Exchange Economy and Ricardian Model

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1 A Simple Model of Trade

In order to understand why trade can generate higher welfare, it is useful to start from a basic model that can help us sharpen our arguments in a clean and precise fashion. In such a model, we will see that, under some very mild conditions, trade can indeed be beneficial. Our analysis departs from an environment where an economic agent, representing an entire nation, is in trade autarky, and we then ask if that agent is better off when allowed to trade with a foreign counterpart. Intuitively, we can reason that there must be some gains from trade in the sense that the option of no trade is always possible where the agent would consume the same as in autarky. But if, under a trade regime, the agent decides a consumption bundle that is different from the autarky one, then it must be that he's better off under his choice. Note however, that nothing is said about the question of whether the level of trade is the right one, or if some agents benefit more from trade than others. We will have something to say about these questions later during this course. For now, we are just comparing a non-trade environment with one where trade is allowed. In doing so, we'll be able to represent a general competitive equilibrium and derive equilibrium quantities and prices from stable parameters as technology and preferences. An important price for international trade known as terms-or-trade will be derived endogenously.

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1.1 An exchange economy under autarky

The domestic economy has two goods available, x_1 and x_2. A representative agent of that economy values consumption of both goods accordingly to a standard utility function $u(x_1, x_2)$. Moreover, this agent also starts with two correspondent endowments: e_1 and e_2. We also assume that the agent operates under a **competitive market** environment: consumers/sellers take prices p_1 and p_2 as given, behave optimally, and markets for both goods 1 and 2 clear at those prices. To solve for the equilibrium in this economy we need first to derive the optimality conditions for the consumer and then impose market clearing conditions. It follows that the problem of the agent is given by:

$$\begin{aligned} & \max_{x_1, x_2} u(x_1, x_2) \\ st & \\ & p_1 x_1 + p_2 x_2 \leq p_1 e_1 + p_2 e_2 \end{aligned}$$

The solution for this problem is characterized by the following first order conditions:

$$\begin{aligned} x_1 : \; MU_1 \equiv \frac{\partial u(x_1, x_2)}{\partial x_1} &= \lambda p_1 \\ x_2 : \; MU_2 \equiv \frac{\partial u(x_1, x_2)}{\partial x_2} &= \lambda p_2 \end{aligned}$$

where λ is a Lagrange multiplier associated with the agent's budget constraint. Dividing one equation into the other yields:

$$MRS_1^2 \equiv \frac{MU_1}{MU_2} = \frac{p_1}{p_2} \tag{1}$$

In economic jargon this means that the **marginal rate of substitution** of good 1 in terms of 2 is equal to the **relative price** of good 1. In other words, we require that rate at which this agent is willing to substitute away from goods 2 in terms of good 1 must be equal to good 1 relative price (that establishes the rate of substitution of goods 2 for each good 1). To see that this is an equilibrium consider instead that we had $MRS_1^2 > p_1/p_2$; then, because the agent is willing to foregone more of goods 2 that what the market asks in terms of good 1, a trade at the market will occur. That is, on one hand the agent's utility falls by $MU_2 (p_1/p_2)$, but on the other hand increases by MU_1, thus implying an increase in utility[1].

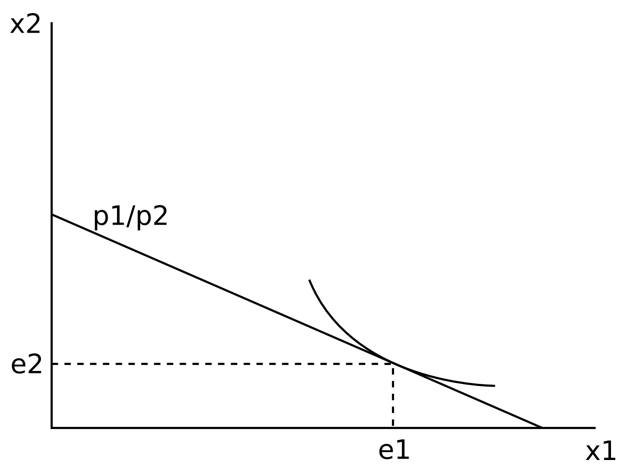
Note that equation (1) is not enough to describe the equilibrium. In order to fully describe that equilibrium we also need:

