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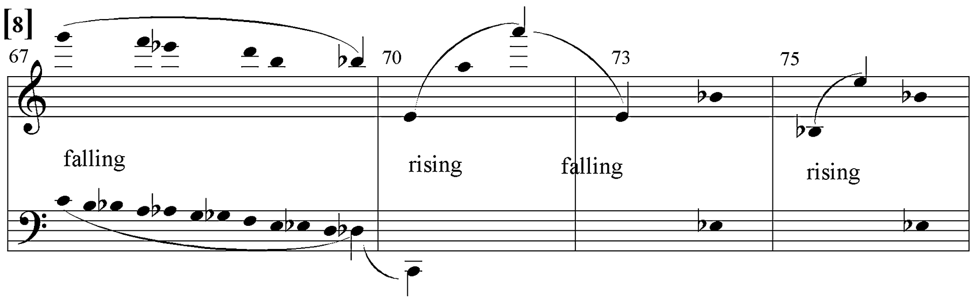
Statistical Form amongst the Darmstadt School

Stockhausen's *Jeux*

One evening in 1954 Stockhausen gave a talk on a Westdeutscher Rundfunk (WDR) *Nachtprogramm* in which he presented an analysis of statistical form in Debussy's *Jeux* (Stockhausen 1963). Audiences and critics had initially been ambivalent about the work, and its lacklustre premiere on 15 May 1913 was overshadowed by the riotous *Rite of Spring* event just two weeks later (Orledge 1982, pp. 168–72). However, *Jeux* became a something of a cult favourite of the Darmstadt composers.[1] Boulez remarked in 1958, 'Since this first failure, there has been a kind of "curse" on *Jeux* and it was rarely played until it was noticed, quite recently, that it is one of Debussy's most remarkable works' (Boulez [1958] 1991, p. 274). Ligeti too suggests that *Jeux* was very much in the air in the late 1950s: 'I cannot recall the occasion when I heard *Jeux* for the first time but in Cologne people kept speaking about it' (Ligeti 1983, p. 42, and Borio 2009). The near absence of a reception history at the time of the premiere meant that *Jeux* was relatively unburdened by its past. Stockhausen in particular capitalised upon *Jeux*'s status as a blank slate when he gave his statistical analysis of the piece. Though he appropriated scientific and technological terms that were obscure to some, he posited highly influential ideas.[2]

In his analysis Stockhausen identified and described rising and falling arches in pitch, loudness and speed. He suggested that these gestures were more important than motives, harmonies or traditional notions of form. Ex. 1 is a score reduction crafted from Stockhausen's prose, which reveals the architectural outlines for rising-falling arches in pitch register in bars 67–76. In addition to these large-scale shapes, Stockhausen was particularly concerned with changing degrees of orchestral density. As he said, 'Between very large densities and pointillist tone dispersions lie continually all variations of density. I am thinking of a *row of graded densities*[,] for vertical as well as for horizontal density' (Stockhausen 1963, p. 78).[3] By 'vertical density', Stockhausen may have meant the thickness of the orchestration; by 'horizontal density', he may have meant the number of attacks in a time frame. However, these specific interpretations are extrapolated from Stockhausen's ambiguous discussions of vertical and horizontal density. His analysis of *Jeux* is rather speculative; in order to gauge what he means, readers are left to make quite a few inferences and assumptions. Just as important as the analytical content is the historical lineage which Stockhausen traces. When he elaborates on a continuum for orchestral sound that extends

Ex. 1 Reduction of Debussy's Jeux, bars 67–76, showing Stockhausen's falling and rising shapes



from pointillism to fluctuating densities, he positions Webern and Debussy as dual fountainheads (see Boulez [1955] 1999, Grant 2001 and Goldman 2011) for his and the Darmstadt composers' collective interest in integral serialism and statistical masses.

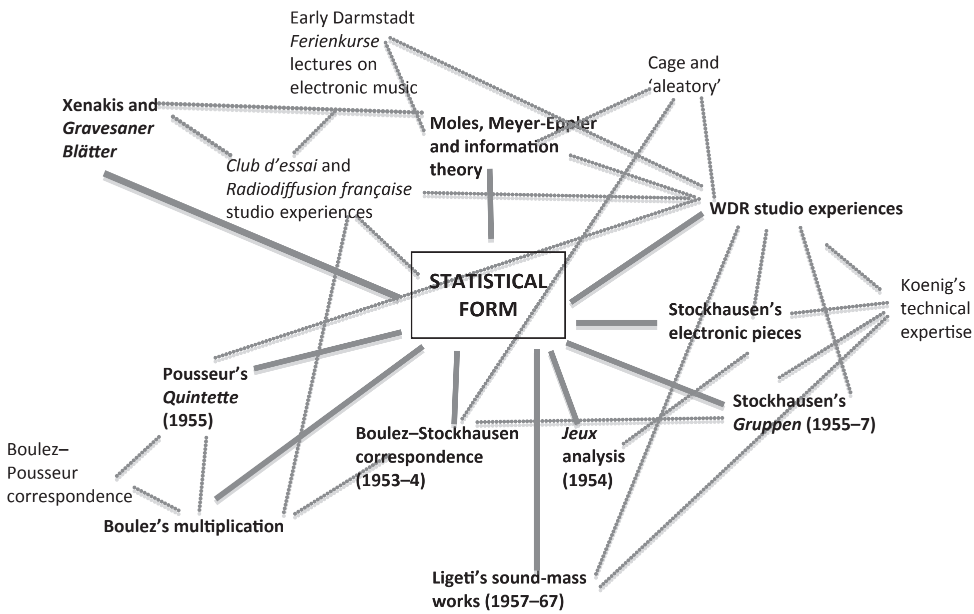
'Statistical form' and similar terms began appearing in the discourses of the Darmstadt composers soon after Stockhausen's talk. Henri Pousseur, writing in 1955 about additive synthesis in electronic sound, suggested that inner partials were 'statistic phenomena, a kind of source of probabilities', in contrast to the global and perceptible *Gestalt* of the composite sound or timbre (Pousseur [1955] 1958, pp. 32–3). Herbert Eimert suggested that form in electronic music depended upon 'statistic structures' that organise sound according to densities (Eimert [1955] 1958, p. 7), and Werner Meyer-Eppler entitled his article in the same volume of *Die Reihe* 'Statistic and Psychologic Problems of Sound' (Meyer-Eppler [1955] 1958, p. 55). Luciano Berio discussed *statistico* and *probabilistico* techniques in a 1956 *Incontri musicale* article, where he suggested that statistical conglomerations could lead composers to consider larger-scale proportions, which were more perceptually relevant, in their serial designs (Berio 1956, pp. 65–6).[4] The growing interest in statistical form is apparent, but the actual definition of statistical form needed to be further clarified.

Stockhausen's use of statistical terminology in his analysis of *Jeux* fits well within the basic premises of descriptive statistics. Instead of dealing in probabilities and predictions, descriptive statistics defines data sets through distribution (range of values), central tendency (mean, median and mode) and dispersion (standard deviation) (Trochim, n.d.). In a 1971 interview, in which Jonathan Cott asked, 'What exactly is this statistical, aleatoric structure?', the premises of descriptive statistics appeared in Stockhausen's reply: 'It's a random distribution of elements within given limits. ... If there's a tendency, then it's a directional statistical one – going upward or downward, becoming thinner, thicker, brighter, or darker. [...] A mass has a certain density, it has certain tendencies, it has shape' (Stockhausen 1973, p. 73).

Notably, Stockhausen was more forthcoming about the definitions of statistical form in the 1970s than he was in the 1950s, when the concept was compelling but also loosely defined. Amongst some of the Darmstadt-affiliated composers, including Stockhausen, Ligeti, Boulez and Pousseur, one finds much discussion and debate of the concept. In what follows, I read scores against sketches, correspondence and writings to investigate the multiple sites where statistical form is invoked, defined and applied. I argue that Stockhausen's noticeably opaque terminology, as well as the appropriation of Debussy's *Jeux* as his main example for statistical processes, disguises a much deeper and more pertinent connection to electronic music technology and information theory. Along the way I question the roles of Xenakis and Cage: how does statistical form relate to Xenakis's stochastic music or Cage's contemporaneous innovations in aleatory composition? Throughout I emphasise controversy amongst the Darmstadt composers, and I do not seek to soften the intense debates amongst those strong personalities. In suggesting that many of them used statistical form, I do not mean to imply that they all meant exactly the same thing, understood the concept in the same way or used the tools towards the same ends. Rather, there is a remarkable diversity of approaches to incorporating statistical form, which each composer realised within his own aesthetic and technical vocabulary. This shows, however, that the composers likewise were engaged in an evolving, collaborative discourse around statistical form. In conclusion, I shall suggest that statistical form represents a second stage of serialism, during which many of the Darmstadt composers used tools from electronic music and information theory to move away from pointillism and towards denser textures and more perceptible *Gestalten*.

Fig. 1 is a visual rendering of the discourse around the concept of statistical form. The solid lines, which lead to bold text, show the main contributors or direct paths to the concept of statistical form. The dotted lines show the secondary factors and multiple dialogues between the composers and their works: composers, pieces, correspondence and institutions are allowed to occupy similar roles as main contributors, though it will become clear that humans animate the network. 'WDR studio experiences', for instance, acknowledges the institutional commonality that the Darmstadt composers could draw upon, because many of them visited and learned the basics of electronic composition at this studio. As the secondary dotted lines show, however, this institution is deeply shaped by Eimert's intellectual leadership, Koenig's practical tutorials and innovative work-arounds for the technological equipment, Meyer-Eppler's teachings on information theory as conveyed via Stockhausen, and so on. I am sensitive to chronology, but I also re-order the topics conceptually, since in some cases certain connections (for instance between Ligeti's sound-mass works and Stockhausen's *Gruppen*) are compelling. I begin by making the case that the electronic music studio, especially via Koenig's technical expertise, was the real locus of the concept of statistical form. I then investigate Stockhausen's *Gruppen* and Ligeti's sound-mass works, showing how their experience at the WDR may

Fig. 1 Main and secondary contributors to the discourse around statistical form



have prompted statistical experiments in their acoustic works. I then delve more deeply into Meyer-Eppler's transmission of information theory, which raises the question of Xenakis's role. I evaluate whether the debate over Cage and aleatory composition impacts statistical form. From there I suggest that Boulez's understanding of pitch multiplication was shaped by the statistical form concept, and finally I show that Pousseur's relatively unknown *Quintette a la mémoire d'Anton Webern* uses statistical concepts.

Electronic Music at the Westdeutscher Rundfunk

If Stockhausen's definition of statistical form was somewhat ambiguous in his analysis of *Jeux*, he also could have been more forthright about the electronic context in which these ideas became relevant for him. Stockhausen's analysis may have first been broadcast under the title 'Von Webern zu Debussy: Formprobleme der elektronischen Musik' – at least, this is the title that appears in Herbert Eimert's introduction to the broadcast.[5] However, in the published version in Stockhausen's *Texte*, the subtitle is altered to 'Bermerkungen zur statistischen Form', a change that disguises the significance of the electronic studio for the evolving concept of statistical form. Throughout the essay as published in the *Texte*, Stockhausen avoids discussing the electronic music studio directly and instead couches his approach to statistical form within the relative security of the Debussy-Webern lineage. As I will argue, the electronic music studio was the true genesis of the concept, but perhaps because of the relative newness and instability of the technology it may have been difficult, precarious or undesirable for Stockhausen to reveal electronic inspirations directly.[6]

It is well-known that the concepts and methodology in use at the WDR studio in the 1950s relied strongly on serial thinking (Borio 1993, Decroupet 1994, Ungeheuer 1994, Ungeheuer and Decroupet 1996 and Decroupet and Ungeheuer 2002). However, the working procedures for electronic music in this era mandated many compromises to the composers' serial schemes. The sine-tone generators and recorders were primitive and unwieldy, and much of the production work had to be done by hand, which was extremely gruelling and time-consuming (Toop 1979 and 1981, and Ligeti [1980] 2007). For instance, the composer and his multiple technical assistants spent much of their time splicing tape and controlling knobs, which introduced imprecision into a rigorously serial conceptual sketch (Pousseur [1955] 1958). Stockhausen explains that while producing some of the mixtures for *Gesang der Jünglinge*, he and two assistants simultaneously controlled the knobs for pulses, filter frequency and volume envelopes during recording, approximating Stockhausen's hand-drawn curves: '[T]his resulted in an aleatoric layer of individual pulses which, in general, speeded up statistically. But you could never at a certain moment say, "*This* pulse will now come with *that* pitch." This was impossible to predetermine' (Stockhausen 1973, p. 72). Gottfried Michael Koenig, who collaborated with Stockhausen to realise *Gesang* and was an expert on the WDR technology, was probably the one to suggest the technical work-arounds that approximated the effect of Stockhausen's detailed plans, but were much more practical for the studio equipment (Koenig [1955] 1958). During the composition of *Gesang der Jünglinge*, Konrad Boehmer says that

> Koenig was convinced that it would not be at all sensible to record thousands of centimeter-long particles of sinus tones and to then measure, cut, and finally glue them together. For this reason, he proposed a quasi-aleatoric production process in which he began with [...] a sinus-glissando that would be automatically divided into distinct, small particles. If several such tapes are synchronized (with different 'rhythms' and glissandi), one hears a 'cloud' of tiny sound-particles with an all-embracing global direction. (Boehmer 2002, pp. 62–3)

Very quickly, serial composers realised that they had to make concessions to accommodate the limits of the studio technology; in addition to the imprecision of manual work, there were problems such as excessive tape noise. Thus, Koenig says, one must supplement the musical imagination with technical know-how: 'One solution for this problem [excessive tape noise] can be provided by a thought-out plan of realization, which translates the musical structure into a technical one. In doing so the necessity for simplification should not be enforced at the expense of the score' (Koenig [1955] 1958, p. 53). Koenig's comment suggests that aesthetic considerations were still paramount. When the serial plan had to be amended and simplified, Koenig seems to have been the expert in teaching the other WDR composers to find shortcuts – from approximate curves to superimposed mixtures – that did not really affect the aesthetic results of the composition that much.

This pragmatic approach was something both Koenig and Stockhausen may have learned from Werner Meyer-Eppler, the scientist and early founder (with Eimert and Robert Beyer) of the WDR studio. Stockhausen had heard Meyer-Eppler lecture at Darmstadt in 1951, encountered him at the studio in 1952 and 1953 (Fess 2004, p. 117) and began to study with him regularly, meeting with him two or three times a week in 1954 (Maconie 1990, pp. 46 and 58). The very first sentence of Meyer-Eppler's 1955 *Die Reihe* article suggests that he was oriented towards statistical processes in electronic music: 'A process is said to be aleatoric if its course is determined in general, but depends on chance in detail' (Meyer-Eppler [1955] 1958, p. 55). Meyer-Eppler used 'aleatoric' as a synonym for 'statistical', a terminological substitution that I shall address later; but we can grasp immediately that he was proposing exactly the same kinds of compromises and approximations in the studio as Stockhausen and Koenig acknowledged.

In fact, Meyer-Eppler probably taught these younger composers where they could and should make compromises in their rigid serial designs. In an article in a 1954 technical manual, he dealt extensively with how perception meets composition. His research in phonetics and psychoacoustics led to the admission that '[a]n arbitrarily fine gradation cannot be attained with respect to either pitch intensity or sound intensity' (Meyer-Eppler 1954, p. 6). In other words, he researched the limits of human perception and went on to suggest ways to exploit these limits; there was no sense, for instance, in insisting on mixtures with frequencies that were too close to be distinguished by the human ear. As Robin Maconie suggests, 'Meyer-Eppler's involuntary benediction freed Stockhausen from the theoretical constraint under which he had been suffering' (Maconie 2005, p. 133). It is likely that Meyer-Eppler's research and Koenig's technical tutoring helped Stockhausen understand how to use statistical interpolations in the serial structure while composing electronic music.

I have thus far emphasised the role of the WDR studio personnel in devising statistical compromises in composers' serial schemes. This is not to say that composers working at the Paris studio led by Pierre Schaeffer would have been ignorant of these procedures. Though the Germans and French made much of their differences in the very early 1950s – the sine-tone generators of *elektronische Musik* versus the sampled real-life sounds of *musique concrète* (see Beyer [1951] 1999, Meyer-Eppler [1951] 1997 and Schaeffer [1951] 1997) – by the mid-1950s these distinctions had softened considerably. For instance, *Gesang der Jünglinge* prominently uses both *concrète* samples of a boy's voice and sine-tone mixtures and pulses. The distinction may have quickly softened because, despite the disagreement about the appropriate source material for electronic music, the two studios used similar equipment to process and manipulate the sound. Thus composers working in any electronic music studio in the mid-1950s would have been proficient in splicing magnetic tape by hand, adding reverberation, manipulating tape speed, using ring and amplitude modulators and high-, low- and band-pass filters, gating the sound electronically or manually and adding volume curves by hand (Chadabe 1997, Manning 2004 and Holmes 2008). Furthermore, Boulez and Xenakis, who did not work at the WDR but who did experience electronic music at Schaeffer's Paris studio, also were exposed to many of the technical limitations that induced statistical compromises in electronic music.

Stockhausen's *Gruppen*

Following on the heels of his early experiences in the studio, Stockhausen applied some of the insights of statistical processes in *Gruppen* (1955–7).[7] Fig. 2 reproduces the sketch for group 131, which matches up with rehearsal number 156 in the score (as Stockhausen notes on the sketch) owing to earlier interpolations of non-serial, freely composed material. This drawing represents the relationship of orchestral thickness on the vertical x-axis and attack frequency or acceleration on the horizontal y-axis. The marimba and woodblocks (marked

Fig. 2 Stockhausen, sketch for *Gruppen*, rehearsal number 156. Reproduced courtesy of the Archive of the Stockhausen Foundation for Music, Kürten (www.karlheinzstockhausen.org). Used with permission

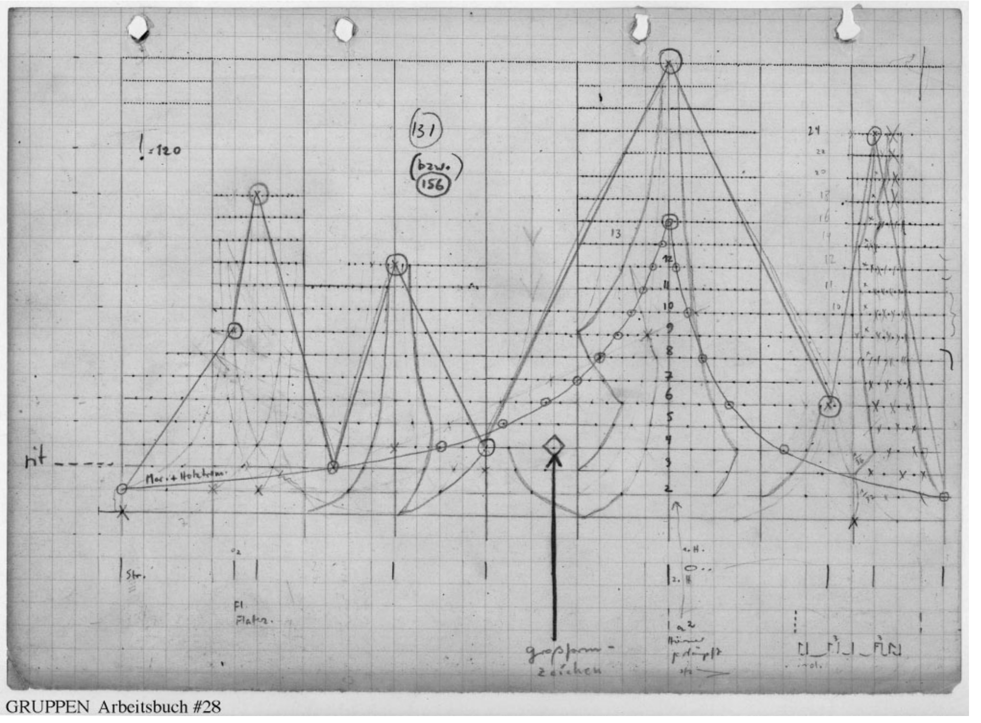
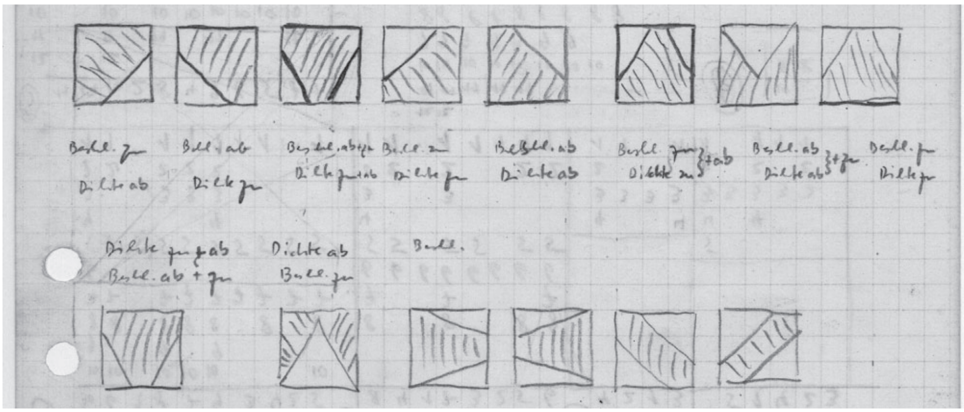


Fig. 3a Stockhausen, sketch for *Gruppen*, 'Dichteverhältnisse der Formanten'. Reproduced courtesy of the Archive of the Stockhausen Foundation for Music, Kürten (www.karlheinzstockhausen.org). Used with permission

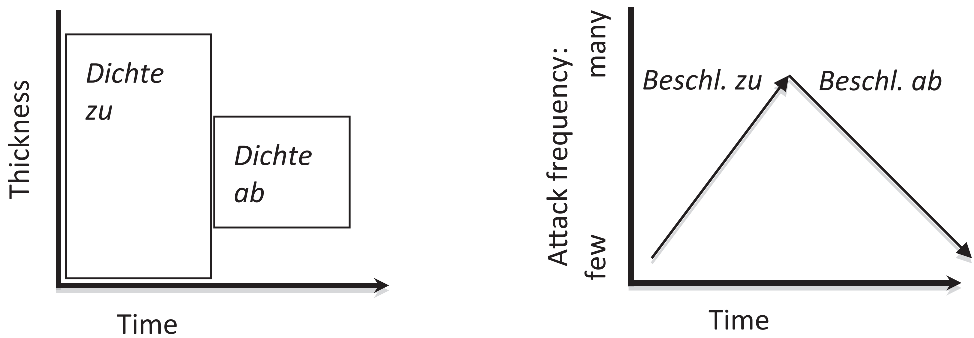


'Mar.' and 'Holztrom.') are shown on the exponential curve. The attack frequency of these wooden percussion sounds increases to a peak and then quickly dies away, as shown visually by the circled articulations on the curve's trajectory. The mountain-like sharp peaks above the curve map out density, or orchestral thickness. The densest vertical texture coincides with the most frequent percussion articulations. Perhaps counter-intuitively, these textures do not necessarily coincide with the loudest moments; the peaks simply represent number of instruments playing at one time.

Stockhausen's attention to the relationship between the number of attacks and the density of orchestration recalls his parallel concern with changes in vertical and horizontal density in *Jeux*. The relationship between thickness and attack frequency was one that Stockhausen worked on in a conceptual, graphic sense as well. In Fig. 3a we see Stockhausen imagining the multiple ways in which thickness (*Dichte*) and attack frequency (*Beschleunigung*) can interrelate.[8] Fig. 3b clarifies that Stockhausen is actually superimposing two x–y coordinate planes in this sketch – one for thickness and one for attack frequency. In this way he can show changes in density (which are perhaps more intuitive in the x–y coordinate plane) at the same time as the linear boundaries of the mass suggest increasing or decreasing attack frequency.

Consider the sixth term in the top row of Fig. 3a, which best corresponds to the curves and peaks modelled in Fig. 2. The attack frequency crescendos to a peak before dying away (*Beschleunigung zu + ab*), and, with a few more intermediate fluctuations, the thickness does the same (*Dichte zu + ab*). Boulez uses very similar notation to describe these phenomena in *Boulez on Music Today* (Fig. 4). Though his accompanying prose addresses only issues of rhythmic distributions, it is clear that the shapes in the figure are made by superimposed

Fig. 3b Superimposed x–y coordinate planes for thickness and attack frequency used in Fig. 3a



layers of durations. In this way, his diagram takes into account both density (texture) and frequency of attack and is akin to Stockhausen's *Gruppen* sketch.

The sketches in Figs 2, 3a and 4 relate to electronic music in a number of ways. If one studies contemporaneous sketches for electronic compositions, one sees many conceptual sketches of this type: for instance, the sketch from Ligeti's *Artikulation* shown in Fig. 5, which shows some of the general sound envelopes that Ligeti envisaged for the piece. The generalised, shape-based figures in Fig. 5 may have been applied to volume or density of sound, or they may have been mapped onto a frequency chart or sound mixture to give fluctuations in pitch or timbre. The multiple applications suggested by these sketches are almost certainly a reflection of the studio technology discussed above. Exact serial precision was either impossible or too time-consuming, so composers had to prioritise the general outlines of figures. As Ligeti says about *Artikulation*, '[D]ifferent statistical distributions are arranged together and grouped [so that] imaginary words and sentences emerge. The whole composition is constructed according to serial principles, though they have largely been applied undogmatically' (Ligeti [1958] 2007, pp. 167–8).[9] Since conventional music notation was of at best marginal use for electronic music, composers might just as well have sketched in shapes or numbers that required somewhat less translation into the realisation score or technical score. They may have actually begun to prefer, or at least appreciate the value of, these conceptual sketches when it came to imagining the overall shape of a passage. As Meyer-Eppler's research and recommendations suggest, the serial schemes were good as far as they went, but when it came to producing electronic music, one had to take care that the major architecture of the piece would be audible. We find Stockhausen undertaking a parallel procedure in *Gruppen*. No doubt many serial calculations are deeply embedded in the work (Misch 1998 and 1999), but the sketches show also a gestural, conceptual approach to the material.

The concept of statistical form allowed Stockhausen to move away from the pointillist textures characteristic of early integral serialism and to directly address

