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A new model for acoustic wave propagation and scattering in the vocal tract

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Abstract

A new and efficient numerical model is proposed for simulating the acoustic wave propagation and scattering problems due to a complex geometry. In this model, the linearized Euler equations are solved by the finite-difference time-domain (FDTD) method on an orthogonal Eulerian grid. The complex wall boundary represented by a series of Lagrangian points is numerically treated by the immersed boundary method (IBM). To represent the interaction between these two systems, a force field is added to the momentum equation, which is calculated on the Lagrangian points and interpolated to the nearby Eulerian points. The pressure and velocity fields are then calculated alternatively using FDTD. The developed model is verified in the case of acoustic scattering by a cylinder, for which the exact solutions exist. The model is then applied to sound wave propagation in a 2D vocal tract with area function extracted from MRI data. To show the advantage of present model, the grid points are non-aligned with the boundary. The numerical results have good agreements with solutions in literature. A FDTD calculation with boundary condition directly imposed on the grid points closest to the wall cannot give a reasonable solution.

Index Terms: sound scattering, sound propagation, vocal tract, FDTD, IBM

1. Introduction

Acoustic characteristics of the vocal tract have been investigated for many years. Among others, the scattering of sound due to complex boundary is one of the basic problems [1, pp. 400–466]. This phenomenon also widely appeared in other fields of science and engineering [2, 3, 4]. Many numerical analysis methods that discretize the computational domain with grids have been used to simulate the sound wave propagation, such as finite element method (FEM) [5], boundary element method (BEM) [6], and finite-difference time-domain (FDTD) method [7, 8]. Among these methods, FDTD plays a vital role in computational electromagnetics and (aero)acoustics due to its accuracy and high efficiency. Takemoto et al. (2010) [9] proved that acoustic phenomena within the vocal tract below 10 kHz can be well simulated by 3D FDTD method.

However, complex geometries and moving boundaries often result in a difficult situation for finite-difference methods. To impose boundary conditions, transformation of grid points close to the boundary is inevitable, which will lead the computation to a difficult and time-consuming process. Moreover, the frequent transformation and interpolation may give rise to the degeneration of numerical accuracy and the divergence problem [10]. The immersed boundary method (IBM), proposed by Peskin [11, 12] for the simulation of blood flow in the heart valves, provides a possible solution to deal with these difficulties. The main idea is to add a force field to the momentum equation to represent the immersed boundary. In this method, two different grid systems are employed. A regular Eulerian mesh is used to simulate the fluid dynamics, a Lagrangian representation is used for the boundary, and the interaction between these two grids is modeled using a Dirac delta function.

In this paper, a novel model which integrates the FDTD method and IBM is developed to simulate the scattering of acoustic wave by complex geometries. The paper is organized as follows. The governing equations, the FDTD method and IBM are briefly described in Section 2. In Section 3, to verify the proposed model, a benchmark problem for acoustic scattering by a cylinder is simulated and compared with analytical solutions. Section 4 presents the application to sound propagation in a 2D vocal tract, for which the area function is extracted from the MRI data. Discussion and concluding remarks are given in Section 5.

2. Numerical formulism

2.1. Governing equations

The problem considered here is the sound field scattering in the vocal tract. Acoustic problems are usually considered as a non-viscous process and hence governed by the Euler equations or even the linearized Euler equations [13]. The governing equations for sound wave in a homogeneous lossy acoustic medium are [9]

$$\begin{aligned} -\kappa\frac{\partial p}{\partial t} - \alpha p &= \nabla\cdot\boldsymbol{u} \\ -\rho\frac{\partial \boldsymbol{u}}{\partial t} - \alpha^*\boldsymbol{u} &= \nabla p \end{aligned} \tag{1}$$

where p is the pressure, $\boldsymbol{u}$ is the particle velocity, ρ is the density of the medium, κ (κ=1/ρc^2) is the compressibility of the medium with the sound speed c, α is the attenuation coefficient associated with compressibility of the medium, and α^* (α^*=$\alpha\rho/\kappa$) is the attenuation coefficient associated with density of the medium and is generally set to zero. After normalization of the variables including ρ, c and κ, Eq. (1) can be written as

$$\begin{aligned} &\frac{\partial p}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \\ &\frac{\partial u}{\partial t} + \frac{\partial p}{\partial x} = 0 \\ &\frac{\partial v}{\partial t} + \frac{\partial p}{\partial y} = 0 \end{aligned} \tag{2}$$

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2.2. Numerical methodology

2.2.1. FDTD method

Finite-difference time-domain (FDTD) method is a numerical approach proposed by Yee [14] in 1966 and was originally designed for the simulation of electromagnetics. Recently, it has been successfully applied to acoustics [9, 15].

