Five Parameters of Interest in the Evaluation of Social Programs

Advanced Microeconometrics
ITAM

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Background Paper

- Heckman, Tobias, and Vytlacil (2001)

Treatment Parameters in a Canonical Roy Model

Consider the following model with two potential outcomes:

$$\begin{aligned} Y^1 &= X\beta^1 + U^1 \\ Y^0 &= X\beta^0 + U^0 \end{aligned}$$

And a decision rule represented by a *latent index-model*:

$$\begin{aligned} D^* &= Z\theta + U^D \\ D(Z) &= 1\left[D^*(Z) \geq 0\right] \end{aligned}$$

Commonly, D^* is interpreted as the net utility of choosing sector 1

The Roy model assumes, $D^* = Y^1 - Y^0$.

In the extended Roy model, $D^* = Y^1 - Y^0 - C$, with C the (invariant over individuals) costs associated to sector 1 (e.g. college tuition).

- We also define the variable $D(z) = 1\left[z\theta \geq U^D\right]$
- $D(z)$ indicates whether or not the individual would have received treatment had her value of Z been externally set to z, holding her unobserved U^D constant.
- We require and exclusion restriction and denote by Z_k some element of Z that is not contained in X
- By varying Z_k, we can manipulate an individual's probability of receiving treatment without affecting the potential outcomes

Assumptions:

- $(U^1, U^0, U^D) \perp\!\!\!\perp (X, Z)$
- $U^1 \not\perp\!\!\!\perp U^0 \not\perp\!\!\!\perp U^D$
- We don't observe the pair (Y^1, Y^0) for each person, but only one of them, according to:

$$Y = DY^1 + (1 - D)Y^0$$

- The gain of participating in the program:

$$\Delta \equiv Y^1 - Y^0$$

With this model, we can define various means of treatment parameters. In what follows, we will define ATE, TT, TUT, LATE, MTE.

1. **ATE (Average Treatment Effect)** gives us the expected gain for a randomly selected individual

$$ATE(x) = E(\Delta | X = x) = x\left(\beta^1 - \beta^0\right)$$

The unconditional ATE is:

$$\begin{aligned} ATE &= E(\Delta) = \int ATE(x)\, dF(x) \simeq \frac{1}{n}\sum_{i=1}^{n} ATE(x_i) \\ &= \overline{x}\left(\beta^1 - \beta^0\right) \end{aligned}$$

2.1. TT (Treatment on the Treated) is the average gain from treatment for those that actually select into treatment

$$
\begin{aligned}
TT(x, z, D[z]=1) &= E(\Delta | X = x, Z = z, D[z] = 1) \\
&= x\left(\beta^1 - \beta^0\right) + E\left(U^1 - U^0 | X = x, Z = z, U^D \geq -z\theta\right) \\
&= x\left(\beta^1 - \beta^0\right) + E\left(U^1 - U^0 | U^D \geq -z\theta\right)
\end{aligned}
$$

with the unconditional TT:

$$
\begin{aligned}
TT &= E(\Delta | D[z] = 1) \\
&= \int TT(x, z, D[z] = 1)\, dF(x, z, D[z] = 1) \\
&\simeq \frac{1}{n_1}\sum_{i=1}^{n} D_i\, TT(x_i, z_i, D[z_i] = 1)
\end{aligned}
$$

2.2. TUT (Treatment on the Untreated) is the average gain from treatment for those that select out from treatment

$$
\begin{aligned}
TUT\left(x, z, D\left[z\right]=0\right) &= E\left(\Delta | X=x, Z=z, D\left[z\right]=0\right) \\
&= x\left(\beta^1-\beta^0\right)+E\left(U^1-U^0 | X=x, Z=z, U^D<-z\theta\right) \\
&= x\left(\beta^1-\beta^0\right)+E\left(U^1-U^0 | U^D<-z\theta\right)
\end{aligned}
$$

with the unconditional TUT:

$$
\begin{aligned}
TUT &= E\left(\Delta | D\left[z\right]=0\right) \\
&= \int TUT\left(x, z, D\left[z\right]=0\right) dF\left(x, z, D\left[z\right]=0\right) \\
&\simeq \frac{1}{n_0}\sum_{i=1}^{n}(1-D_i)\, TUT\left(x_i, z_i, D\left[z_i\right]=0\right)
\end{aligned}
$$

3. **LATE (Local Average Treatment Effect)** is the expected outcome gain for those induced to receive treatment through a change in the instrument from $Z_k = z_k$ to $Z_k = z'_k$ (also called compliers)

The variable Z_k is assumed to affect the treatment decision, but not to affect the outcomes Y^1 and Y^0.

