

Problem Set 1

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The Model

Assume that the economic model generating the data for potential outcomes is of the form:

$$\begin{aligned}Y_1 &= \alpha + \varphi + U_1, \\Y_0 &= \alpha + U_0,\end{aligned}$$

where U_1 and U_0 represent the unobservables in the potential outcome equations and φ represents the benefit associated with the treatment ($D = 1$). Individuals decide whether or not to receive the treatment ($D = 1$ or $D = 0$) based on a latent variable I :

$$I = Z\gamma + V,$$

where Z and V represent observables and unobservables, respectively. Thus, we can define a binary variable D indicating treatment status,

$$D = \mathbb{1}[I \geq 0].$$

Finally, we assume that the error terms in the model are not independent even conditioning on the observables, i.e. $U_1 \not\perp U_0 \mid Z$, but $(U_1, U_0, V) \perp Z$.

1. Treatment Parameters

Let $\beta = Y_1 - Y_0 = \varphi + U_1 - U_0$ such that, in regression notation, $Y = \alpha + \beta D + \varepsilon$ where $\varepsilon = U_0$.

Average Treatment Effect.

$$\begin{aligned}ATE &= E[\beta] \\&= E[\varphi + U_1 - U_0] \\&= \varphi\end{aligned}$$

Treatment on the Treated.

$$\begin{aligned}TT &= E[\beta \mid D = 1] \\&= E[\varphi + U_1 - U_0 \mid Z\gamma + V \geq 0] \\&= \varphi + E[U_1 - U_0 \mid V \geq -Z\gamma]\end{aligned}$$

Treatment on the Untreated.

$$\begin{aligned}TUT &= E[\beta \mid D = 0] \\&= E[\varphi + U_1 - U_0 \mid Z\gamma + V < 0] \\&= \varphi + E[U_1 - U_0 \mid V < -Z\gamma]\end{aligned}$$

Marginal Treatment Effect.

$$\begin{aligned}MTE &= E[\beta \mid I = 0, V = v] \\&= E[\varphi + U_1 - U_0 \mid I = 0, V = v] \\&= \varphi + E[U_1 - U_0 \mid Z\gamma = -V, V = v]\end{aligned}$$

Instrumental Variables.

Ordinary Least Squares.

Local Average Treatment Effect.

$$\begin{aligned}
LATE &= E[\beta \mid D(z) = 0, D(z') = 1] \\
&= E[\beta \mid z\gamma < -V \leq z'\gamma] \\
&= \varphi + E[U_1 - U_0 \mid -z'\gamma \leq V < -z\gamma]
\end{aligned}$$

2. Some closed form expressions

Suppose that the error terms in the model have the following structure:

$$\begin{aligned}
U_1 &= \sigma_1 \epsilon, \\
U_0 &= \sigma_0 \epsilon, \\
V &= \sigma_V^* \epsilon, \\
\epsilon &\sim \mathcal{N}(0, 1).
\end{aligned}$$

So, we can say that

$$\begin{aligned}
TT &= \varphi + E[U_1 - U_0 \mid V \geq -Z\gamma] \\
&= \varphi + E\left[\epsilon(\sigma_1 - \sigma_0) \mid \epsilon \geq \frac{-Z\gamma}{\sigma_V^*}\right] \\
&= \varphi + (\sigma_1 - \sigma_0)E\left[\epsilon \mid \epsilon \geq \frac{-Z\gamma}{\sigma_V^*}\right] \\
&= \varphi + (\sigma_1 - \sigma_0) \frac{\phi(-Z\gamma/\sigma_V^*)}{1 - \Phi(-Z\gamma/\sigma_V^*)},
\end{aligned}$$

and

$$\begin{aligned}
TUT &= \varphi + E[U_1 - U_0 \mid V < -Z\gamma] \\
&= \varphi + E\left[\epsilon(\sigma_1 - \sigma_0) \mid \epsilon < \frac{-Z\gamma}{\sigma_V^*}\right] \\
&= \varphi + (\sigma_1 - \sigma_0)E\left[\epsilon \mid \epsilon < \frac{-Z\gamma}{\sigma_V^*}\right] \\
&= \varphi - (\sigma_1 - \sigma_0) \frac{\phi(-Z\gamma/\sigma_V^*)}{\Phi(-Z\gamma/\sigma_V^*)} \\
&= \varphi + (\sigma_0 - \sigma_1) \frac{\phi(-Z\gamma/\sigma_V^*)}{\Phi(-Z\gamma/\sigma_V^*)}.
\end{aligned}$$

