Solving and Estimating an Incomplete Information Entry Game

Carlos Enrique Lezama Jacinto

Instituto Tecnológico Autónomo de México

clezamaj@itam.mx

Suppose there are two firms, A and B, that simultaneously decide whether to enter in the market $m \in \{1, \ldots, M\}$ or not. If firm $i \in \{A, B\}$ decides to enter, its profits are given by

$$\Pi_{i,m} = X_{i,m}\beta - D_{j,m}\alpha + \nu_m + u_{i,m}, \quad \forall\ i, j \in \{A, B\},\ i \neq j$$

where $X_{i,m}$ is an observable characteristic of firm i that boosts profits in market m, $D_{j,m} \in \{0, 1\}$ is j's decision of entering the market, ν_m is a market fixed effect, and $u_{i,m}$ captures unobserved idiosyncratic shocks to profits.

On the other hand, if firm i decides not to enter market m, its profits are zero.

Notice that $u_{i,m}$ is observed by i but not by its opponent j. Consequently, the firms are playing an incomplete information entry game described below.

		Firm B	
		Enter	Do not enter
Firm A	Enter	$(\Pi_{A,m}, \Pi_{B,m})$	$(X_{A,m}\beta + \nu_m + u_{A,m}, 0)$
	Do not enter	$(0, X_{B,m}\beta + \nu_m + u_{B,m})$	$(0, 0)$

Thus, the expected profits of firm A if it decides to enter market m are

$$\begin{aligned} \mathrm{E}\left(\Pi_{A,m}\right) = {} & \left(D_{A,m}\right)\left(p(D_{B,m} = 1)\left(X_{A,m}\beta - \alpha + \nu_m + u_{A,m}\right)\right. \\ & \left. +\ \left(1 - p(D_{B,m} = 1)\right)\left(X_{A,m}\beta - \alpha + u_{A,m}\right)\right) \\ & + \left(1 - D_{A,m}\right) \cdot 0 \end{aligned}$$

where, $\forall\ i, j \in \{A, B\}$, $i \neq j$, the condition for firm i to enter the market in terms of its expected profits is defined as follows:

$$\begin{aligned} D_{i,m} = \mathbf{1}\,(& p(D_{j,m}=1)\,(X_{i,m}\beta-\alpha+\nu_m+u_{i,m}) \\ & +(1-p(D_{j,m}=1))\,(X_{i,m}\beta+\nu_m+u_{i,m})>0)\,. \end{aligned}$$

Namely, the probability that firm A enters market m is

$$\begin{aligned} p\left(D_{A,m=1}\right) &= p\left(X_{A,m}\beta-\alpha p\left(D_{B,m}=1\right)+\nu_m+u_{A,m}>0\right) \\ &= p\left(u_{A,m}>-\left(X_{A,m}\beta-\alpha p\left(D_{B,m}=1\right)+\nu_m\right)\right) \\ &= \Phi\left(X_{A,m}\beta-\alpha p\left(D_{B,m}=1\right)+\nu_m\right) \end{aligned}$$

where $\Phi=\mathcal{N}(0,1)$. Recall that we assume $u_{i,m}\sim\mathcal{N}(0,1)$, $\forall\ i,j\in\{A,B\}$, $i\neq j$.

Similarly,

$$p\left(D_{B,m}=1\right)=\Phi\left(X_{B,m}\beta-\alpha p\left(D_{A,m}=1\right)+\nu_m\right).$$

Furthermore, we propose the following parametrization to simulate our model: