What is a Wavelet?

The easiest place to start in understanding what a wavelet is is the simple Fourier transform. A Discrete Fourier Transform (DFT), purely mathematically, is just a linear transformation from the original basis set to a new, more mathematically powerful one. In the DFT, this new basis set is a combination of periodic solutions to the unit circle. This is great when you are working with systems that are easily modeled by multiple sine and cosine functions, but DFT analysis does not provide a way to deal with signals that have non-periodic elements like step-functions or random spikes in the signal. This need for a new type of transform that could deal both with a general trend of a signal as well as being able to handle random events gave rise to "wavelets".

Just like a DFT, a discrete wavelet transform (DWT), is just a linear transformation taking a signal from its original basis to a new, more useful, linear span. A wavelet transform separates a signal (x) into two separate vectors, a signal and a detail (s and d). This is done through matrix multiplication as shown in Equation 1 where T_a is the matrix multiplied to the signal in order to produce the signal and the detail. Note, vectors are all given the simple arrow above them and matrices are given a tilde above their symbol.

$$\widetilde{T_a}\,\vec{x} = \vec{y} = \begin{bmatrix} \vec{s} \\ \vec{d} \end{bmatrix}$$

The simplest wavelet transform is the Haar DWT. The Haar DWT splits the x signal into a moving average signal (s) and a difference from this average (d). This works by a series of matrix multiplications via upper triangular (U), lower triangular(P), and normalization matrices (D), These three matrices are all 2 by 2 block matrices made up of scalars, an even and odd entry separation matrix called *split,* and identity matrices (I). For the Haar DWT, Equation 2 gives the equation used to determine T_a.

$$T_{a\,Haar} = \widetilde{DUP}\,\widetilde{split} = \begin{bmatrix} I & O \\ -I & I \end{bmatrix}\begin{bmatrix} I & -0.5\,I \\ 0 & I \end{bmatrix}\begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}\widetilde{split}$$

On the opposite end, the inverse of the wavelet transform will take a vector made of the signal and detail and transform it back into the original signal. For the Haar transform, the inverse of T_a (T_s) is given by Equation 3. Note that the inverse of *split* is *merge*.

$$T_{a\,Haar}{}^{-1} = T_{s\,Haar} = \widetilde{merge}P^{-1}\widetilde{U^{-1}}D^{-1}$$

However, the Haar transform is a somewhat primitive method for performing DWT's as it does not return and orthonormal basis set in T_a. Furthermore, Haar DWT's have trouble detecting discontinuities in signals. Another DWT will be analyzed in this project. The equations for the Daubechies 4 wavelet is given below. Note that S is a matrix that shifts a vector's values one position and wraps the last value around to be the first value.

$$T_{a\,Daub4} = DU_2PU_1\,\widetilde{split} = \frac{1}{4\sqrt{2}}\begin{bmatrix} (aI + cS^{-1}) & (bI + dS^{-1}) \\ -(bI + dS) & (aI + cS) \end{bmatrix}\widetilde{split}$$

Equations 1-4 give equations for level 1 DWT's, but it is much more powerful to use multi-scale transforms. This occurs were a DWT is the applied to the signal vector part of the y vector. Multi-scale DWT's are given the notation as in Equation 5. Note that the signal and detail vectors are referred to as the level 1, and level 2 details respectively.

$$\widetilde{W_{a\,2}}\,\vec{x}\begin{bmatrix} T_{a\,2} & 0 \\ 0 & I \end{bmatrix}\widetilde{T_{a\,1}}\,\vec{x} = \vec{y} = \begin{bmatrix} T_{a\,2} & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} \overrightarrow{s_1} \\ \overrightarrow{d_1} \end{bmatrix} = \begin{bmatrix} \overrightarrow{s_2} \\ \overrightarrow{d_2} \\ \overrightarrow{d_1} \end{bmatrix}$$

The most common analysis method used with wavelets is called thresholding. This is where detail values below a certain level are discarded, making a new, modified, y vector which can then be multiplied by T_s (the transpose of T_a) to obtain a compressed signal of x.

Note: Wavelet theory was developed with the help of "Discrete Fourier and Wavelet Transforms" by Dr. Roe W. Goodman [1].