Fig. 4 Blocks of duration, from Boulez (1971), p. 56

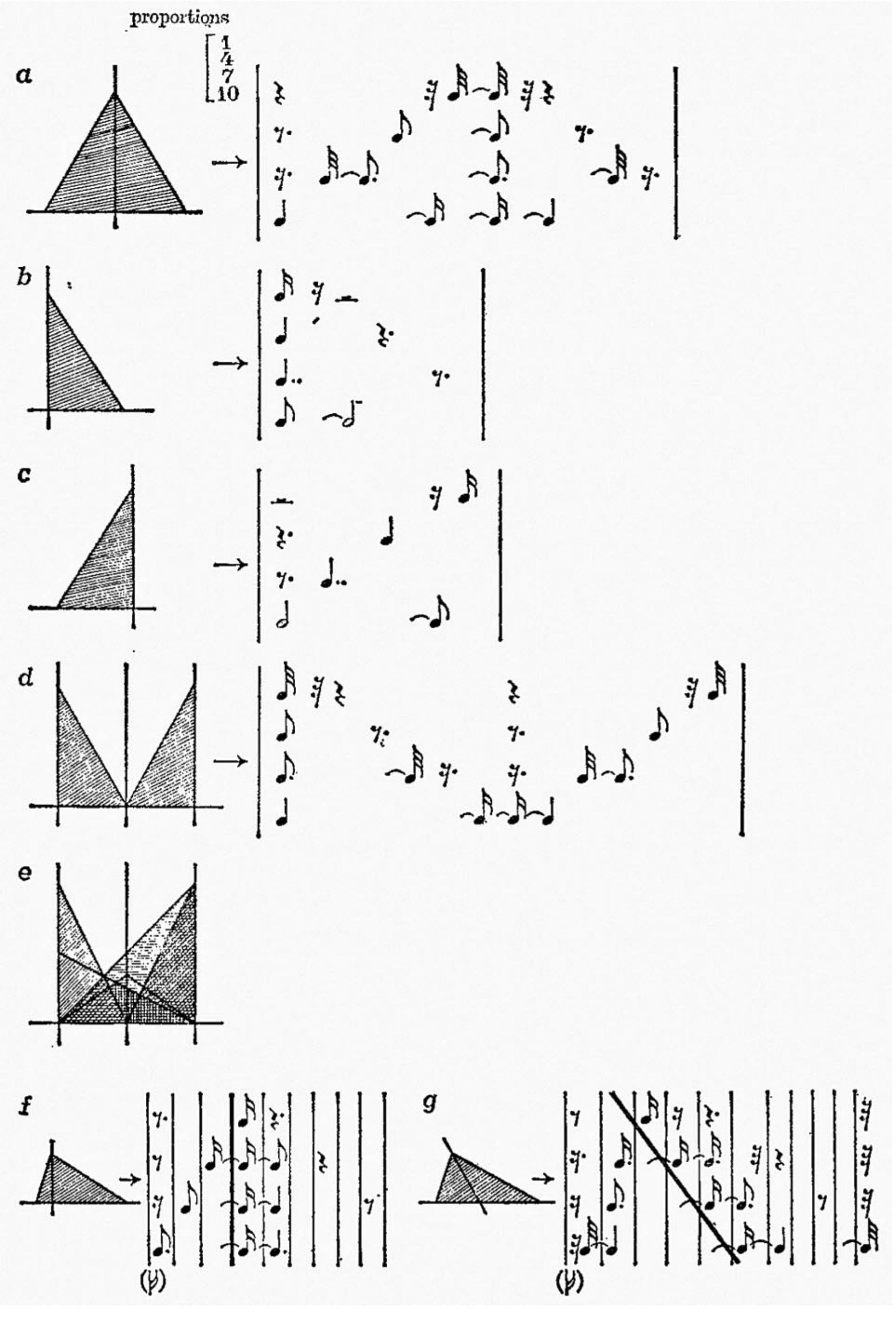
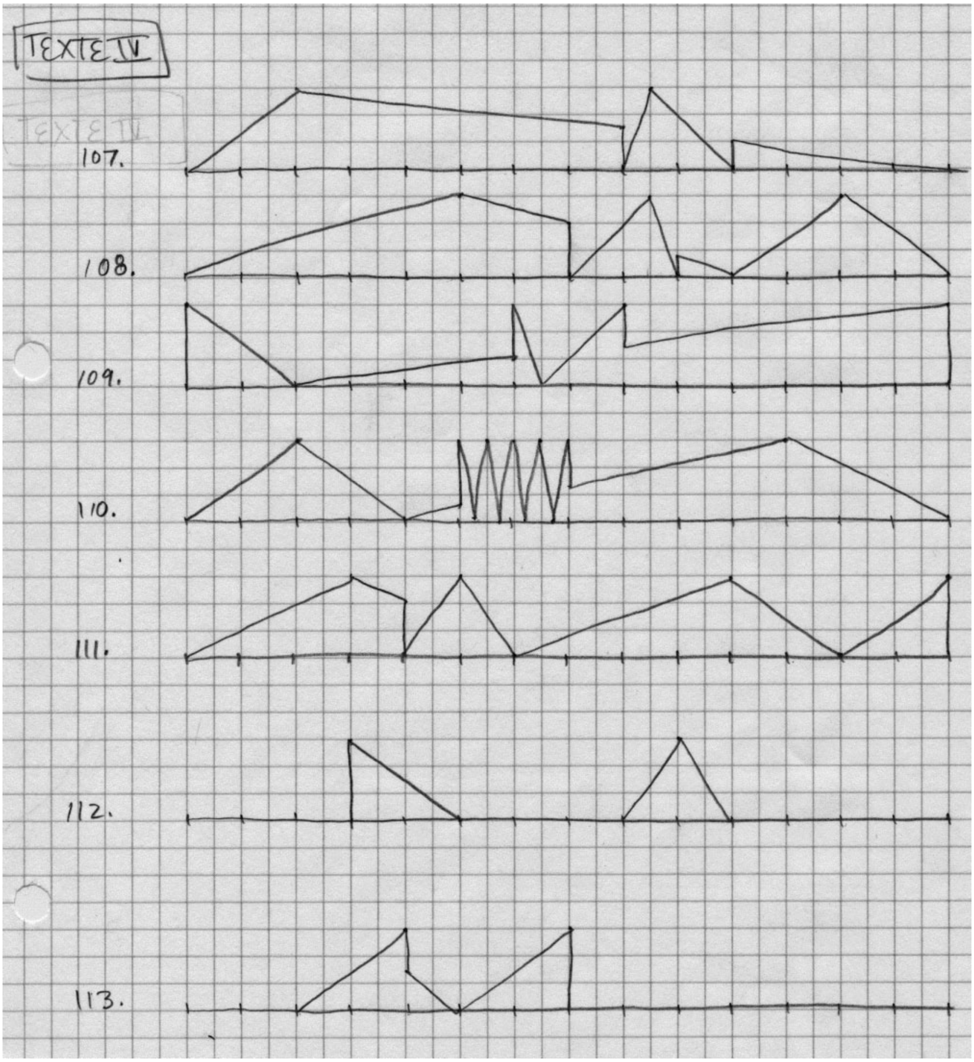


Fig. 5 Ligeti, *Artikulation*, planning sketch. Author's transcription of document from the György Ligeti Collection, Paul Sacher Foundation, Basel. Used with permission



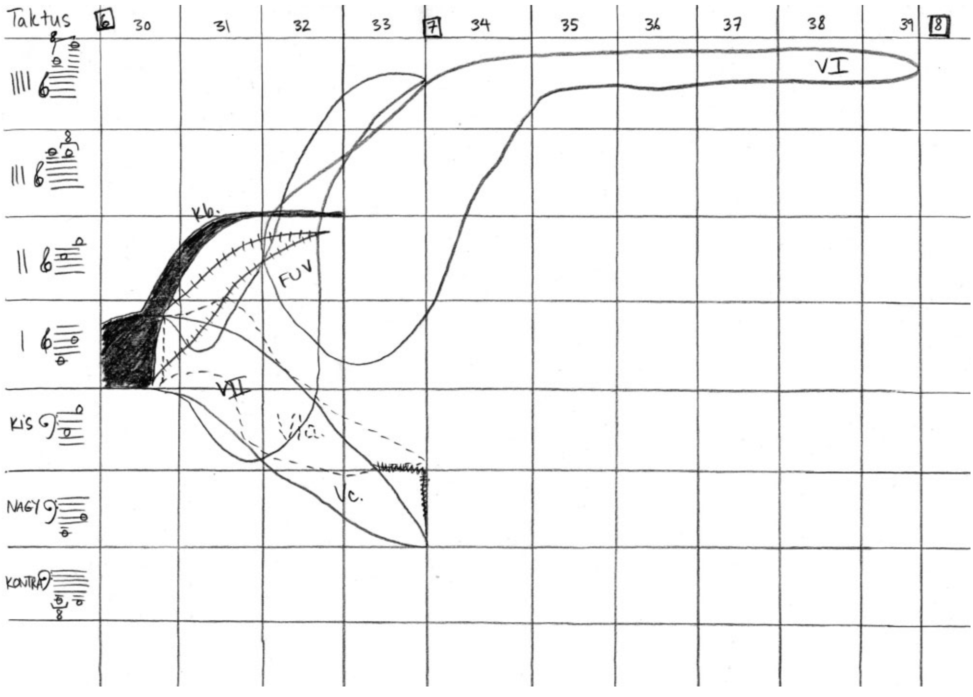
changes in texture and density as compositional considerations. In *Gruppen* in particular, Stockhausen kept using precise, serial designs to define the boundaries of shapes, but he began to experiment with ways of interpolating less controlled events within those boundaries. This had the aural effect of moving the sonic continuum towards groups, gestures and masses, rather than isolated events or points. Once he had clarified these compositional techniques in his mind and his work, he turned his language away from the studio and back into compelling metaphor; he speaks about statistical form as the transition of points into groups, a swarm of bees or the distribution of leaves on a tree (Stockhausen 1963, 1989 and 1973). Remarkably, he thoroughly co-opted the statistical methods, concepts and terminology of the electronic studio – especially adopting Meyer-Eppler and Koenig's technical innovations – and turned them into acoustic analogues. His opacity in both the definitions and origins of statistical form suggests that he advanced his own account of innovations in electronic and serial music, a fitting reflection of the well-known history of contested intellectual leadership at the WDR studio (see Stockhausen 1971 and Eimert 1972).

Ligeti and Sound-Mass Music

The electronic studio is the strongest common bond between Stockhausen and Ligeti's concepts of statistical form. Ligeti's exposure to *elektronische Musik* (Iverson 2009 and 2010, and Levy 2009) and to statistical-form concepts – though a few years later than the initial forays of Stockhausen, Boulez and Pousseur – is thorough and well-documented. Ligeti stayed at Stockhausen's apartment upon first arriving in Cologne from Hungary in early 1957, studied sketches for *Gruppen* with him and immediately began an internship in electronic composition with him and Koenig at the WDR electronic studio (Stockhausen 1989, p. 71; Stockhausen 1973, pp. 71–2; Ligeti 1983, p. 34; and Steinitz 2003, pp. 75–82). Ligeti's sound masses easily reflect the principles of statistical form, since they are necessarily defined by their outer limits. Recent graphic analyses also testify to the fact that Ligeti's sound masses are amenable to spatial representations that show boundaries and shape (Bauer 1997, Bernard 1987 and 1994 and Clendinning 1995). An early sketch for *Atmosphères*, transcribed in Fig. 6, shows that Ligeti initially thought of the work in terms of shape. The sketch shows octave registers in the left margin and bar numbers across the top of the chart. The free-form shapes represent the register and general direction that Ligeti imagined for each group of strings. Though it gives a sense of the direction of the passage, the sketch is fairly imprecise in terms of musical detail. Ligeti was to go even further in this direction in his *Volumina* for organ (composed 1961–2, rev. 1966), which contains no conventional notation or micropolyphony but instead focuses entirely on large-scale shapes and their texture, timbre and density (Fig. 7).

By studying pitch and voice-leading sketches for *Atmosphères*, *Apparitions* and *Lontano*, and also from the careful analyses of Bernard, Clendinning and others, we know that Ligeti was in most cases extremely precise about the inner composition of his sound masses, focused as he was on the micropolyphonic threads. Ironically, in the 1971 interviews with Cott, Stockhausen criticised Ligeti for focusing too much on weaving the parts and for 'avoiding all the *Gestalten*, the clearly defined figures' (Stockhausen 1973, p. 72). To many, some of the most striking features of Ligeti's works are in fact the large sound-mass *Gestalten*, and the sketches show that Ligeti often began there. It is thus difficult to agree with

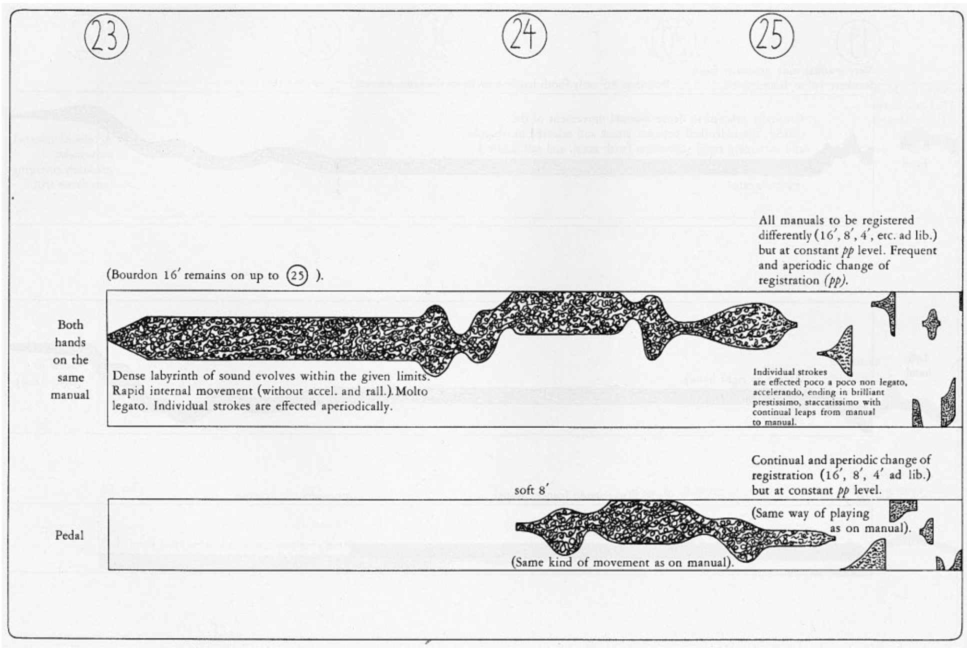
Fig. 6 Ligeti, *Atmosphères*, planning sketch, bars 30–39. Author's transcription of document from the György Ligeti Collection, Paul Sacher Foundation, Basel. Used with permission. Original document reproduced in facsimile in Steinitz (2003), p. 109, and Duchesneau and Marx (2011), p. 172



Stockhausen that Ligeti 'avoids' them. However, Stockhausen correctly points to a difference between himself and Ligeti: while Stockhausen preferred to randomise events within boundaries, Ligeti was exacting in his control and manipulation of the inner parts of the sound masses.

Stockhausen's criticism of Ligeti has a way of manufacturing controversy, of emphasising difference even when similarity is more compelling. Ligeti's emphasis on the global architecture of a gesture, along with considerations about density and internal movement, seems to map directly onto Stockhausen's own working definition of statistical form, but Stockhausen focuses on their difference. Thus, although Ligeti and Stockhausen may have both understood the usefulness of statistical form via their experiences in the electronic studio, they used those tools for slightly different aesthetic and technical ends in their acoustic music. While Stockhausen introduces randomised interpolations within serially controlled boundaries, Ligeti usually prefers to maintain control at the micro-level of compositional surface; but in the late 1950s and early 1960s both composers emphasised large, globally perceptible shapes and masses.

Fig. 7 Ligeti, *Volumina*, score, p. 14. Copyright © 1973 by Henri Litolff's Verlag. Used by permission of C. F. Peters Corporation. All Rights Reserved



Meyer-Eppler, Moles and Information Theory

So far, we have seen that Stockhausen and Ligeti both leveraged their electronic music experiences towards their own interpretations of statistical form. It could be argued that Meyer-Eppler, though remaining apart from the WDR studio infrastructure, was a motivating intellect in shaping these discoveries. A mathematician and scientist based in Bonn and the 'most renowned German representative of information theory' (Eimert [1962] 1968, p. 5), he showed many of the Darmstadt composers how concepts from physics, acoustics and information theory applied to (electronic) music composition (Sirker 1975, Ungeheuer 1992 and 1996 and Kautny 2001). Abraham Moles was another scientist with connections to acoustics, physics and psychology. He sat on the editorial board of the *Gravesaner Blätter*, the Swiss journal edited by Hermann Scherchen, which published many research papers and documents relating to Scherchen's plan to build a cutting-edge electronic music studio. Both Moles and Meyer-Eppler wrote important books that conveyed and translated central insights of information theory for use in musical composition (Meyer-Eppler 1949 and 1959 and Moles 1958 and 1966).