$$\frac{p_1}{p_2}x_1 + x_2 = \frac{p_1}{p_2}e_1 + e_2 \quad (2)$$

$$x_1 = e_1 \quad (3)$$

Where equation (2) states that in the equilibrium the agent consumes all his available endowment, and equation (3) is a goods market market clearing condition[2]. Note the equilibrium implied by equations (1)-(3), that is $\{p_1/p_2, x_1, x_2\}$, has a simple diagrammatical representation:

Figure 1: Competitive market outcome of a country under autarky



Finally it should be noted that (1), (2) and (3) imply that equilibrium prices are given by:

$$\frac{p_1}{p_2} = \frac{MU_1(e_1, e_2)}{MU_2(e_1, e_2)}$$

These prices have the standard interpretations. Let's say that the endowment of e_1 increases: that should imply that the MU_1/MU_2 should fall implying also a fall in prices. On

[1]Formally, note that the change in total utility of this trade is given by

$$\Delta U = MU_1 \Delta x_1 + MU_2 \Delta x_2 = MU_1 - MU_2 \cdot p_1/p_2 > 0$$

whenever $\Delta x_1 = 1$ and using the fact that $x_1p_1 + x_2p_2 = m \Rightarrow \Delta x_1 p_1 + \Delta x_2 p_2 = 0 \Rightarrow \Delta x_2 = -\Delta x_1 p_1/p_2$.

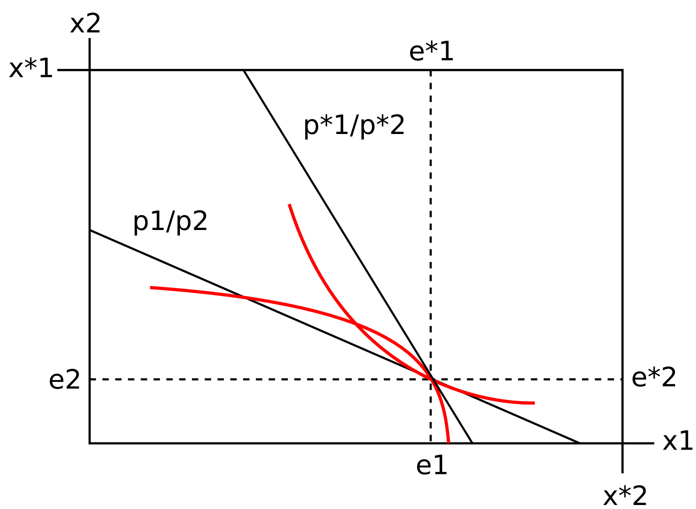
[2]Note that we don't need the equation $x_2 = e_2$ to characterize the equilibrium as that is already implied by equations (2) and (3). This is a simple application of the Walras's law.

the other hand, imagine that preferences suddenly value more x_1: in that case MU_1/MU_2 should increase with a increase also of the relative price p_1/p_2.

1.2 An exchange economy under trade

Lets consider now a foreign economy characterized by a utility function $u^*(x_1^*, x_2^*)$ and endowments e_1^* and e_2^*. Reasoning in the same way as before we get an autarky equilibrium given by $\{p_1^*/p_2^*, x_1^*, x_2^*\}$. In order to compare the market equilibrium of the domestic and foreign economy, one can make use of an Edgeworth box which bounds together the diagram 1 for the domestic economy with the one from the foreign economy by inverting the axis. Figure 2 shows the representation.

