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Pitch-Class Set Analysis: An Evaluation

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That I have a problem with "pitch-class set analysis," as formulated and developed by Allen Forte and a vast number of dedicated first-generation disciples and, by now, their students as well, will come as no surprise to anyone in this audience.* But the real nature of my problem cannot help but be camouflaged in the reasoned, logical, and objective statements that argument requires and that it is my intention to offer. My critique of the Forte system does not begin with an objective and reasoned appraisal of it, and I think it would be worthless if it did. My critique begins with the subjective, intuitive, and spontaneous experience of one who has spent a lifetime listening to music, composing it, playing it, and thinking about it, and then finds himself confronted with ways of talking about and analyzing music that have nothing whatever to do with what I would call this "common sense" experience. But at such a fundamental level the act of musical judgment is a private one, a matter that must be left to one's self and one's conscience.

I do not, however, think it inappropriate for me to point out that I am not alone in finding Forte's system of pitch-class set analysis altogether irrelevant at this most basic level. According to the preface of Forte's book, *The Structure of Atonal Music*,[1] it is his intention "to provide a general theoretical framework, with reference to which the processes underlying atonal music may be systematically described." The key figure in the revolutionary change from the traditional tonal system to atonality is Arnold Schoenberg. Schoenberg's role in the

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[1] Allen Forte, *The Structure of Atonal Music* (New Haven, 1973).

transition from atonal to 12-tone composition is equally crucial. Schoenberg has given us his own formulations of the basic principles that govern both his tonal music and his 12-tone music, but nothing of the sort for the music of the intervening atonal period, and in the little that he did have to say about it there is nothing whatever that shows the slightest awareness of the special analytical categories and compositional relations described in the writings of Allen Forte. And this deficiency is not made up in any of the writings and statements that have come down to us through any of Schoenberg's pupils and disciples. Here is the best argument the Forteans have been able to muster with respect to this question. I quote from Schmalfeldt's book on *Wozzeck:* "In light of a recently published letter to Schoenberg in which Berg describes his painstaking twelve-tone calculations in preparation for the *Lyric Suite*, it is not inconceivable that the same composer submitted himself to similar kinds of pitch-structural manipulations in his earlier work."[2] Let me propose a less tendentious formulation: in the light of Berg's description of "his painstaking twelve-tone calculations in preparation for the *Lyric Suite*," and, may I add, of copious similar evidence of his painstaking calculations in preparation for every one of his other 12-tone works, the absence of any evidence whatever "that the same composer submitted himself to similar kinds of pitch-structural calculations in his earlier work" makes it inconceivable that he did so. Why should these composers have made it so amply clear that they were aware of the structure and functions of the 12-tone row, but never have dropped even the vaguest hint that they were aware of any aspect of the "general theoretical framework" that Allen Forte has discovered to be the basis for their atonal music?

But my rejection of Forte's system is not based on historical considerations of this sort. It is based, as I've said, on the fact that I find the system irrelevant to my experience as a composer, to my perceptions as a listener, and to my discoveries as an analyst. I will try to give you a compendium of my objections to the basic elements of the system as formulated in Part 1, entitled "Pitch-Class Sets and Relations," of Forte's book, *The Structure of Atonal Music.*

The one-page preface immediately raises two important and obvious problems that are never confronted, or even acknowledged. Nowhere is there any reference whatever to the interesting fact that Schoenberg himself asserted that it was precisely the impossibility of hypothesizing a "structure" for atonal music which led him to formulate "a new procedure in musical construction which seemed fitted to

[2] Janet Schmalfeldt, *Wozzeck* (New Haven, 1983), p. 154.

replace those structural differentiations provided formerly by tonal harmonies," namely, the 12-tone system."[3]

The second problem is raised by the only titles cited in the preface as examples of "major works in this [atonal] repertory," namely "Schoenberg's Five Pieces for Orchestra Op. 16 (1909), Webern's Six Pieces for Large Orchestra Op. 6 (1910), Stravinsky's *The Rite of Spring* (1913), and Berg's *Wozzeck* (1920)." Forte's next sentence shows that he is not unaware that one of these entries is unexpected: "The inclusion of Stravinsky's name in the list above suggests that atonal music was not the exclusive province of Schoenberg and his circle, and that is indeed the case." But surely such an inclusion has some other novel implications as well. The atonal period of Schoenberg and his circle terminates with their adoption of the 12-tone method around 1923. The very different direction followed by Stravinsky would need to be reconsidered and reevaluated by historians, in the light of a reinterpretation that would assign *Le Sacre* to the same repertory as the atonal works of Schoenberg, Berg, and Webern. What is known as "atonal" music had been around for more than six decades by the time Forte came to write his book, and it had never before occurred to anyone, least of all the composers themselves, that the Stravinsky work belonged in the company of the others, as part of that special repertory. Why not? This is a legitimate and significant question for the theorist, and an analysis that is oblivious to it will be, in my view, seriously deficient in what it has to say about the music under scrutiny. The listener, of course, answers this question in an intuitive way, by somehow sensing a categorical distinction between, for example, the bassoon melody that opens *Le Sacre* and the first two chords of *Wozzeck*, or the first phrase of Schoenberg's Opus 11. Much of my own work as a theorist and teacher has been particularly concerned with the discovery and explanation of shared structural elements in the music of composers whom we generally recognize as representing disparate stylistic tendencies—Bartók and Berg, for example. But precisely what is momentous about these connections disappears, if we fail to take note of the different musical contexts in which these shared elements occur.

