Midterm

Advanced microeconometrics

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MTE, Treatment Effects Parameters, and Weights

In the framework of Heckman and Vytlacil (1999, 2005, 2007), and Heckman, Urzua, and Vytlacil (2006):

1. Show that any discrete choice model, $I = \eta(Z) - V$, may be written in equivalent form as,

$$I \geq 0 \iff \eta(Z) > V$$

or,

$$P(Z) > U_D,$$

where $P(Z) = F_V(\eta(Z)), U_D = F_V(V)$, F_V is the cdf of V (assumed strictly increasing), and U_D is uniformly distributed.

Given our choice model, we know that

$$\begin{aligned} D &= 1[I \geq 0] \\ &= 1[\eta(Z) \geq V] \end{aligned}$$

Since F_V is strictly increasing, we can rewrite

$$\begin{aligned} D &= 1[F_V(\eta(Z)) \geq F_V(V)] \\ &= 1[P(Z) \geq U_D] \end{aligned}$$

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2. Define the Marginal Treatment Effect ($MTE(X = x, U_D = u_D)$).

Let $\Delta = Y_1 - Y_0$.

$$MTE(X = x, U_D = u_D) = E(\Delta \mid X = x, U_D = u_D)$$

Namely, the expected effect of treatment conditional on observed characteristics and unobservables.

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3. Define the LIV (Local Instrumental Variables) estimator. Assume Z is continuous conditional on X (variables in outcome equations). Show that LIV identifies the MTE if the Z are continuous.

Given our initial assumptions, we can write $E(Y \mid Z = z) = E(Y \mid P(Z) = p)$.

Thus,

$$\begin{aligned} E(Y \mid P(Z)=p) &= E(DY_1 + (1-D)Y_0 \mid P(Z)=p) \\ &= E(Y_0) + E(D\Delta \mid P(Z)=p). \end{aligned}$$

Note that

$$\begin{aligned} E(D\Delta \mid D) &= 1 \times \Delta \times \Pr(D=1) + 0 \times \Delta \times \Pr(D=0) \\ &= p \times \Delta \\ &= E(\Delta \mid D=1), \end{aligned}$$

such that

$$\begin{aligned} E(E(\Delta \mid D=1) \mid P(Z)=p) &= E(\Delta \mid D=1)p \\ &= E(E(\Delta \mid P(Z)=p) \mid D=1). \end{aligned}$$

This way,

$$\begin{aligned} E(Y \mid P(Z)=p) &= E(Y_0) + E(\Delta \mid D=1)p \\ &= E(Y_0) + E(\Delta \mid P(Z)=p \geq U_D)p \\ &= E(Y_0) + \frac{1}{p}\left(\int_0^p \Delta \times f_{U_D}(\Delta) du_D\right)p \\ &= E(Y_0) + \int_0^p \Delta \times f_{U_D}(\Delta) du_D \\ &= E(Y_0) + \int_0^p E(\Delta \mid U_D = u_D) du_D. \end{aligned}$$

As a consequence,

$$\frac{\partial}{\partial p} E(Y \mid P(Z)=p)\bigg|_{P(z)=p} = E(\Delta \mid U_D = p).$$

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4. Now, suppose that the Z are discrete instruments (i.e. are discrete valued variables). What does IV identify?

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5. Prove that,

$$\begin{aligned} ATE(x) &= \int_0^1 MTE(x, u_D) du_D, \\ TUT(x) &= \int_0^1 MTE(x, u_D)\omega^{TUT} du_D, \end{aligned}$$

where $\omega^{TUT}(u_D) \geq 0$ for all u_D in $[0,1]$, and $\int_0^1 \omega^{TUT}(u_D) du_D = 1$. Derive the expression for ω^{TUT}.