In FDTD, the pressure and particle velocity are calculated on a staggered grid with central difference for spatial derivatives and leap-frog scheme for temporal derivatives [7, 8]. After discretization, Eq. (2) becomes

$$
\begin{aligned}
&\frac{p^{n+1/2}(i,j)-p^{n-1/2}(i,j)}{\Delta t} \\
&\qquad+\frac{u^{n}(i+1/2,j)-u^{n}(i-1/2,j)}{\Delta x} \\
&\qquad+\frac{v^{n}(i,j+1/2)-v^{n}(i,j-1/2)}{\Delta y}=0 \\
&\frac{u^{n+1}(i+1/2,j)-u^{n}(i+1/2,j)}{\Delta t} \\
&\qquad+\frac{p^{n+1/2}(i+1,j)-p^{n+1/2}(i,j)}{\Delta x}=0 \\
&\frac{v^{n+1}(i,j+1/2)-v^{n}(i,j+1/2)}{\Delta t} \\
&\qquad+\frac{p^{n+1/2}(i,j+1)-p^{n+1/2}(i,j)}{\Delta y}=0
\end{aligned}
\tag{3}
$$

where $p^{n+1/2}(i,j)$ is the sound pressure on the grid of (i,j) at the time step of $n+1/2$, $u^n(i+1/2,j)$ and $v^n(i,j+1/2)$ represent the components of particle velocities in the x, y direction, respectively, Δt is time sampling interval, and Δx and Δy are the spatial sampling intervals in the x and y direction, respectively.

2.2.2. IBM

Immersed boundary method (IBM) was proposed by Peskin [11, 12] in 1970s and now is used in various complex flow simulations [16]. A definition sketch of IBM is shown in Figure 1. Using this method, any geometry such as complex or moving boundary can be simulated easily. To better treat the moving boundary, a modified IBM was developed by Deng et al. [17] and recently applied to study the mechanism of tandem flapping wings [18]. In this approach, the flow field is discretized by a regular Cartesian grid (the corresponding points are called Eulerian points), the complex geometry or moving boundary is represented by a series of Lagrangian points (Figure 1(a)). To construct the relationship between these two grids, a force field is added to the momentum equation, which is calculated on the Lagrangian points and interpolated to the nearby Eulerian points. Then, Eq. (2) reads as

$$
\begin{aligned}
&\frac{\partial p}{\partial t}+\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \\
&\frac{\partial u}{\partial t}+\frac{\partial p}{\partial x}=F_x \\
&\frac{\partial v}{\partial t}+\frac{\partial p}{\partial y}=F_y
\end{aligned}
\tag{4}
$$

where F_x and F_y are the added body forces on the Eulerian points in the x and y direction, respectively. There are two steps for the calculation of the added forces. According to the given boundary condition, the forces on the Lagrangian points are

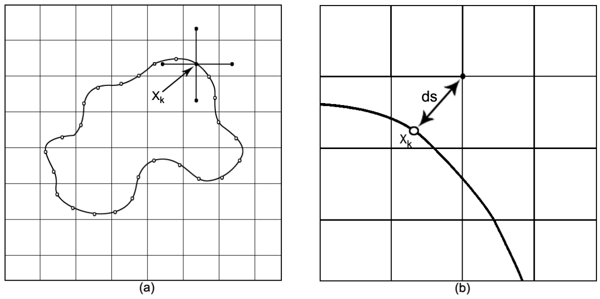

Figure 1: *(a) Sketch of the immersed boundary. (b) Interpolation procedure.*

firstly calculated, which are then interpolated to the Eulerian points nearby. See below for details.