Figure 2: Edgeworth box for the domestic and foreign economy under autarky



In the figure we can see that $p_1/p_2 < p_1^*/p_2^*$. Suppose now that allow the foreign economy to trade. To see that both prices will be equalized, note that the foreign economy would prefer to buy good 1 at the domestic economy's price while the foreign economy would prefer to buy good 2 from the foreign economy. The flow of goods induced by these relative price difference means that eventually prices are equalized.

To formalize these ideas, lets allow now for trade. As before we use a competitive market structure, but now the market clearing conditions are given by:

$$
\begin{aligned}
x_1 + x_1^* &= e_1 + e_1^* \qquad (4) \\
x_2 + x_2^* &= e_2 + e_2^*
\end{aligned}
$$

meaning that consumption for each good needs to be equal to the available resources in that same good. As before the equilibrium condition for the agent in the domestic economy is given by:

$$\frac{MU_1}{MU_2} = \frac{p_1^T}{p_2^T} \tag{5}$$

$$\frac{p_1^T}{p_2^T}x_1 + x_2 = \frac{p_1^T}{p_2^T}e_1 + e_2 \tag{6}$$

where p_1^T/p_2^T is the relative price under free trade. Similarly for the foreign economy:

$$\frac{MU_1^*}{MU_2^*} = \frac{p_1}{p_2} \tag{7}$$

$$\frac{p_1^T}{p_2^T}x_1^* + x_2^* = \frac{p_1^T}{p_2^T}e_1^* + e_2^* \tag{8}$$

Thus, equations (4)-(8) characterize the equilibrium, that is, they determine the allocations and relative prices $\{x_1, x_2, x_1^*, x_2^*, p_1^T/p_2^T\}$. Note that under this equilibrium, exports must equal imports at market values for each country, that is, the **trade balance** must be equal to 0 in each economy[3]. This can be seen from rearranging equations (6) and (8):

$$\underbrace{\frac{p_1^T}{p_2^T}(x_1 - e_1)}_{Imports} = \underbrace{e_2 - x_2}_{Exports}$$

$$\underbrace{\frac{p_1^T}{p_2^T}(e_1^* - x_1^*)}_{Exports^*} = \underbrace{x_2^* - e_2^*}_{Imports^*}$$

and, because the exports of a country must be the imports of the other, the right hand side of both equations are equal, thus verifying the market clearing condition.

Figure 3 depicts the equilibrium graphically. Two comments are in order. First, we can see that a movement from autarky to free trade increases the utility for both countries (both countries are in higher indifference curves). Second, in the picture, one can see that the domestic economy is an exporter of good 1 and importer of good 2. Using the definition of **terms-of-trade**, given by the ratio of the price of exports with respect to imports, the

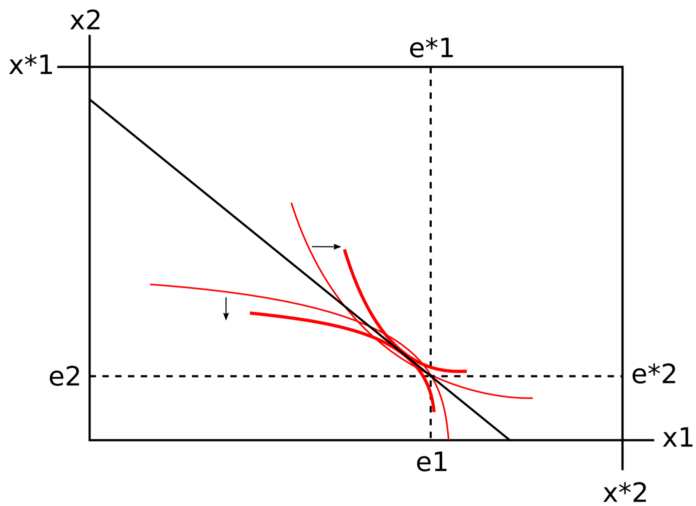
[3]Trade balance is defined as a country's exports minus imports at market prices. A deficit or surplus can be achieved in these economies if we allow for international financial transactions (to be studied later in this course).

following is true:

$$tot \equiv \frac{p_{exports}}{p_{imports}} = \frac{p_1^T}{p_2^T}$$

For a given level of world endowments, it is easy to see that any increase in the terms of trade implies that the domestic economy is better off.