Biological Background

The body communicates with itself through electrical signals that are created through an imbalance of ions across cell membranes. Measuring these electrical signals gives physicians crucial insight into body which allows for diagnosis of disease on a fundamental level. In an electrocardiogram (EKG) the electrical signals in the heart are measured using a set of ten electrodes and electrocardiograph to amplify the measured potentials. These electrodes are placed on the limbs and across the chest and work together in pairs electrode to create twelve leads, or perspectives on the hearts electrical activity [2]. The theoretical line connecting the pairs of electrodes forms the lead axis. Additionally, the electrode pairs work in reference to a third reference electrode, located on the right leg, that is arbitrarily taken as the zero potential [3]. The twelve perspectives occur in two planes: a frontal plane with anterior to posterior view of the hearts electrical activity and a horizontal plane with an inferior to superior view [3]. The output from the EKG is a patterned line graph of measured voltage representative of the depolarization and repolarization of the heart muscle versus time. Electrical activity towards the positive electrode or away from a negative electrode causes an upward deflection in the graph.

In a normal sinus rhythm (NSR), the electrical signal in the heart starts with depolarization of the pacemaker cells in the sinoatrial node (SAN). A depolarization wave then travels to the atrioventricular node (AVN), causing the depolarization and subsequent contraction of the heart's atria. The wave of depolarization continues from the AVN through the bundle of His. At the bundle of His, the depolarization splits into two paths and travels to the base of the heart then to the Purkinje System and eventually to the ventricular walls causing a ventricular contraction [3]. The ventricular contraction occurs through the activation of three muscles, the interventricular septum, the right and left ventricular free walls, and then remaining small areas of the ventricles located at the base of the heart. After the depolarization, the heart muscles, the muscle must repolarize before it can depolarize beforw the next cardiac cycle can occur.

An EKG readout is broken into three sections, the depolarization of the atria, depolarization of the ventricles, and the repolarization of the ventricles **(Figure 1)**. The depolarization of the atria is depicted as a P-wave on an EKG readout. Typically, the P-wave is an upward-rounded deflection lasting 0.06-0.12 seconds [4]. The absence or large variance in the presence P-waves in the data indicates the depolarization of the heart originates in a place other than the SA node, like in the ventricles, and represents an atrial fibrillation. Other abnormities in

P-waves, such as having peaked and enlarged waves, indicate diseases such as heart failure and atrial hypertrophy. The time between P-waves in consecutive cardiac cycles represents an atrial rhythm and should have little variation. The time length of the atrial depolarization is measured as the PR-interval, the time between start of the P-wave to the start of the QRS complex. A PR-interval is dependent on the age of the patient, but for an adult is typically between 0.12-0.20 seconds. Short PR-intervals also indicates the depolarization of the heart is occurring outside of the SA node and suggests a preexcitation syndrome and/or a junctional arrhythmia [4]. A PR-interval is greater than 0.20 seconds advocates for conduction tissue disease or ischemia (inadequate blood supply to the heart).

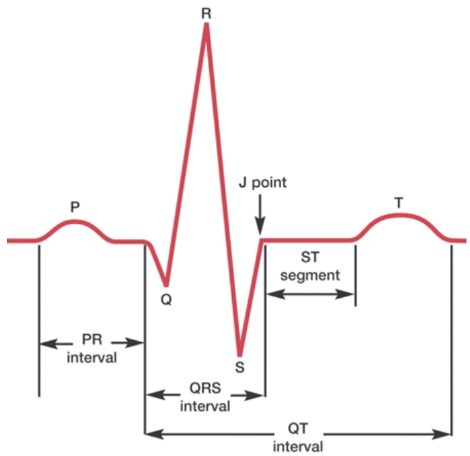

Figure 1. An EKG readout of a single cardiac cycle. [1]

The second section of the EKG readout contains the QRS complex. The QRS complex consists of three separate waves, Q, R, and S-waves. The combination of these waves depict the depolarization of the ventricles. The Q-wave is a downward deflection representing the activation of the interventricular septum [3]. The R-wave, occurs via the depolarization of the right and left ventricle free walls and is depicted as an upward deflection. The R-wave, the largest of the three waves, represents the depolarization of the largest amount of ventricular mass. Often the R-wave relates to breathing rates; the time between consecutive R-waves in consecutive cardiac cycles represents a ventricular rhythm and should have little variation. However, variations up to 0.04 seconds in the ventricular rhythm are considered normal. Lastly, the S-wave depicts the depolarization of the remaining ventricle muscles at the base of the heart. It is normal for QRS complex to not contain all three waves **(Figure 2a)**. Occasionally, a second, distinct R-wave, often labeled as R', will appear in the QRS complex **(Figure 2b)**. A QRS interval, measured from the start to end of the QRS complex QRS signal will last 0.06 to 0.10 seconds in length or half of the PR-interval in an NSR. A signal lasting longer than 0.12 seconds suggests a ventricular conduction delay. Abnormalities in the Q wave indicate myocardial infarctions (heart attacks) [3]. In a myocardial infarction, the Q wave is both deepened and widened. Numerically, the Q wave is greater than 25% of the R-wave amplitude and lasts for longer than 0.04 seconds when a myocardial infarction has occurred. Additionally, the R-wave can have two peaks in the same deflection, a pattern that is termed a notched R-wave **(Figure 2c)**. A notched R-wave suggests a bundle branch block may be occurring.