Force calculation. The added force to the Lagrangian points (x_k, $k=1,...,N$) can be represented by:

$$
\boldsymbol{f}(x_k)=\boldsymbol{f}_a(x_k)+\boldsymbol{f}_p(x_k) \tag{5}
$$

where $\boldsymbol{f}_a(x_k)$ and $\boldsymbol{f}_p(x_k)$ are the acceleration force and pressure force of the Lagrangian point x_k, respectively corresponding to the two terms on the LHS of momentum equation, which are written as [17, 18]:

$$
\begin{aligned}
\boldsymbol{f}_a(x_k)&=\frac{\partial \boldsymbol{u}}{\partial t} \\
\boldsymbol{f}_p(x_k)&=\nabla p
\end{aligned}
\tag{6}
$$

It is noted that for the calculation of the adding force, interpolation of field information from Eulerian points to Lagrangian points is necessary. In this procedure, the spatial derivatives of the velocity and pressure are obtained using the Lagrangian point x_k and the adjacent four auxiliary points which are located on the top, bottom, left and right of x_k with a distance of Δx. The velocity and pressure on the five points are obtained by a bi-linear interpolation from their adjacent four Eulerian points.

Interpolation procedure. The calculated force on the Lagrangian points needs to be interpolated back to the Eulerian points. This procedure is described below.

Once the adding force is calculated, a closest Lagrangian point is found for each Eulerian point around the immersed boundary. If the distance between these two points is greater than the diagonal distance of the grid, this Eulerian point is rejected. Otherwise, the adding force $\boldsymbol{f}(x_k)$ will be interpolated to this Eulerian point using the following formula:

$$
\boldsymbol{F}(i,j)=(1-ds/h)\boldsymbol{f}(x_k) \tag{7}
$$

where $h=\sqrt{\Delta x^2+\Delta y^2}$ and ds is the distance between these two points as shown in Fig. 1(b).

2.2.3. Full discretized model

In the present model, both the FDTD and IBM algorithms are employed. A direct force is first added to the linearized Euler equations to replace the complex or immersed boundary, and the governing equations are then discretized by the FDTD method.

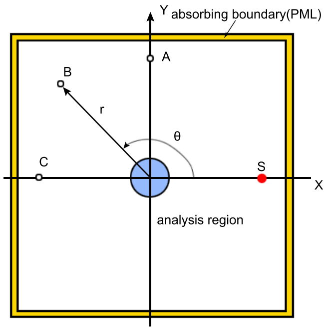

Figure 2: *(Color online) Model of the acoustic scattering.*

The fully discretized model is written as:

$$\begin{aligned}
&\frac{p^{n+1/2}(i,j)-p^{n-1/2}(i,j)}{\Delta t} \\
&\quad+\frac{u^n(i+1/2,j)-u^n(i-1/2,j)}{\Delta x} \\
&\quad+\frac{v^n(i,j+1/2)-v^n(i,j-1/2)}{\Delta y}=S(i,j) \\
&\frac{u^{n+1}(i+1/2,j)-u^n(i+1/2,j)}{\Delta t} \\
&\quad+\frac{p^{n+1/2}(i+1,j)-p^{n+1/2}(i,j)}{\Delta x}=F_x(i,j) \\
&\frac{v^{n+1}(i,j+1/2)-v^n(i,j+1/2)}{\Delta t} \\
&\quad+\frac{p^{n+1/2}(i,j+1)-p^{n+1/2}(i,j)}{\Delta y}=F_y(i,j)
\end{aligned} \tag{8}$$

where $S(i,j)$ is a source term on the point of (i,j). The pressure and velocity on an Eulerian point are then formulated as:

$$\begin{aligned}
p^{n+1/2}(i,j)&=p^{n-1/2}(i,j)+S(i,j)\Delta t \\
&\quad-\left[\frac{u^n(i+1/2,j)-u^n(i-1/2,j)}{\Delta x}\right. \\
&\quad\left.+\frac{v^n(i,j+1/2)-v^n(i,j-1/2)}{\Delta y}\right]\Delta t \\
u^{n+1}(i+1/2,j)&=u^n(i+1/2,j)+F_x(i,j)\Delta t \\
&\quad-\frac{p^{n+1/2}(i+1,j)-p^{n+1/2}(i,j)}{\Delta x}\Delta t \\
v^{n+1}(i,j+1/2)&=v^n(i,j+1/2)+F_y(i,j)\Delta t \\
&\quad-\frac{p^{n+1/2}(i,j+1)-p^{n+1/2}(i,j)}{\Delta y}\Delta t
\end{aligned} \tag{9}$$

2.2.4. *Perfectly matched layer*

In computational acoustics, a very important consideration is the proper truncation of the finite simulation domain to approximate an infinite space. Perfectly matched layers (PML) [19, 20], which can absorb the outgoing waves near the boundary, are employed around the analysis domain to achieve this goal.

2.2.5. *Numerical implementation*

Given the values at time step n, the procedure for calculating the values of pressure and velocity at time step $n+1$ can be listed as follows.

