

# **Aiyagari (1994). Uninsured Idiosyncratic Risk and Aggregate Saving, QJE 109(3)**

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# Introduction

# Heterogeneous Agent Models in Continuous Time

This paper is part of a literature of *heterogeneous agent models in continuous time*, that studies general equilibrium with incomplete markets and uninsured idiosyncratic labour income risk.

market incompleteness + borrowing constrained in future periods,  
→ agents accumulate excess capital, to smooth consumption in the face of uncertain individual labor incomes.

# Research Question

Two main goals

- Provide an exposition of models whose aggregate behavior is the result of market interaction among a large number of agents subject to idiosyncratic shocks.
- Use such a model to study the quantitative importance of individual risk for aggregate saving.

# Main Takeaways

The two results arise:

- **Aggregate saving:** Differences between the saving rates with and without insurance are quite small for moderate and empirically plausible values of  $\sigma, \rho, \mu$ .
- **Variability:** Consumption varies about 50-70 percent as much as income. Saving and assets are much more volatile than income. Saving varies about three times as much as income, and assets vary about twice as much as income.

# Main Contributions

- This model, jointly with Huggett (1993, 1997), is considered the workhorse of income and wealth distribution in macroeconomics.
- A large strand of literature is built upon these.

# **The Environment**



# The Environment

There is a continuum of agents each maximizing an infinite flow of discounted utility

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t U(c_t) \right] \quad (1)$$

subject to  $c_t + a_{t+1} = w/l_t + (1 + r)a_t$ ;

where  $b$  is the limit on borrowing and  $l_t$  captures idiosyncratic risk in labor earnings and could be interpreted as unemployment risk.

We also make the assumptions:  $c_t \geq 0, a_t \geq -b_t$

## Borrowing Limit

A borrowing constraint is necessarily implied by nonnegative consumption. The borrowing limit can be specified as:

$$a_t \geq -\phi, \text{ where}$$

$$\phi \equiv \min[b, wl_{\min}/r], \text{ for } r > 0; \phi \equiv b, \text{ for } r \leq 0$$

we define  $\hat{a}_t$  and  $z_t$  as follows:

$$\hat{a}_t = a_t + \phi,$$

$$z_t = wl_t + (1 + r)\hat{a}_t - r\phi$$

where  $z_t$  can be thought as the total resources of the agent.

Thus, the we can rewrite the constraints as:

$$\begin{aligned}c_t + \hat{a}_{t+1} &= z_t, c_t \geq 0, \hat{a}_t \geq 0, \\ z_{t+1} &= wl_{t+1} + (1 + r)\hat{a}_{t+1} - r\phi\end{aligned}$$

# Bellman Equation

Let  $V(z_t, b, w, r)$  be the optimal value function for the agent with total resources  $z_t$ .

This is the unique solution to:

$$V(z_t, b, w, r) \equiv \max \left[ U(z_t - \hat{a}_{t+1}) + \beta \int V((z_{t+1}, b, w, r) dF(l_{t+1}) \right]$$

where the maximization is over  $\hat{a}_{t+1}$  subject to the (re-written) constraints.

## Solution

The *optimal asset demand rule* for an agent is obtained by solving the previous maximization problem.

$$\hat{a}_{t+1} = A(z_t, b, w, r)$$

we substitute into our re-written constraints to obtain:

$$z_{t+1} = w l_{t+1} + (1 + r)A(z_t, b, w, r) - r\phi$$

# Calibration

Specification and parameterization consistent with postwar US economy.

