Applied Macroeconometrics
Midterm

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1. Please, make sure your codes run.
2. This exam has 120 points. The extra points earned in this midterm (20 in total) will add up to your score in the final exam for a maximum of 100 points.

1. Linear and Non-Linear System of Equations Combo - 60 points

Consider the following function $h : \mathcal{R}^2 \to \mathcal{R}$

$$h(x_1, x_2) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

In this problem, you will see the equivalence between minimizing a function and finding the zeros of a function. Suppose we want to minimize function $h(x_1, x_2)$. The first-order conditions will be

$$f(x_1, x_2) = \begin{pmatrix} -400x_1(x_2 - x_1^2) - 2(1 - x_1) \\ 200(x_2 - x_1^2) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

where function $f : \mathcal{R}^2 \to \mathcal{R}^2$. In this problem, you will apply the Newton and chord methods, discussed in class, to find the roots of function f, that is, to minimize function h.

1. (20 pts) Write a Matlab script FUNC0.M that takes as an input a vector $\mathbf{x} \in \mathcal{R}^2$ and delivers as outputs the function f (a vector **fx** of 2×1) evaluated at x and the derivative of f (a matrix **dfx** of 2×2) with respect to x_1 and x_2. You can calculate **fx** and **dfx** by hand or using the Symbolic toolbox of Matlab. **Hint:** Use the accompanying script FUNC0.M and modify it so we can use it. You are supposed to fill in the dots (...) in the script.
2. (20 pts) Implement both the Newton and chord methods in scripts that will be called NEWTON.M and CHORD.M. Write those scripts so that they deliver three outputs, namely, the number of iterations, the vector-solutions, and $\frac{\|F(x^k)\|_\infty}{\|F(x^0)\|_\infty}$, starting at iteration 0, that is, the initial condition. Slightly modify those scripts so we can use the linear system of equations (thus, the combo). Recall that the Newton iteration equation is

$$\nabla F(x^k)s^k = -F(x^k)$$

Let $A = \nabla F(x^k)$ and $b = F(x^k)$ and solve $As^k = b$. Use the Gauss-Jordan direct method to solve for s^k in each Newton iteration. As you know, in the chord method, $A = \nabla F(x^0)$ for every iteration.

3. (20 pts) Write a Matlab script that calls NEWTON.M and CHORD.M and report iteration statistics in a single plot. Use a tolerance of 10^{-5} (and CRIT=100) and use as initial values $x^0 = (1.2; 1.2)^T$ and $x^0 = (-1.2; 1)^T$ (a single plot for each initial value, that is, two plots). Explain your results.

1. Chronic Laziness - 60 points

Consider the following AR(1) process

$$z_{t+1} = \rho z_t + u_{t+1}, \quad u_{t+1} \sim \mathcal{N}(0, \sigma_u^2)$$

that is supposed to characterize a productivity shock in Mexico well.

1. (15 pts) Discretize this process using the Rouwenhorst method, assuming $\rho = 0.9$, $\sigma_u^2 = 0.06$, and $N = 50$ as the size of the grid.

2. (15 pts) Use the state-space and the stochastic kernel to simulate the Markov process for a sample size of $n = 250$.

3. (15 pts) Suppose recessions are periods of chronic laziness. Let l be a leisure shock that makes you value leisure more. Specifically, let

$$l_t = -0.5z_t + \varepsilon_t \quad \varepsilon_t \sim \mathcal{N}(\mu, \sigma^2)$$

with $\mu = 0.01$ and $\sigma^2 = 0.25$.

4. (15 pts) Calculate the Pearson correlation coefficient ($\hat{\theta}$) between z and l (using the command CORRCOEF) and save it. Use the stationary bootstrap with 2000 replications to calculate an Efron confidence interval with $\alpha = 0.1$ around $\hat{\theta}$. Implement the stationary bootstrap so that the mean of block is 12. Report the Efron interval.