Figure 2: Examples of different potential normal and abnormal of QRS signals. **a.** QRS signal without all three QRS waves **b.** an extra R wave peak in the QRS signal. **c.** a notched R wave [4].

The third section of EKG readout contains the T-wave, a representation of the electrical activity associated with the repolarization of the heart muscles between cardiac cycles. Normally the T-wave looks similar to a normal P-wave—an upward-rounded deflection. Sometimes the P-wave for the next cardiac cycle will nest itself in the T-wave, as a result the T-wave will appear bumpy. Moreover, a pointed T-wave is indicative of myocardial injury or hyperkalemia and, in adults, pericarditis [5]. The time scale of the T-wave is broken into two separate time intervals, an ST segment and a QT interval (Figure 1). The ST segments measures a period of zero polarization between the end to the S-wave to the start of the T-wave. An abnormal ST-segment is categorized as either depressed or elevated. A depressed segment will appear 0.5 mm below the baseline on the EKG readout. The elevated segment, moreover, will have an ST-segment 1 mm above the base-line, zero-potential. The QT interval represents the time between the start of the QRS wave to the end of the T-wave. The QT interval usually ranges from 0.36-0.44 seconds but is dependent on the patient. A more general rule of thumb is that the QT interval should not be longer than half the time between consecutive R-wave peaks [4].

The analysis of an EKG readout depends greatly on the ability to distinguish between the peaks in the various section of the EKG. Running a DWT, as described in the above section, on the EKG data allows for a more complete automated analysis of the EKG data.

Machine Learning

Now that EKG's and wavelets have been explained, the technology that will make this project tick, machine learning, needs to be discussed as well. The type of machine learning that will be employed in this scenario is called "Supervised Learning". This is a type of learning in which a set of labeled "training data" is given to an algorithm in order to try and discern different groups based on the characteristics in the training data. The data is said to be labeled because there is a set of known outcomes in the training set. In this example, it is known whether the EKG signal is from a healthy or an unhealthy heart, and it is passed into an algorithm in order to form a model applicable to new signals. These new signals are called the "test set" or "learning set". This is the set of signals that has an unknown type, and the machine learning algorithm then tried to determine what type the signal is based on the training set data [6]. In this example, the learning set will be a number of EKG signals that are either from healthy or unhealthy patients, but they are not labeled. The algorithm will then send back a response of whether the EKG matches the traits that are characteristic of a healthy or unhealthy heart.

Supervised learning has many different specific algorithms that can be used in order to derive a model for how to sort new data. Three main algorithms will be tested in this project:

logistic regression, n-nearest neighbor, naïve Bayes, and decision trees. These can all be very complex algorithms, so a small description of the basics of each one are given below [6].

Logistic Regression:
This is an algorithm which uses linear modeling in order to split data into two sets. This is done by fitting the training data to a logistic curve which provides a probability density function for a new single test to either be in one category or another. In this project's case, different traits of the signals would be fit to logistic curves, then the unknown signal's traits would be tested on each of these curves in order to give a probability of which type of heart the signal was from. **Figure 3** illustrates how logistic modeling allows for a model to be implemented.