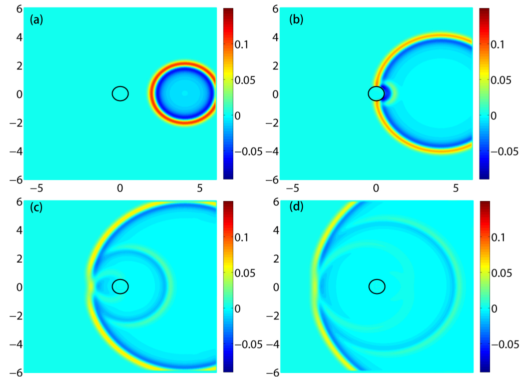

Figure 3: *(Color online) Snapshots of pressure field at four different times. (a) t=2, (b) t=4, (c) t=6 and (d) t=8.*

- For every Eulerian point, find the closest Lagrangian point.
- Interpolate the pressure and velocity at this Lagrangian point from the given values.
- Calculate the corresponding Lagrangian force using Eq. (5) and (6).
- Interpolate the force to this Eulerian point by Eq. (7).
- Solve the pressure and velocity at the Eulerian point at time step $n+1$ using Eq. (9).

3. Model validation

For validation, the proposed model is applied to the two-dimensional benchmark problem of wave scattering by a circular cylinder. The definition of the problem is shown in Figure 2.

In a square domain $-6.0 \le (x,y) \le 6.0$, a Gaussian type sound pulse

$$p=\exp\left\{-\ln 2\left[\frac{(x-x_s)^2+(y-y_s)^2}{b^2}\right]\right\} \tag{10}$$

is initially located around point S (see Figure 2) with $(x_s,y_s)=(4,0)$ and $b=0.2$. A circular cylinder with diameter 1 is placed at $(x,y)=(0,0)$. The initial velocity is zero. The same problem has been theoretically [3] and numerically [4, 13] studied. For easy comparison, three measuring points A, B and C at $r=5$, and $\theta=90^\circ$, 135° and 180° are specified (see Figure 2).

In the computation, to absorb the sound wave propagating to the boundary, PML is placed around the square domain. The spatial intervals between two Eulerian points in x and y direction are both 0.02, i.e. $\Delta x=\Delta y=0.02$, although they are not necessarily the same. The time interval is $\Delta t=0.005$ and the total simulation time is 10.

The snapshots of the computed pressure field at four time levels $t=2,4,6$ and 8 are shown in Figure 3. The sound scattering due to the cylinder is clearly seen, and the outgoing acoustic waves are perfectly absorbed by PML. These all demonstrate the efficiency of the developed model.

The time histories of pressure at three measuring points are shown in Figure 4 together with the analytical solutions given in [3]. Excellent agreements are clearly observed. Therefore, the present model, combining FDTD and IBM to solve the linearized Euler equations, is a feasible tool for simulating sound wave propagation and scattering problems with complex geometry.

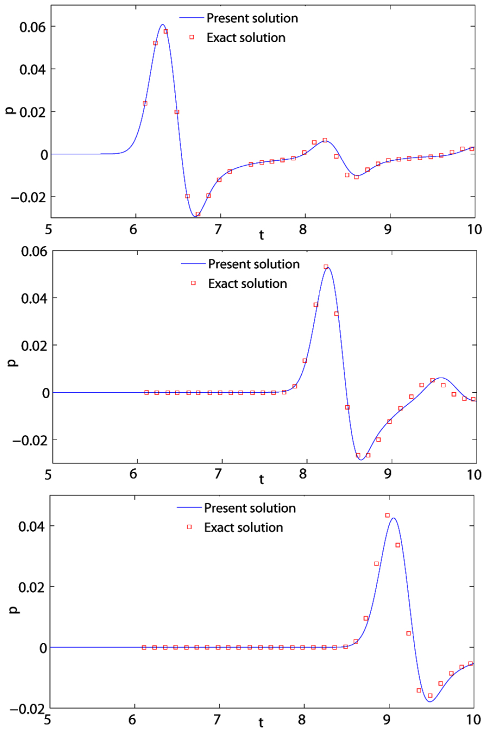

Figure 4: *(Color online) Time histories of pressure at three measuring points. (a) Point A ($r = 5$, $\theta = 90^\circ$), (b) Point B ($r = 5$, and $\theta = 135^\circ$) and (c) Point C ($r = 5$, and $\theta = 180^\circ$).*

