Final Exam

Carlos Lezama

Instituto Tecnológico
Autónomo de México

clezamaj@itam.mx

In this exam we investigate a static continuous game with complete information in the supply side. We consider two scenarios: one where the firms have constant marginal costs, and another one where firms have increasing marginal costs. We study the solution and estimation of this game.

1 Constant Marginal Costs

Suppose that we have already estimated a demand model, thus we have estimates for the demand function of the firm, $q_j(p)$. Suppose, too, that the profit function of the firm is

$$\Pi_j(p) = p_j q_j(p) - c_j\left(q_j(p)\right),$$

where $c_j\left(q_j(p)\right) = c_0 + c_1 q_j(p) + Z_j\gamma + \nu_j$, in which c_0, c_1, and γ are unknown (to the econometrician) parameters to be estimated, Z_j is a vector of cost shifters, and ν_j is a mean zero unobserved (to the econometrician) determinant of costs.

Note that the marginal cost function is

$$c_j'(q_j) = c_1 q_j'(p).$$

Recall that $q_j = M \cdot s_j$ where M is the market size and s_j corresponds to the firm share.

In general, given J firms and their marginal cost functions,

$$\frac{\partial \sum_{j \in J} \Pi_j}{\partial p_j} = s_j(p) + \sum_{k \in J}(p_k - c_k')\frac{\partial s_k}{\partial p_j}.$$

This is equivalent to $s(p) + \Omega(p)(p - c')$ in matrix form, where $\Omega(p)$ is the matrix $\mathbf{1}_J \cdot \frac{\partial s_k}{\partial p_j}$.

In our setting, this implies the following first order condition

$$s_j(p) + \sum_{k \in J} (p_k - c_1 M s_k') \frac{\partial s_k}{\partial p_j} = 0.$$

Using the estimated demand function, $\hat{q}_j(p)$, the expression for the firm's first order condition, and assuming no endogeneity concerns, we way follow Hackmann (2019) to estimate $\theta = (c_0, c_1, \gamma)$ through GMM. Under this suitable conditions this estimator is consistent, asymptotically normal, and, potentially, also asymptotically efficient.[1]

Notice that we would need to adjust our standard errors, because we are using an estimate of $q_j(p)$, which induces measurement error in your estimation. To fix this, we may easily apply a two-stage estimation such 2SLS. The advantage of 2SLS estimators over other IV estimators is that 2SLS can easily combine multiple instrumental variables, and it also makes including control variables easier.

However, while easy to implement, the main drawback of two-stage models has been that the estimation of standard errors from the second stage alone are incorrect because they ignore the measurement error that carries over from using the predictions of one model in the next model.

According to Wooldridge (2014), the model described before can also be fit using quasi maximum likelihood. As described in the article, for cases in which the models are linear (2SLS), and under the assumption that the errors of the models distribute normal and independent, the joint maximum log-likelihood function can be written as

$$\ell(\theta_1, \theta_2) = \ell_1(\theta_1) + \ell_2(\theta_2, \hat{\theta}_1).$$

This way, the estimation of this model would not require additional adjustments on the estimation of the standard errors, because the measurement error from θ_1 is already accounted for in the model.

Now, suppose that we have our supply side estimates at hand, i.e. $\hat{c}_0$, $\hat{c}_1$, $\hat{\gamma}$. Suppose, too, that there are only two firms in the market, $j = 1, 2$, and that you want to simulate new equilibrium prices, p^*, that correspond to new values of the vector of cost shifters, Z'. This way, we may easily solve for

$$p^* = \hat{\Omega}^{-1}(p^*)\hat{s}(p^*) + \hat{c}'.$$

[1] With right choice of a positive-definite weighting matrix.

2 Increasing Marginal Costs

Now, suppose that the cost function is quadratic, and of the form

$$c_j\left(q_j(p)\right) = c_0 + c_1 q_j(p) + c_2 q_j(p)^2 + Z_j\gamma + \nu_j,$$

where c_0, c_1, c_2, and γ are unknown (to the econometrician) parameters to be estimated.

Note that the marginal cost function is

$$c_j'(q_j) = c_1 q_j'(p) + 2c_2 q_j(p) q_j'(p).$$

This way, our first order condition yields to

$$s_j(p) + \sum_{k \in J} (p_k - M s_k'(c_1 + 2c_2 M s_k)) \frac{\partial s_k}{\partial p_j} = 0.$$

Using the estimated demand function, $\hat{q}_j(p)$, the expression for the firm's first order condition, and assuming no endogeneity concerns, we may easily apply maximum likelihood estimation on θ with

$$\ell(p;\theta) = \sum_j \log(\Pi_j(p)).$$

References

Hackmann, Martin B. 2019. Incentivizing better quality of care: The role of medicaid and competition in the nursing home industry. *American Economic Review* 109(5). 1684–1716. https://doi.org/10.1257/aer.20151057. https://www.aeaweb.org/articles?id=10.1257/aer.20151057

Wooldridge, Jeffrey M. 2014. Quasi-maximum likelihood estimation and testing for nonlinear models with endogenous explanatory variables. *Journal of Econometrics* 182(1). 226–234. https://doi.org/https://doi.org/10.1016/j.jeconom.2014.04.020. https://www.sciencedirect.com/science/article/pii/S0304407614000797. Causality, Prediction, and Specification Analysis: Recent Advances and Future Directions