Information theory was a hot topic in the 1950s. An evolving line of scientific inquiry, it was concerned with the transmission of messages of any kind. This was on the one hand a technological problem; early telephone technology, for instance, had to confront the problem of changing sound waves into electricity, a process which frequently resulted in garbled messages on account of inadequate bandwidth, poor control of amplitude and frequency modulation, or noisy interference in the communication channel (Gleick 2011). Claude Shannon, an engineer at Bell Telephone Laboratories who is widely known as the founder of information theory, devised mathematical equations that defined bandwidth or channel capacity and showed how to calculate information content (Shannon and Weaver [1949] 1964). These equations were to be useful to engineers who were inventing and improving communication technologies and were also important in early digital computing.

On the other hand, information theory also had philosophical and phenomenological implications for all kinds of communications. Shannon was particularly careful to calculate the degree of redundancy or predictability in messages, because successful information transmission was directly related to how predictable messages were. As Moles summarised in his psychological brief on information theory, the most difficult messages to transmit were information-dense messages. If the message contained the maximum elements or was not segmented or grouped, it could appear to the listener as '*indistinguishable* from *background noise*, with a uniform probability distribution for its elements. It loses all interest because it lacks intelligible meaning' (Moles 1966, p. 61, emphasis in original). The receiver of the complex, information-dense message needs to be able to guess or predict what will come next on the basis of previous experience.

Shannon argued that, since language was predictable, its redundant structures could be described through probability and statistical analysis. This grew out of his ciphering and cryptography research during World War II, where as a code breaker he observed that not all letters in the alphabet occur with the same frequency (Gleick 2011, p. 216). For example, the most frequently occurring letter in a code is likely to stand for *E*, as this is the most frequently occurring letter in English (Gleick 2011, p. 225; Moles 1966, p. 41; and Shannon and Weaver [1949] 1964, p. 39). This might be termed the intrinsic or 'first order' probability of an event. Letters are also delimited by events occurring around them, or contextual 'second order' probabilities; for instance, when the letter *Q* occurs in English, it is very likely to be followed by *U*. Words and sentences are further constrained by grammatical structures (Shannon and Weaver [1949] 1964, p. 43). One could use these probabilities to reduce the size of a message by trimming redundancies; or, in another approach, one could mitigate interference in an otherwise intelligible message by adding redundancies or 'expanding the verbosity of [the] discourse' (Gleick 2011, p. 224).

Because information theory addresses messages of any kind, the theory was extremely generalisable (Both 1995). Its basic equations were articulated first in mathematical terms, but the field quickly became highly interdisciplinary. As Warren Weaver, one of the early proponents of information theory, writes, 'This is a theory so general that one does not need to say what kinds of symbols are being considered – whether written letters or words, or musical notes, or spoken words, or symphonic music, or pictures. The theory is deep enough so that the relationships it reveals indiscriminately apply to all these and to other forms of communication' (Shannon and Weaver [1949] 1964, p. 25). Thus information-theoretic concepts quickly began appearing in fields such as psychology, genetics, sociology, linguistics, philosophy and music (see Joel Cohen's translator's note in Moles 1966).[10]

Mid-century music has at least three obvious overlaps with information theory. First, the problem of noisy channels – interference in the transmission of a message – was a pragmatic, technological problem in the electronic music studio. For instance, composers had to mitigate the excessive tape noise that resulted from recording and re-recording sine tones from just a few generators during additive synthesis.[11] Likewise, amplification introduced noise into the signal. The development of information theory had a deep impact on the engineering, technology and working procedures at the WDR and other electronic music studios. Meyer-Eppler's first book, *Elektrische Klangerzeugung*, dealt extensively with the technical details of acoustics in electronic sound, sine-tone and impulse generators, amplitude and frequency modulation and recording technology (Ungeheuer 1992); all of the same technologies were implicated in mid-century communication systems. Thus the development of early electronic music was deeply linked to scientific and technological innovations carried beyond World War II along with the evolving information theory.

Second, because information theory was concerned with communication, there was an implied extension of research into human perception. Though Shannon had focused on the mathematical definitions of bandwidth and information content, the perceptual abilities of the receiver were a crucial part of the communication circuit. Sound is often periodic in both the pitch and temporal dimensions: consider sine waves and periodic rhythms, for instance. Sound waves are also continuous. This initially posed a problem in information theory because it was difficult to use continuous waveforms in algebraic calculations and to create enough bandwidth to handle continuous waves. Shannon solved this problem by sampling continuous waveforms at regular intervals, thus assembling a picture of the wave from discrete bits (Gleick 2011, p. 202). As it turns out, the human perceptual mechanism does the same thing (Meyer-Eppler 1959). Both Ligeti and Moles noted that a film, for instance, is assembled from still shots that are played fast enough to give the impression of continuity (Moles 1966 and Ligeti [1980] 2007). Stockhausen discussed the transition between discrete impulses and continuous sound at length in *... how time passes ...* (Stockhausen [1957] 1959). The premises of information theory suggested that composers may treat sound as discrete particles that can be grouped into *Gestalten* or organised into continuums (Meyer-Eppler [1955] 1958 and Both 1995). Thinking of sound in terms of continuums relates to research into phonetics carried out by Meyer-Eppler, who showed that speech sounds can be turned into music, music into noise, and so on (Ungeheuer 1992). As M. J. Grant says, 'The importance of the continuum in serial thinking cannot be overemphasised' (Grant 2001, p. 98; see also pp. 135–9). From an information-theoretic perspective, all sound is discrete particles that are reassembled in the perceiver's imagination into a continuous entity.

Third, musical concerns interfaced well with information theory's focus on predictability in linguistic structure. The always familiar comparison between music and language is a philosophical conundrum: while music is not semantic in the same sense as language, there is a concern for syntax, redundancy and predictability in musical structure (Eimert [1957] 1959). Moles suggested that periodicity contributed to whether listeners would receive complex messages, because expectation plays an enormous role in perception (Moles 1966). Predictability is an important feature of music, since it aids the perception of *Gestalten* and can make the syntactical or formal structure of the music clear to the listener (Meyer-Eppler [1962] 1968, pp. 9–10, and Moles 1966). In the first volume of *Die Reihe*, Meyer-Eppler suggested that musical events are predictable by intrinsic or 'first-order' probabilities, or the frequency with which that event is likely to occur on its own. Though he gives no specific musical examples, one might hypothesise that in tonal music, the tonic has the intrinsic 'first-order' probability of occurring most frequently. He also discusses the contextual or 'second-order' probabilities, which can be extended into higher levels of contextual probability (e.g. third order, fourth order, etc.) through Markoff chains (Meyer-Eppler [1955] 1958, pp. 57–8). For instance, in tonal music, when a dominant seventh chord appears, it is very likely to be followed by a tonic. In dodecaphonic and serial music, we can consider the predictability of the series; as the row proceeds, there are gradually fewer and fewer choices for pitch classes that have not yet appeared. The series is constrained by the progression through the twelve tones and thus becomes more predictable as the available pitch choices are limited. Whether or not the next event is very predictable, somewhat predictable or totally random depends on both the intrinsic and contextual probabilities for that event. Via information theory, the statistical nature of music may thus be tied to the probability of certain events. In this sense, statistical analysis moves beyond the descriptive tools we have so far discussed because it implicates probabilities, which can be usefully leveraged for music composition. As Meyer-Eppler says, 'Up to the present, mathematical results exist only for literary works but there is no reason why the same procedures should not be applied to musical compositions' (Meyer-Eppler [1955] 1958, p. 58). It is a view that Xenakis shared.

Xenakis and the *Gravesaner Blätter*

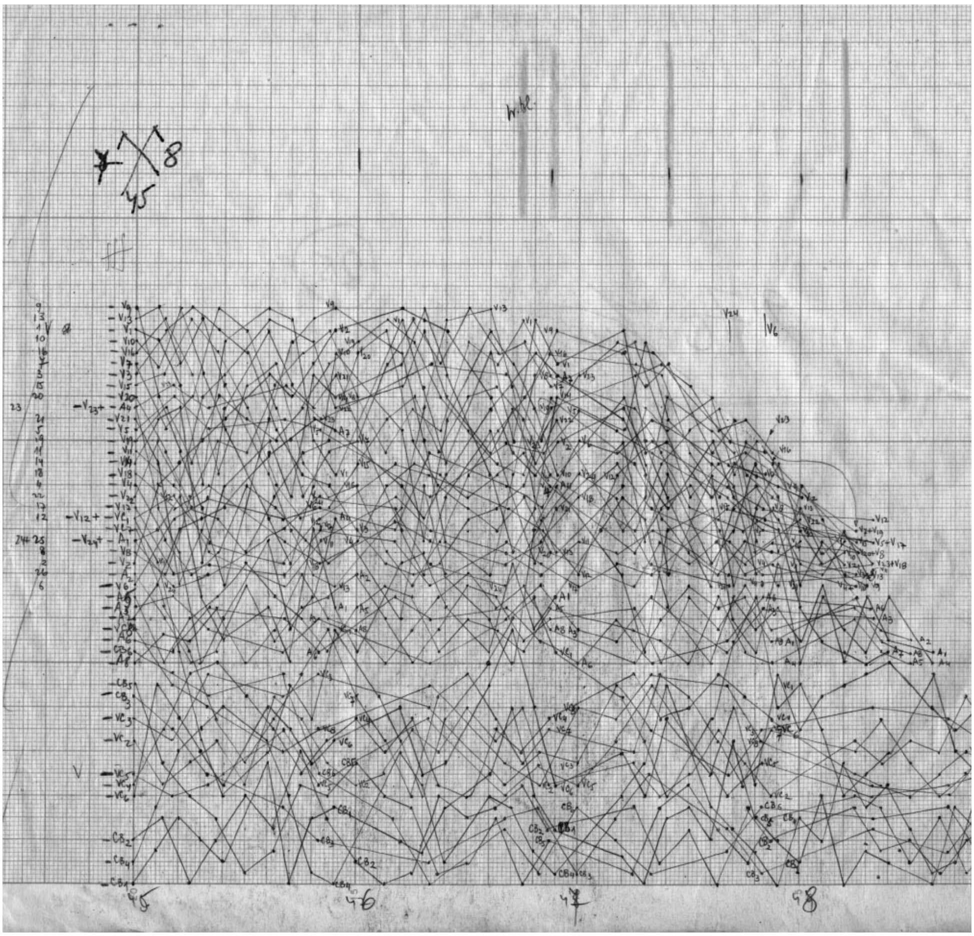
Xenakis is known for his stochastic music (beginning with *Metastasis* [1953–4] and *Pithoprakta* [1955–6]), or music that uses probability theory to determine the course of events. Whereas Stockhausen's use of statistics tended more towards the descriptive, Xenakis explicitly engaged with probability and prediction in his statistical structures, formalising the stochastic ideas in a series of articles in early 1960s (Xenakis 1960).[12] However, his work as Le Corbusier's architectural assistant, which he started in 1949, and his training in mathematics and engineering laid the foundation for his spatial and textural use of sound in the early to mid-1950s (Harley 2004 and Squibbs 1996). As Xenakis said, 'In the Philips pavilion, I realized the basic ideas of *Metastasis:* as in the music, here too I was interested in the question of whether it was possible to get from one point to another without breaking the continuity. In *Metastasis*, this problem led to glissandos, while in the pavilion it resulted in the hyperbolic parabola shapes' (Varga 1996, p. 24).[13] There is an overlap here between Xenakis's architectural interest in the parabola and information theory's use of continuities assembled from many discrete points. Either way he approached the concept, Xenakis applied the idea to continuity in musical pitch when he composed the parabolic glissandos of *Metastasis*.

Xenakis was exposed to developments in information theory especially via summits organised by Scherchen at his home in Gravesano, Switzerland. These interdisciplinary summits brought together leading scientists (Meyer-Eppler and Moles), electronic music studio directors and instrument builders (Trautwein, Enkel, Springer and Winkel) and new music conductors and impresarios (Scherchen, Souvtchinsky, and Steinecke). For example, Moles's 1956 colloquium on information theory and music was published in volume 6 of the *Gravesaner Blätter*, the same volume in which Xenakis's 'Probability Theory and Music' was also published (Xenakis 1956).

In 'Probability Theory and Music' Xenakis seeks to free serial music from its exceedingly linear, geometrical conceptions. He devises equations to handle the relative separation or overlap of articulations in time (*Zeitdauern*), the relative thickness or pointillist texture in the vertical dimension (*Frequenzen*) and the relative velocity of glissandos (*Schnelligkeiten*). In a figure that accomplishes aims similar to those of Stockhausen's thickness-attack frequency sketches for *Gruppen* (Figs 3a and 3b), Xenakis shows the possible correlations between density, tempo and frequency. The article also includes a graph-paper sketch from *Pithoprakta* that shows individual string instruments playing glissandos in varying velocities and ranges. A similar sketch is shown in Fig. 8. Xenakis explains that he has used an equation that adapts from physics the Maxwell-Boltzmann distribution, which describes the speed and movement of particles in a gas. Xenakis uses an equation to calculate the musical motion of the 'particles' or individual instruments as molecules, which change their velocity according to the 'temperature' of the sonic atmosphere (*Klangatmosphäre*).

Xenakis's individualistic treatment of the string players resonates deeply with Ligeti's sound-mass techniques in pieces such as *Atmosphères*. Likewise, his use of graph paper correlates with the density/time planning sketches of Stockhausen (*Gruppen*) and Ligeti (*Pièce électronique Nr. 3*; *Atmosphères*). Each of these composers ends up with a sound mass defined by its outer boundaries, though they use slightly different methods to define the inner composition of the mass.

Fig. 8 Xenakis, *Pithoprakta* (1955), facsimile of a detailed graphic sketch. Courtesy Famille Xenakis. Used with permission



Stockhausen worked graphically, showing that layers of independent, differentiated curves and continuums could fit within the boundaries of the mass (see again Fig. 2). Xenakis built his sound masses by deriving each instrument's range and speed variations from equations and calculations (Xenakis 1956 and 1960; see again Fig. 8). Ligeti seemingly thought about the inner composition of the masses more in musical terms, creating micropolyphony by layering a melody with intuitively varied rhythms in the individual string instruments.