3. Parametrization 1

Now, to add more structure to the problem, suppose that $Z = (1, Z_1, Z_2)$, and $\gamma = (\gamma_0, \gamma_1, \gamma_2)$. Also, suppose that

$$\begin{array}{lll}
\gamma_0 = 0.2, & \gamma_1 = 0.3, & \gamma_2 = 0.1, \\
\sigma_1 = 0.012, & \sigma_0 = 0.05, & \sigma_V^* = 1, \\
\alpha = 0.02, & \varphi = 0.2. &
\end{array}$$

Finally,

$$\begin{aligned}
Z_1 &\sim \mathcal{N}(-1, 9), \\
Z_2 &\sim \mathcal{N}(1, 9),
\end{aligned}$$

where $Z_1 \perp Z_2$. Since we only observe either Y_1 or Y_0 , the econometrician observes the outcome described as follows:

$$Y = DY_1 + (1 - D)Y_0.$$

```

set.seed(1234)

N <- 5000 # random sample observations

alpha <- 0.02
phi <- 0.2

g <- matrix(c(0.2, 0.3, 0.1))
sigma1 <- 0.012
sigma0 <- 0.05
sigmaV <- 1

eps <- rnorm(N)

U1 <- sigma1 * eps
U0 <- sigma0 * eps
V <- sigmaV * eps

Z <- cbind(rep(1, N),
            rnorm(N, -1, 9),
            rnorm(N, 1, 9))

I <- (Z %*% g) + V
D <- ifelse(I >= 0, 1, 0)

Y1 <- alpha + phi + U1
Y0 <- alpha + U0

Y <- (D * Y1) + ((1 - D) * Y0)

df <- data.frame('beta' = Y1 - Y0,
                 'decision' = D,
                 'y1' = Y1,
                 'y0' = Y0,
                 'net utility' = I)

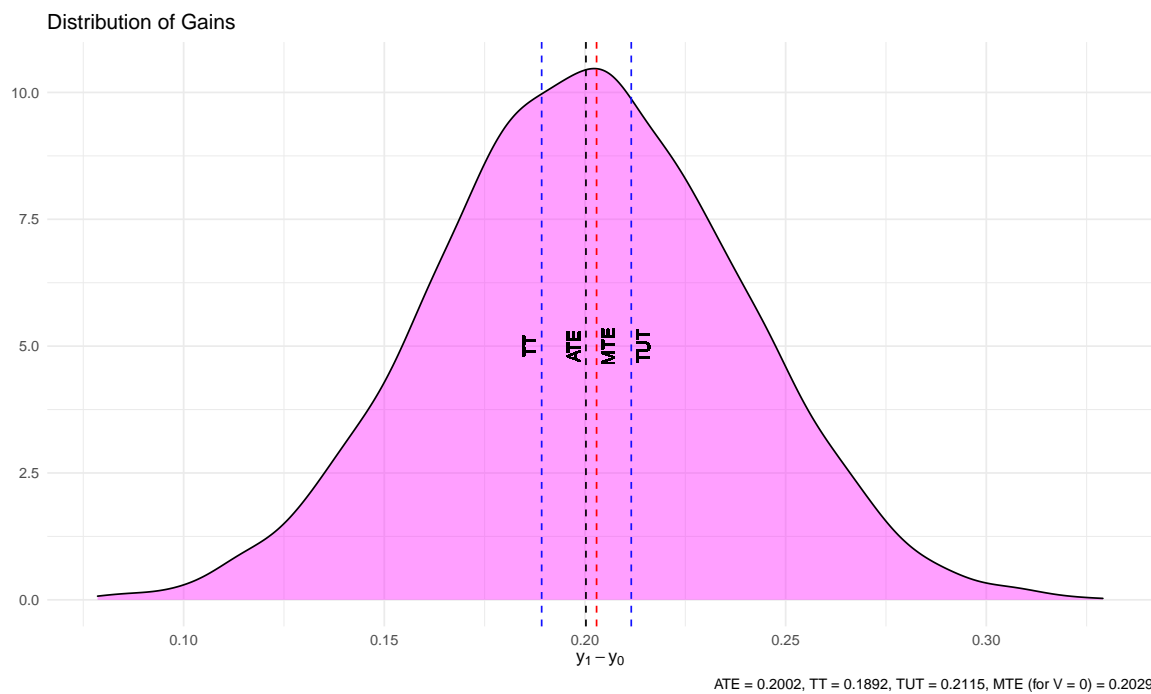
ATE <- df %>%
  pull(beta) %>%
  mean()

TT <- df %>%
  filter(decision == 1) %>%
  pull(beta) %>%
  mean()

TUT <- df %>%
  filter(decision == 0) %>%
  pull(beta) %>%
  mean()

MTE <- df %>%
  filter(abs(net.utility - V) <= 0.01) %>%
  pull(beta) %>%
  mean()

```



4. Parametrization 2
5. GMM
6. Likelihood Function
7. Discussion
8. Maximum Likelihood