Some extensions of the Krugman Model

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1 Some extensions of the Krugman model

The basic model presented in Krugman (1979) and Krugman (1980) has the natural advantage of being quite tractable and easy to work with. Is is therefore a small step to include several extensions to the basic model: trade among countries of different dimensions; assumption of a general utility function to generate a non-constant markup; extension to a multi-sectoral economy to explain both intra and inter-industry trade; multi-country extension with a derivation and interpretation of a gravity equation. For recent papers covering the last two examples we have Bernard, Redding, and Schott (2007) that integrate an Heckscher–Ohlin model with a Krugman kind of model that allows the study of resources reallocation from trade shocks occurring along both intra and inter sectoral levels; in Arkolakis, Costinot, and Rodríguez-Clare (2012), the authors show an equivalence result that allows for a unified interpretation of gravity equations that are implied by several type of models, including the Krugman model.

A detailed analysis of most of these extensions is usually confined to the study at a graduate level and falls outside the scope of this course. Nevertheless, in this handout, we analyze the impact in the Krugman model of using an utility function that is more general than the CES. We will see that with some utility functions we can generate a markup that is decreasing with the quantity produced, thus, in such environment, when an economy opens up to free trade, firms increase their optimal prices leading also to higher output levels per variety produced. This in turn generates an additional gain from international trade related with an increased scale of production.

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1.1 The Krugman's model with an exponential utility function

This extension departs from the model introduced in the previous handout by specifying a different utility function. As before, we'll start by describing the model equilibrium for an autarky economy and then analyze the new equilibrium once trade is allowed.

Household Let an economy have a fixed supply of L workers, each supplying a single unit of labor. Also, each worker enjoys consumption of N varieties of the same good following utility function:

$$C = \left[\sum_{i=1}^{N} u\left(c_i\right)\right]^2 /2 = \left[\sum_{i=1}^{N} -\exp\left(-c_i/\gamma\right)\cdot\gamma\right]^2 /2 \tag{1}$$

Here γ is a preference parameter that determines the curvature of the utility function. Note that the utility function has the usual properties summarized on the fact that $u'\left(c\right) \equiv \frac{\partial u(c)}{\partial c} > 0$ and $u''(c) \equiv \frac{\partial^2 u(c)}{\partial c^2} < 0$.

Expenditure minimization for a particular consumer amounts to solve the following problem:

$$\min_{\{c_i\}_{i=1}^{N}} \left\{\sum_{i=1}^{N} p_i c_i \quad st \quad \left[\sum_{i=1}^{N} u\left(c_i\right)\right]^2 /2 \geq \bar{C}\right\}$$

First order conditions imply the following implicit demand functions for each $i = 1, ..., N$:[1]

$$p_i = \lambda u'\left(c_i\right)\cdot C^{1/2} \tag{2}$$

where λ is the Lagrangian multiplier associated with the constraint of the expenditure minimization.

To check that the demand function implied by the expenditure minimization is a decreasing function of p_i, we can apply the implicit function theorem to equation (2) by taking the total derivative with respect to p_i:

$$\begin{aligned} 1 &= \lambda C^{1/2} u''\left(c_i\right)\frac{dc_i}{dp_i} \\ \Rightarrow \tfrac{dc_i}{dp_i} &= \frac{1}{\lambda C^{1/2} u''\left(c_i\right)} < 0 \end{aligned}$$

The right hand side of the last equality must be negative as the second derivative of the

[1]For the derivations of the demand functions, refer to the Armington model handout.

utility function u is negative. Therefore, c_i should decrease when p_i increases, as we would expect from well behaved utility funcitons

Firms As with the previous model, we specify a firms' technology displaying increasing returns to scale. That is, in terms of labor requirements to produce q_i units of variety i:

$$l_i = q_i/\varphi + f \tag{3}$$

Profit maximization implies that firms choose the optimal price that maximizes the following function:

$$\begin{aligned}\pi_i &= p_i q_i - w l_i \\ &= p_i q_i - w\left(f + q_i/\varphi\right) \\ &= p_i c_i L - w\left(f + c_i L/\varphi\right)\end{aligned}$$

where the last equation comes from the fact that a firm producing variety i has to supply the entire market composed by L consumers, each demanding c_i: $q_i = Lc_i$. First order conditions imply the following:

$$\begin{aligned}&\frac{\partial \pi_i}{\partial p_i}=0 \\ \Rightarrow\quad & Lc_i + p_i L\frac{dc_i}{dp_i} - \frac{w}{\varphi}L\frac{dc_i}{dp_i}=0 \\ \Rightarrow\quad & Lc_i + p_i L\frac{1}{\lambda C^{1/2}u''\left(c_i\right)} - \frac{w}{\varphi}L\frac{1}{\lambda C^{1/2}u''\left(c_i\right)}=0\end{aligned}$$

