(\Delta^{IV}_{Z_1|Z_2=z_2}\right)$ and Conditional Local Average Treatment Effect $\left(\Delta^{LATE}(p_l, p_{l+1} \mid Z_2 = z_2)\right)$ when Z_1 is the Instrument (given $Z_2 = z_2$)

The Extended Roy Economy

	$Z_2 = -1$	$Z_2 = 0$	$Z_2 = 1$
$P(-1, Z_2) = p_3$	0.7161	0.6306	0.5379
$P(0, Z_2) = p_2$	0.6306	0.5379	0.4432
$P(1, Z_2) = p_1$	0.5379	0.4432	0.3516
λ_1	0.8402	0.5375	0.2871
λ_2	0.1598	0.4625	0.7129
$\Delta^{LATE}(p_2, p_3)$	-0.7455	-0.2479	0.2498
$\Delta^{LATE}(p_1, p_2)$	-0.2479	0.2498	0.7474
$\Delta^{IV}_{Z_1\|Z_2=z_2}$	-0.3274	0.0196	0.3927

Theoretical Example 2: Continuous and Normal Instruments

Outcomes

$$Y_1 = \alpha + \varphi + U_1$$

$$Y_0 = \alpha + U_0$$

Choice Model

$$D = \begin{cases} 1 \text{ if } D^* > 0 \\ 0 \text{ if } D^* \leq 0 \end{cases}$$

$$\text{with } D^* = Y_1 - Y_0 - \gamma Z$$

Parametrization$^{(*)}$

$$(U_1, U_0) \sim N(0, \Sigma) \text{ and } Z \sim (\mu_Z, \Sigma_Z)$$

$$\Sigma = \begin{bmatrix} 1 & -0.9 \\ -0.9 & 1 \end{bmatrix}, \ \mu_Z = (2, -2) \text{ and } \Sigma_Z = \begin{bmatrix} 9 & -2 \\ -2 & 9 \end{bmatrix}$$

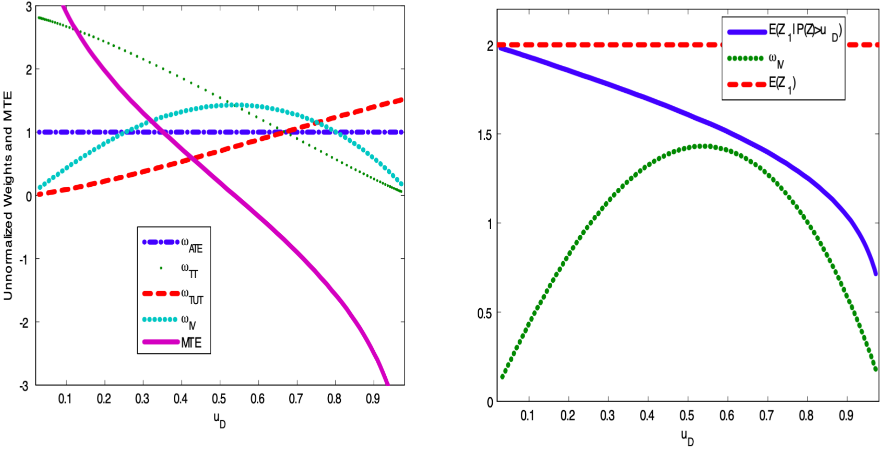
$$\alpha = 0.67, \ \varphi = 0.2, \ \gamma = (0.5, 0.5)'$$

Parameter	Under Assumptions(*)
ATE	0.2
TT	1.1878
TUT	-0.9132
IV_{Z_1}	0.0924

Figure 6. Treatment Weights, *IV* Weights using Z_1 as the Instrument and the Marginal Treatment Effect

A. Weights and *MTE*

B. *IV* Weights, $E(Z_1 \mid P(Z) > u_D)$ and $E(Z_1)$



Theoretical Example 3: Continuous and Non-Normal Instruments

Outcomes

$$Y_1 = \alpha + \varphi + U_1$$

$$Y_0 = \alpha + U_0$$

Choice Model

$$D = \begin{cases} 1 \text{ if } D^* > 0 \\ 0 \text{ if } D^* \leq 0 \end{cases}$$

$$\text{with } D^* = Y_1 - Y_0 - \gamma Z$$

Parametrization

$$(U_1, U_0) \sim N(0, \Sigma),\ \Sigma = \begin{bmatrix} 1 & -0.9 \\ -0.9 & 1 \end{bmatrix},\ \alpha = 0.67,\ \varphi = 0.2$$