Below, we define the LATE parameter as a change in the index from $Z\theta = z\theta$ to $Z\theta = z'\theta$, where $z'\theta > z\theta$ and z and z' are identical except for the kth coordinate.

$$
\begin{aligned}
LATE\left(D[z] = 0, D\left[z'\right] = 1, X = x\right) &= E\left(\Delta | D(z) = 0, D\left(z'\right) = 1, X = x\right) \\
&= x\left(\beta^1 - \beta^0\right) + E\left(U^1 - U^0 | -z'\theta \leq U^D \leq -z\theta, X = x\right) \\
&= x\left(\beta^1 - \beta^0\right) + E\left(U^1 - U^0 | -z'\theta \leq U^D \leq -z\theta\right)
\end{aligned}
$$

with the unconditional LATE:

$$
\begin{aligned}
LATE &= E\left(\Delta | D(z) = 0, D\left(z'\right) = 1\right) \\
&= \int LATE\left(D[z] = 0, D\left[z'\right] = 1, X\right) dF(X) \\
&\simeq \frac{1}{n} \sum_{i=1}^{n} LATE\left(D[z] = 0, D\left[z'\right] = 1, X = x_i\right)
\end{aligned}
$$

4. MTE (Marginal Treatment Effect) is the treatment effect for individuals with a given value of U^D

$$\begin{aligned} MTE\left(x, u^D\right) &= E\left(\Delta | X = x, U^D = u^D\right) \\ &= x\left(\beta^1 - \beta^0\right) + E\left(U^1 - U^0 | U^D = u^D X = x\right) \\ &= x\left(\beta^1 - \beta^0\right) + E\left(U^1 - U^0 | U^D = u^D\right) \end{aligned}$$

with the unconditional MTE:

$$\begin{aligned} MTE\left(u^D\right) &= \int MTE\left(X, u^D\right) dF(X) \\ &\simeq \frac{1}{n} \sum_{i=1}^{n} MTE\left(X = x_i, u^D\right) \\ &= \overline{x}\left(\beta^1 - \beta^0\right) + E\left(U^1 - U^0 | U^D = u^D\right) \end{aligned}$$

The MTE parameter can also be expressed as the limit form of the LATE parameter,

$$\begin{aligned} \lim_{z\theta \to z'\theta} LATE\left(x, D[z] = 0, D\left[z'\right] = 1\right) &= x\left(\beta^1 - \beta^0\right) + \lim_{z\theta \to z'\theta} E\left(U^1 - U^0 | -z'\theta \leq U^D \leq -z\theta, X = x\right) \\ &= x\left(\beta^1 - \beta^0\right) + E\left(U^1 - U^0 | U^D = -z'\theta\right) = MTE(x, -z'\theta) \end{aligned}$$

The MTE measures the average gain for those individuals who are just indifferent to the receipt of treatment when the $z\theta$ index is fixed at $-u^D$

Treatment Parameters in the Gaussian Selection Model

Assumption: The error terms are jointly distributed according to a trivariate normal distribution,

$$\begin{bmatrix} U^D \\ U^1 \\ U^0 \end{bmatrix} \sim N\left(0, \begin{bmatrix} 1 & \sigma_{1D} & \sigma_{0D} \\ \sigma_{1D} & \sigma_1^2 & \sigma_{10} \\ \sigma_{0D} & \sigma_{10} & \sigma_0^2 \end{bmatrix}\right)$$

We then have,

$$\begin{aligned}
ATE(x) &= x\left(\beta^1-\beta^0\right)\\
TT\left(x,z,D\left[z\right]=1\right) &= x\left(\beta^1-\beta^0\right)+\left(\rho_1\sigma_1-\rho_0\sigma_0\right)\frac{\phi\left(z\theta\right)}{\Phi\left(z\theta\right)}\\
TUT\left(x,z,D\left[z\right]=0\right) &= x\left(\beta^1-\beta^0\right)+\left(\rho_1\sigma_1-\rho_0\sigma_0\right)\frac{\phi\left(z\theta\right)}{1-\Phi\left(z\theta\right)}\\
LATE\left(x,D\left[z\right]=0,D\left[z'\right]=1\right) &= x\left(\beta^1-\beta^0\right)+\left(\rho_1\sigma_1-\rho_0\sigma_0\right)\frac{\phi\left(z'\theta\right)-\phi\left(z\theta\right)}{\Phi\left(z'\theta\right)-\Phi\left(z\theta\right)}\\
MTE\left(x,u^D\right) &= x\left(\beta^1-\beta^0\right)+\left(\rho_1\sigma_1-\rho_0\sigma_0\right)u^D
\end{aligned}$$

Estimation

The parameters of the model can be consistently (and efficiently) estimated by maximum likelihood. They can also be consistently estimated using nothing more than the output from a two-step procedure. We revise both.