Despite articulating ideas similar to those of Stockhausen, Ligeti and others and producing works comparable to theirs, Xenakis wavered between being *persona non grata* and a marginal figure in the Darmstadt avant-garde, for reasons that are still not fully explained. Xenakis seems to have been poised for entry into the group at any point. He studied under Messiaen from 1951 to 1953, saw his

Metastasis premiered by Hans Rosbaud at Donaueschingen in 1955 and had *Pithoprakta* premiered under Scherchen's baton at Musica Viva Munich in 1957; Scherchen performed *Pithoprakta* again at Darmstadt in 1958 (Harley 2004, pp. 4–16; and Varga 1996, pp. 33–7). Furthermore, Xenakis studied and composed *musique concrète* extensively at Pierre Schaeffer's Paris studio between 1955 and 1962. Yet he did not become integrated into the discourses of the Darmstadt composers, publishing in Scherchen's Swiss journal instead of the more visible German journal *Die Reihe*. He was very much on a parallel track to Stockhausen, and he was exposed to contemporary developments in information theory perhaps to an even greater degree than Stockhausen, given that his mathematical training gave him greater capabilities with respect to the foundations of information theory. But his primary points of reference – Schaeffer, Scherchen and the *Gravesaner Blätter* – were in France and Switzerland, not Cologne and Darmstadt.

Why Xenakis never became integrated into the Darmstadt School remains an open question, but we can hazard a guess as to the answer. As recorded in a 1996 interview, he was never welcomed into it:

> [*Varga:*] Did it not occur to you that you might work in the electronic studio of the *Westdeutscher Rundfunk*?
>
> [*Xenakis:*] No, because electronic music [as opposed to *musique concrète*] left me completely cold – and besides, Stockhausen was the absolute master. He had never invited me there. Pretending to ignore someone's work is a way of fighting it. (Varga 1996, p. 43)

As a second example of this *persona non grata* status, consider Xenakis's comments on participating in a special seminar on the future of music at the 1955 Donaueschingen festival, where *Metastasis* was premiered:

> Nobody invited me. The chairman of the conference was the convinced serialist Antoine Goléa, who didn't like *Metastasis*. [...] The French, the Germans and the Italians had formed an influential and exclusive club, that of serial music. Scherchen was then the only one who liked and supported what I was doing and who invited me to Gravesano to attend the meetings and give lectures. (Varga 1996, pp. 35–6)

In a series of interviews late in his life, Ligeti says that he developed his sound-mass ideas independently of Xenakis, since he had little exposure to Xenakis's music until 1961 or 1962 and even then did not understand the mathematical calculations in *Formalized Music* very well (Ligeti 2003, p. 98). Ligeti says his underexposure to Xenakis's music was due to 'Stockhausen's endless self-centeredness [*unendliche Selbstbezogenheit*]. [...] He and Iannis Xenakis were the two composers who would absolutely not be known as colleagues. [...] For Stockhausen and Boulez he [Xenakis] did not exist' (Ligeti 2003, pp. 95 and 98).[14] Though Ligeti expresses his own struggles against

Stockhausen and Boulez's perceived dominance in those interviews, he also confirms the *persona non grata* reputation that Xenakis had complained about.

Why might the more powerful Darmstadt composers have ostracised Xenakis in this way? I have not found evidence for a definitive answer to the question; but since Xenakis also distilled and translated the central ideas of information theory, he may have simply presented too much of a challenge to Stockhausen's self-proclaimed authority on statistical form. We have some evidence of this in an interview Stockhausen gave in 1994 in which he claimed all the credit for propagating statistical form and 'aleatory' ideas amongst the mid-century avant-gardists:

> I learned the term *aleatory* from him [Meyer-Eppler] and it was I who transferred it into music. A later use of this term (for example in an article 'Alea' by the Frenchman Boulez), as well as in American music as 'chance operations' or 'indeterminacy' (Cage), has been *directly* mediated by me from information theory through personal interviews or indirectly through publications. (Letter to Christoph Both, 1995, pp. 275–6; emphasis in original)

Here Stockhausen makes the astonishing claim that all the variations on the term 'aleatory' – a topic that I will address shortly – are mediated through his own publications and teachings. Thus, Stockhausen claims to have taken Meyer-Eppler's teachings on phonetics and information theory and translated them directly into music, showing all of his colleagues how to do interesting musical things with statistical ideas. Stockhausen's ideas were no doubt influential, but his claim in the above quote is much too all-encompassing. Even so, we can glean from his narrative that there was no room for others. Xenakis was deeply engaged with statistical compositional tools in music composition, but he devised them by marrying his undergraduate and professional training in engineering and architecture with contemporary developments in electronic music and information theory – not by following Stockhausen's lead.

While we are questioning to what degree marginal figures in the Darmstadt circle shaped the concept of statistical form, we must take up the question of Cage. Is statistical form a re-appropriation of some of Cage's contemporaneous innovations? Why do Stockhausen and Meyer-Eppler use 'statistical form' and 'aleatory' as synonyms?

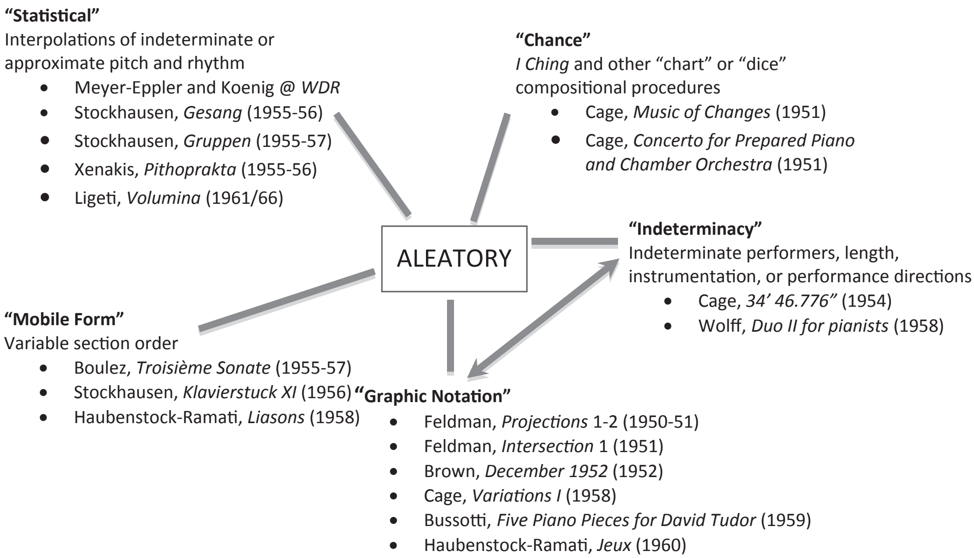
Cage and the Multiplicity of Aleatory Composition

Cage's aleatory techniques were famously and contentiously debated amongst the European avant-garde. Cage was at Donaueschingen for a performance of his *34′46.776″ for Two Pianists* in 1954, and David Tudor played Cage's *Music of Changes* at Darmstadt in 1956. Cage himself made an incendiary visit to Darmstadt in 1958 (Beal 2006; Borio and Danuser 1997, vol. 2, pp. 198–202; Shultis 2002, pp. 29–40; and Iddon 2013b, pp. 196–228). We might begin by asking whether Meyer-Eppler's and Stockhausen's substitutions of 'aleatory' for 'statistical' are a way of co-opting or neutralising Cage's ideas. Stockhausen was clearly intrigued by Cage and his ideas, but also perhaps threatened, as Joan Peyser relates in an anecdote told to her by Hans Werner Henze: 'Stockhausen "embraced" Cage. Henze extends his arms in a wide circle, then brings [his arms] together as though to caress a friend, and finally crushes them tightly against his chest' (Peyser 1976, p. 141). The anecdote suggests that Stockhausen initially signalled a friendly relationship with Cage, as is demonstrated early in the Boulez-Cage correspondence (Nattiez 1993 and Piencikowski and Nattiez 2002), but eventually tried to subsume Cage's ideas or neutralise his influence. Regardless of how Stockhausen felt about Cage, we must acknowledge that discussions about how to incorporate aleatory techniques into composition had been taking place since the early 1950s, and not all of them involved Stockhausen. It is well-known that Cage and Boulez met in Paris in 1949 and maintained a lively correspondence in the early 1950s but had a falling-out in early 1954 or thereabouts (Nattiez 1993 and Piencikowski and Nattiez 2002). Their feud became much more public when Boulez published the article 'Alea' in 1957 and Cage made his incendiary visit to Darmstadt in 1958, but these public disagreements came at the end of a long-standing debate amongst Cage and some of the Darmstadt composers about aleatory techniques (Grant 2001, pp. 144–6).

One of the reasons the aleatory debates flared up in the 1950s was the slippery meaning and multiple definitions of the term.[15] For instance, in his lecture 'Indeterminacy', the second given at Darmstadt in 1958, Cage discusses three ways in which chance can be incorporated into the composition (Fig. 9). Incorporating chance at the level of the compositional technique results in a piece such as Cage's *Music of Changes*, which uses random or probability-based methods such as the *I Ching* to determine the musical material: pitch, register, duration, articulation, dynamics, and so forth. Writing about this 'chance' technique in 'Alea', Boulez does not mince words: '[T]he adoption of a quasi-oriental philosophy [...] conceals a fundamental weakness in compositional technique' (Boulez [1957] 1991, p. 26). Ironically, Boulez had praised Cage's *Music of Changes* in letters to Cage in 1951 and 1952. But by mid-1954 (and possibly as early as 1952) he had changed his mind; the chance composer, he thought, does not have enough discernment or discipline to make crucial compositional decisions.[16] As Boulez acknowledges, this is only one way (and perhaps the crudest way) of incorporating chance.

For both Cage and Boulez, 'indeterminacy' is a term best applied when a general schema is laid out for the work, but the details of the realisation – from number of performers to instrumentation to ambiguity in performance directions – are left up to the performer(s). As Cage says, 'This is a lecture on composition which is indeterminate with respect to its performance. The *Klavierstück XI* by Karlheinz Stockhausen is an example. *The Art of Fugue* by Johann Sebastian Bach is an example' (Cage 1973, p. 35). Cage's apparent non sequitur calls attention to the fact that indeterminacy in performance occurs on

Fig. 9 Multi-faceted nature of the concept of 'aleatory' in the late 1950s



a continuum, where the composer may leave only the small details of timbre and dynamic markings to chance (Bach), or may leave the large-scale ordering of even the form up to the performer (Stockhausen). In 'Alea' Boulez remains critical of indeterminacy, suggesting that it still allows the composer to shirk his responsibility for the finished product: '[O]ne escapes choice, not by numbers, but through the performer. One transfers one's choice to him' (Boulez [1957] 1991, p. 28). In 'Indeterminacy' Cage ends his lecture by discussing graphic notation, especially as employed by Morton Feldman in *Intersection 3* (1953) and Earle Brown in *Four Systems* (1954), which introduces new complexities and new indeterminate parameters that the performer must manage. In 'Alea' Boulez elides all the many dimensions of indeterminacy that Cage carefully ferrets out, essentially dismissing graphic notation by failing to discuss it at all.[17] For Boulez, the composer is responsible for making compositional choices; this work simply cannot be pushed onto the performer (Goldman 2011).

Boulez does, however, make allowances for composer choice or accident within the work's structure, which relates to the 'statistical' category included at the top left of Fig. 9. As he explains, 'As a function of duration, its unfolding in physical time, musical development could admit "accidents" at various compositional stages or levels. The net result would be a rather high-probability succession of aleatoric events with a certain duration, itself indeterminate' (Boulez [1957] 1991, p. 30). His terminology ('high-probability successions of aleatoric events') coincides neatly with statistical approximations that Meyer-Eppler and Stockhausen had found so necessary in the electronic music studio: the general course of a gesture is determined, but its exact events within boundaries are flexible, improvised or indeterminate. That Boulez is comfortable with this limited application of statistical chance is unsurprising, since, as Stockhausen and Meyer-Eppler also emphasised, the global serial architecture of the piece is not compromised by the local indeterminate interpolations. For Boulez, the composer must retain control of the structure and the aesthetics of the composition on the global level.

Boulez's imagination is also clearly piqued by the possibility for mobile forms, since they can introduce subtle, dialectical relationships between indeterminate and fixed elements in the score. He might as well be discussing Stockhausen's *Klavierstuck XI* or his own *Troisième sonate* when he speaks of 'the desire to create a self-renewing kind of mobile complexity, specifically characteristic of music that is played and interpreted, in contrast to the rigid and non-self-renewing complexity of the machine'. Here he alludes to the uncomfortable rigidity of a piece such as *Structures* (1951–2), now in the late 1950s envisaging something more organic, more 'self-renewing', an 'evolving form which rebels against its own repetition' (Boulez [1957] 1991, p. 29). Here and elsewhere in 'Alea', Boulez shows that he is sensitive to the paradoxical relationship between serial automatism and absolute chance.

This oft-noted paradox can be clarified from an information-theoretic perspective, since both serial and chance music are extremely information-dense (see Grant 2001, pp. 131–64). In a pointillist piece such as *Structures Ia* or *Music of Changes*, each event is defined by its own pitch, dynamics, articulation and duration. There is very little that is predictable about the pieces, since there are very few redundancies in the musical languages of either total serialism or chance composition. Each bit of information is necessary to the integrity of the message. Whether or not listeners can comprehend a message with so much information, however, is a question that can almost certainly be answered in the negative. The problem of listener comprehension, in part, spurred the Darmstadt composers away from the pointillist first stage of serialism. As Ligeti famously wrote in his 1960 critique of serialism in *Die Reihe*:

> The finer the network of operations with pre-ordered material, the higher the degree of levelling-out in the result. Total, consistent application of the serial principle negates, in the end, serialism itself. There is really no basic difference between the results of automatism and the products of chance; total determinacy comes to be identical with total indeterminacy. This is the place to seek the parallelism (mentioned earlier) between integral-serial music and music governed by chance (John Cage). The following is characteristic of both types: pause–event–pause–event–pause, etc. (Ligeti [1960] 1965, p. 10)

Though Ligeti's formulation is elegant and rhetorically compelling, Boulez had made a similar observation as early as 1954: 'Perpetual variation – on the surface – produced a total absence of variation on a more general level. An exasperating monotony took hold of the musical work in which at every moment all means of renewal were in play' (Boulez [1954a] 1991, p. 17; see also Boulez [1957] 1991). Xenakis made a similarly incisive critique:

> Linear [serial] polyphony undoes itself through its very complexity. That which one hears, in reality, is only a mass of notes in various registers. The enormous complexity prevents the ear from following the tangle of lines and has, like a macroscopic effect, an irrational and random dispersion of sounds across the entire range of hearing. There is thus a contradiction between the linear polyphonic system and the expected result[,] which is surface, mass. (Xenakis 1955, p. 3)[18]

Ligeti's, Boulez's and Xenakis's remarks suggest that the problem really is one of hearing. Despite the complaints of conservative critics and audience members, the Darmstadt composers actually did care very much about listeners' experience of their works (see Boulez [1956b] 1991).