Figure 3: Edgeworth box for the domestic and foreign economy under free trade



Finally, we are now able to show that opening up to trade is beneficial for both countries. The argument will make use of the principle of revealed preferences stating that if a particular choice is taken over an available alternative, then that particular choice must be at least as good as the first one. Applying this idea to our problem, lets assume that a free trade equilibrium generates p_1^T/p_2^T. Because this is an equilibrium condition, total utility of the domestic economy is given by:

$$\begin{aligned} U^T = & \max_{x_1,x_2} u(x_1, x_2) \\ st & \\ & p_1^T x_1 + p_2^T x_2 = p_1^T e_1 + p_2^T e_2 \end{aligned}$$

Because e_1, e_2 is feasible (it's an available alternative), it must be that

$$U^T \geq U(e_1, e_2)$$

But since the utility of autarky is given by $U^A = U(e_1, e_2)$, it must be that $U^T \geq U^A$.

Similar arguments also apply for the foreign country.

This essentially proves that under **exchange** provides **gains from trade** for both domestic and foreign agents. The next section will show that specialization can be other source of gains from trade.

2 Specialization and the Ricardian model

In this section, instead of an endowment economy, we allow for production in the sense that output of each good uses labor under some specific productivity:

$$\begin{aligned} y_1 &= z_1 l_1 \\ y_2 &= z_2 l_2 \end{aligned} \tag{9}$$

where z_1 is the productivity of labor at the production of good 1 and l_1 is the quantity of labor used in the production of good 1.

Now, let's consider the classical example given by David Ricardo at the beginning of the XIX century. Let two countries, domestic and foreign, be characterized by the following productivities of labor across sectors:

Productivity	good 1	good 2
domestic	5	2
foreign	1	1

$\Leftrightarrow$

Labor Cost	good 1	good 2
domestic	1/5	1/2
foreign	1	1

Where we make use of the fact that the labor cost is just the inverse of the productivity, that is, if with one unit of labor the domestic country can produce 5 units of good 1, then one unit of good 1 can be produced with 1/5 units of labor. Because the domestic economy has a higher labor productivity in both goods, we say that the domestic economy has an **absolute advantage** at producing both goods. That however doesn't mean that allowing for free trade is not beneficial for the foreign economy. In fact it can be beneficial to both. To see this, suppose for instance that each country has a 10 units available of labor for production and under autarky each allocates 5 units to the production of each good, implying the following:

	Labor allocation				Output	
	good 1	good 2			good 1	good 2
domestic	5	5	$\Leftrightarrow$	domestic	25	10
foreign	5	5		foreign	5	5
				total	30	15

Note now that although the domestic economy productivities are higher in both goods, the same is not true in relative terms, that is:

$$\frac{z_1}{z_2} = 2.5 \quad > \quad \frac{z_1^*}{z_2^*} = 1$$
$$\frac{z_2}{z_1} = 0.4 \quad < \quad \frac{z_2^*}{z_1^*} = 1$$

So the domestic country has a **relative advantage** at producing good 1, while the foreign country has a relative advantage at producing good 2. In other words, the foreign country is worse other in all sectors, but in sector 2 he is less worse. By allowing these countries to specialize in the production of goods in which they are relatively more productive, the world output can increase. For example, a different labor allocation:

	Labor allocation				Output	
	good 1	good 2			good 1	good 2
domestic	7	3	$\Leftrightarrow$	domestic	35	6
foreign	0	10		foreign	0	10
					35	16

would increase world output would increase in both goods: both countries can potentially benefit from **specialization**. We will see later that international trade can induce countries to specialize, but before that, it is useful to characterize a production economy of this type under autarky.