Since our common understanding of what is included in the "atonal" repertory doesn't coincide with Forte's, doesn't he owe us, first of all, a definition of that term, as he understands it? This is provided as follows by the first paragraph of his text proper, which I quote in its entirety: "The repertory of atonal music is characterized by the occurrence of pitches in novel combinations, as well as by the

[3] "Composition with Twelve Tones," in *Style and Idea: Selected Writings of Arnold Schoenberg*, ed. Leonard Stein (New York, 1975), p. 218.

occurrence of familiar pitch combinations in unfamiliar environments." As an example of an unfamiliar pitch combination we are shown the final chord of Schoenberg's Opus 15, No. 1, which "could occur in a tonal composition only under extraordinary conditions, and even then its meaning would be determined by harmonic-contrapuntal constraints. [Since in this instance] such constraints are not operative, one is obliged to seek other explanations." Atonal music, then, is that music to which these "other explanations," Allen Forte's pitch-class set theory, apply. In sum, this is all we are offered by way of a definition of atonal music. In defining atonal music, or delimiting the extent of its repertory, Forte never takes us beyond the succinct formulation he provides in his preface: "Any composition that exhibits the structural characteristics that are discussed, and that exhibits them throughout, may be regarded as atonal." We have been offered "a general theoretical framework, with reference to which the processes underlying atonal music may be systematically described." And what, please, is "atonal music"? Atonal music is that music whose underlying processes "may be systematically described" by this "general theoretical framework."

12-tone pitch-class set theory borrows the term "set" from the theory of 12-tone music. According to a footnote in Janet Schmalfeldt's book on *Wozzeck*,[4] "Perle was the first to use the term *set* to refer to unordered collections of fewer than twelve pitch classes." I didn't know this before but am pleased to accept the credit as a license to refer to some of the implications of this usage.

The foundational requirement for a "12-tone system" is a means of precompositionally differentiating representations of the universal set of twelve pitch classes from one another. Schoenberg solved this problem by assigning a specific ordering to the twelve pitch classes; Hauer, around the same time, by partitioning them into unordered segments. Sets of less than twelve pitch classes, however—or of less than eleven if we assume transpositional equivalence, as both Schoenberg and Hauer did—may be differentiated from one another in terms of content alone. Thus there is a radical difference in the meaning of the term "set" for 12-tone music and in the meaning of the same term for other kinds of music.

Forte assumes that the concept of the pitch-class set has the same universality for all of non-12-tone atonal music as the concept of the 12-tone set has for music in the 12-tone system. He does this by defining every single pitch combination in all of atonal music as a

[4] Schmalfeldt, op. cit., p. 247.

representation of one or another set, all of which are reduced to certain standard forms and catalogued in a numbered index in an appendix to his book. This is Allen Forte's own special contribution to the theory of atonal music. Though others beside myself have discussed the use of pitch-class sets in atonal music, no one before Forte had ever suggested that *everything* that happens in an atonal piece must unfold a pitch-class set statement, just as everything that happens in a 12-tone piece unfolds a 12-tone set statement. The pitch-class set, in Forte's system, has the same universality for atonal music as the triad has for tonal music. In fact, if you will excuse the locution, the pitch-class set is *more* universal than the triad, for in tonal music we have passing notes and other non-triadic elements. There is nothing in Forte's theory of atonal music that is to be construed as a non-set element. What is not a component of one set is a component of another, possibly overlapping, set.

The referability of the pitch-class sets in Forte's catalog to the pitch combinations of atonal music depends on certain carefully defined criteria of equivalence. The first of these, transpositional equivalence, is fundamental to our comprehension of any kind of music whatever. If a melody, or a theme, or a composition is literally transferred by the same one-to-one transformation of pitches to a different pitch level, we recognize it as the "same" melody, the "same" theme, the "same" composition. In this sense, "transpositional equivalence" is simply another name for "relative pitch." This is our most common understanding of what is meant by transposition, and perhaps it is its very familiarity that explains Forte's failure to represent this most obvious kind of transposition in his three examples from the literature. One of his examples (Example 1) "shows the opening clarinet figure in Berg's Op. 5/1 (A) and the opening piano figure in the third piece of the same composition (B). Comparison of A and B reveals that B is transpositionally equivalent to A and that t [the transposition number] = 4." B, however, preserves neither the intervallic ordering nor the contour of A, but only its relative pitch-class content. This is something very different from our understanding of transposition in the 12-tone system, which doesn't necessarily preserve contour but does preserve order. We cannot transpose a 12-tone set in terms of its unordered pitch-class content because all 12-tone sets have the *same* pitch-class content. When a 12-tone set is transposed the corresponding pitches may be effected by octave displacement, as in Example 2, which revises the contour but not the ordering of Example 1A.

