Graphical Economics (with Resale)

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General Equilibrium

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Introduction

General Equilibrium Theory

- Long history and deep mathematical grounding
- We attempt to explain supply, demand, and prices
- Arrow–Debreu Model is **central**

Setting and Background

We consider economies consisting of:

- A set $[\ell] := \{1, \ldots, \ell\}$ of **divisible goods**
- A set $[m] := \{1, \ldots, m\}$ of **agents embedded as nodes** in some graph $G = ([m], E)$, whose edges E describe who may trade with whom
- A **bundle of goods** $\mathbf{e}^i \in \mathbb{R}^\ell_+$ that agent $i \in [m]$ enters the market with
- A **utility function** $u_i : \mathbb{R}^\ell_+ \to \mathbb{R}_+$ that encodes agent i's preferences over bundles of goods

Graphical Economy

A **graphical economy** is an undirected graph G over agents $[m]$ with neighbor relation $\simeq$, utilities $\{u_i : \mathbb{R}_+^\ell \to \mathbb{R}_+\}_{i\in[m]}$, and endowments $\{\mathbf{e}^i \in \mathbb{R}_+^\ell\}_{i\in[m]}$, where ℓ is an integer denoting the number of goods being traded.

For ease of exposition, G is assumed undirected — all results can be easily extended to directed graphs.

To discuss equilibria in a graphical economy, we also need:

- Local price vectors $\mathbf{p}^i \in \mathbb{R}^\ell_+$ for each agent $i \in [m]$
- The bundle of goods $\mathbf{x}^{ij} \in \mathbb{R}^\ell_+$ agent i purchases from agent j for consumption
    - To enforce the condition that trade must traverse edges, $\mathbf{x}^{ij} = 0$ for $j \neq i$

Agent i buys an amount x_k^{ij} of good k from agent j for consumption

$$i \neq j \in [m]$$
$$k \in [\ell]$$

The Arrow–Debreu (AD) Exchange Model

The graphical economies are generalizations of AD which retain AD as a special case.

AD Equilibrium

An **AD Equilibrium** is a pair $(\mathbf{p}, \mathbf{x})$ of a set of price vectors $\mathbf{p}$ and set of consumption plans $\mathbf{x}$ such that, if the underlying graph is complete, we have $\mathbf{p}^i = \mathbf{p}^j$ for all $i, j \in [m]$, and the following conditions are satisfied:

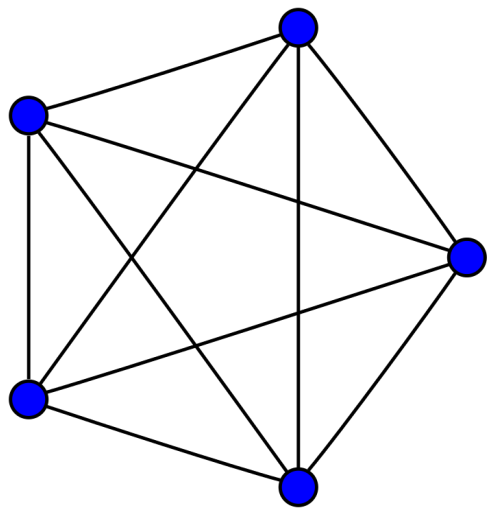
- *Market Clearing.*

$$\sum_{i,j \in [m]} \mathbf{x}^{ij} = \sum_{i \in [m]} \mathbf{e}^i$$

- *Individual Rationality.* For all agents $i \in [m]$, setting $\hat{\mathbf{x}}^i = \mathbf{x}^i$ maximizes their utility $u_i \left(\sum_{j \simeq i} \hat{\mathbf{x}}^{ij} \right)$ over all $\hat{\mathbf{x}}^i \in \mathbb{R}^\ell_+$ satisfying

$$\sum_{j \simeq i} \mathbf{p}^j \cdot \hat{\mathbf{x}}^{ij} \leq \mathbf{p}^i \cdot \mathbf{e}^i$$

- Agents sell endowments at market prices
- They spend profits on goods maximizing their utility
- There is a single global market
  - Single price for each good
  - Every pair of agents can trade
  - Markets are cleared when the demand of all agents is equal to the supply of all agents



We would like something more ***local***

The Kakade, Kearns, Ortiz (KKO) Exchange Model

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Allowing Resale

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