Maximum Likelihood

$$L\left(\beta^0, \beta^1, \sigma_0, \sigma_1, \rho_0, \rho_1\right) =$$
$$\Pi\left[\int_{-\infty}^{-z\theta} f\left(Y^0 - X\beta^0, u^D\right) du^D\right]^{1-D_i} \left[\int_{-z\theta}^{\infty} f\left(Y^1 - X\beta^1, u^D\right) du^D\right]^{D_i}$$

which, can be expressed as

$$\begin{aligned}
L &= \Pi\left[\int_{-\infty}^{-z\theta} f\left(u^0, u^D\right) du^D\right]^{1-D_i} \left[\int_{-z\theta}^{\infty} f\left(u^1, u^D\right) du^D\right]^{D_i} \\
&= \Pi_{1-D_i} \int_{-\infty}^{-z\theta} f\left(u^D | u^0\right) f\left(u^0\right) du^D \Pi_{D_i} \int_{-z\theta}^{\infty} f\left(u^D | u^1\right) f(u^1) du^D \\
&= \Pi_{1-D_i} f\left(u^0\right) \int_{-\infty}^{-z\theta} f\left(u^D | u^0\right) du^D \Pi_{D_i} f(u^1) \int_{-z\theta}^{\infty} f\left(u^D | u^1\right) du^D
\end{aligned}$$

Note that,

$$
\begin{aligned}
u_D | u_1 &\sim N\left(u_1 \frac{\sigma_{1D}}{\sigma_1^2}, \sigma_D^2 (1 - \rho_1^2) \right) \\
&\sim N\left(\left(Y^1 - X' \beta^1 \right) \frac{\sigma_{1D}}{\sigma_1^2}, \sigma_D^2 (1 - \rho_1^2) \right)
\end{aligned}
$$

Thus,

$$
\left(u_D - u_1 \frac{\sigma_{1D}}{\sigma_1^2} \right) \frac{1}{\sqrt{\sigma_D^2 (1 - \rho_1^2)}} \sim N(0, 1)
$$

Therefore,

$$\begin{aligned}
L \quad = \quad & \Pi_{1-D_i}\left[\frac{1}{\sigma_0\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{Y_0-X'\beta_0}{\sigma_0}\right)^2\right)\right]\left[1-\Phi\left(\left(-Z\theta-\left(Y_0-X'\beta_0\right)\frac{\sigma_{0D}}{\sigma_0^2}\right)\frac{1}{\sigma_D\sqrt{(1-\rho_0^2)}}\right)\right] \\
& \Pi_{D_i}\left[\frac{1}{\sigma_1\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{Y_1-X'\beta_1}{\sigma_1}\right)^2\right)\right]\left[\Phi\left(\left(-Z\theta-\left(Y_1-X'\beta_1\right)\frac{\sigma_{1D}}{\sigma_1^2}\right)\frac{1}{\sigma_D\sqrt{(1-\rho_1^2)}}\right)\right] \\
= \quad & \Pi_{1-D_i}\left[\frac{1}{\sigma_0\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{Y_0-X'\beta_0}{\sigma_0}\right)^2\right)\right]\left[\Phi\left(\left(Z\theta+\left(Y_0-X'\beta_0\right)\frac{\rho_{0D}^2}{\sigma_0}\right)\frac{1}{\sqrt{(1-\rho_0^2)}}\right)\right] \\
& \Pi_{D_i}\left[\frac{1}{\sigma_1\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{Y_1-X'\beta_1}{\sigma_1}\right)^2\right)\right]\left[1-\Phi\left(\left(Z\theta+\left(Y_1-X'\beta_1\right)\frac{\rho_{1D}^2}{\sigma_1}\right)\frac{1}{\sqrt{(1-\rho_1^2)}}\right)\right]
\end{aligned}$$

where $\rho_i^2 = \sigma_{iD}/\sigma_D\sigma_i \;\; i = 0, 1$

Two-step Procedure

- Obtain $\widehat{\theta}$ from a probit model on the decision to take the treatment
- Compute the appropriate selection correction terms evaluated at $\widehat{\theta}$. These are, $\frac{\phi(z\widehat{\theta})}{\Phi(z\widehat{\theta})}$ when $D_i = 1$, and $\frac{\phi(z\widehat{\theta})}{1-\Phi(z\widehat{\theta})}$ when $D_i = 0$
- Run treatment-outcome-specific regressions, with the inclusion of the appropriate selection-correction terms obtained from previous step

$$
\begin{aligned}
Y^1 &= x\beta^1 + \rho_1\sigma_1 \frac{\phi\left(z\widehat{\theta}\right)}{\Phi\left(z\widehat{\theta}\right)} + V^1 \\
Y^0 &= x\beta^0 + \rho_0\sigma_0 \frac{\phi\left(z\widehat{\theta}\right)}{1-\Phi\left(z\widehat{\theta}\right)} + V^0
\end{aligned}
$$

- Given $\widehat{\beta^0}$, $\widehat{\beta^1}$, $\widehat{\rho_1\sigma_1}$ y $\widehat{\rho_0\sigma_0}$ obtained from previous step, and $\widehat{\theta}$ from the first step, compute point estimates of the conditional and/or unconditional versions of the treatment parameters. Bootstrap standard errors.