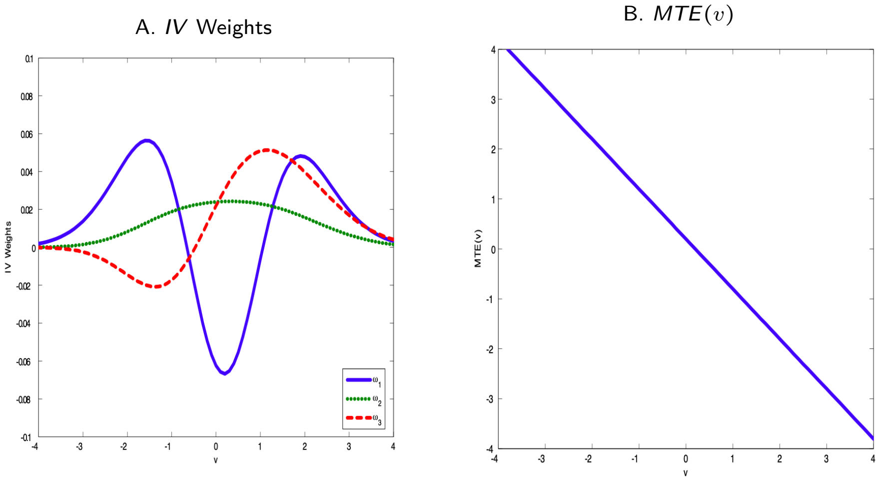
$$Z = (Z_1, Z_2) \sim p_1 N(\mu_1, \Sigma_1) + p_2 N(\mu_2, \Sigma_2)$$

$$p_1 = 0.45,\ p_2 = 0.55;\ \Sigma_1 = \begin{bmatrix} 1.4 & 0.5 \\ 0.5 & 1.4 \end{bmatrix}$$

Table 3. *IV* estimator and $COV(Z_2, \gamma' Z)$ associated with each value of Σ_2

Weights	Σ_2	μ_1	μ_2	*IV*	*ATE*	*TT*	*TUT*	$Cov(Z_2, \gamma Z) = \gamma \Sigma_2^1$
ω_1	$\begin{bmatrix} 0.6 & -0.5 \\ -0.5 & 0.6 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \end{bmatrix}$	0.367	0.2	1.544	-1.327	-0.58
ω_2	$\begin{bmatrix} 0.6 & 0.1 \\ 0.1 & 0.6 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \end{bmatrix}$	-0.216	0.2	1.522	-1.299	0.26
ω_3	$\begin{bmatrix} 0.6 & -0.3 \\ -0.3 & 0.6 \end{bmatrix}$	$\begin{bmatrix} 0 & -1 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \end{bmatrix}$	-2.348	0.2	1.454	-1.101	-0.30

Figure 7. Marginal Treatment Effect and *IV* Weights using Z_1 as the Instrument when $Z = (Z_1, Z_2) \sim p_1 N(\mu_1; \Sigma_1) + p_2 N(\mu_2; \Sigma_2)$ for different values of Σ_2



Empirical Example: The GED Effect

- GEDs versus Dropouts

$$
\begin{aligned}
Y_1 &= \alpha + \varphi + U_1 \\
Y_0 &= \alpha + U_0
\end{aligned}
$$

- Choice model: $D = 1[\mu_D(Z) - V > 0]$

- The empirical model

$$Y = \alpha + \beta D + \varepsilon$$

  where, due to the endogeneity of D (selection bias), $D \not\perp \varepsilon$.

- Z = family background variables, local labor variables at age 17, and the propensity score.

Table 4. Instrumental Variables Estimates
NLSY79 - Sample of GED and Dropouts - Males age 25

Instruments	*IV*
Number of Siblings	-0.052
	(0.160)
Dropout's local unemployment rate at age 17	0.443
	(1.051)
Propensity Score	-0.058
	(0.164)

