Aiyagari (1994). Uninsured Idiosyncratic Risk and Aggregate Saving, QJE 109(3)

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Introduction

Heterogeneous agent models in continuous time

This paper is part of a literature of *heterogeneous agent models in continuous time*, that studies general equilibrium with incomplete markets and uninsured idiosyncratic labour income risk.

Research question

Two main goals

- Provide an exposition of models whose aggregate behavior is the result of market interaction among a large number of agents subject to idiosyncratic shocks.
- Use such a model to study the quantitative importance of individual risk for aggregate saving.

Main takeaways

The two results arise:

- *Aggregate saving:* Differences between the saving rates with and without insurance are quite small for moderate and empirically plausible values of σ, ρ, μ.
- *Variability:* Consumption varies about 50-70 percent as much as income. Saving and assets are much more volatile than income. Saving varies about three times as much as income, and assets vary about twice as much as income.

Main contributions

- This model, jointly with Bewley (1986) and Huggett (1993), is considered the workhorse of income and wealth distribution in macroeconomics.
- A large strand of literature is built upon these.

The environment

The environment

There is a continuum of agents each maximizing an infinite flow of discounted utility

$$E_0 \left[\sum_{t=0}^{\inf} \beta^t U(c_t) \right] \tag{1}$$

subject to $c_t + a_{t+1} = wl_t + (1+r)a_t$;

where b is the limit on borrowing and l_t captures idiosyncratic risk in labor earnings and could be interpreted as unemployment risk.

We also make the assumptions: $c_t \geq 0, a_t \geq -b_t$

Borrowing limit

A borrowing constraint is necessarily implied by nonnegative consumption. The borrowing limit can be specified as:

$$a_t \geq -\phi, \text{ where}$$

$$\phi \equiv min[b, wl_{min}/r], \text{ for } r > 0; \ \phi \equiv b, \text{ for } r \leq 0$$

we define $\hat{a}_t$ and z_t as follows:

$$\hat{a}_t = a_t + \phi,$$

$$z_t = wl_t + (1 + r)\hat{a}_t - r\phi$$

where z_t can be thought as the total resources of the agent.

Thus, the we can rewrite the constraints as:

$$\begin{aligned} c_t + \hat{a}_{t+1} &= z_t, c_t \geq 0, \hat{a}_t \geq 0, \\ z_{t+1} &= wl_{t+1} + (1+r)\hat{a}_{t+1} - r\phi \end{aligned}$$

Bellman equation

Let $V(z_t, b, w, r)$ be the optimal value function for the agent with total resources z_t.

This is the unique solution to:

$$V(z_t, b, w, r) \equiv max \left[U(z_t - \hat{a}_{t+1}) + \beta \int V((z_{t+1}, b, w, r) dF(l_{t+1}) \right]$$

where the maximization is over $\hat{a}_{t+1}$ subject to the (re-written) constraints.

Solution

The *optimal asset demand rule* for an agent is obtained by solving the the previous maximization problem.

$$\hat{a}_{t+1} = A(z_t, b, w, r)$$

we substitute into our re-writen constraints to obtain:

$$z_{t+1} = wl_{t+1} + (1 + r)A(z_t, b, w, r) - r\phi$$

Calibration

Specification and parameterization consistent with postwar US economy.

- Period 1 year, $\beta = 0.96$
- Production function $f(\cdot)$ is Cobb-Douglas, with $\alpha = 0.36$,
- Depreciation rate $\delta = 0.08$
- Utility function $U(c) = [c^{1-\mu} - 1]/(1-\mu)$, where $\mu \in [1, 3, 5]$
- Labor endowment shocks: Markov chain with seven states to math AR for the log of labor endowment shock:

$log(l_t) = \rho log(l_{t_1}) + \sigma(1-\rho^2)^{1/2}\epsilon$, $\epsilon_t \sim N(0,1)$

$\sigma \in [0.2, 0.4]$, $\rho \in [0, .3, .6, .9]$

- σ and ρ are based on Kydland (1984)
- $b = 0$, i.e. no borrowing allowed

Algorithm for approximating the SS

- Start with some value r close to but less than the rate of time preference.
- Then compute the asset demand function of total resources by a continuous piece-wise linear function over an interval.
- Generate 10,000 sumulations of Markov chain for the labor endowment shock.
- Obtain simulated series of assets. Take sample mean E_a
- Then calculate r_2, such that $K(r_2)$ equals E_a.
- If r_2 exceeds the rate of time preference, it is replace by the rate of time preference.
- The define $r_3 = (r_1 + r_2)/2$ and calculate E_a corresponding to r_3.

Results

results 1

results 2

Conclusion

This class of models can also differ from the infinite-lived agent complete markets model on some important policy issues. e.g. - Chamley (1986) with complete markets, dynamic optimal factor taxation leads to the result that the capital income tax should be zero in the long run, *vs* Aiyagari (1994) shows that with idiosyncratic shocks and incomplete markets the capital income tax is strictly positive even in the long run.