- Period 1 year,  $\beta = 0.96$
- Production function  $f(\cdot)$  is Cobb-Douglas, with  $\alpha = 0.36$ ,
- Depreciation rate  $\delta = 0.08$
- CRRA Utility function  $U(c) = [c^{1-\mu} - 1]/(1 - \mu)$ , where  $\mu \in [1, 3, 5]$
- Labor endowment shocks: Markov chain with seven states to match AR for the log of labor endowment shock:

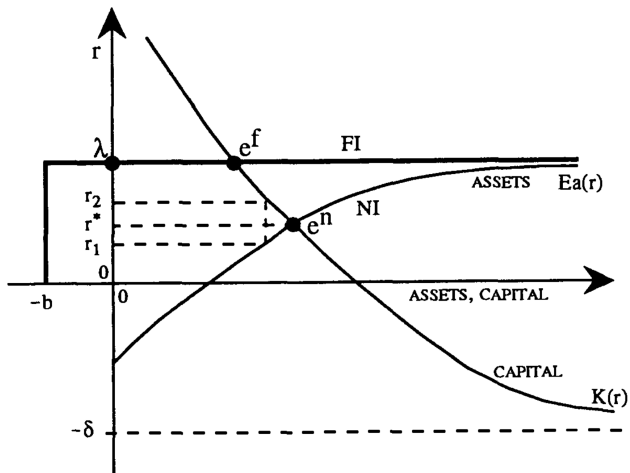
$$\log(l_t) = \rho \log(l_{t-1}) + \sigma(1 - \rho^2)^{1/2} \epsilon_t, \epsilon_t \sim N(0, 1)$$

$$\sigma \in [.2, .4], \rho \in [0, .3, .6, .9]$$

- $\sigma$  and  $\rho$  are based on Kydland (1984)
- $b = 0$ , i.e. no borrowing allowed

# Algorithm for Approximating the SS

- Start with some value  $r$  close to but less than the rate of time preference.
- Then compute the asset demand function of total resources by a continuous piece-wise linear function over an interval.
- Generate 10,000 simulations of Markov chain for the labor endowment shock.
- Obtain simulated series of assets. Take sample mean  $E_a$
- Then calculate  $r_2$ , such that  $K(r_2)$  equals  $E_a$ .
- If  $r_2$  exceeds the rate of time preference, it is replaced by the rate of time preference.
- Then define  $r_3 = (r_1 + r_2)/2$  and calculate  $E_a$  corresponding to  $r_3$ .





## Results

# Aggregate Saving

## A. Net return to capital in %/aggregate saving rate in % ( $\sigma = 0.2$ )

$\rho \backslash \mu$	1	3	5
0	4.1666/23.67	4.1456/23.71	4.0858/23.83
0.3	4.1365/23.73	4.0432/23.91	3.9054/24.19
0.6	4.0912/23.82	3.8767/24.25	3.5857/24.86
0.9	3.9305/24.14	3.2903/25.51	2.5260/27.36

## B. Net return to capital in %/aggregate saving rate in % ( $\sigma = 0.4$ )

$\rho \backslash \mu$	1	3	5
0	4.0649/23.87	3.7816/24.44	3.4177/25.22
0.3	3.9554/24.09	3.4188/25.22	2.8032/26.66
0.6	3.7567/24.50	2.7835/26.71	1.8070/29.37
0.9	3.3054/25.47	1.2894/31.00	-0.3456/37.63

## **Variabilities**

Consumption variability rises with persistence in earnings and falls with risk aversion. Variability of consumption relative to income behaves similarly.

## **Importance of Asset Trading**

Consumers are able to accomplish a significant amount of consumption smoothing by accumulating and decumulating assets and, hence, enjoy significant welfare benefits from participating in asset markets.

## **Cross-Section Distributions and Inequality Measures**

The model does generate empirically plausible relative degrees of inequality. Consumption exhibits the least inequality, followed by net income, gross income, and then capital, and saving exhibits the greatest inequality. However, the model cannot generate the observed degrees of inequality.

## Conclusion

This class of models can also differ from the infinite-lived agent complete markets model on some important policy issues such as **Chamley (1986)** where dynamic optimal factor taxation leads to the result that the capital income tax should be zero in the long run, **vs Aiyagari (1994)** shows that with idiosyncratic shocks and incomplete markets the capital income tax is strictly positive even in the long run.