2.1 A production economy under autarky

Lets consider as an household that consumes both x_1 and x_2 and supplies a fixed amount of labor L at a wage rate w. Given the wage, a firm in the sector for goods 1 hire l_1 units of labor and produces y_1 units of good 1 (and similarly for a firm in the sector for goods 2). A competitive equilibrium is one where, given prices, $(p_1/p_2, w/p_2)$, optimal allocations,

(x_1, x_2, y_1, y_2), clear all the markets:

$$\begin{aligned} x_1 &= y_1 \\ x_2 &= y_2 \\ l_1 + l_2 &= L \end{aligned}$$

Where the first two equations refer to the goods market and the last one to the labor market. To characterize such equilibrium, it is useful to look at the household and firm's problem separately.

Household The household problem is given by the following utility maximization (it is useful for our derivations to use a particular utility function):

$$\max_{x_1, x_2} u(x_1, x_2) = \alpha \log x_1 + (1 - \alpha) \log x_2 \tag{10}$$

$$st$$

$$p_1 x_1 + p_2 x_2 \leq wL$$

Implying the following optimality conditions:

$$\alpha / x_1 = \lambda p_1 = \frac{p_1}{wL} \tag{11}$$

$$(1 - \alpha) / x_2 = \lambda p_2 = \frac{p_2}{wL} \tag{12}$$

where $\lambda = 1/wL$ is derived from replacing the optimal conditions back in the household budget constraint with equality. These equations give us the demand for both goods 1 and 2.

Firm The firm problem consists in the maximization of profit, given by the difference of revenues and costs:

$$\begin{aligned} profit_1 \equiv \pi_1 &= p_1 y_1 - w l_1 \\ &= p_1 z_1 l_1 - w l_1 \end{aligned}$$

Three situations can happen, either the marginal revenue, given by $p_1 z_1$ is larger than the marginal cost, w, and the firm hires all the labor that is available, or the marginal revenue is lower that the marginal cost an no labor is hired at all, or the marginal cost is just equal to the marginal and any amount of labor can be sustained:

$$l_1 = \begin{cases} 0 & if\ z_1 < w/p_1 \\ [0, L] & if\ z_1 = w/p_1 \\ L & if\ z_1 > w/p_1 \end{cases} \tag{13}$$

and the analogous for the the firm in the goods 2 sector:

$$l_2 = \begin{cases} 0 & if\ z_2 < w/p_2 \\ [0, L] & if\ z_2 = w/p_2 \\ L & if\ z_2 > w/p_2 \end{cases} \tag{14}$$

Equilibrium The first thing we need to realize in the equilibrium is that a corner solution, that is $l_1 = 0$ or $l_1 = L$, is not admissible in the equilibrium because that would imply a zero output for one of the goods and therefore $-\infty$ utility (given the utility function (10)). In other words, full specialization is not admissible in trade autarky. This implies that:

$$\begin{aligned} w/p_1 &= z_1 \\ w/p_2 &= z_2 \end{aligned}$$

Substituting these prices on the goods demand equations gives:

$$\begin{aligned} \alpha/x_1 &= \frac{1}{z_1 L} \Rightarrow x_1 = \alpha z_1 L \\ (1-\alpha)/x_2 &= \frac{1}{z_2 L} \Rightarrow x_2 = (1-\alpha) z_2 L \end{aligned}$$

Because goods market clear $x_1 = y_1$ and $x_2 = y_2$:

$$\begin{aligned} x_1 = &\quad \alpha z_1 L &= z_1 l_1 \Rightarrow l_1 = \alpha L \\ x_2 = &\quad (1-\alpha) z_2 L &= z_2 l_2 \Rightarrow l_2 = (1-\alpha) L \end{aligned}$$

Finally these allocations imply the following prices:

$$\frac{p_1}{p_2} = \frac{1/z_1}{1/z_2}$$
$$\frac{w}{p_2} = z_2$$

The interpretation of these prices and wages (using the good 2 as a unit of account or numeraire) is the following: the price of good 1 relative to good 2 is equal to the labor cost of producing one unit of good 1 relative to good 2 (remember that the labor cost of good 1 is given by $1/z_1$), and the real wage is equal to the productivity of good 2.