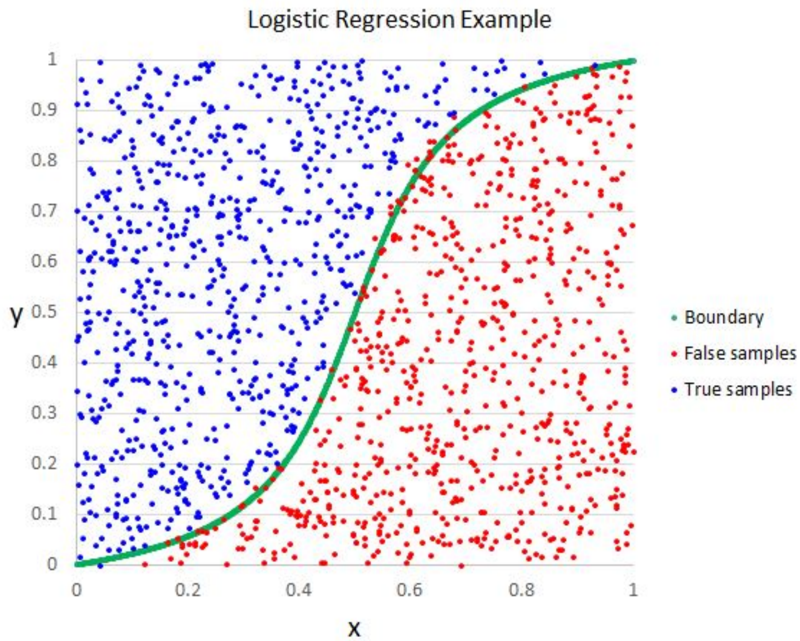

Figure 3. A logistic regression of two-type data used in logistic machine learning [7].

N-Nearest Neighbor:
The nearest neighbor algorithm is probably the simplest machine learning algorithm for supervised learning. This algorithm plots the training set on a "map" then places the test sample on the map and calculates the Euclidean distance from always from the nearest training points on the map which allows for an estimate of the type of data that the test sample is. **Figure 4** shows a good visual representation of how this algorithm creates a map for different types of data based on the training set.

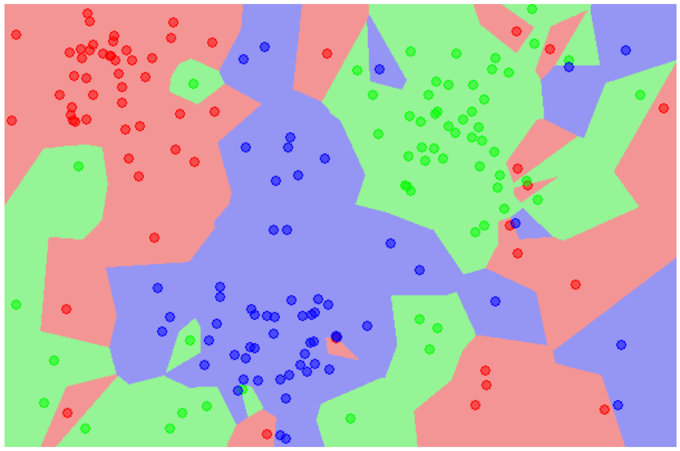

Figure 4. An illustration of how to classify data using the nearest neighbor algorithm [8].

Naïve Bayes:

Naïve Bayes algorithm takes an independent probability approach to sorting data. The algorithm is considered "naïve" because it assumes that all features in a set are independent, thus providing the same weight to the overall outcome. For example, in EKG signals, Naïve Bayes would place the same amount of emphasis on the heart rate and fluctuations in heart rate for determining if a heart was healthy or not. It is obvious that fluctuation in heart rate is much more dangerous than a fast heart rate, so Naïve Bayes may not work well in this case. However, it has been shown to perform well in many real-world applications and is therefore worth trying.

Decision Trees:

Decision trees are a broader type of algorithm which uses the training data to devise a logical stepwise rule system which will lead to the correct classification of a test sample. This algorithm is simple and effective if the training data is reasonably stable. This lends it to most likely be a good contender for an algorithm that will be able to sort EKG signals. **Figure 5** shows what a sample decision tree might look like.

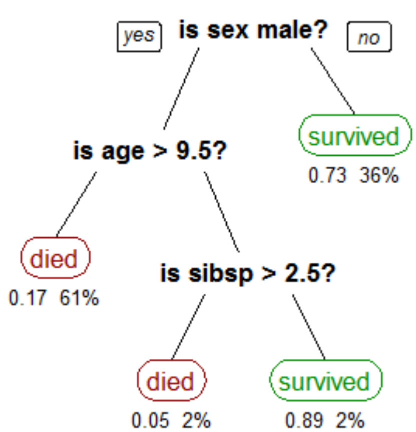

Figure 9. An example of a decision tree used to classify simple death rates in a population [9].

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