Observable 2 (self-interaction)

1D

```
> restart :
  d := 1;
  m := 0.5 :
  k := 1/(m^2 + 2·d) :
```

$$d := 1 \tag{1.1.1}$$

```
> L := 128 :
```

```
> S := [ ]
```

$$S := [\,] \tag{1.1.2}$$

```
> for i from 0 by 1 to L − 1 do
  S := [op(S), 2·Pi/L ·i] :
  end do:

  S :
```

```
> f(X) := 1/(4·k· sin^2(X) + 1 − 2·k);
```

$$f := X \mapsto \frac{1}{4\,k\,\sin(X)^2 + 1 - 2\,k} \tag{1.1.3}$$

```
> total := 0;
```

$$total := 0 \tag{1.1.4}$$

```
> for i from 1 by 1 to numelems(S) do
  total := total + f(S[i]) :
  end do:
> total :

  total := total/(4·L) :
> evalf(total);
```

$$0.5457051560 \tag{1.1.5}$$

2d

```
> restart;
  d := 2; m := 0.5;
```

$$d := 2$$
$$m := 0.5 \tag{1.2.1}$$

```
> k := 1/(m^2 + 2·d) :
> L := 128 :
```

```
> S := [ ]:
> for i from 0 by 1 to L − 1 do
  S := [op(S), (2·Pi/L)·i]:
  end do:
> S:
  total := 0:
  f(X, Y) := 1/(4·k·(sin^2(X) + sin^2(Y)) + 1 − 4·k);
```

$$f := (X, Y) \mapsto \frac{1}{4\,k\left(\sin(X)^2 + \sin(Y)^2\right) + 1 - 4\,k} \qquad \textbf{(1.2.2)}$$

```
> for x from 1 by 1 to L do
  for y from 1 by 1 to L do
  total := total + f(S[x], S[y]);
  end do:
  end do:

  total := total/(4·L^2):
  evalf(total);
```

$$0.4004087262 \qquad \textbf{(1.2.3)}$$

3D

```
> restart;
  d := 3:
  m := 0.5;
```

$$m := 0.5 \qquad \textbf{(1.3.1)}$$

```
> k := 1/(m^2 + 2·d);
```

$$k := 0.1600000000 \qquad \textbf{(1.3.2)}$$

```
> L := 128:
> S := [ ]:
> for i from 0 by 1 to L − 1 do
  S := [op(S), (2·Pi/L)·i]:
  end do:
> S;
  total := 0:
  f(X, Y, Z) := 1/(4·k·(sin^2(X) + sin^2(Y) + sin^2(Z)) + 1 − 6·k);
```

$$\left[0, \frac{\pi}{64}, \frac{\pi}{32}, \frac{3\,\pi}{64}, \frac{\pi}{16}, \frac{5\,\pi}{64}, \frac{3\,\pi}{32}, \frac{7\,\pi}{64}, \frac{\pi}{8}, \frac{9\,\pi}{64}, \frac{5\,\pi}{32}, \frac{11\,\pi}{64}, \frac{3\,\pi}{16}, \frac{13\,\pi}{64}, \frac{7\,\pi}{32},\right.$$
$$\left.\frac{15\,\pi}{64}, \frac{\pi}{4}, \frac{17\,\pi}{64}, \frac{9\,\pi}{32}, \frac{19\,\pi}{64}, \frac{5\,\pi}{16}, \frac{21\,\pi}{64}, \frac{11\,\pi}{32}, \frac{23\,\pi}{64}, \frac{3\,\pi}{8}, \frac{25\,\pi}{64}, \frac{13\,\pi}{32},\right.$$