Because the Darmstadt composers recognised so quickly that both total serialism and total chance were aesthetically unsatisfying, many – in fact, all the composers discussed here – showed an interest in moving away from pointillist textures regardless of the compositional technique behind them (Griffiths 1995 and Osmond-Smith 2004). When it comes to statistical form, the strongest intellectual foundations are found in adaptations from the electronic studio and information theory. This is not to minimise the significant debates about Cage's particular use of chance or indeterminacy amongst the Darmstadt circle: composers struggled over where and how to incorporate indeterminate or chance elements in their compositions, since there were multiple ways of doing so. 'Statistical form' was only one way among many of incorporating aleatory elements. Statistical form is first and foremost a move away from pointillism, a textural device driven in part by concern for the listener's ability to perceive *Gestalten* in the musical structure. This was an idea that attracted both Boulez and Pousseur.

Boulez and Multiplication

In the early to mid-1950s, when Stockhausen was intensively developing his ideas on statistical form, Boulez was ensconced in finding a new way to extend his own serial thinking through pitch multiplication. Even so, the two composers frequently exchanged ideas about their most recent working methods and thoughts. In a letter to Boulez dating from November 1953, Stockhausen wrote: 'I find myself more and more taken up with statistical composition: serial "improvisation" within limits of the serial spaces of time, of pitch, of intensity. [...] The serial groups of intervals (pitch – equally intensity and duration) divide the statistical spaces. We keep ourselves within limits [of these statistical spaces], while working in the directions of the intervals.'[19]

Boulez, however, apparently struggled to grasp what Stockhausen could be proposing:

> Dear Karlheinz,
>
> Your last letter is really quite esoteric. I confess that in spite of all my application, and in spite of my habit of deciphering the ellipses, I have been unable to unravel

> all of the enigmas that you put before me in your last letter. [...] What are you calling 'statistical composition'? What do you mean exactly by 'improvisation' within certain limits? What are you calling 'statistical spaces'? What do you mean by 'work in the directions of the intervals'?[20]

Throughout November 1953 the two colleagues corresponded about statistical form, Stockhausen desperately trying to articulate his new ideas in French and Boulez repeatedly claiming that Stockhausen was only becoming more esoteric and opaque. But despite Boulez's initial difficulties in comprehending statistical form, there is evidence that by the early 1960s the concept had penetrated his thinking about pitch multiplication.

Pitch-class multiplication takes the intervals of the series as the primary generator of pitch materials but thickens the series so that it contains chords. Boulez parses the row into chords and transposes or 'multiplies' them at various levels. Thus, multiplication is already related to statistical form in that it compounds serial materials into groups rather than the isolated objects of the pointillist aesthetic. Boulez used this technique to generate compositional material as early as 1952 for the withdrawn piece *Oubli signal lapidé*, reusing the same pre-compositional chart in *Le Marteau sans maître* (1953) and 'Tombeau' from *Pli selon pli* (1959–60); he continued to use multiplication to generate materials for a number of other pieces extending well into the 1970s. It is clear that he was working on multiplication before he encountered statistical form via Stockhausen. However, there is evidence that Boulez extended the implications of this density-building technique to the musical surface after he encountered the statistical form discourses. In other words, I argue, statistical form helped Boulez think differently about multiplication and its musical ramifications.

Boulez wrote about multiplication in *Boulez on Music Today*, but he did not fully explain the operation (Boulez 1971, pp. 79–80). This leads Ciro Scotto to liken multiplication to a 'black box', where the inputs and outputs may be known – from Boulez's sketch material if from nothing else – but the exact operation that connects the inputs and outputs is vague (Scotto forthcoming, p. 4). The transcription in Ex. 2 shows some of Boulez's sketch material for the 'l'artisanat furieux' movements in *Le Marteau sans maître*; he began with a twelve-tone row, making five groups of pitch classes in the row according to rotations of the factor series 2–4–2–1–3. This procedure forms five rows, Alpha (24213), Beta (42132), Gamma (21324), Delta (13242) and Epsilon (32421). Ex. 2 shows the partitions only for row Alpha, and shows only two rows of multiplied objects below. Boulez always leaves the first of the Alpha, Beta, Gamma, Delta and Epsilon rows unmultiplied – that is, there are no aa, ab, ac, ad, or ae terms in Ex. 2.[21] Multiplication begins when the 'b' object in row Alpha is multiplied with a, b, c, d and e and transposed; the 'c' object in row Alpha is multiplied with a, b, c, d and e and transposed; and so on. This procedure continues for the remaining rows and results in a table of pre-compositional material (refer ahead to Fig. 10).

Since Lev Koblyakov began to decode Boulez's multiplication (Koblyakov 1977 and 1990), clarity about the procedure that happens inside the 'black box'

Ex. 2 Boulez, sketch for pitch-class set multiplication for *Le Marteau sans maître* and 'Tombeau', in row Alpha. Author's transcription and annotations. Original document in Pierre Boulez collection, Paul Sacher Foundation, Basel. Used with permission

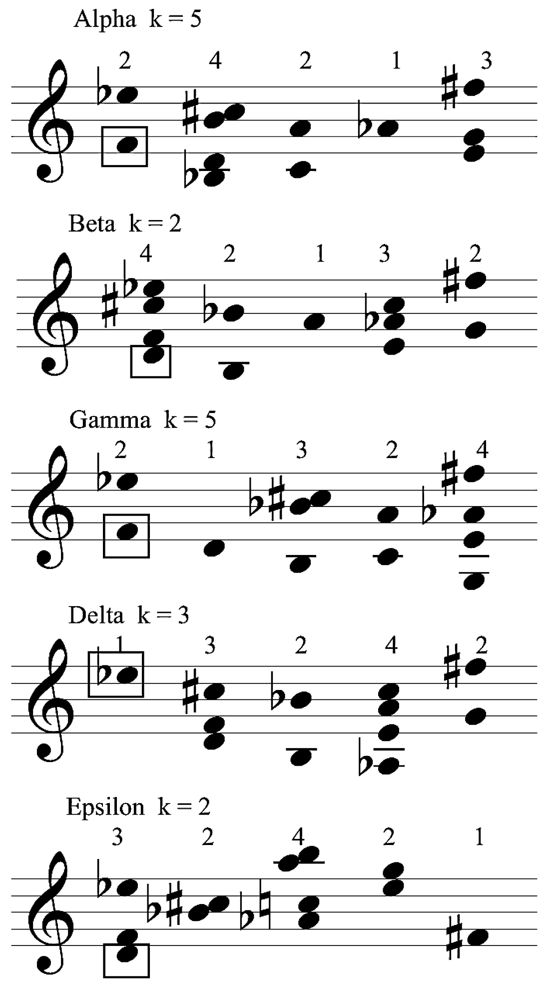
Table 1 Boulezian TC or complex multiplication formula, after Scotto (forthcoming) and Heinemann (1998); a and b are input sets; k and T_i are transpositional operators that are contextually determined

	Boulezian TC (Scotto) or complex multiplication (Heinemann)
Equivalent formulas	$T_i(a) + b$ OR $a + T_i(b)$ OR $T_i\,(a + b)$ $(a + b) - k$ OR $(a + b) + k^{-1}$

has gradually emerged. Heinemann divides the procedure into simple, compound and complex types of multiplication (Heinemann 1998). Scotto goes even further to simplify and clarify the operation when he shows that multiplication is formally and functionally equivalent to transpositional combination (TC) as theorised by Richard Cohn and Robert Morris (Scotto forthcoming, Cohn 1987 and Morris 1995). As Table 1 shows, Scotto's Boulezian TC produces results equivalent to those of Heinemann's complex multiplication: in both cases the pitch classes of the input sets are added and a transpositional operator 'k', which is contextually determined, is subtracted (or its mod-12 complement is added). One can also transpose either of the input sets by T_i, in this case the mod-12 complement of k, before performing the addition (Scotto forthcoming).

Mathematically, any k or T_i value will produce a product that is of the same set class, but Heinemann (1998) and Losada (2008 and 2014) have confirmed

Ex. 3 *Le Marteau sans maître* and 'Tombeau' row with rotating segmentation, showing k value or transpositional operator as lowest sounding pitch in the first segmentation. Author's transcription and annotations. Pierre Boulez collection, Paul Sacher Foundation, Basel. Used with permission



that specific compositional logic explains Boulez's particular transposition levels.[22] In sum, the transposition levels are intimately connected to the pitch classes and/or intervals of the series and produce specific formal and structural conditions in the completed pre-compositional multiplication tables and in the works that use them. As Losada (2014) demonstrates, Boulez uses a transpositional matrix for the *Marteau* table that calculates the intervals between 'roots' or lowest sounding pitches in the partition of the row. Ex. 3 presents a simplified logic for the transpositional operator in the 'l'artisanat' *Marteau* table, where k is drawn from the lowest sounding pitch class in the first partition of each row

(Heinemann 1998). Thus, the k value for all products multiplied from row Alpha partitions is 5, because pitch class 5 (F) is the lowest sounding note in the first partition of row Alpha. In the Beta and Epsilon rows, the k value is 2 after the pitch class D; in Gamma, the k value is also 5 (F); in Delta, the k value is 3 (E♭). It should be noted that thinking of the k value or transpositional operator as a pitch class is a convenient simplification that probably does not correspond with Boulez's compositional logic. Losada (2008 and 2014) shows how the transpositional operator relates to *intervals* inherent in the partitioned series, which is likely much more faithful to Boulez's own process: 'The internal structure of a series is crucial in the development of its organizational potential' (Boulez 1971, p. 70).

In Tables 2 and 3 I have summarised Boulezian TC or multiplication and generated the pitch-class content for the sketch portion shown in Ex. 2 using Scotto's TC matrices and the contextual k operators as described by Heinemann. Note that the k value (simplified as a pitch-class integer) must be subtracted from the preliminary product (a + b). If we think of the corresponding transposition, which is by convention calculated with addition rather than subtraction, then we must use the mod-12 complement of k. Thus T_7 is the operator connected to the k value 5 in row Alpha. Whether one uses Heinemann's simplified formula (a + b) – k, where in this case k = 5, or Scotto's transpositional combination T_7(a + b) is inconsequential for our limited

Table 2 Boulezian TC or complex multiplication generates pitch-class products for Alpha *ba–be*

Adding objects (mod-12)					Gives preliminary output (a + b)	–k (or +T_i)	Gives final output
ba	T	E	1	2	[1234567]	–5 (or +7)	[89TE012] = *ba*
3	1	2	4	5			
5	3	4	6	7			
bb	T	E	1	2	[89TE01234]	–5 (or +7)	[3456789TE] = *bb*
T	8	9	E	0			
E	9	T	0	1			
1	E	0	2	3			
2	0	1	3	4			
bc	T	E	1	2	[78TE12]	–5 (or +7)	[235689] = *bc*
0	T	E	1	2			
9	7	8	T	E			
bd	T	E	1	2	[679T]	–5 (or +7)	[1245] = *bd*
8	6	7	9	T			
be	T	E	1	2	[23456789]	–5 (or +7)	[9TE01234] = *be*
4	2	3	5	6			
6	4	5	7	8			
7	5	6	8	9			

Table 3 Boulezian TC or complex multiplication generates pitch-class products for Alpha *ca–ce*

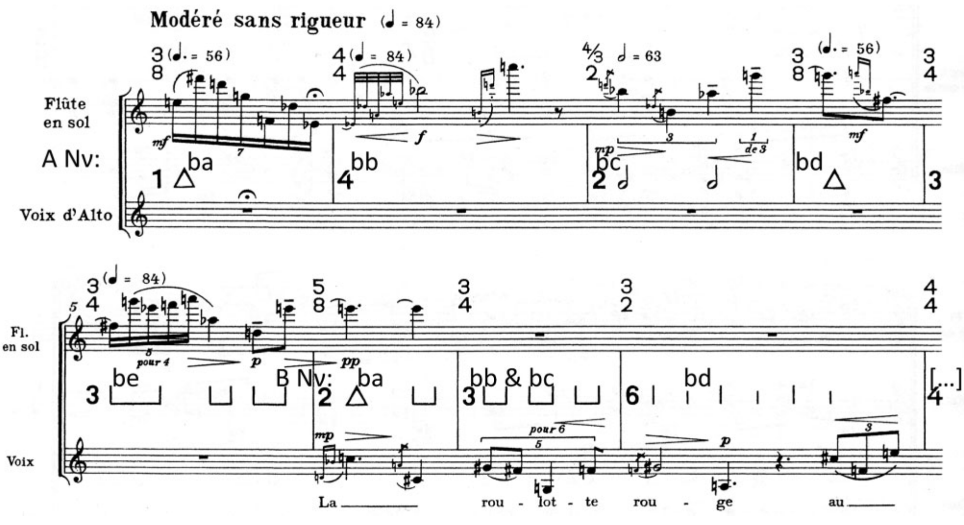
Adding objects (mod-12)			Gives preliminary output (a + b)	−k (or +T_i)	Gives final output
ca	0	9	[0235]	−5 (or +7)	[79T0] = *ca*
3	3	0			
5	5	2			
cb	0	9	[78TE12]	−5 (or +7)	[235689] = *cb*
T	T	7			
E	E	8			
1	1	T			
2	2	E			
cc	0	9	[690]	−5 (or +7)	[147] = *cc*
0	0	9			
9	9	6			
cd	0	9	[58]	−5 (or +7)	[03] = *cd*
8	8	5			
ce	0	9	[13467]	−5 (or +7)	[8TE12] = *ce*
4	4	1			
6	6	3			
7	7	4			

purposes. These operations have different implications with regard to their mathematical formalisms, but both generate the correct pitch-class content of Boulez's sketch.[23] Losada (2008 and 2014) is particularly strong in showing the way contextual intervallic relationships produce specific products in *pitch space*, that is, with particular registral dispositions. Interested readers may wish to consult Losada (2008 and 2014) for more on the underlying logic of Boulez's transpositional schemes.