In order to get rid of λ we substitute in (2) and rearrange:

$$\begin{aligned}& Lc_i + p_i L\frac{1}{\lambda C^{1/2}u''\left(c_i\right)} - \frac{w}{\varphi}L\frac{1}{\lambda C^{1/2}u''\left(c_i\right)}=0 \\ \Rightarrow\quad & Lc_i + p_i L\frac{u'\left(c_i\right)}{u''\left(c_i\right)}\cdot\frac{1}{p_i} - \frac{w}{\varphi}L\frac{u'\left(c_i\right)}{u''\left(c_i\right)}\cdot\frac{1}{p_i}=0 \\ \Rightarrow\quad & 1 + \frac{u'\left(c_i\right)}{c_i \cdot u''\left(c_i\right)} - \frac{w}{\varphi}\frac{u'\left(c_i\right)}{c_i \cdot u''\left(c_i\right)}\cdot\frac{1}{p_i}=0 \\ \Rightarrow\quad & 1 + \frac{u'\left(c_i\right)}{c_i \cdot u''\left(c_i\right)} - \frac{w}{\varphi}\frac{u'\left(c_i\right)}{c_i \cdot u''\left(c_i\right)}\cdot\frac{1}{p_i}=0 \\ \Rightarrow\quad & p_i=\frac{u'\left(c_i\right)/\left(c_i \cdot u''\left(c_i\right)\right)}{1+u'\left(c_i\right)/\left(c_i \cdot u''\left(c_i\right)\right)}\cdot\frac{w}{\varphi}\end{aligned}$$

Finally, let's define the following function:

$$\sigma(c_i) = -\frac{u'(c_i)}{c_i \cdot u''(c_i)}$$

If we substitute this function in the optimal price we then get:

$$\begin{aligned} p_i &= \frac{-\sigma(c_i)}{1-\sigma(c_i)} \cdot \frac{w}{\varphi} \\ \Rightarrow \quad p_i &= \frac{\sigma(c_i)}{\sigma(c_i)-1} \cdot \frac{w}{\varphi} \end{aligned} \tag{4}$$

This last expression has a form and interpretation that is very similar to the one that we derived before: at the optimum, firms charge a markup over the marginal cost. The main difference now is that the markup is no longer constant, but instead depends on the level demanded c. Using the definition, note that $\sigma(c)$ equals to:

$$\begin{aligned} \sigma(c_i) &= -\frac{u'(c_i)}{c_i \cdot u''(c_i)} \\ &= -\frac{\exp(-c_i/\gamma)}{-\exp(-c_i/\gamma)/\gamma} \cdot \frac{1}{c_i} \\ &= \frac{\gamma}{c_i} \end{aligned}$$

That is, the term $\sigma(c_i)$, akin to the elasticity of substitution in case when we have a CES utility function, is decreasing with c_i. But this also implies that the term $\sigma(c_i)/(\sigma(c_i)-1)$ is increasing with c_i. Intuitively, firms can compensate the fact that consumers demand less of a variety, by decreasing the markup they charge over the marginal cost, thus sustaining demand. We will see that there'll be some important implications of such pricing behavior in the general equilibrium of the model.

Free entry The free entry condition doesn't change with respect to the previous model. In particular, we still impose that firms will continue to enter into the market by producing new varieties until profits are driven completely to zero, that is, revenues equal costs:

$$\begin{aligned}
&\pi_i = q_i p_i - w l_i = 0 \\
\Rightarrow &\pi_i = q_i p_i - w\left(f + \frac{q_i}{\varphi}\right) = 0 \\
\Rightarrow &\frac{p_i}{w} = \frac{f}{q_i} + \frac{1}{\varphi} \quad \forall i = 1, ..., N
\end{aligned} \tag{5}$$

Market clearing conditions As before we need to impose clearing conditions to two set of markets:

$$q_i = Lc_i \tag{varieties markets}$$

$$L = \sum_{i=1}^{N} l_i = Nf + \sum_{i=1}^{N} q_i/\varphi \tag{labor market}$$

A symmetric equilibrium In a symmetric equilibrium, where all firms behave equivalently, the following is verified:

$$p_i = p; \quad q_i = q; \quad c_i = c \quad \forall i = 1, ..., N$$

To characterize such equilibrium, we can make use of two equations: one for the fact that firms charge optimal prices (4) and the other that states that firms operate under zero profits (5):

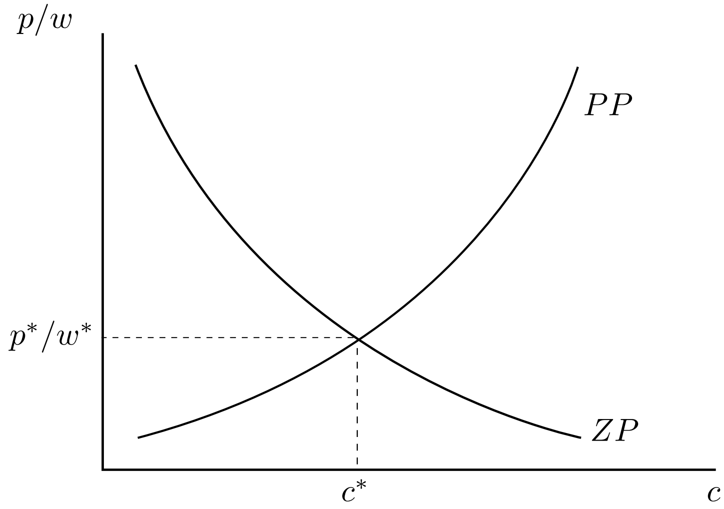
$$\frac{p}{w} = \frac{\sigma(c)}{\sigma(c) - 1} \cdot \frac{1}{\varphi} \tag{PP}$$

and:

$$\frac{p}{w} = \frac{f}{q} + \frac{1}{\varphi} = \frac{f}{cL} + \frac{1}{\varphi} \tag{ZP}$$