$$\frac{27\pi}{64}, \frac{7\pi}{16}, \frac{29\pi}{64}, \frac{15\pi}{32}, \frac{31\pi}{64}, \frac{\pi}{2}, \frac{33\pi}{64}, \frac{17\pi}{32}, \frac{35\pi}{64}, \frac{9\pi}{16}, \frac{37\pi}{64}, \frac{19\pi}{32},$$
$$\frac{39\pi}{64}, \frac{5\pi}{8}, \frac{41\pi}{64}, \frac{21\pi}{32}, \frac{43\pi}{64}, \frac{11\pi}{16}, \frac{45\pi}{64}, \frac{23\pi}{32}, \frac{47\pi}{64}, \frac{3\pi}{4}, \frac{49\pi}{64}, \frac{25\pi}{32},$$
$$\frac{51\pi}{64}, \frac{13\pi}{16}, \frac{53\pi}{64}, \frac{27\pi}{32}, \frac{55\pi}{64}, \frac{7\pi}{8}, \frac{57\pi}{64}, \frac{29\pi}{32}, \frac{59\pi}{64}, \frac{15\pi}{16}, \frac{61\pi}{64},$$
$$\frac{31\pi}{32}, \frac{63\pi}{64}, \pi, \frac{65\pi}{64}, \frac{33\pi}{32}, \frac{67\pi}{64}, \frac{17\pi}{16}, \frac{69\pi}{64}, \frac{35\pi}{32}, \frac{71\pi}{64}, \frac{9\pi}{8}, \frac{73\pi}{64},$$
$$\frac{37\pi}{32}, \frac{75\pi}{64}, \frac{19\pi}{16}, \frac{77\pi}{64}, \frac{39\pi}{32}, \frac{79\pi}{64}, \frac{5\pi}{4}, \frac{81\pi}{64}, \frac{41\pi}{32}, \frac{83\pi}{64}, \frac{21\pi}{16},$$
$$\frac{85\pi}{64}, \frac{43\pi}{32}, \frac{87\pi}{64}, \frac{11\pi}{8}, \frac{89\pi}{64}, \frac{45\pi}{32}, \frac{91\pi}{64}, \frac{23\pi}{16}, \frac{93\pi}{64}, \frac{47\pi}{32}, \frac{95\pi}{64},$$
$$\frac{3\pi}{2}, \frac{97\pi}{64}, \frac{49\pi}{32}, \frac{99\pi}{64}, \frac{25\pi}{16}, \frac{101\pi}{64}, \frac{51\pi}{32}, \frac{103\pi}{64}, \frac{13\pi}{8}, \frac{105\pi}{64}, \frac{53\pi}{32},$$
$$\frac{107\pi}{64}, \frac{27\pi}{16}, \frac{109\pi}{64}, \frac{55\pi}{32}, \frac{111\pi}{64}, \frac{7\pi}{4}, \frac{113\pi}{64}, \frac{57\pi}{32}, \frac{115\pi}{64}, \frac{29\pi}{16},$$
$$\frac{117\pi}{64}, \frac{59\pi}{32}, \frac{119\pi}{64}, \frac{15\pi}{8}, \frac{121\pi}{64}, \frac{61\pi}{32}, \frac{123\pi}{64}, \frac{31\pi}{16}, \frac{125\pi}{64}, \frac{63\pi}{32},$$
$$\frac{127\pi}{64}\Big]$$

$$f := (X, Y, Z) \mapsto \frac{1}{4k\left(\sin(X)^2 + \sin(Y)^2 + \sin(Z)^2\right) + 1 - 6k} \quad \textbf{(1.3.3)}$$

```
> for x from 1 by 1 to L do
  for y from 1 by 1 to L do
  for z  from 1 by 1 to L do
  total := evalf(total + f(S[x], S[y], S[z]));
  end do:
  end do:
  end do:

  total := total/(4·L^3):
  evalf(total);
```

0.3299306241 **(1.3.4)**

```
> ``
```

(1.3.5)

```
>
```

Observable 1 (nearest neighbour interaction)

1D

```
>
> restart :
  d := 1;
  m := 0.9 :
  k := 1/(m^2 + 2·d) :
```

$$d := 1 \tag{2.1.1}$$

```
> L := 100 :
> S := [ ]
```

$$S := [\,] \tag{2.1.2}$$

```
> for i from 0 by 1 to L − 1 do
  S := [op(S), (2·Pi/L)·i] :
  end do:

  S :
```

```
> f(X) := 1/(4·k·sin^2(X) + 1 − 2·k);
```

$$f := X \mapsto \frac{1}{4\,k\,\sin(X)^2 + 1 - 2\,k} \tag{2.1.3}$$

```
> total := 0;
```

$$total := 0 \tag{2.1.4}$$

```
> for i from 1 by 1 to numelems(S) do
  total := total + f(S[i]) :
  end do:
> total :

  total := total/(4·L) :
```

```
> total := evalf(total);
  total_2 := 0.
```

$$total := 0.3559026985$$
$$total\_2 := 0. \tag{2.1.5}$$

```
> g(X) := exp(-I•X)/(4·k·sin^2(X) + 1 − 2·d·k);
  for i from 1 by 1 to numelems(S) do
  total_2 := total_2 + g(S[i]) :
  end do:
```

$$g := X \mapsto \frac{e^{-I \cdot X}}{4\,k\,\sin(X)^2 + 1 - 2\,d\,k} \tag{2.1.6}$$

```
> total_2 := total_2/(d·L^d) :
  total_2 := evalf(total_2);
  total + total_2;
```

$$total\_2 := -5.36\ 10^{-10} - 1.6\ 10^{-11}\ I$$

$$0.3559026980 - 1.6\ 10^{-11}\ \mathrm{I} \tag{2.1.7}$$

```
> latex table
```

latex table **(2.1.8)**

```
>
```