For our purposes, we can suffice with this basic understanding of the operation of multiplication and now reflect, as Losada and Scotto do, on the results of multiplication. One obvious consequence is that harmonies instead of single pitch classes form the serial pre-compositional material. Boulez himself confirms that with the technique of multiplication, a move away from the pointillist style was one of his primary concerns. In fact, his 1963 reflections on multiplication have definite resonances of statistical form: '[T]here is no need to keep *defined* objects; the concept of serial generation can equally well be applied to *fields*, provided that they obey the fundamental laws stated above' (Boulez 1971, p. 41; emphasis in original). A few pages later he gives a summary that is closely aligned with the terms of statistical form articulated by Stockhausen in the *Jeux* article and in the correspondence between the two composers:

> This method [of permuting serial intervals through multiplication] gives us, so to speak sound 'surfaces' using either the true continuum or a rough approximation

Ex. 4 'L'artisanat furieux', *Le Marteau sans maître*, bars 1–8, after Koblyakov (1990). Pierre Boulez, *Le Marteau sans maître für Alt und 6 Instrumente.* © Copyright 1954 by Universal Edition (London) Ltd., London/UE 34133. Used with permission



of this continuum by the aggregation of all the unitary intervals included within the given limits; these are called clusters in the vertical sense, or glissandi in the diagonal sense. [...] My previous remarks about defined objects and fields must be recalled in this connection; the frequency band represents a field completely filled in with amorphous material. (Boulez 1971, p. 43)

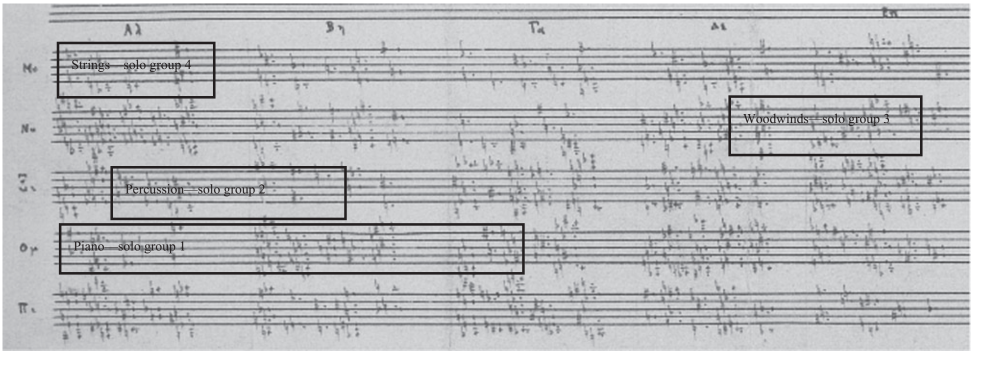
From this extended quotation it is clear that by 1963 Boulez understands the terms of statistical form that Stockhausen was so desperate to describe in the early 1950s. He has begun to conceptualise music in terms of sound spaces, clusters and fluctuations of density. Boulez's vocabulary, which refers to frequency bands, glissandi and continuums, also overlaps with the electronic studio techniques and possibly information-theoretic ways of thinking.

However, the textural, aesthetic ramifications of multiplication seemingly evolved slowly in Boulez's compositional output in the ten years between 1953 and 1963. Consider the musical surface of the 'l'artisanat furieux' movement of *Le Marteau* (1935–55), one of Boulez's earliest pieces to use multiplication. Lev Koblyakov's analysis of the beginning of this movement is reproduced in Ex. 4. Boulez is clearly using a sequence of the multiplied objects similar to those we calculated above. In some ways the gestural flourishes of notes concede a vaguely harmonic implication. Boulez very quickly stepped away from the harsh pointillism of *Structures Ia*; *Structures II* and *Le Marteau* – frequently described as 'sensual' (Ligeti [1958] 1960, p. 62) – are much more gestural than pointillist. However, for a realisation of the essentially harmonic device of multiplication, *Le Marteau* is still exceedingly linear. The main aesthetic sensibility of the music is melodic, not harmonic or textural.

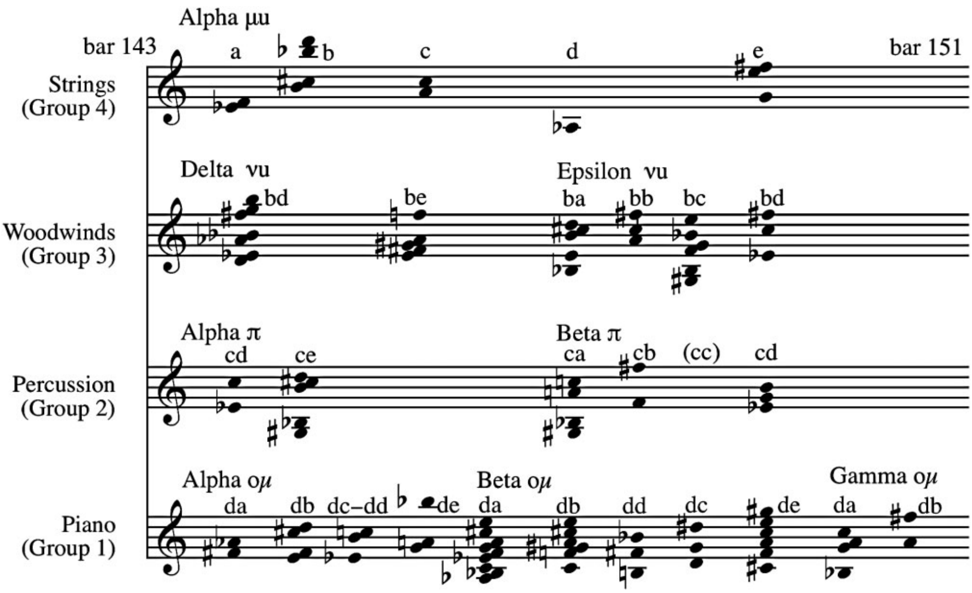
It would be another four years before Boulez capitalised upon the harmonic, textural implications of multiplication on the musical surface in 'Tombeau' (1959–60). As Robert Piencikowski details, the piece is divided into six sections, with an instrumental family being designated as primary in each section (solo piano, percussion, woodwinds, strings, plucked strings, brass) (Boulez 2010, pp. 28–30). After each instrument or group is introduced, it remains in the texture in an accompanimental role. This textural accretion of groups is mirrored in the pitch material, as each new principal group states the unmultiplied rows (Alpha, Beta, Gamma, Delta and Epsilon) that make up the first row in Boulez's pre-compositional chart. Then, as the group remains in the texture as accompaniment, it moves through the multiplied rows of the table below (Boulez 2010, p. 30). That is to say, the new groups enter at the top left of the chart and progress through each row from left to right.

Fig. 10 shows this schematically using Boulez's entire multiplication table for the piece. Though Boulez's tiny handwriting cannot really be read, the annotations show the progress of the instrumental groups in bars 143–151, from the middle of the fourth section.[24] The strings, being the fourth principal group, have entered the texture most recently and are playing from the unmultiplied rows at the top of the chart. The woodwind (group 3), percussion (group 2) and solo piano (group 1) have all been in the texture longer and have thus processed, reading the rows horizontally from left to right and top to bottom, more deeply into the chart. New instrumental groups tread the same paths as their predecessors, but at a delay. Piencikowski compellingly notes that this repeated procession of groups resembles a funeral march, in keeping with the nature of the commission, which honoured the deceased patron Prince Max Egon zu Fürstenberg (Boulez 2010, pp. 28–9). One consequence of this

Fig. 10 Boulez, *Le Marteau sans maître* and 'Tombeau' multiplication table sketch; my annotations show location for instrumental groups in 'Tombeau', bars 143–151. Pierre Boulez Collection, Paul Sacher Foundation, Basel. Used with permission. Original document reproduced in facsimile in Boulez (2005), p. 83 and (2010), p. 50



Ex. 5 Reduction of 'Tombeau', bars 143–151, showing pitch-class material in each instrumental group and schematic temporal alignment amongst layers



quasi-passacaglia or spiral form is that the texture thickens considerably throughout the movement. As instrumental groups are superimposed, the layers of melodic material thicken the texture. Furthermore, as groups process more deeply into the chart, the pitch language is increasingly drawn from the multiplied rows, which have denser pitch material.[25]

Ex. 5 illustrates this increasing density of texture and pitch material. Bars 143–151 have been reduced to show the pitch classes played in each of the groups and their temporal relationships with one another. The newest principal group, the strings, plays the sparsest material, from the unmultiplied top row (see again Fig. 10), and also is allowed the most space between articulations. The wind, percussion and piano are all playing denser sonorities from multiplied rows. The piano, in particular, plays frequently and cycles through pitch material much more quickly. Whereas *Le Marteau* did not capitalise on the harmonic and textural implications of the multiplication procedure, 'Tombeau' exploits them on the musical surface.

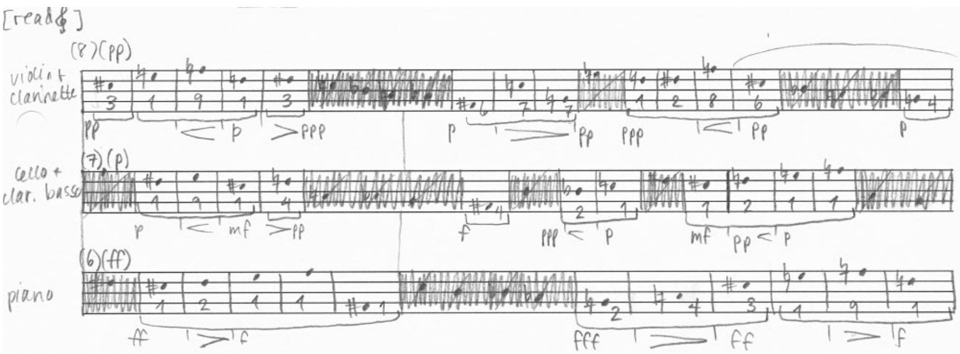
The density of the texture was a point of commentary for contemporary reviewers at the premiere, who were polite but could not quite make aural sense of Boulez's 'noticeably opaque textures' (quoted in Boulez 2010, p. 28). As Piencikowski writes, by the sixth section the layering of pitch material 'progressively saturates the acoustic space to the point of sound asphyxia' (Boulez 2010, p. 30). The progressive thickening of texture was definitely a perceptible element of the composition's vocabulary, whether or not it was judged aesthetically successful. If we want more confirmation that Boulez was indeed exploring the spatial and textural aspects of the multiplication procedure, we need look no further than his sketch on millimetre graph paper (the same type Ligeti used to plan *Pièce électronique Nr. 3*), which visualises and plans the textural counterpoint between the various durational layers (Boulez 2010, p. 60). This is something like a composition-specific rendering of Fig. 4. In another sketch Boulez contemplated the rearrangement of players in the hall, which suggests that he was bearing in mind the spatial effects Stockhausen had explored in *Gruppen* (Boulez 2010, p. 55). These sketches confirm that Boulez's thinking had not only turned towards using sound blocks in serial pitch design, but that he carried this orientation towards texture and density into duration, orchestration and spatial arrangement. Statistical form produced larger-scale shapes than the pointillist objects of early serialism. Despite Boulez's considerable stylistic differences from Stockhausen and Ligeti, we can see 'Tombeau' in dialogue with pieces like *Gruppen* and *Atmosphères* based on his orientation towards density and texture in many aspects of this composition.

Pousseur's *Quintette*

It is arguable that both Boulez and Pousseur began contemplating the statistical density or textural thickening of their works through a concern for the listener's perception. As Boulez says in his 1954 essay '... Near and Far', '[P]erception may not be as universal as one would like to imagine', since 'varying acuteness depends on the listener and on the work itself' (Boulez [1954b] 1991, p. 148). Boulez advocates using analysis alongside hearing: the listener must be educated in the formal process because musical form carries the aesthetic ideas. As listeners become more and more astute and educated, they will rise to the challenge of the newest innovations in contemporary music (Boulez [1952] 1991). The Belgian composer Henri Pousseur was likewise oriented towards phenomenology and often addressed listener's problems in perceiving and receiving the Darmstadt composers' works. Though Pousseur might now be considered a lesser Darmstadt composer, his writings in *Die Reihe* were compelling and influential in their time (Koenig [1958] 1960 and Neidhöfer 2009, p. 340 n. 34).[26] Pousseur's experiences with electronic music were similar to those of Stockhausen and Boulez, and he was also in close correspondence and collaboration with them in the 1950s.[27] There are clear conduits for the exchange of statistical form ideas amongst these composers, and Pousseur's 1957 *Die Reihe* article 'Outline of a Method' shows him reckoning with some of the same issues at the same time (Pousseur [1957] 1959).

In 'Outline' Pousseur begins with phenomenology, essentially arguing, as had many of his colleagues, that the unpredictable texture of pointillism is highly problematic for the listener. Like Boulez and Stockhausen, Pousseur sought to incorporate shapes, trends and patterns into his serial structures, with a special focus on the insights of gestalt psychology. About his *Quintette a la mémoire*

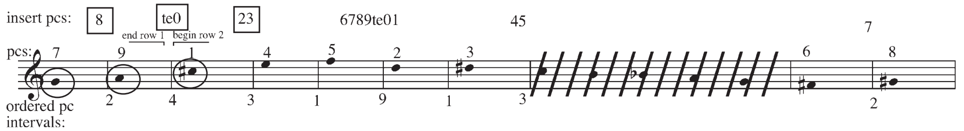
Fig. 11 Pousseur, *Quintette a la mémoire d'Anton Webern*, sketch. Author's transcription and annotation for clef. Henri Pousseur Collection, Paul Sacher Foundation, Basel. Used with permission



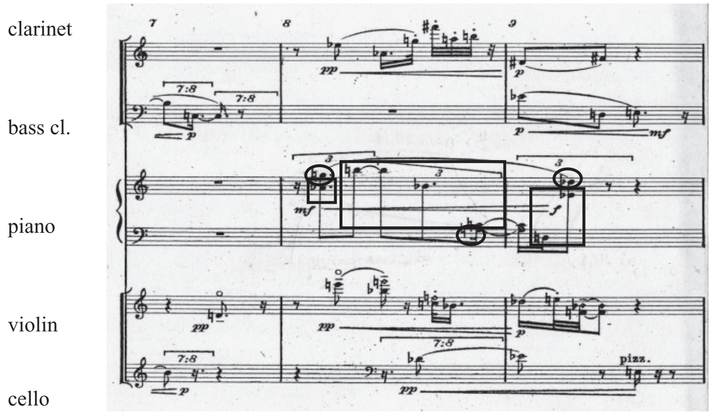
d'Anton Webern (1955), he wrote: 'The two middle sections are thus welded together in a single broad movement to and fro, in a rise and fall of the statistical density, while the outer sections build a firmer, stationary frame around this process of development' (Pousseur [1957] 1959, p. 52). The remark is important for its invocation of the watchword 'statistical', but also because it reveals Pousseur's desire to create a macroscopic architecture in the piece. Already his rhetoric displays many similarities to that advanced by Stockhausen and Boulez: a focus on shapes and trends over time and a strong desire to move away from a texture of isolated points.