Does the composer do anything between the beginning of the first movement and the beginning of the third to keep us in mind of the pitch-class content of the initial statement of the so-called set? For example, does its transposed pitch-class content ever occur in conjunction with the order and contour assigned to that initial statement? What about the pitch class, *c*, in Example 1B, which is represented at two different points and at two different octave positions? Doesn't that influence our perception of B as being the transpositional equivalent of A? Aren't we more justified in construing the variant that I've provided in Example 3, which eliminates the octave duplication, as a transpositional equivalent of A than what Forte has deduced as its equivalent in Example 1? And if so, what does this imply for pitch-class set theory, which eliminates every compositional consideration that relates to multiple representations of the same pitch-class? Aren't we more justified in construing Example 3 as a transpositional equivalent than the version of the same pitch-class content that I give you in Example 4, in which the relative pitch-class ordering is completely revised, and in which the successive pitches, except for the last, unfold intervals that occur nowhere between successive pitches in the original? If Example 1B is the transpositional equivalent of A, so is Example 5, which shows another transposition of the same pitch-class collection. One of them cannot be more equivalent than the other. Nevertheless, Example 1B *sounds* more like Example 1A, because it duplicates four of its pitch-classes—*c*, *g*♯, *e* and *g*—rather than only one, *a*, as in Example 5. It does not, however, sound more like A than does Example 6, which shares the same four pitches. But Example 6 is a *new* set, no longer transpositionally equivalent to Example 1A, since it replaces the required pitch class *b*♮ by *b*♭.

The reduction of the diatonic scale to nothing but a pitch-class set would severely restrict its usefulness as an analytical tool in the study of tonal music. Perhaps the reduction of the pitch combinations of atonal music to pitch-class sets has similar limitations. If A and B in Example 1 are to be construed as compositional representations of the same set, it is up to the composer to establish this, and to establish that they are functioning as sets at all. Professor Forte has not presented any evidence to show that this is the case. The mere fact that we can trace both of them to a pitch-class collection labelled No. 6-Z44 in the catalog of sets in the appendix of Forte's book is irrelevant. Does the traditional and conventional meaning of the term "transposition" play no role in this music? And if so, why doesn't Forte say so?

That would spare us a problem posed by his next example (Example 7). On the lookout for transpositional equivalence, we might suppose that the first three notes of A are transposed at the tritone in the first three notes of B, since ordering and direction are similar to that point. Thus the leap from *d* to *g* at A is reflected at B in the extended span from the low *g*♯ in the piano to the high *c*♯ in the violin

EXAMPLE 1. Berg, Four Pieces for Clarinet and Piano, Op. 5.

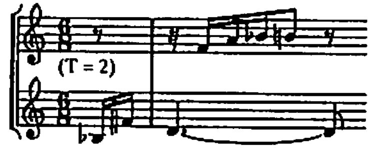
6 → 4

EXAMPLE 2.

EXAMPLE 3.

EXAMPLE 4.

EXAMPLE 5.

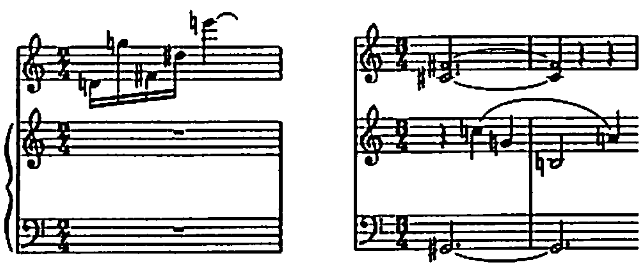


EXAMPLE 6.

harmonic, and the descent of an octave and a semitone from *g* to *f*♯ at A is reflected at B in the relation between the *c*♯ in the violin and the *c* in the piano. This takes us 3/5ths of the way through the pitch-class set at A, but now we discover that this promising beginning won't carry us through the two remaining notes, neither of which transposes to what we had prematurely hypothesized as the fourth note of the set at B. We have been too hasty and should have postponed any conjecture respecting the relation of A and B until we had come to the end of the phrase in the right hand of the piano part. We would then not have made the mistake of neglecting a basic principle of pitch-class set comparison which Forte had presented two pages earlier: "the only requirement necessary for comparison is that the pitch configurations be reducible to pitch-class sets of the same cardinal number." This requirement is not met if we include the note in the violin part in the pitch configuration at B. It turns out that what we have at B is not a direct note-for-note transposition at interval 6 but—not counting the note in the violin part, and assuming the permuted ordering of A that I show in Example 8—a transposition at interval 5. These are the kinds of construals Forte's understanding of the transposition operation would impose—not only on the analyst, but, since there is nothing in the listening experience that is more fundamental and more elementary than the recognition of transpositional equivalence, on the listener as well.

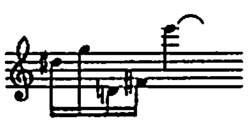
Perhaps Forte's interpretation of transposition as referring only to pitch-class content is more relevant to the concept of key-change in tonal music. When we move from one key to another we transpose a collection of pitch and pitch-class relations, even where the respective musical content in the two keys is very different. Is this how we should understand Forte's use of the term "transposition"? But when we move from one key to another in tonal music, we transpose a clearly defined and immediately perceptible complex of functional relations.
158 It has been suggested that a troop of Martian musicologists, not knowing that the *Eroica* is a tonal composition, might be satisfied with a

EXAMPLE 7. Webern, Four Pieces for Violin and Piano, Op. 7/4.



A: {2,3,4,6,7} B:{7,8,9,11,0}

EXAMPLE 8.



