Applied Macroeconometrics
Problem Set 1

Due on Monday 26 23:59pm

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1. Newton Solver (Kelley, 1995) - 40 points

Write a Matlab script that solves single nonlinear equations with Newton's method and the *chord* method (see below). Apply your program to the following functions/initial iterate combinations, plot iteration statistics (i.e., $\frac{\|F(x^k)\|_\infty}{\|F(x^0)\|_\infty}$, starting at iteration 0, that is, the initial condition), and explain your results:

1. $f(x) = \cos(x) - x$, $x_0 = 0.5$
2. $f(x) = \tan^{-1}(x)$, $x_0 = 1$
3. $f(x) = \sin(x)$, $x_0 = 3$
4. $f(x) = x^2$, $x_0 = 0.5$
5. $f(x) = x^2 + 1$, $x_0 = 10$

Hint: The chord method is like Newton's method except for the step in which $\nabla F(\cdot)$ is evaluated at $x = x^0$, instead of $x = x^k$, at the k-th iteration. More precisely, the chord method is defined as follows:

Algorithm Chord Method

Objective: Find x such that $F(x) = 0$

1. Set an initial value x^0 and $\nabla F(x^0)$
2. For iteration $k = 0, 1, 2 \ldots$ until convergence do:
   Find s^k which satisfies:
$$\nabla F(x^0) s^k = -F(x^k) \qquad (\textit{Newton equation})$$
3. Set $x^{k+1} = x^k + s^k$

2. Implementation of the Jacobi Method - 20 points

Write a Matlab script that performs the Jacobi method. Apply your program to the script LINEARSOLVE_EX, compare it to the Gauss-Seidel method (discussed in class), plot iteration statistics (the one that appear by default in the script LINEARSOLVE_EX), and explain your results.

3. Linear System of Equations and the Mexican Labor Market (Leyva and Urrutia, 2020) - 40 points

Consider the following characterization (in the steady-state) of the Mexican labor market:

$$
\begin{aligned}
(f^{UE} + f^{UO})U &= Ef^{EU} + Of^{OU} \\
(f^{EU} + f^{EO})E &= Uf^{UE} + Of^{OE} \\
E + U + O &= k
\end{aligned}
$$

where E, U, and O denote the number of employed, unemployed, and people out of the labor force. Moreover, f^{AB} denotes the labor market gross flow from state A to B, and k is the size (fixed) of the working-age population. Gross flows calculated by Leyva and Urrutia (2020) for Mexico are available in the companion dataset LM_GROSSFLOWS.XLS. The problem could be expressed as a linear system of equations,

$$
\begin{pmatrix} -b & a & -c \\ d & -e & -f \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ k \end{pmatrix}
$$

where $(x_1, x_2, x_3)^T \equiv (E, U, O)^T$ and $k > 0$.

1. Find expressions for a, b, c, d, e, and f in terms of f^{AB}, where $A, B \in \{E, U, O\}$.

2. Solve for the linear system of equations in each quarter of data (t=2005.Q2:2016.Q4) and find $(x_1^*, x_2^*, x_3^*)^T$ for each quarter t. Provide an economic interpretation of the solution. **Hint:** Assume $k = 100$.

3. Use the solution to calculate the (time series of the) unemployment rate $\textbf{urate}_t$, i.e., unemployed workers divided by the labor force.

4. Repeat 2 by fixing all gross flows to their average period value except for f^{UE}.

5. Repeat 3 by fixing all gross flows to their average period value except for f^{UE}. Call this rate the counterfactual unemployment rate $\textbf{urate}_t^{cf}$; the rate one would observe had f^{UE} been the only gross flow allowed to change over time.

6. **Bonus (4 points):** Perform the following regression:

$$\text{urate}_t^{cf} = \beta_0 + \beta_1 \text{urate}_t + \beta_2 \tau + \varepsilon_t, \quad \text{with} \quad \varepsilon_t \sim (0, \sigma_\varepsilon^2)$$

where τ denotes a linear trend. Estimate β_1 by Ordinary Least Squares. Interpret the value of $\beta_1 \times 100$. **Hint:** Recall the definition of the OLS estimator of β_1.

References

Kelley, C. T. (1995): *Iterative Methods for Linear and Nonlinear Equations*, Society for Industrial and Applied Mathematics.

Leyva, G. and C. Urrutia (2020): "Informality, labor regulation, and the business cycle," *Journal of International Economics*, 126, 103340.