Relative demand and supply An alternative way of representing the equilibrium in this economy is to equate the relative supply of goods with the relative demand.

The relative supply is just defined as y_1/y_2, which imply, using (13) and (14) and the fact that $w = z_2 p_2$:

$$\frac{y_1}{y_2} = \begin{cases} 0 & if \ \frac{p_1}{p_2} < \frac{1/z_1}{1/z_2} \\ [0, \infty] & if \ \frac{p_1}{p_2} = \frac{1/z_1}{1/z_2} \\ \infty & if \ \frac{p_1}{p_2} > \frac{1/z_1}{1/z_2} \end{cases}$$

where the first thing we should note is that the conditions are rearranged so as to highlight the comparison of the price ratio p_1/p_2 with the labor cost of producing good 1 with good 2: $\frac{1/z_1}{1/z_2}$. The second one is to understand how the ratio is derived. Suppose a firm needs to choose where to produce. Irrespectively of the sector choice, that firm has a cost of w per unit of labor used in the production. At the same time the revenue per unit of labor is given by $p_1 z_1$ for sector 1, and $p_2 z_2$ for sector 2. Since the cost is the same across sectors, if $p_1 z_1 < p_2 z_z$, then no firm will produce in sector 1 since sector 2 generates larger revenues. But then the output in sector 1 is zero $y_1 = 0$, while in sector 2 is the maximum possible $y_2 = z_2 L$. If we take the ratio of output in sector 1 to output in sector 2, we have a relative supply of $y_1/y_2 = 0/(z_2 L) = 0$. This explanation gives the intuition for the first branch of the relative supply function.

It follows that in the first branch, when relative price of good 1 is below the relative cost of producing that same good, then all the production shifts to good 2, implying a 0 relative supply; in the second brach, when $\frac{p_1}{p_2} = \frac{1/z_1}{1/z_2}$, firms in both sectors are willing to demand any amount of labor which implies that if production in sector i is 0, the production in the other sector is $z_{-i}L$ with a relative supply of $y_i/y_{-i} = 0$ or $y_{-i}/y_i = \infty$; and finally in the last branch, the relative price of good 1 is larger than

the relative cost of producing it, so no firm in sector 2 will choose to produce, and hence the ∞ relative supply. The relative supply has the standard property that it increases with prices.
As for the relative demand we just divide (11) in (12) and rearrange to get:

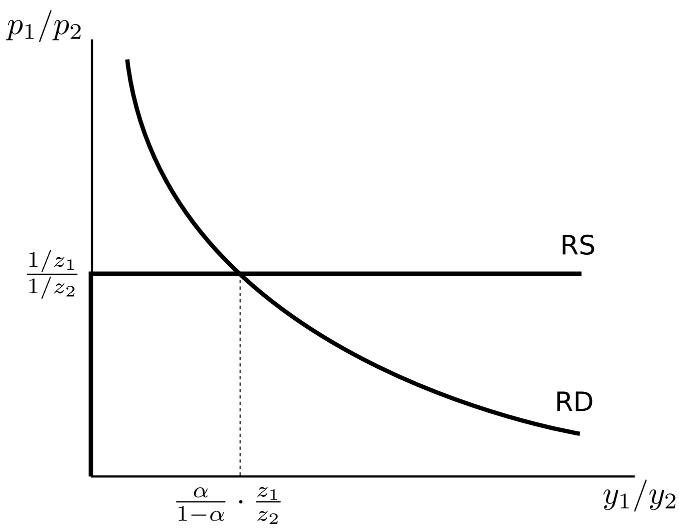
$$\frac{x_1}{x_2} = \frac{\alpha}{1-\alpha} \cdot \frac{1}{p_1/p_2} \tag{15}$$