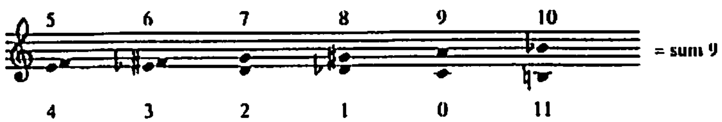
pitch-class set analysis of it.[5] For the rest of us, the same conditions that are necessary to our recognition of a key center in the opening bars are sufficient for our recognition of subsequent transpositions of that key center. These are precisely the "harmonic-contrapuntal constraints" that are "inoperative" and for which Allan Forte has felt "obliged to seek other explanations" in atonal music. Moreover, we don't have to fish through 220 different pitch-class sets to find the operative one. We are concerned with only one pitch-class set, namely the one that is identified as No. 7–35 in Forte's catalog—i.e., the diatonic scale.

But we must not suppose, because of Forte's failure to demonstrate its relevance, that the concept of transpositionally equivalent unordered pitch-class collections plays no role in atonal music. On the contrary, it plays a role of the first importance, not only in atonal music but in other post-diatonic music as well, such as *The Rite of Spring* and the Bartók string quartets. Since I've already discussed this role at some length in my writings on this music, both before Forte and since, I will not go into it here.

The second type of equivalence relation that Forte presents is inversion. Where transpositional equivalence is basic to our under- 159
standing of any sort of music, strict inversional complementation is a new type of equivalence relation, which arises as a consequence of the replacement of a diatonic scale of unequal intervals and functionally differentiated notes by a twelve-tone scale of equal intervals and functionally undifferentiated notes. Where each pair of corresponding elements of two transpositionally equivalent pitch-class collections will show the same interval (i.e., difference), each pair of corresponding elements of two inversionally equivalent pitch-class collections will show the same sum, as in Example 9. What is known as "contrapuntal inversion" in diatonic music is not to be equated with inversional equivalence in post-diatonic music. The latter is a precompositional means of symmetrically partitioning the 12-tone scale and thus generating a given harmonic context, as in Example 9, which shows the principal collection of inversionally related dyads of both the first movement of Bartók's Fourth Quartet and the first movement of Berg's *Lyric Suite*. Contrapuntal inversion in diatonic music is a *compositional* procedure, a means of motivic elaboration in a harmonic context that is already given. The respective harmonic and tonal *functions* of the inversionally corresponding elements in diatonic tonal music are not invertible into one another. A Martian musicologist, like Joseph Schillinger, will not understand the difference between these

[5] Richard Taruskin, "Reply to van den Toorn," *In Theory Only* X/3 (October 1987).

EXAMPLE 9.



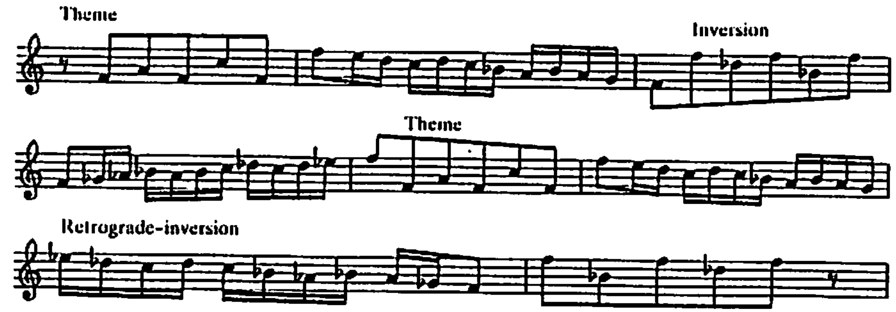
two concepts of inversion. As Schillinger himself admits, if only Bach had been familiar with the Schillinger System it might have occurred to him to exploit the subject of the F-major two-part invention in the transformations shown in Example 10.[6] Martian musicology is not aware of the harmonic consequences of the substitution of a Phrygian scale for the major scale and of a B♭-minor triad, with F presumably functioning as its root, for the tonic F-major triad.

One of the three examples with which Allen Forte illustrates his exposition of inversional equivalence (Example 11) raises some doubt as to the relevance of such questions for him. If we look at the string parts alone of Ives's *The Unanswered Question* we see "familiar pitch combinations"—mainly major and minor triads—in a "familiar," i.e., diatonic "environment." If the prevailing pitch-class collections are "familiar," it is not only because of their content, but also because of the way in which that content is deployed, through voice-leading, pitch doublings, and so on. The harmonic implications of Examples 12A and 12B will not be the same in such an "environment," even though both are representations of the same relative pitch-class combination. It would seem, then, that the music which Ives has assigned to the strings is not, in itself, part of "the repertory of atonal music" as defined by Forte in his opening sentence. Against this diatonic music in the strings there is a recurrent non-diatonic figure in the trumpet. The programmatic significance of this figure, which should be self-evident even without the clue provided by the title, is dependent on our recognition of the disparate character of the respective harmonic domains of strings and trumpet.

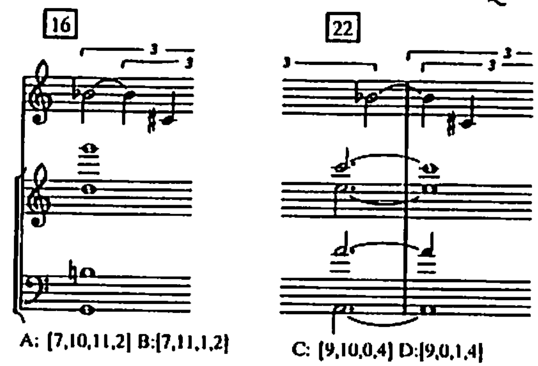
Allen Forte, however, has found an answer to "The Unanswered Question." Where in Example 7B he chose to *subtract* the sustained note in the violin part from the notes that appear in the same two bars in the piano part, and to deduce his pitch-class set from the latter alone, in Example 11 he chooses to *add* each of the notes in the trumpet part to the chord simultaneously sustained in the strings. He thus arrives at the four pitch-class sets indicated in the example, and discovers the requisite inversional relation between the second set,

[6] *The Schillinger System of Musical Composition*, ed. L. Dowling and A. Shaw (1941; reprint ed., New York: Da Capo Press, 1976), pp. 193f.