Clearly,

$$\begin{aligned} ATE(x) &= E(\Delta \mid X = x) \\ &= \int_0^1 E(\Delta \mid X = x, U_D = u_D) du_D \\ &= \int_0^1 MTE(x, u_D) du_D. \end{aligned}$$

Furthermore, we know that

$$(1-P(z))TUT(x, P(z)) = \int_0^1 E(\Delta \mid X = x, U_D = u_D) du_D.$$

Hence,

$$
\begin{aligned}
TUT(x) &= \int_0^1 \left[\int_p^1 E(\Delta \mid X=x, U_D=u_D)du_D\right] dF_{P(Z)|X,D}(p \mid x, 0) \\
&= \frac{1}{1-p}\int_0^1 \int_0^p E(\Delta \mid X=x, U_D=u_D)du_D dF_{1-P(Z)|X,D}(p \mid x, 0) \\
&= \frac{1}{1-p}\int_0^1 \int_0^1 1(u_D \leq 1-p)E(\Delta \mid X=x, U_D=u_D)du_D dF_{1-P(Z)|X}(p \mid x) \\
&= \frac{1}{\int F_{1-P(Z)|X}(u_D)dt}\int_0^1 \int_0^1 1(p \leq 1-u_D)E(\Delta \mid X=x, U_D=u_D) dF_{1-P(Z)|X}(p \mid x)du_D \\
&= \int_0^1 E(\Delta \mid X=x, U_D=u_D)\left[\frac{F_{1-P(Z)|X}(u_D)}{\int F_{1-P(Z)|X}(u_D)dt}\right]du_D \\
&= \int_0^1 MTE(x, u_D)\omega^{TUT}(u_D)du_D.
\end{aligned}
$$

Moreover, using Bayes' rule, we get

$$
\begin{aligned}
\omega^{TUT}(u_D) &= \frac{F_{1-P(Z)|X}(u_D)}{\int F_{1-P(Z)|X}(u_D)dt} \\
&= \frac{\Pr(P(Z) \leq 1-u_D \mid X=x)}{E(1-P(Z) \mid X=x)} \\
&= \left[\int_0^{u_D} f(p \mid X=x)dp\right]\frac{1}{E(1-P(Z) \mid X=x)}.
\end{aligned}
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6. Define and derive the Policy Relevant Treatment Effect in terms of the MTE and compare it to the ATE.

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7. Derive the IV weights for $IV(Z_1)$, where the vector of instruments is $Z=(Z_1, \ldots, Z_k)$, $k \geq 2$, and Z_1 is continuously distributed.

Semiparametric Estimation of the MTE

Consider the following model:

$$
\begin{aligned}
Y_1 &= \alpha_0 + \varphi + U_1, \\
Y_0 &= \alpha_0 + U_0,
\end{aligned}
$$

where U_0 and U_1 represent the unobservables in the potential outcome equations, and φ represents the benefit associated with the treatment ($D=1$). Individuals decide whether or not to receive the treatment ($D=1$ or $D=0$) based on a latent variable I:

$$
I = Z\gamma - V,
$$

where Z and V represent observables and unobservables, respectively. Thus, we can define a binary variable D indicating treatment status,

$$
D = 1[I \geq 0].
$$

Finally, we assume that the error terms in the model are not independent even conditioning on the observables, i.e. $U_1 \not\perp\!\!\!\perp U_0 \not\perp\!\!\!\perp V \mid Z$, but $(U_1, U_0, V) \perp\!\!\!\perp Z$.

1. Show that,

$$E(Y \mid P(Z) = p) = \alpha + K(p),$$

where $Y = DY_1 + (1 - D)Y_0$, $P(Z)$ is the propensity score or probability of selection, p is a particular evaluation of the propensity score, and

$$K(p) = \varphi p + E(U_0 \mid P(Z) = p) + E(U_1 - U_0 \mid D = 1, P(Z) = p)p.$$

2. Use these two expressions to explain the steps involved in the semiparametric estimation of the Marginal Treatment Effect as presented in Heckman, Urzua, and Vytlacil (2006).

3. Explain in detail why you can approximate $K(p)$ with a polynomial on p. Explain how you can estimate the Marginal Treatment Effect using this approximation.