Applied Macroeconometrics
Final

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This exam (100 points) should be handed in individually, not in groups. Please make sure your codes run. This exam has four problems; you must choose **2 only**. The solution to the first problem should be handed in as F_1.M, the solution to the second problem as F_2.M, and so on.

1. Value Function with Endogeneous Labor Choice - 50 points

Modify the problem discussed in class to include labor choice. The representative individual decides how much to consume but also how much labor to offer in the market. Consider the new problem:

$$v(k) = \max_{c,l,k'} \{u(c,l) + \beta v(k')\},$$

where l denotes time allocated to leisure activities giving pleasure. The budget constraint now reads

$$c + k' = F(k,n),$$

where $F(k,n) = k^{\alpha} n^{1-\alpha} + (1-\delta)k$. There is another constraint (time-budget constraint) establishing that time allocated to work (n) and leisure activities (l) must be exhausted in a given period, say, 24 hours in a day. We normalize the total time available to 1, thus $l + n = 1$.

Assume that $u(c,l) = \theta \log(c) + (1-\theta)\log(l)$, where $\theta \in (0,1)$. In a nutshell, in this problem, the only state is still capital in the current period (k). The choice variables are now c, k', and l (or n). Furthermore, notice that the labor choice is an intra-temporal decision, unlike the consumption choice, which is an inter-temporal decision (involving two periods). Assume the following parameters: $\beta = 0.95$, $\alpha = 0.3$, $\underline{\delta = 0.05}$, and $\theta = 0.6$.

1a. Use the value function iteration method to solve the Bellman equation, i.e., in a single graph of 4 panels, report policy functions for k', c, and n, and the value function $v(k)$ on a grid for capital $[0.01, \bar{k}]$ of size $N_k = 1000$.

1b. Find $\bar{k}$, the minimum value of k such that $n = 1$. Recall $n \in (0,1)$. **Hint:** Find $\bar{k}$ from the Euler equation evaluated at the steady-state and use the codes shared in class.

1c. How does the policy function for n depend on k in this setup, and why? Explain.

2. NGM with investment constraints - 50 points

Consider the neoclassical growth model discussed in class. Write a script that performs the value function iteration method to solve the following dynamic programming problem:

$$v(k) = \max_{c,k'} \{\log(c) + \beta v(k')\}$$

subject to

$$c + k' = F(k) \equiv k^{\alpha} + (1-\delta)k$$

and the non-negativity constraint for investment

$$i \geq 0,$$

where, by definition, $i = k' - k + \delta k \equiv k' - (1-\delta)k$. Assume the following parameters: $\beta = 0.95$, $\alpha = 0.3$, and $\delta = 0.05$.

2a. In a single graph of 4 panels, report the policy functions for k', c, i, and the value function $v(k)$ on a grid for capital $[0.01, 20]$ of size $N_k = 1000$. **Hint:** Use a variant of the script VI_IMPROVED_0.M.

2b. Compare your results, using the same single graph, to the case with no constraints for investment. Explain differences regarding the policy function for k'. Notice the differences at high values of k on the grid.

2c. Explain your results regarding the policy function for c. Are there any differences between the NGM with and without constraints? Explain.

3. NGM with productivity shocks - 50 points

In this problem, you will implement the value function iteration for the stochastic neoclassical growth model. The Bellman equation now reads

$$v(k,z) = \max_{c,k'}\{u(c) + \beta \mathrm{E}_{z'}\left(v(k',z')|z\right)\} \tag{1}$$

subject to

$$\begin{aligned} c + k' &= F(k,z) \equiv e^z k^\alpha + (1-\delta)k & (2)\\ z' &= \rho z + \varepsilon', \quad \varepsilon' \sim (0,\sigma_\varepsilon^2), \quad 0<\rho<1 & (3)\end{aligned}$$

where $\mathrm{E}_{z'}(\cdot|z)$ is the expectation of the argument "$\cdot$" conditional on the realization of z, which is the productivity shock. Write a script that implements the value function iteration method for the stochastic growth model. Construct a grid for capital that starts at 0.01 and ends at 5, of size $N_k = 1000$. Also, consider the following values for the parameters: $\beta = 0.95$, $\alpha = 0.3$, $\underline{\delta = 0.05}$. Finally, assume the utility function is logarithmic.

3a. Use the Rouwenhorst method to discretize the process in (3) for $N_z = 5$ states, $\rho = 0.95$ and $\sigma_\varepsilon^2 = (0.01)^2$.

3b. In a single graph of 3 panels, report the policy functions for k', c, and the value function $v(k)$ for every value of the grid for capital and the productivity process.

3c. Use the policy functions reported in (3a) to simulate artificial data on $\{z_t\}_{t=0}^{19}$, $\{k_t\}_{t=0}^{19}$, and $\{c_t\}_{t=0}^{19}$ in a horizon of 20 periods. In a single graph of 3 panels, report the time series simulated for these variables. Explain your results. **Hint:** Take z as exogenous, which is the case, and see how c and k respond to changes in z. **Another hint:** Use the script MCSIMUL.M, shared in class, and recall that the first output is the simulated series, and the second output is the index that the simulated series takes on the grid of the discretized process.

4. Incomplete Markets the Mexican Way - 50 points

In this problem, you will implement the script that solves the incomplete market economy with Mexican data. You will use the script discussed in class, modify it, and use the data from PS1 to feed the stochastic process of the endowment.

4a. In the accompanying file F_4.XLS, identify the transition probabilities corresponding to 2007.Q4, just before the global financial crisis. Use those transition probabilities to construct a Markov transition matrix and find the ergodic vector of probabilities. What is the interpretation of this vector for the Mexican labor market? **Hint:** Construct the matrix so that employment goes first and inactivity third.

4b. We have the stochastic kernel (the Markov transition matrix). We also need the state space of the process to characterize it fully. Assume that this space is given by $(1.0; 0.5; 0.1)'$, where the first input corresponds to the income of the employed, the second to the unemployment insurance (half of the employee's income), and the third is the reservation wage of the inactive. Compute the policy functions for $a'(a, e)$, $c(a, e)$, the value function $v(a, e)$ for a price $q = 1.0338$. Use a grid for assets $[-2, 6]$ of size $N_k = 500$ and the following parameters: $\beta = 0.96$ and $\sigma = 2$ [SIG in the code] (the utility function is not longer logarithmic). Use the value function iteration method that <u>interpolates linearly</u> over the value function (VI_INTERPOL_1_RA.M).

4c. In a single graph of 4 panels, report four subplots containing the policy function for $a'(a, e)$, $c(a, e)$, the value function $v(a, e)$, and the stationary distribution $F(a, e)$. To iterate over the latter, use the same grid but of size $N_F = 1000$, as in the code discussed in class.