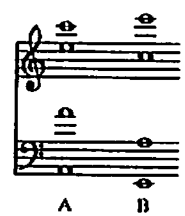
EXAMPLE 10.



EXAMPLE 11. Ives, *The Unanswered Question.*



EXAMPLE 12.



Example 11B, and the third, Example 11C. Again, as with transpositional equivalence, differences in the respective number of representations of the supposedly corresponding pitch classes, and their respective distribution, play no role in making this determination for the analyst. But perhaps they do so for the composer? The chord at bar 16 in Example 11 has the familiar look of a G-major triad in root position with the root doubled, and the chord at bar 22 the familiar look of an analogously deployed A-minor triad. Had this second chord been laid out as shown in Example 12B, I would have felt constrained to accept Example 11C as the inversional equivalent of 11B. But in that case I would have supposed this to be a compositional *idea*, and I would have looked for other manifestations of the same idea, and for some sort of meaningful and coherent exploitation of this same idea in the movement as a whole. Forte's interpretation of the relation between Examples 11B and 11C is even more extraordinary, but he is content to regard it as a matter of routine compositional practice, and to ignore the fact that it is totally destructive of the whole motivational basis of the piece.

Perhaps nothing so clearly reveals the problem that Forte has in distinguishing between what he may find it convenient to formulate as a theoretical precept and what one can actually discover in the composition at hand than the summary assertion, utterly incomprehensible in itself, that precedes his examples of inversional equivalence: "It is important to observe that whereas transposition does not imply prior inversion, inversion always implies subsequent transposition." In other words, inversion, unlike transposition, always implies prior inversion. In Example 13 the chord at B is the inversion of the chord at A. Professor Forte assigns a transposition number, 11, to it, thus implying a prior inversion (Example 14). But where is that prior inversion? Only in Professor Forte's head. He assumes a standard model for inverse mapping according to which the inverse-related pitch classes always show the sum of 0. *C* is thus always inverted into *c*, and, correspondingly, *b* into *c*♯, *b*♭ into *d*, and so on. But why should we suppose that Schoenberg, on his way from A to B, had first of all to convert A into the inversionally complementary form shown in Example 14, and then to transpose the latter? The concept of inver-

EXAMPLE 13.

A: [3,4,7,10] B: [1,4,7,8]

EXAMPLE 14.

sional equivalence has been a basic component of my own compositional language for at least fifty years, and it has never occurred to me that I could not convert a given pitch-class collection directly into its inversional complement at any one of the twelve transpositional levels of the latter. Why should I suppose that such a direct conversion would have posed a problem for Schoenberg? Forte has confused the sum of inversely complementary pitch-class numbers that defines the relation between a given pitch-class collection and a given transposition of its inversion with the transposition number that defines the relation between a given transposition of the inversion and another transposition of the inversion. We find the transposition number of an inverted set by identifying the interval that separates it from a referential statement of the inverted set, just as we find the transposition number of a prime set by identifying the interval that separates it from a referential statement of the prime set, and just as we find the interval of transposition of a subject in tonal music by referring it to the principal statement of that subject. The perception of Example 13B as a T(11) transposition would be analogous to the perception of key relations in tonal music only with reference to the key of C, so that one would be constrained to identify the tonality of the *Eroica* as a transposition of C major to the minor third above.

The two remaining criteria of equivalence invoked by Forte, though they are supposed to be as basic to atonal music as the familiar concepts of transposition and inversion, have never before been exploited as analytical tools. Forte, however, is not the first to have observed what he calls the "Z-relation" between pitch-class collections. It is self-evident that pitch-class collections that are transpositionally or inversionally equivalent will also be equivalent as to interval content, but it is not self-evident that the converse should not invariably hold—that there should be certain pitch-class collections that are neither transpositionally nor inversionally equivalent, but that are nevertheless equivalent as to interval content. In fact, however, there are such "Z-related" pairs of pitch-class collections, provided that we equate "interval content" with what Forte calls the "interval vector," the specification of the interval content of a set by a series of integers showing the number of representations of each interval class contained in the set. Howard Hanson and David Lewin, in 1960, were the first to publish descriptions of pitch-class collections that share the same interval vector but are neither transpositionally nor inversionally equivalent.[7] I had investigated the same phenomenon a few years before this, but only as it relates to hexachordal collections. I had

[7] Howard Hanson, *Harmonic Materials of Modern Music: Resources of the Tempered Scale* (New York, 1960); David Lewin, "Re: The Intervallic Content of a Collection of Notes," *Journal of Music Theory* IV/1 (1960).

noticed that the two hexachords that make up a 12-tone set always share the same interval content, regardless of whether or not the hexachords are inversionally related, as they are in all of Schoenberg's later 12-tone compositions.