Figure 8. *MTE* of the GED with Confidence Interval

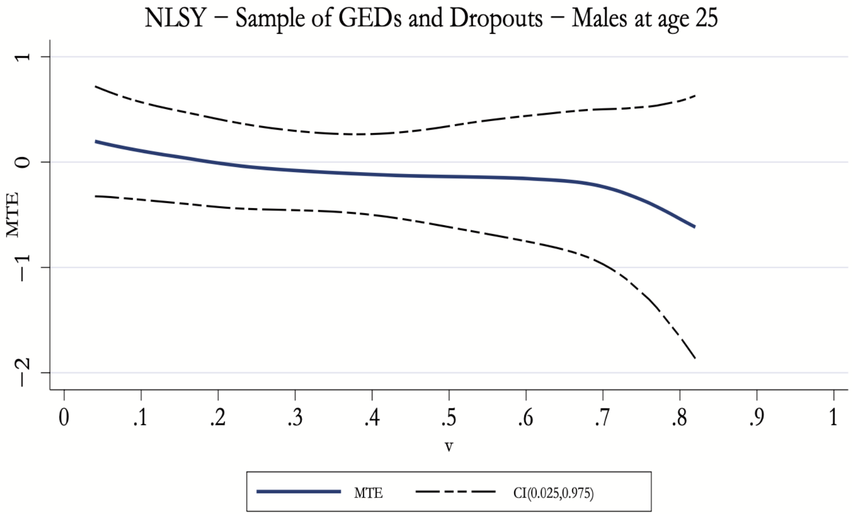


Figure 9. *IV* Weights

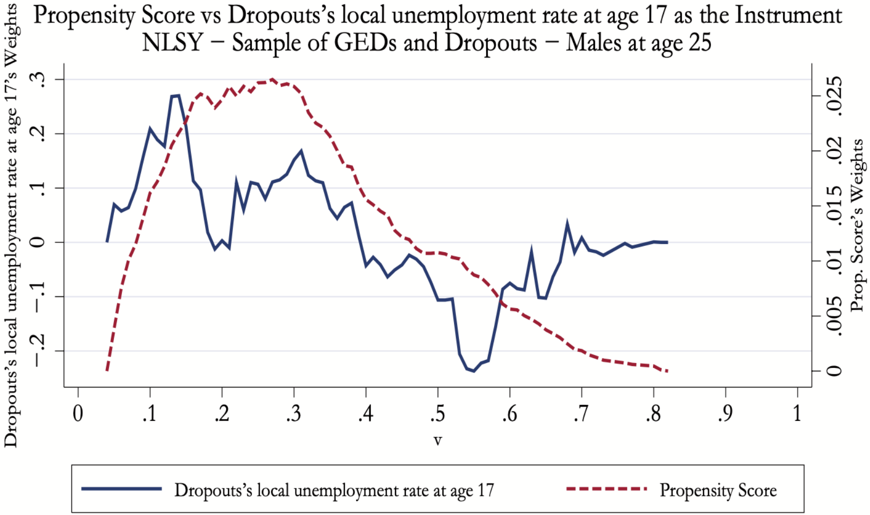
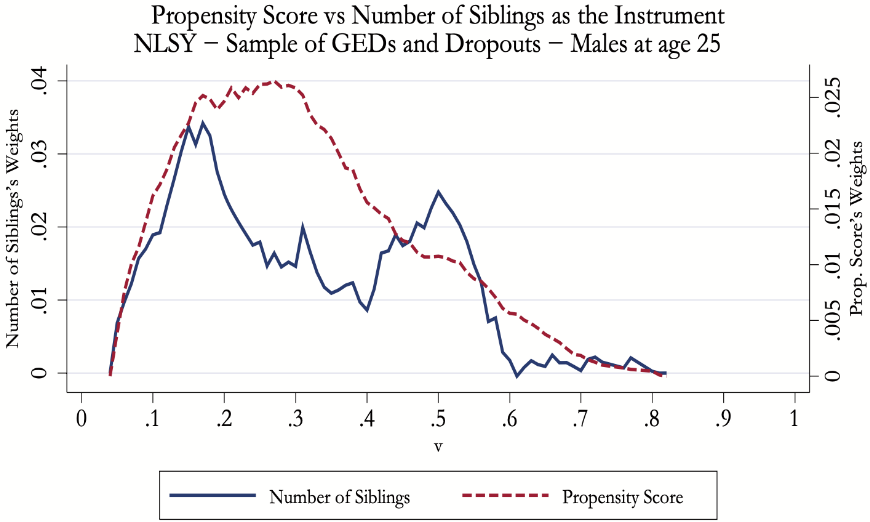


Figure 10. *IV* Weights



Conclusions

- We consider the application of the method of instrumental variables to models where agents make treatment choices based in part on heterogeneous gains and some components of heterogeneity are unobserved by the economist.
- In this framework, two economists using the same valid instrument and the same outcome equations but maintaining different models of economic choice will interpret the same point estimate differently.
- The agnostic and robust features of IV in its classical setting disappear in a model with essential heterogeneity.
- In general, IV does not estimate a treatment effect. Positivity of weights is required to interpret IV estimates as treatment effects.
- However, many interesting policy questions do not require treatment effects. Policy effects and treatment effects are distinct.