By allowing us to delve further into the construction of the *Quintette*, the sketch material can clarify Pousseur's discussion in his article for *Die Reihe*. As he wrote, 'The intervals of Webern's [Quartet Op. 22] series, subjected to particular permutations of order, and constantly developed further in the same direction, were accordingly "completed" through all the intervening steps, i.e. through the fragment of the chromatic scale that they contained' (Pousseur [1955] 1958, p. 51). In the sketch fragment shown in Fig. 11, we see repeated twelve-note rows in all three staves. Notice that Pousseur has crossed out some notes of the rows. Below the notes that remain, he writes the ordered pitch-class interval or ascending distance to the next note of the row. Ex. 6a transcribes the piano stave, including Pousseur's pitch-class interval information, and adds pitch-class labels above the stave. As he explains, he interpolates chromatic segments to 'fill in' the interval gaps – for instance, between pitch classes 7 and 9 there is a gap (a whole step), into which Pousseur inserts pitch class 8. The gap between pitch classes 9 and 1 is filled by pitch classes 10, 11 and 0. The score fragment of the piano part shown in Ex. 6b clarifies how these chromatic interpolations are actualised in the musical texture. Notably, Pousseur treats the chromatic interpolations somewhat like Boulez does his multiplied objects: the registers of both serialised and interpolated pitch classes are flexible and spacious in the final composition. He

Ex. 6a Pousseur, *Quintette*, sketch reduction. Author's transcription and annotations, including words, pitch-class integers, circles and squares. Henri Pousseur Collection, Paul Sacher Foundation, Basel. Used with permission



Ex. 6b Pousseur, *Quintette*, bars 7–9, showing pcs of the row (circled) and interpolated pcs (boxed) on piano stave



even refers to pitches as 'partials', a term that indicates the importance of the electronic studio in his acoustic writing.

Though Pousseur's approach to statistical form seems to deal exclusively with pitch – whereas in many cases the works of Stockhausen, Ligeti, Xenakis and Boulez have more obvious textural implications – he is definitely engaged with concepts similar to those that concern his contemporaries, as always, however, translating them into his own compositional and aesthetic vocabulary.

* * *

This article has investigated the communications between Stockhausen, Ligeti, Koenig, Eimert, Meyer-Eppler, Moles, Xenakis, Cage, Boulez and Pousseur around the concept of statistical form. I have given attention to their writings, scores, sketches, compositional techniques and correspondence, which show their collective attempts to define and exploit the concept of statistical form. It is well-known that the pointillist aesthetic of early integral serialism was short-lived. We might consider the concepts around statistical form as a post-pointillist, second stage of serialism, in as much as a number of the Darmstadt composers found it necessary to move towards groups and more indeterminate elements in their work. Although not all of them used the same terminology, a range of Darmstadt composers found statistical concepts – whether encountered via electronic music, information theory, aleatory debates, or discussions with each other – useful in the move away from pointillism.

I have often referred to the 'Darmstadt composers', but we must acknowledge that they are a differentiated group of strong, independent thinkers (Fox 2007a and b; Borio and Danuser 1997, vol. 3, pp. 333–40; and Iddon 2011, 2013a and 2013b). Throughout this article I have emphasised the controversies and disagreements between them in their accounts of statistical form. Hermann Danuser suggests that we understand 'school' in the construction 'Darmstadt School' as dynamic and verb-like, where the making of a school depends on an active network of associations and connections (Borio and Danuser 1997, vol. 3, p. 346). We can dispense with the idea of a fixed Darmstadt School; it is not a monolithic entity full of dogma, ideology and internal consistency. At the same time, we can acknowledge that the contested discourse around statistical form is evidence of strongly shared foundations and deep-seated connections. Many of the Darmstadt composers had similar experiences, especially considering the formative role of the new electronic music technology. Based on these experiences, composers communicated and worked together to solve technological, conceptual and philosophical problems. It is here, in this alternation between collaboration and controversy, that we can see the Darmstadt composers drawing from the same reference points. At the same time, the various approaches to statistical form in their music make it clear that they differentiated ideas into their own compositional and aesthetic vocabularies. In this sense, statistical form is a rich network of inspirations and connections that are the foundation of the second stage of serial thought.

NOTES

1. *Jeux* was performed, presumably music only, at Darmstadt on 14 July 1956. (Stockhausen 2001, p. 120; and Borio and Danuser 1997, vol. 3, p. 579). Mid-century analyses of *Jeux* include Eimert ([1959] 1961) and Boulez (1956a).
2. The Darmstadt composers' appropriation of quasi-scientific terminology is often noted; see Grant (2001). Perle (1960) and Backus (1962) lash out at Stockhausen and other writers in *Die Reihe* for their faux-scientism and impenetrable terminology.
3. 'Zwischen sehr grosser Dichte und punktueller Tonverteilung liegen kontinuierlich alle Dichtevarianten. Ich denke an *Reihen von Dichtegraden*; sowohl für vertikale als auch für horizontale Dichten.' Translations are mine throughout unless otherwise indicated.
4. I am grateful to Matthew Schullman for pointing me toward this citation and for sharing his draft of an English translation of 'Aspetti'.

5. *Musikalisches Nachtprogramm*, 23 December 1954. WDR Historical Archive, fol. 15199.

6. For more on this, see Decroupet in Borio and Danuser (1997), vol. 1, p. 323, and Grant (2001), pp. 120–8.

7. Stockhausen actually paused composing *Gesang der Jünglinge* mid-way to depart for Switzerland and begin *Gruppen* (Decroupet and Ungeheuer 2002, p. 9). Stockhausen ([1957] 1959) is a good demonstration of the overlap between electronic music insights such as harmonic and inharmonic spectra, the continuum between rhythmic pulses and timbre and their application in the acoustic realm in *Gruppen*.

8. *Beschleunigung* (abbreviated 'Beschl.' in Stockhausen's sketch) translates more usually as 'momentum'. I substitute 'attack frequency' here because it is more indicative of Stockhausen's actual concern with number of attacks in a certain time span; this creates the perception of changes in momentum or velocity due to snowballing or slackening attacks.

9. 'Filtrierte Impulse und sehr kurzes filtriertes Rauschen bilden die "Konsonanten", Sinustonkomplexe und längeres filtriertes Rauschen bilden die "Vokale", die – nach verschiedenen Verteilungsstatistiken aneinandergereiht und gruppiert – imaginäre Wörter und Sätze ergeben. Die ganze Komposition ist nach seriellen Prinzipien entworfen, die jedoch weitgehend undogmatisch angewendet werden.'

10. Wiener also wrote about many of the same ideas as Shannon contemporaneously, publishing the first edition of his *Cybernetics* in 1948 (Wiener 1961). The two acknowledged each other's work and said that they arrived at their ideas independently; Shannon's theory tended toward mathematics and engineering, while Wiener's tended toward biological and computing applications. Shannon, Weaver, Wiener and other notable scholars met annually in the 1950s at the Macy Conferences, a meeting which fuelled the explosive interdisciplinary growth of information theory and cybernetics. For more, see Hayles (1999).

11. Ligeti's *Piece èlectronique Nr. 3*, for instance, was abandoned because the 48 individual sine-tone layers Ligeti imagined simply could not be mixed down without excessive tape noise. The piece was realised in 1996 using digital technology by Kees Tazelaar at the Institut of Sonology at The Hague. *His Master's Noise*, BVHAAST Records 06/0701.

12. *Gravesaner Blätter*, Nos. 11–12 and 18–24; later expanded into *Musiques formelles* (1963) and translated as *Formalized Music* ([1971] 1992).

13. Xenakis is now recognised as the primary designer of the Philips pavilion for the 1958 World's Fair, which exhibited Varèse's *Poème électronique* and

Le Corbusier's accompanying images (Xenakis 1957, pp. 51–4; Varga 1996, p. 24; Harley 2004, pp. 17–18; and Squibbs 1996, pp. 4–6).

14. 'Meine Schwierigkeiten sind die Schwierigkeiten, die alle Leute mit ihm haben: Stockhausens unendliche Selbstbezogenheit. Das wurde mit dem Alter immer extremer. Er und Iannis Xenakis sind die zwei Komponisten, die absolut nichts von Kollegen wissen wollten' (p. 95). 'Für Stockhausen und Boulez existierte er nicht' (p. 98).

15. Borio and Danuser have conceptualised the multifaceted nature of aleatory in a helpful three-termed rubric. In the first position, 0/1 describes fixed or mobile form; in the second position, 0/1 describes fixed or aleatory musical details; in the third position, 0/1 describes specific or aleatory instrumentation. Thus *Music of Changes* is coded 000, for fixed form, fixed musical details and fixed instrumentation; *Klavierstück XI* and the *Troisième sonate* are both 100, for open form with fixed musical details and instrumentation; Cage's *Variations* (1958) is coded 111, for open form, open musical details and open instrumentation. See Borio and Danuser (1997), vol. 2, pp. 192–8.

16. Boulez shows his interest in Cage's chance experiments in letter no. 35 (December 1951) and letter no. 39 (October 1952). The seeds of Boulez's discontent are also sown in letter no. 35 and reinforced in letter no. 45 (July 1954).

17. Cage's *Variations* (1958), a highly indeterminate piece with a graphic score, was performed at Darmstadt in 1958 and was not well-received. As Shultis writes, 'It should be remembered that it was indeterminacy, not chance, that caused the split between Cage and his European contemporaries' (2002, p. 34).

18. 'La polyphonie linéaire se détruit d'elle même par sa complexité actuelle. Ce qu'on entend n'est en réalité qu'amas de notes à des registres variés. La complexité énorme empêche à l'audition de suivre l'enchevêtrement des lignes et a comme effet macroscopique une dispersion irraisonnée et fortuite des sons sur toute l'étendue du spectre sonore. Il y a par consequent [*sic*] contradiction entre le système polyphonique linéaire et le résultat entendu qui est surface, masse.'

19. Paul Sacher Foundation, Pierre Boulez collection, Stockhausen-Boulez correspondence, letter 35. Transcribed by Robert Piencikowski, translation mine; grammatical and spelling errors reflect the author's orthography. 'Je me trouve de plus en plus dans la composition statistique: "improvisation" serielle entre limites des espaces serielles du temps, de l'hauteur, de l'intensité. Le propoportions [*sic*] logarithmic (fonctionnelle) des series de frequence et db [*sic*] sont liées au temps. Les groups serielles des intervalles (d'hauteur, – egalement d'intensité et durée) divisent les

espaces statistiques. On se tient entre les limites travaillant sur les directions des intervalles. Permutations dans les groupes silences et superpositions (aspect polyphon et egalement du timbre non stationär) sont resultat de cette rotation en valves.'

20. Paul Sacher Foundation, Pierre Boulez collection, Stockhausen-Boulez correspondence, letter 36. Transcribed by Robert Piencikowski, translation mine; grammatical and spelling errors reflect the author's orthography. 'Votre dernière lettre est vraiment très ésotérique. Et j'avoue que malgré toute mon application, et malgré mon habitude à déchiffrer les ellipses, je n'ai pu résoudre toutes les énigmes que me proposait votre dernière lettre. [...] Qu'est-ce que vous appelez composition "statistique"? Qu'est-ce que vous entendez exactement par "improvisation" entre les différentes limites? Qu'est-ce que vous appelez "espaces statistiques"? Qu'est-ce que vous entendez par "travailler sur les directions des intervalles"? Qu'est-ce que vous appelez "rotations en valves"?'

21. Heinemann (1998, p. 87) speculates that this is because the transposition-determining constant 'k' (to be discussed momentarily) comes from the 'a' field. The 'a' field thus plays an intrinsic role in the result of the complex multiplication. Boulez may eliminate the *aa* ... *ae* multiplied objects to avoid over-emphasising pitch classes from the 'a' field in the precompositional chart.

22. As Scotto (forthcoming) has shown, TC is commutative in a certain sense, because the product of multiplication will always give members of the same set class. However, Boulez specifies particular pitches (not pitch classes) in his products, and set-class isography is not enough for Losada and Heinemann to consider the operation commutative. Thus, the transpositional indices control whether the pitch content is reproduced as in Boulez's sketches or only set-class isography is maintained.

23. Scotto (forthcoming) has shown that the operations may produce the same results but are not really mathematically equivalent. Scotto argues that there is really no such thing as simple, compound and complex multiplication; rather, they are just different ways of applying T_i indices.

24. Pressed for time, Boulez ended the first version of the piece (1959–60) at the end of this fourth section (bar 174) and added a coda. This version can be heard in a recording of the premiere on Col Legno 20509. The final version of the piece (as the fifth movement in *Pli selon pli* [1971]) carries the process described above through all six instrumental groups and appends the coda (Boulez 2010, pp. 27–8).

25. Keep in mind that the topmost row of the table contains *unmultiplied* objects: Boulez never multiplied his first row of objects (that is, there is no *aa*, *ab*, *ac*, *ad* or *ae*). This first row is simply the twelve-note series seg-

mented five different ways. Boulez probably declined to multiply his first row of objects to avoid over-saturating the texture with primary intervals from the series, since these intervals also determine the transpositional levels for the rest of the table (see n. 20 above, Heinemann 1998 and Losada 2008).

26. Karel Goeyvaerts and Michel Fano are two others whose innovations in the early 1950s were extremely influential but whose legacy has been outweighed by Stockhausen and Boulez. See Toop (1974) and the special issue of *Revue belge de musicologie* 48 (1994) dedicated to Goeyvaerts.

27. There is a voluminous correspondence between Pousseur and Boulez and also between Pousseur and Stockhausen in 1952–6 (Paul Sacher Foundation), including many complex discussions of electronic music and other contemporary topics. See also Pousseur ([1955] 1958).

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NOTE ON CONTRIBUTOR

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ABSTRACT

This article analyses the Darmstadt composers' discourse around the concept of 'statistical form' in the mid- to late 1950s and early 1960s. Stockhausen introduced this terminology in a 1954 analysis of Debussy's *Jeux*, but the real inspiration lies in the nascent electronic music studio. Using score and sketch analysis, published writings and correspondence, I show how Ligeti, Boulez and Pousseur also made critical contributions to the definition and application of statistical form in music. Along the way I introduce foundational concepts from information theory and show how they were disseminated in Moles's and Meyer-Eppler's teachings and writings. Via information theory, I investigate how Xenakis's early stochastic sound-mass music and Cage's aleatory techniques are related to statistical form. Ultimately, I show that a network of shared ideas underlies statistical form, even when debate characterises the discourse and each composer's musical rendering of the concept is distinct. In closing, the article suggests that statistical form represents a second stage of serialism, precipitating a move towards density and texture in sound that was shared by many in the Darmstadt circle.