Applied Macroeconometrics
Problem Set 4

Due on Saturday July 22, 23:59pm

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1. Interpolation and linear equations (again) - 30 points

Consider the value function iteration method that refines the solution of the Bellman equation using a quadratic spline interpolation, like the variant discussed in class (VI_INTERPOL_2.M). You will modify the script GOLDEN2.M, line 30.

1. (20 pts) Instead of using the function COEFS(x, fx), you will create your own function (call it SOLE_SPLINE.M) that takes the same inputs but that implements a system of linear equations solver (you are free to use any of the methods discussed in class but be aware of the conditions that justify their use). **Hint:** You may also want to change the function POLYEVAL.M to suit your case.

2. (10 pts) Report the policy functions for k', c, and the value function $v(k)$ for every value of the grid for capital in a single graph (use subplots). **Hint:** Use the capital grid and the values for the parameters that appear as default in the script VI_EX1.M, discussed in class.

2. The Stochastic NGM - 40 points

In this problem, you will implement the value function iteration for the stochastic neoclassical growth model. The Bellman equation now reads

$$v(k,z) = \max_{c,k'} \{u(c) + \beta \mathrm{E}_{z'} \left(v(k',z') | z \right)\} \tag{1}$$

subject to

$$c + k' = F(k,z) \equiv e^z k^\alpha + (1-\delta)k \tag{2}$$

$$z' = \rho z + \varepsilon', \quad \varepsilon' \sim (0, \sigma_\varepsilon^2), \quad 0 < \rho < 1 \tag{3}$$

where $\mathrm{E}_{z'}(\cdot|z)$ is the expectation of the argument "$\cdot$" conditional on the realization of z, which is the productivity shock.

1. (30 pts) Write a script that implements the value function iteration method for the stochastic growth model. Use the Rouwenhorst method to discretize the process in (3) for $N_z = 3$ states, $\rho = 0.95$ and $\sigma_\varepsilon^2 = (0.01)^2$. Construct a grid for capital that starts at 0.01 and ends at 5, of size $N_k = 250$. Also, consider the following values for the parameters: $\beta = 0.95$, $\alpha = 0.3$, $\delta = 1$. Finally, assume the utility function is logarithmic.

2. (10 pts) Report in a single graph the policy functions for k', c, and the value function $v(k)$ for every value of the grid for capital and the productivity process.

3. Parametric Approximation - 30 points

Implement the parametric approximation of the policy function using a quadratic instead of a linear function (like in class). **Hint:** Use the grid for capital and values for the parameters that appear as default in the script ROFT_0.M, discussed in class.

1. (15 pts) Report the policy functions for k', c, and the value function $v(k)$ in a single graph.

2. (15 pts) Compare your results with those obtained under the linear approximation (seen in class), the value function iteration (improvement due to Judd, 1998), and the analytical solution in a single graph.