That is, relative demand is a downward sloping function of relative prices.
As usual, equilibrium is achieved when relative supply equals relative demand:

$$y_1/y_2 = x_1/x_2$$

with the, also usual, graphical representation:

Figure 4: Market equilibrium under autarky



2.2 Trade

Allowing for trade in this economy can be easily analyzed using a relative supply and demand diagram. To be more specific, let's allow for a foreign economy with the same preferences as the domestic (given by (10)), but with a lower relative productivity in the sector for goods 1 and a different labor supply L^*:

$$\frac{z_1^*}{z_2^*} < \frac{z_1}{z_2}$$

Note that while in autarky, the relation between domestic and foreign prices is given by:

$$\frac{p_1^*}{p_2^*} > \frac{p_1}{p_2}$$

But this implies that at home prices foreign suppliers will specialize on the production of good 2 (as they wish to sell at a higher price), while, at foreign prices domestic suppliers specialize in good 1. Also note that the relative world supply when both countries specialize is given by:

$$\frac{y_1 + y_1^*}{y_2 + y_2^*} = \frac{z_1 L}{z_2^* L^*}$$

This should give us a world relative supply equal to:

$$\frac{y_1 + y_1^*}{y_2 + y_2^*} = \begin{cases} 0 & if\ \frac{p_1}{p_2} < \frac{1/z_1}{1/z_2} \\ \left[0, \frac{z_1 L}{z_2^* L^*}\right] & if\ \frac{p_1}{p_2} = \frac{1/z_1}{1/z_2} \\ \frac{z_1 L}{z_2^* L^*} & if\ \frac{1/z_1}{1/z_2} < \frac{p_1}{p_2} \leq \frac{1/z_1^*}{1/z_2^*} \\ \infty & if\ \frac{p_1}{p_2} > \frac{1/z_1^*}{1/z_2^*} \end{cases}$$

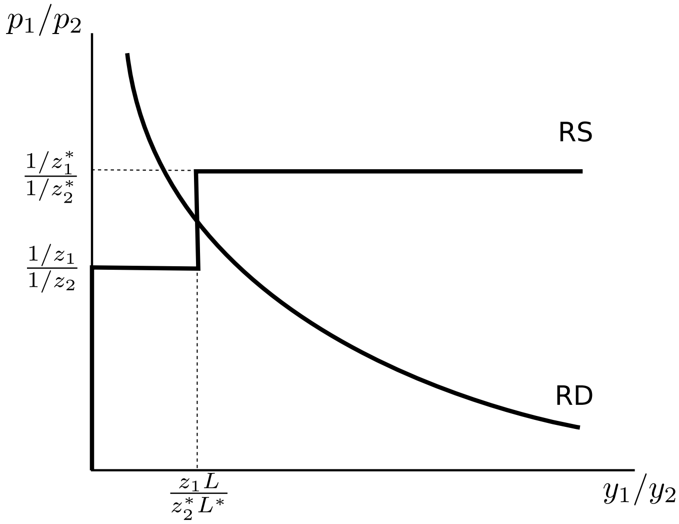
where the first branch implies that $p_1 z_1 < p_2 z_2$, that is the revenue of producing in sector 1 is smaller than in sector 2, implying also that $p_1 z_1^* < p_2 z_2^*$ since $z_1^*/z_2^* < z_1/z_2$. The remaining branches are derived in a similar manner.

