Aiyagari (1994). Uninsured Idiosyncratic Risk and Aggregate Saving, QJE 109(3)

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Introduction

Heterogeneous agent models in continuous time

This paper is part of a literature of *heterogeneous agent models in continuous time*, that studies general equilibrium with incomplete markets and uninsured idiosyncratic labour income risk.

Research question

Two main goals

- Provide an exposition of models whose aggregate behavior is the result of market interaction among a large number of agents subject to idiosyncratic shocks.
- Use such a model to study the quantitative importance of individual risk for aggregate saving.

Main takeaways

The two results arise:

- *Aggregate saving:* Differences between the saving rates with and without insurance are quite small for moderate and empirically plausible values of σ, ρ, μ.
- *Variability:* Consumption varies about 50-70 percent as much as income. Saving and assets are much more volatile than income. Saving varies about three times as much as income, and assets vary about twice as much as income.

Main contributions

- This model, jointly with Bewley (1986) and Huggett (1993), is considered the workhorse of income and wealth distribution in macroeconomics.
- A large strand of literature is built upon these.

The environment

The environment

Based on Brock-Mirman [1972] growth model with a large number of agents.

$$E_0 \left[\sum_{t=0}^{\inf} \beta^t U(c_t) \right] \tag{1}$$

subject to $c_t + a_{t+1} = wl_t + (1+r)a_t$; $c_t \geq 0, a_t \geq -b_t$

where b is the limit on borrowing and l_t is assumed to be *i.i.d.* with bounded support given by $[l_{min}, l_{max}]$, with $l_{min} > 0$.

Model

The borrowing limit can be specified as $a_t \geq -\phi$ where $\phi \equiv min[b, wl_{min}/r]$, for $r > 0$; $\phi \equiv b$, for $r \leq 0$

we define $\hat{a}_t$ and z_t as follows: $\hat{a}_t = a_t + \phi$,
$z_t = wl_t + (1 + r)\hat{a}_t - r\phi$ where z_t may be thought as of the total resources of the agent at date t.

Thus, the we rewrite the constraints as:

- $c_t + \hat{a}_{t+1} = z_t$, $c_t \geq 0$, $\hat{a}_t \geq 0$,
- $z_{t+1} = wl_{t+1} + (1 + r)\hat{a}_{t+1} - r\phi$

Bellman equation

Let $V(z_t, b, w, r)$ be the optimal value function for the agent with total resources z_t. This is the unique solution to:

$$V(z_t, b, w, r) \equiv max \left[U(z_t - \hat{a}_{t+1}) + \beta \int V((z_{t+1}, b, w, r) dF(l_{t+1}) \right]$$

where the maximization is over $\hat{a}_{t+1}$ subject to the (rewriten) constraints.

Calibration

Specification and parameterization consistent with postwar US economy.

- Period 1 year, $\beta = 0.96$
- Production function $f(\cdot)$ is Cobb-Douglas, with $\alpha = 0.36$,
- Depreciation rate $\delta = 0.08$
- Utility function $U(c) = [c^{1-\mu} - 1]/(1-\mu)$, where $\mu \in [1, 3, 5]$
- Labor endowment shocks: Markov chain with seven states to math AR for the log of labor endowment shock:

$log(l_t) = \rho log(l_{t_1}) + \sigma(1-\rho^2)^{1/2}\epsilon$, $\epsilon_t \sim N(0, 1)$

$\sigma \in [0.2, 0.4]$, $\rho \in [0, .3, .6, .9]$

- σ and ρ are based on Kydland (1984)
- $b = 0$, i.e. no borrowing allowed

Algorithm for approximating the SS

- Start with some value r close to but less than the rate of time preference.
- Then compute the asset demand function of total resources by a continuous piece-wise linear function over an interval.
- Generate 10,000 sumulations of Markov chain for the labor endowment shock.
- Obtain simulated series of assets. Take sample mean E_a
- Then calculate r_2, such that $K(r_2)$ equals E_a.
- If r_2 exceeds the rate of time preference, it is replace by the rate of time preference.
- The define $r_3 = (r_1 + r_2)/2$ and calculate E_a corresponding to r_3.

Results

results 1

results 2

Conclusion

This class of models can also differ from the infinite-lived agent complete markets model on some important policy issues. e.g. - Chamley (1986) with complete markets, dynamic optimal factor taxation leads to the result that the capital income tax should be zero in the long run, *vs* Aiyagari (1994) shows that with idiosyncratic shocks and incomplete markets the capital income tax is strictly positive even in the long run.