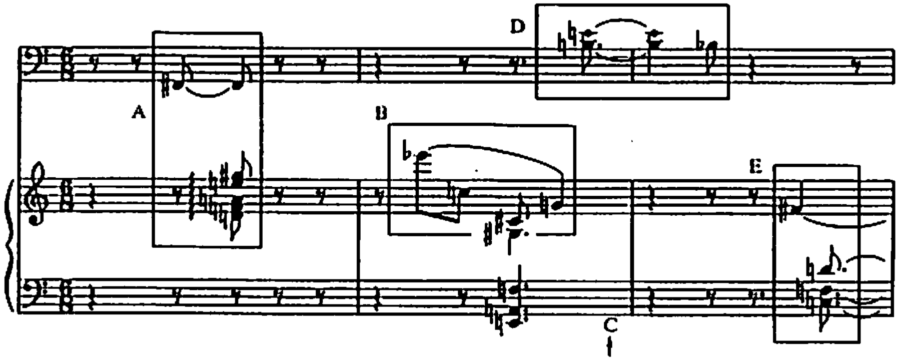
Initially Forte assumed that two sets of the same interval vector must be regarded as equivalent, whether or not they could be transposed or inverted into each other. By the time he wrote his book, however, he was dissuaded from this extraordinary notion and accepted the argument that "an equivalence based on inversion and transposition has the advantage of being 'a simple venerable concept, of preeminent musical relevance.' "[8] He nevertheless discovers "a primary structural role" for Z-related pitch-class sets, by means of analytical construals analogous to those by which he has already established transpositional and inversional equivalence.

Forte's demonstration of the compositional employment of the Z-relation commences with Examples 15–17. In Example 15 the pitch-class set at A, No. 6-Z10, finds its Z-related correspondent, 6-Z39, in B and D combined. The two instrumental parts are joined here to generate the requisite Z-related hexachordal sets. In the next example, however, the note on the middle staff does *not* join A and B in the formation of a Z-type hexachord. The identification of C as its eventual Z-related correspondent is justified by invoking "factors other than identical interval-content [that] associate the two sets." We are asked to observe "that they occur in the same register." In the example which immediately follows, however (Example 17), "an association of temporally remote events through Z-related sets" is offered with *no* such justification: "Pitch-class set 6-Z37 is stated as a continuous melodic configuration at the outset by the second violin. The Z-correspondent, 6-Z4, subsequently is distributed among all four instruments at the close of the movement." Not only are the two Z-related sets maximally separated by their respective occurrence in the first three and the last two bars of the piece, but they are radically differentiated in every other way as well—registrally, rhythmically, and timbrally. The listener is called upon to perceive the first three bars retrospectively, for only when he arrives at the conclusion of the movement will he learn that the three instrumental parts omitted from the citation of the opening bars were not to be construed as forming pitch combinations with the second violin.

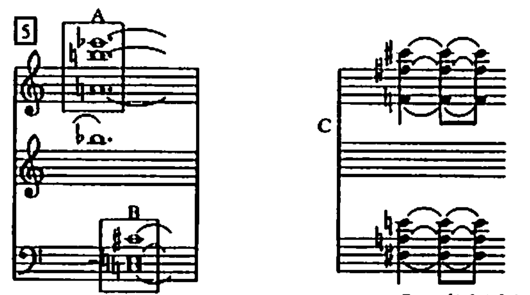
Nowhere does Forte explain why, of the twelve transpositions of a Z-related correspondent set, the composer should have selected one

[8] Schmalfeldt, op. cit., p. 21, quoting John Clough, "Pitch-Set Equivalence and Inclusion (A Comment on Forte's Theory of Set-Complexes)," *Journal of Music Theory* IX/1 (1965).

EXAMPLE 15. Webern, Three Short Pieces, Op. 11/1.



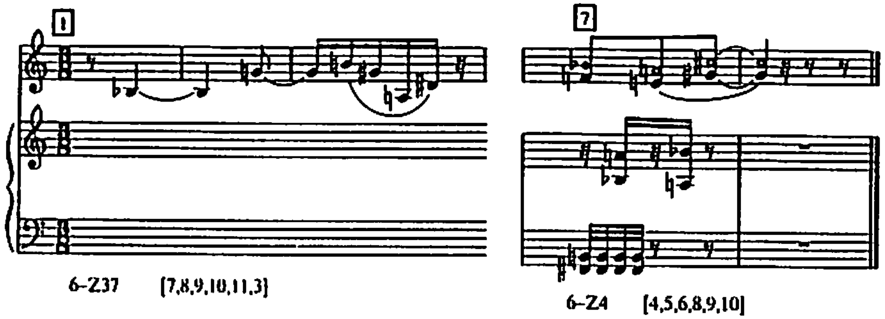
EXAMPLE 16. Varèse, *Intégrales.*



rather than another transposition. I think I can explain this for him with respect to the Z-related pair 6-Z10/6-Z39 in Example 15. In one of his posthumously published lectures on "the new music" Webern describes his intuitive anticipation, in his early atonal compositions, of the concept of the 12-tone aggregate:

> What happened? I can only relate something from my own experience; about 1911 I wrote the "Bagatelles for String Quartet" (Op. 9), all very short pieces, lasting a couple of minutes—perhaps the shortest music so far. Here I had the feeling, "When all twelve notes have gone by, the piece is over." Much later I discovered that all this was a part of the necessary development. In my sketchbook I wrote out

EXAMPLE 17. Webern, Six Bagatelles, Op. 9/4.