At the same time, from (15) relative demand is given by:

$$\frac{x_1 + x_1^*}{x_2 + x_2^*} = \frac{\alpha}{1 - \alpha} \cdot \frac{1}{p_1^T / p_2^T}$$

This should clarify how the world relative supply and demand diagram is constructed:

Figure 5: Market equilibrium under Trade



In the figure, the equilibrium trade relative price is above the domestic and below the foreign relative price of good 1 that would prevail under autarky. This implies that specialization will be complete for both countries. However, a different structure for this economy (different preferences parameter α; relative productivities z_1/z_2, z_1^*/z_2^*; or labor supplies L, L^*) could push the intersection to the the first horizontal segment of supply curve, thus implying a price of z_2/z_1. Note at this price, the domestic economy wouldn't fully specialize (some labor would be affected in both sectors), nevertheless the foreign would still fully specialize on sector 2.

2.2.1 General world equilibrium

To solve for the general equilibrium, we just need to realize that either neither country specializes, or both specialize or only one specialize.

neither specialize The only way we have for both countries not to specialize is if:

$$\frac{p_1}{p_2} = \frac{p_1^T}{p_2^T} = \frac{p_1^*}{p_2^*}$$

That is to say:

$$\frac{z_1}{z_2} = \frac{z_1^*}{z_2^*}$$

or that both countries must have exactly the same relative productivity.

both specialize When both countries specialize (without loss of generality say domestic economies specialize in good 1 and the foreign in 2: $p_1^*/p_2^* > p_1/p_2$), the world supply is given by:

$$y_1 = z_1 L \quad and \quad y_2 = 0$$
$$y_2^* = z_2^* L^* \;\; and \quad y_1^* = 0$$

With this supply, note that domestic budget constraint becomes:

$$x_2 = \frac{p_1^T}{p_2^T}\left(z_1 L - x_1\right)$$

Also note that (11) and (12) implies:

$$\frac{x_1}{x_2} = \frac{\alpha}{1-\alpha} \cdot \frac{1}{p_1^T/p_2^T} \tag{16}$$

So, substituting in these equations yields:

$$x_1 = \alpha z_1 L$$

But then, the fact that world goods market must clear, *i.e.*, $x_1 + x_1^* = z_1 L$, implies that:

$$x_1^* = (1-\alpha) z_1 L$$

Proceeding in the same way for the foreign economy gives that:

$$\begin{aligned} x_2^* &= (1-\alpha) z_2^* L^* \\ x_2 &= \alpha z_2^* L^* \end{aligned}$$

And, using for example (16), yields the following equilibrium world prices:

$$\frac{p_1^T}{p_2^T} = \frac{\alpha}{1-\alpha} \cdot \frac{z_2^* L^*}{z_1 L}$$

domestic doesn't specialize In this case, world prices must be equal to domestic prices under autarky (or otherwise the domestic economy would have specialized):

$$\frac{p_1}{p_2} = \frac{p_1^T}{p_2^T} < \frac{p_1^*}{p_2^*}$$

But then prices must be equal to the relative cost of labor in the domestic economy:

$$\frac{p_1^T}{p_2^T} = \frac{1/z_1}{1/z_2}$$

With these prices, it is straightforward to derive the equilibrium quantities (using the relative demands and the households budget constraints). Because the formulas won't be very pretty, or informative, solving for them should be left as an exercise for the interested reader:

$$\begin{aligned}
x_1^* &= \alpha z_1 \frac{z_2^*}{z_2} L^* \\
x_2^* &= (1-\alpha) z_2^* L^* \\
x_1 &= \alpha z_1 L \\
x_2 &= (1-\alpha) z_2 L
\end{aligned}$$

Note, however, that the consumption allocation of the domestic economy will be exactly the same as the one under autarky, but the foreign economy is now better off by specializing. The idea is that by specializing, the foreign economy ships their goods to the domestic economy, allowing it to free labor into the sector where it is relatively more productive. That increases production that is subsequently imported into the foreign country:

$$\begin{aligned}
l_1^* &= 0 \\
l_2^* &= L^* \\
l_1 &= \alpha L + \alpha \frac{z_2^*}{z_2} L^* \\
l_2 &= (1-\alpha) L - \alpha \frac{z_2^*}{z_2} L^*
\end{aligned}$$