Applied Macroeconometrics
Problem Set 3

Due on Monday July 10, 23:59pm

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1. Value Function Iteration - 100 points

1. Consider the model discussed in class, that is, the recursive problem of the representative individual, whose sole choice variable is how much to consume (and save for the future), and whose budget constraint could be written as $c + k' = F(k)$, where $F(k) = f(k) + (1 - \delta)k$.

   (a) (20 pts) Solve this problem using the (raw) value function iteration (VI.M and VI_EX0.M) with parameters $\beta = 0.95$, $\delta = 1$, $\alpha = 0.3$. Assume also that utility is logarithmic and consider the values for the capital grid in the code.

   (b) (20 pts) Find sequences of consumption ($\{c_t^*\}_{t=0}^T$) and savings ($\{k^{*\prime}\}_{t=0}^T$) for $T = 10000$ that are consistent with the solution of the Bellman equation. To which value do those sequences appear to converge? **Hint:** You may want to use the script PW_LINEAR.M.

   (c) (20 pts) Take $k_0 = 1.0120$ and verify that $v(k_0) \approx \sum_{t=0}^T \beta^t u(c_t^*)$.

   (d) (40 pts) Modify the problem discussed in class to include labor choice. The representative individual decides how much to consume but also how much labor to offer in the market. Consider the new recursive problem

   $$v(k) = \max_{k'}\{u(c, l) + \beta v(k')\}$$

   where l denotes time allocated to leisure activities giving pleasure. The budget constraint now reads

   $$c + k' = F(k, N)$$

   where $F(k, N) = k^\alpha N^{1-\alpha} + (1 - \delta)k$. There is another constraint (time-budget constraint) establishing that time allocated to work (N) and leisure activities (l) must be exhausted in a given period, say, 24 hours in a day. We normalize the total time available to 1, thus

   $$l + N = 1$$

Assume that $u(c, l) = \theta \log(c) + (1 - \theta) \log(l)$, where $\theta \in (0, 1)$. In a nutshell, in this problem, the only state is still capital in the current period (k). The set of choice variables is now c, k', and l (or N). Furthermore, notice that the labor choice is an intra-temporal decision, unlike the consumption choice which is an inter-temporal decision (involving two periods).

Assume the following parameters: $\beta = 0.95$, $\alpha = 0.3$, $\delta = 1$, and $\theta = 0.6$. Use the value function iteration method to solve the Bellman equation.