> the chromatic scale and crossed off the individual notes. Why? Because I had convinced myself, "This note has been there already."[9]

In the first bar of Example 15 we find a collection of six pitch classes at A. As Webern has just told us, there is a very good chance that we will also find, in close juxtaposition to this, the six remaining elements of the 12-tone aggregate, as we do in this instance. But the two hexachords that make up a 12-tone aggregate *always* have the same interval vector. Of the thirty-five hexachordal "source sets," or "tropes," six fall into two hexachords that are equivalent by both inversion and transposition, one falls into two hexachords that are equivalent by transposition only, and thirteen fall into two hexachords that are equivalent by inversion only. Why should a special value be placed on the remaining fifteen whose two hexachords are neither inversionally nor transpositionally equivalent? Forte tells us that they are characterized by another kind of equivalence, namely equivalence of interval content as defined by their respective interval vectors. But *every* 12-tone aggregate displays this same kind of hexachordal equivalence. Forte has singled out these fifteen hexachordal pairs only because of a negative property—the non-equivalence of the Z-related hexachordal correspondents by either of the familiar criteria of equivalence, transposition or inversion. But no special value can be attributed to the only positive feature they share, the duplication of an interval vector, since this feature is shared by the two hexachordal components of each of the thirty-five hexachordally partitioned 12-tone tropes.

[9] *Webern, The Path to the New Music* (Bryn Mawr, Pa.: Presser, 1963), p. 51.

Arnold Schoenberg, it would seem, did *not* find any special virtue in the structural non-equivalence of Z-related hexachords. From 1927 to the end of his life he based his music on 12-tone sets whose hexachordal components are inversionally related, and Babbitt's further development of Schoenberg's "combinatorial" procedures extends them to sets whose hexachordal components are simultaneously transpositionally and inversionally related. I know of nothing in 12-tone theory or 12-tone practice where the Z-relation plays some special role. But if it plays such a role for hexachordal relations in *non-serial* atonal music, why should that role not have been carried forward into hexachordal relations in 12-tone music?

Am I making too much of only one type of Z-relation, namely the hexachordal type, instead of considering the concept of Z-related equivalence in the context of the whole field of pitch-class sets? But in his exposition of the Z-relation Forte himself gives us examples of hexachordal sets only, and indeed, Z-related pairs are much easier to come by among these than they are among sets of any other cardinal number. As he himself points out, of the nineteen Z-related pairs there is only one of cardinal number 4 (or 8), three of cardinal number 5 (or 7), and fifteen of cardinal number 6. If identity of interval vectors really represents a significant equivalency relation in the absence of transpositional or inversional equivalence, perhaps examples exploiting the unique Z-related pair of cardinal number 4 will offer better evidence for this than the examples that Forte has given us. This is the type of Z-correspondence that Schmalfeldt demonstrates in her exposition of the Z-relation.[10] Her examples pair the all-interval tetrachord *c-a-b-f*, which she derives from the Doctor's *Leitmotiv*, with its Z-correspondent *f♯-a-e-b♭*, which she finds in an excerpt from Act 1, Scene 4 (Example 18). The latter shows the Doctor "haranguing Wozzeck on the subject of man's 'freedom' of will to exert self-control," more specifically, the independence of his "will" from the constraints of Nature, as the missing portion of the text will show if we restore the upbeat (Example 19) that Schmalfeldt has omitted from this passage. What she has read as a collection of four pitch classes is thereby converted into a collection of six.

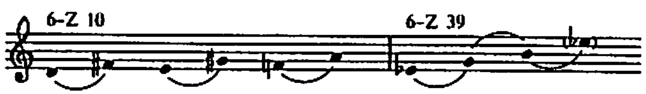
Is identity of interval vectors an appropriate measure of the similarity of interval content of two pitch-class sets? The interval vectors of 6-Z10 and 6-Z39 in Example 15 are identical. Does this really establish, in the absence of the ordinary equivalence relations of transposition and inversion, any special affinity between the two sets? Both sets, for instance, have the same number of major thirds, or interval 4, as shown in Example 20. For the set on the left it takes all six of its

[10] Schmalfeldt, op. cit., pp. 14ff.

EXAMPLE 18. Berg, *Wozzeck*, Act I/Scene 4: 500–02.

EXAMPLE 19.

EXAMPLE 20.



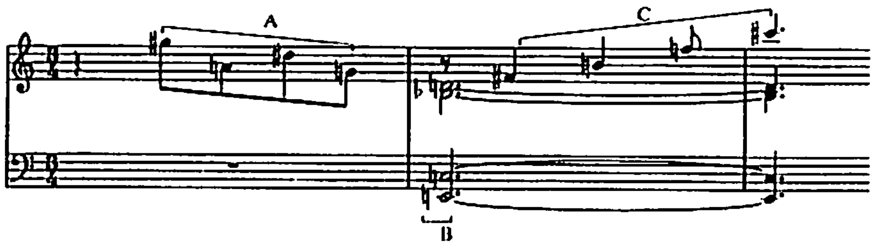
elements to unfold the three representations of interval 4. A compositional statement of these must necessarily entail the total intervallic content of the set. For the set on the right only three elements are involved in the three representations of interval 4, and none of the remaining components of the interval vector are entailed. The identical cardinality of interval 4 in the interval vectors of these Z-related pitch-class sets counts for nothing at all in view of the totally different structures that generate this interval in the two sets, and their totally different compositional implications.