Note that these two equations are similar to the ones that we derived before with the exception that the equation (PP) is now increasing in c. Since the first equation is increasing, and the second is decreasing, the intersection between the two give us the equilibrium as shown in figure 1.

Figure 1: Autarky equilibrium in the Krugman model



It is now apparent that the (PP) is increasing, being this the only difference from introducing an exponential utility function. Comparative statics is done in the same fashion as before. For example, an increase in the parameter γ should shift the (PP) to the right, implying a fall in p/w and an increase in c. Note however that, we cannot find a closed form solution for both p/w and c as we did in the CES utility case. Nevertheless, as emphasized, a numerical solution should exist and should be given by the interception of the two equations.

Number of varieties and firms under a symmetric equilibrium Similarly as before, to determine the number of firms/varieties that will be operating in equilibrium, we make use of the market clearing condition (labor market) :

$$\begin{aligned}
L=&Nf + \sum_{i=1}^{N} q_i/\varphi \\
\Rightarrow L=&Nf + Nq/\varphi \\
\Rightarrow N=&\frac{L}{f + q/\varphi} \\
\Rightarrow N=&\frac{L}{f + Lc/\varphi}
\end{aligned}$$

Where now c is characterized by (PP) and (ZP).

Utility From the definition of the utility function in (1) we have

$$\begin{aligned} C &= \left[\sum_{i=1}^{N} - \exp\left(-c_i/\gamma\right)/\gamma\right]^2 /2 \\ &= \left[\sum_{i=1}^{N} - \exp\left(-c/\gamma\right)/\gamma\right]^2 /2 \\ &= \left[-N \exp\left(-c/\gamma\right)/\gamma\right]^2 /2 \\ &= \left[-\left(\frac{L}{f + Lc/\varphi}\right) \exp\left(-c/\gamma\right)/\gamma\right]^2 /2 \end{aligned}$$

which can be used to make welfare analysis from shocking the model.

1.2 Free trade between two identical economies

Recall as before that opening up to trade for an identical economy can be characterized only by using the (PP) and (ZP) equations. Note that (PP) equation doesn't change:

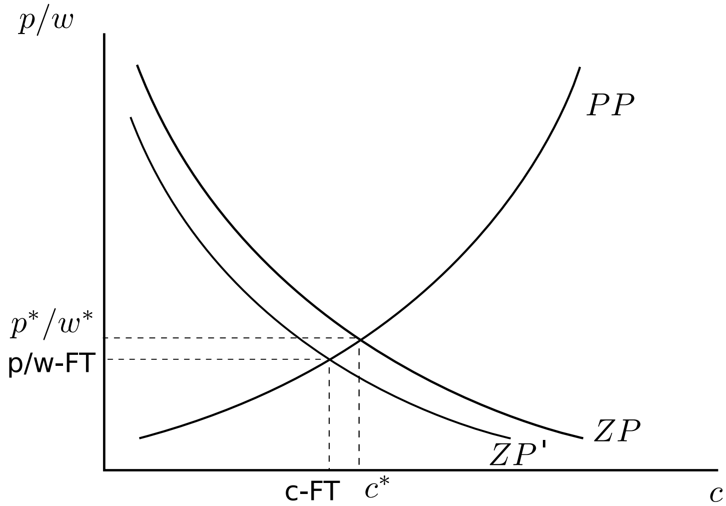
$$\frac{p}{w} = \frac{\sigma(c)}{\sigma(c) - 1} \cdot \frac{1}{\varphi} \tag{PP-FT}$$

While (ZP) becomes:

$$\frac{p}{w} = \frac{f}{q} + \frac{1}{\varphi} = \frac{f}{c2L} + \frac{1}{\varphi} \tag{ZP-FT}$$

Again, this is just a shift to the left side of the (ZP) equation as shown in figure 2.

Figure 2: Free-trade equilibrium in the Krugman model



It is obvious from the figure that under trade, the domestic individual consumption per variety falls, but not as much as the increase in the total population. Moreover, we should note that now the intersection of the two curves imply a lower c and a lower p/w. But if p/w is lower, then, from the equation (ZP-FT), it must be that the quantity produced per variety q increases. That increase reflects the case that under free trade each firm in the home economy will produce more than under autarky. By doing so, it will operate at a larger scale, implying that the average cost is also lower. That is, under an exponential utility function, the gains from trade accrue from the fact that consumers have now access to more varieties and also that firms now produce at a larger scale (at a lower average cost).

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