Problem Set 1

Empirical Industrial Organization

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This exam is inspired by Luco (2019). Individuals in their working age make savings decisions for retirement. We approach the worker's decision problem in a static manner, i.e., the worker makes a one-time decision.

One Level Decision

The worker must choose one of J pension fund administrators (PFA) to manage her retirement savings. The worker is mandated to save 10% of her salary, y_i. Each PFA charges a percentage fee, p_j, over the worker's salary. PFAs differ in their return on investment, R_j. Finally, ε_{ij} is an i.i.d. Type I Extreme Value preference shock. With all this in hand, we can write down the indirect utility that worker i obtains from enrolling in PFA j as

$$u_{ij} = \alpha_i \left(y_i - 0.1 y_i - p_j y_i\right) + \beta_i R_j + \varepsilon_{ij}, \tag{1}$$

where α_i and β_i denote random coefficients. Moreover, let $\gamma_i = (\alpha_i, \beta_i)$, and $\gamma = (\alpha, \beta)$. We assume,

$$\gamma_i = \gamma + \Gamma D_i + \nu_i, \tag{2}$$

where D_i is a $d \times 1$ vector of demographic variables, Γ is a $2 \times d$ matrix of coefficients that measure how taste varies with demographics, and ν_i is a 2×1 vector of unobserved individual characteristics determining taste, where $\nu_i \sim F_\nu(\cdot)$, with $F_\nu(\cdot)$ a distribution function.

Following Train (2009, 3.1), we can write the probability that worker i chooses PFA j, s_{ij}, as follows:

$$\begin{aligned} s_{ij} &= P(u_{ij} \geq u_{ik},\ \forall j \neq k) && (3)\\ &= P(\varepsilon_{ij} - \varepsilon_{ik} \geq \gamma_{ik} - \gamma_{ij}) && (4)\\ &= \frac{\exp(\gamma_{ij})}{\sum\limits_{j \in J} \exp(\gamma_{ik})} && (5) \end{aligned}$$

where $\gamma_{ij} = \beta_i R_j - \alpha_i p_j y_i$.

Additionally, we can express the price elasticity of the demand for PFA j, E_{j,p_j}, and the cross return elasticity of the demand for PFA j with respect to the return of PFA k (with $j \neq k$), E_{j,R_k}, like so

$$\begin{aligned} E_{j,p_j} &= \frac{\partial s_{ij}}{\partial p_j} \cdot \frac{p_j}{s_{ij}} && (6)\\ &= \frac{\partial \gamma_{ij}}{\partial p_j} p_j (1 - s_{ij}) && (7)\\ &= -\alpha_i p_j y_i (1 - s_{ij}) && (8)\\ & && (9)\\ E_{j,R_k} &= \frac{\partial s_{ij}}{\partial R_k} \cdot \frac{R_k}{s_{ij}} && (10)\\ &= -\frac{\gamma_{ik}}{R_k} R_k s_{ik} && (11)\\ &= -\beta_i R_k s_{ik} && (12) \end{aligned}$$

as derived in Train (2009, 3.6).

Subsequently, with this setup, we can describe the log-likelihood of the model, $\mathcal{L}(\cdot)$, such that

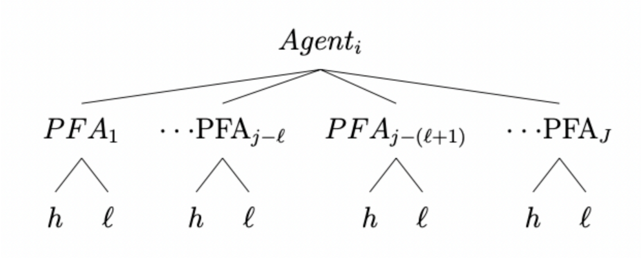
$$\mathcal{L}(\cdot) = \sum_i \sum_j \mathbf{1}_{ij} \log(s_{ij}) \tag{13}$$

$$= \sum_i \sum_j \left(\frac{\exp(\gamma_{ij})}{\sum\limits_{j \in J} \exp(\gamma_{ik})} \right). \tag{14}$$

Two Level Decision

Now, suppose that each PFA offers two portfolios that workers can choose where to invest their savings in: a high-return, high-risk portfolio, h; and low-return, low-risk portfolio, l.

We model the worker's choice of PFA and portfolio as a sequential decision, where individuals first choose the PFA, and then, conditional on the PFA, they choose the portfolio. The following diagram depicts the sequential two-level decision of the worker,



In this context, we model the indirect utility of choosing PFA $j = 1, \ldots, J$ and portfolio $g = h, l$ as,

$$u_{ijg} = \alpha \left(y_i - 0.1 y_i - p_j y_i \right) + \beta R_{jg} + \theta C_{jg} + \varepsilon_{ijg}, \tag{15}$$

where now, for simplicity, α and β are not random but homogeneous coefficients across individuals. The variable R_{jg} denotes the return of portfolio g in PFA j and, similarly, C_{jg} denotes the risk of portfolio g in PFA j. Finally, ε_{ijg} is an i.i.d. GEV preference shock such that the model is a two-level nested logit.