Game Theory

Ultimatum Bargaining

Tetsuya Hoshino

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In this note, we study the model of **ultimatum bargaining**. This is a simple model of bargaining, but its equilibrium is counterintuitive. Indeed, the theoretical predictions have been denied by experiments. Although we do not detail—actually, I do not know—the experiments, we attempt to sketch the boundaries of the theory and experiments.

1 Ultimatum Bargaining

1.1 Model

Two players 1 and 2 bargain over a pie of size 1. Player 1 makes an offer $s \in [0,1]$, which we interpret as player 1 receiving a share s of the pie and player 2 receiving a share $1-s$. Player 2 observes the offer s and then decides whether to accept (A) or reject (R) it. If player 2 accepts the offer then players $1,2$ will divide the pie according to the offer, resulting in payoffs $(s, 1-s)$; if player 2 rejects the offer then neither will get the pie, resulting in payoffs $(0,0)$.

The ultimatum bargaining is represented by the following game tree:[1]

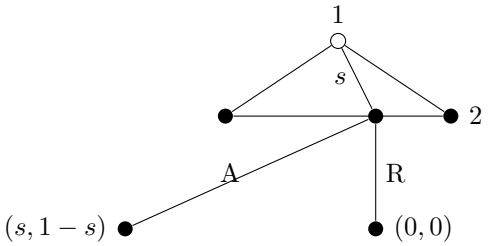

Figure 1: the ultimatum bargaining

1.2 Equilibrium

Since the model of ultimatum bargaining is a finite-horizon extensive-form game of perfect information, we can find subgame perfect equilibria by backward induction.

Proposition 1. *There is a unique subgame perfect equilibrium in the ultimatum bargaining, and it is such that player 1 makes offer $s = 1$ and player 2 accepts offer $s \leq 1$.*

[1] This extensive-form game has infinitely many nodes. It is, therefore, impossible to write down all the nodes. Instead, we draw them with a solid line that connects player 2's nodes.

Proof. We use backward induction. By abuse of notation, let s denote player 2's information set at which player 2 observes offer s. First, we consider period 2, at which player 2 (he) has observed offer s. His optimal (behavioral) strategy β_2^* is such that:

$$\beta_2^*(s) \in \begin{cases} \{\mathrm{A}\} & \text{if } s < 1, \\ \{\mathrm{A}, \mathrm{R}\} & \text{if } s = 1. \end{cases}$$

Second, we consider period 1, at which player 1 (she) makes offer s. What offer s does she make if she expects player 2 to play the strategy β_2^*? Since it is never optimal for her to make offer $s < 1$ (why?), it suffices to consider the case $s = 1$. There are two cases:

1. If $\beta_2^*(1) = \mathrm{A}$ then player 1 has an optimal strategy such that she makes offer $s = 1$.
2. If $\beta_2^*(1) = \mathrm{R}$ then player 1 has no optimal strategy because $\operatorname{argmax}_{s \in [0,1)} s = \varnothing$.

Hence, the strategy profile of this proposition is the unique subgame perfect equilibrium. ■

2 Experiment on Ultimatum Bargaining*

The ultimatum bargaining provides a theoretical prediction that player 1 takes all and player 2 takes nothing. Does this theory explain the actual behavior of people? We can then test the theory with experiments. For example, we may ask a pair of subjects to play the ultimatum bargaining and collect data on their actual play to compare with the theoretical prediction.

2.1 Variant Model and Equilibrium

Before discussing the experiments, we present a variant of ultimatum bargaining with a discrete action space for player 1. Specifically, we assume that player 1 chooses a share s from the set $\{0, 0.01, 0.02, \ldots, 0.99, 1\}$. In words, she has to choose a discretized share for every 1 percent. All the other settings are maintained.

Remark 1. In the original model, player 1 chooses a share s from the unit interval $[0, 1]$; that is, she chooses any *real* number between 0 and 1. □

We can find subgame perfect equilibria by backward induction.

Proposition 2. *There are two subgame perfect equilibria in the variant model of the ultimatum bargaining, and they are such that:*

1. *Player 1 makes offer $s = 1$ and player 2 accepts offer s.*
2. *Player 1 makes offer $s = 0.99$ and player 2 accepts offer $s \leq 0.99$ and rejects offer $s = 1$.*

Proof. Omitted. ■

The variant model has two equilibria, but they are almost the same. In any equilibrium, player 1 takes (almost) all the pie and player 2 will accept the offer. Hence, the implication is qualitatively the same as the implication of the original model.

2.2 Experiment

Suppose that there are 100 pesos, instead of the pie of size 1. If you were to make an offer (as player 1), how much would you offer? Would you demand 100 or 99 pesos? If you were offered only 1 peso for you (as player 2), would you accept it?

Güth et al. (1982) perform the first experiment on the ultimatum bargaining. They report that the behavior of most subjects (students at the University of Cologne, Germany) is different from the theoretical prediction (Proposition 2). The proposers make fairer offers, and the responders reject unfair offers.

Do we observe a similar result in other countries or cultures? Roth et al. (1991) compare experimental results in four cities—Jerusalem, Ljubljana, Pittsburgh, and Tokyo. Henrich et al. (2001) conduct a large-scale comparison across a variety of cultures. They compare fifteen small-scale tribal societies that are not industrialized and continue to live traditional lifestyles such as hunting and gathering, slash-and-burn agriculture, nomadism, and small-scale agriculture. In all these experiments, the proposers make fairer offers, and the responders reject unfair offers.[2]

There are many other experiments on numerous variants of the ultimatum bargaining. They also show that subjects behave in a "fair" way, which is different from the game-theoretic prediction. However, we omit the details of these experiments here.

Coffee Break ☕. The experiment in Tokyo was conducted at Keio University (private), not at the University of Tokyo (public), where Okuno-Fujiwara, one of the coauthors of Roth et al. (1991), was affiliated. In the experiment of the ultimatum bargaining, the prize money that the proposer and responder receive changes as a result of their behavior. The University of Tokyo was afraid that the experiment might be a kind of "gambling" (that is illegal in Japan) and did not permit the experiment. □

References

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Henrich, J., Boyd, R., Bowles, S., Camerer, C., Fehr, E., Gintis, H., & McElreath, R. (2001). In search of homo economicus: behavioral experiments in 15 small-scale societies. *American Economic Review*, *91*(2), 73–78.

[2] Proctor et al. (2013) report that chimpanzees exhibit similar preferences to humans regarding reward division. This suggests a long evolutionary history to the human sense of fairness.

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