4. Sound propagation in the vocal tract

In this section, the validated model is applied to simulate sound wave propagation in a 2D vocal tract with area function extracted from MRI data. Five Mandarin vowels /a/, /i/, /o/, /u/, /e/ have been simulated and only the results for vowel /a/ and vowel /o/ are presented here due to the limit of space. The area functions given in [21] are used. Taking into account the shape of the vocal tract, the computational domain is set to be $(x, y) \in [-2, 20] \times [-5, 5]$ (cm) in these two simulations. The same sound source Eq. (10) is added near the glottis with the center at (0.3,0). The spatial intervals are $\Delta x = \Delta y = 0.1$ cm, the time interval is $\Delta t = 0.001$ s and the total simulation time is 15 s.

In the simulation of sound wave propagation when pronouncing vowel /a/, all the boundaries of the vocal tract are treated with IBM and the right side (lip) is open. The snapshots of the computed pressure at two time levels $t = 3$ and 9 s are shown in Figure 5 (a) and (b). It is clear that no wave penetrates the wall as physically should be.

With condition of $u = v = 0$ directly imposed on the grid points nearest to the boundary, the same case is studied with FDTD. As shown in Figure 5 (c) and (d), unphysical wave penetrations are clearly observed. This simple comparison qualitatively demonstrates the efficiency and accuracy of the proposed model.

The results at four time levels for vowel /o/ are presented in Figure 6. As indicated, IBM works very well on the boundaries of vocal tract.

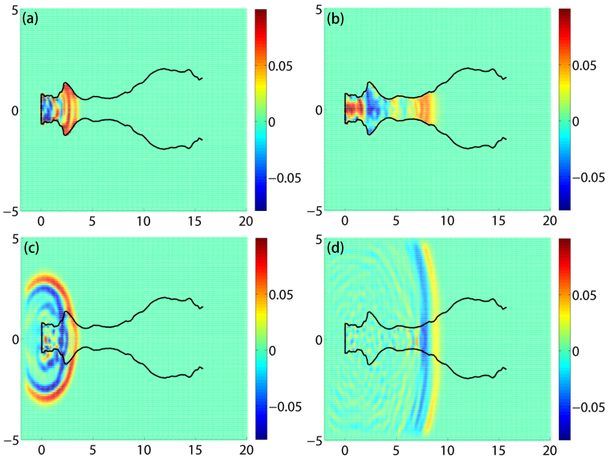

Figure 5: *(Color online) Pressure fields at different times of vowel /a/. (a) t=3 (FDTD combined IBM), (b) t=9 (FDTD combined IBM), (c) t=3 (direct FDTD) and (d) t=9 (direct FDTD).*

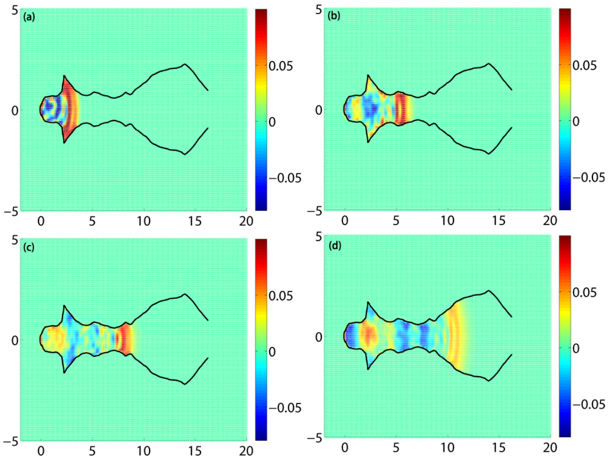

Figure 6: *(Color online) Pressure fields at four different times of vowel /o/ (FDTD combined IBM). (a) t=3, (b) t=6, (c) t=9 and (d) t=12.*

5. Discussion and conclusions

The main contribution of this work is the development of a new model for simulating acoustic wave propagation and scattering with complex geometries. The model is validated against the exact solutions for a benchmark scattering problem due to a cylinder. It has also been applied to Mandarin vowels in 2D vocal tract with some preliminary success. The frequency domain analysis for five vowels is being worked on. The extension of the present model to 3D vocal tract is also planned. Furthermore, the shape of the vocal tract always keeps changing in continuous speech production. It is actually a sound wave propagation problem with moving boundaries, for which the proposed model has high potential to provide complete, accurate and real simulations.

6. Acknowledgements

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7. References

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