But none of these arguments would carry any weight with me if I could only *hear* the correspondences that Forte describes. I can discover these connections between Z-related collections only by subjecting them to an analytical scrutiny that has nothing whatever to do with my intuitive experience as a listener or as a composer. Or, to speak more candidly, by allowing Professor Forte to conduct this analytical scrutiny for me. When I arrive at the pitch-class collection that brings Webern's op. 9, no. 4 (Example 17) to a close, there is, for me, no thrill of recognition, no sense of a goal achieved, induced by the fact that the interval vector of this final collection is exactly the same as the interval vector of the melody in the second violin at the beginning of the piece. Nor can I find anything in the way the composer unfolds these respective collections that points up a formally significant special relation between them, such as one might find in reference to such a relation at the conclusion of a tonal piece. Not to speak of the thrill of recognition, the sense of a goal achieved, that derives from the respective pitch levels of the two collections!

Forte's most remarkable, and entirely original, contribution to the theory of atonal music is his concept of pitch-class set complementation. What Webern described, in the above quotation, as an intuitive tendency toward the 12-tone aggregate—his crossing-off procedure where "this note has been there already"—is extended by Forte into a universal principle governing, in the most remarkably abstract and complex ways, all of atonal composition. As Forte shows in Example 21, the 12-tone aggregate that opens Webern's Op. 12, No. 4, may be reasonably segmented into three four-note collections. He prefaces the example with the following theorem: "The selection of a set of n elements from U [the universal set of 12 pitch-class integers] effectively divides or partitions U into two sets; the set of n elements selected and the set of 12−n elements not selected." As a simple and straightforward, not to say trivial, formulation, it is impossible to take exception to Forte's statement, but the question that interests us is its relevance to the music at hand. The two sets, n and 12−n, are said, in Forte's terminology, to be complements of each other. Thus the set of four pitch classes marked A in the example is said to have as its complement a set of eight pitch classes that consists not of the two 4-element sets B and C but of their combined and unsegmented pitch-class content. Set B stands in the same relation to the combined content of A+C, and Set C in the same relation to Sets A+B. If there is anything in this music to support the view that the complementary relations defined by Forte have any effective implications whatever for Webern, he fails to say what it is. All we are shown is Example 21, with Forte's catalog names for the three 4-element sets and their hypothetical complementary 8-element sets. There is nothing at all here to support the ways in which Forte contradicts the sense of Webern's own description of his rule-of-thumb exploitation of the 12-tone aggregate at the time—"this note has been there already."

A second example (Example 22) provides Forte with the rationale for an ultimate extension of the concepts of complementation and pitch-class set equivalence. The 5-note set marked K that opens the

EXAMPLE 21. Webern, Four Songs, Op. 12/4.



A:	[3,7,8,9]	(4-5)	$\bar{A}$ = +(B,C)	(8-5)
B:	[10,0,2,4]	(4-21)	$\bar{B}$ = +(A,C)	(8-21)
C:	[11,1,5,6]	(4-16)	$\bar{C}$ = +(A,B)	(8-16)

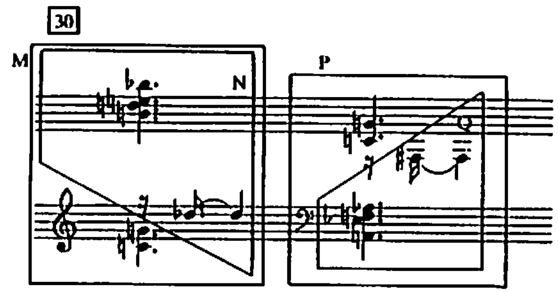
piece is described as complementary to the 7-note set marked P near the close, since the two together add up to a 12-tone aggregate. P is preceded at M by an inversion of the same 7-note set. This leads Forte to what, in an egregious understatement, he describes as a "significant extension of the complement relation." Since the set at P is the complement of K, let us also regard the set at M as the complement of K, even though it contains K and the two together are therefore five notes shy of the requisite pitch-class content of the 12-tone aggregate which was the basis for our definition of the complementary relation in the first place: "We accept as the complement not only the literal pitch-class complement, but also any transposition or any transposition of the inversion of the complement." The Fortean system of analysis has been described by one of its few unfriendly critics as a "fishing expedition."[31] This new definition of complementation vastly enlarges the fishing pond—none too soon, considering the meager support the two examples he has so far mustered have given to his claim that "for atonal music . . . the complement relation plays a fundamental structural role." And as for poor Webern, we have left him far behind, with his mere search for the "next note" which has not "been there already."

Forte assures us that this now vastly expanded definition of complementation will not unduly complicate analytical procedures, specifically, the problem of set identification: "As a practical consequence of the complement relation it is never necessary, for purposes of set

[31] Taruskin, op. cit.

EXAMPLE 22. Schoenberg, Five Piano Pieces, Op. 23/3.

K:	[10,11,1,2,4]	(5-10)
M:	[10,11,0,1,2,4,7]	(7-10)
N:	[10,11,1,2,4]	(5-10)
P:	[0,3,5,6,7,8,9]	(7-10)
Q:	[3,5,6,8,9]	(5-10)



or (3,5,6,8,9)
or (10,11,1,2,4)

identification, to deal with sets of cardinal number greater than 6. To identify a set of cardinal 9, for example, one need only discover the three missing pitch-class integers, determine their prime form, and consult the list of prime forms for cardinal 3." The problem of dealing *perceptually* with a set of nine elements as opposed to a set of three is not so easily resolved, however. A pitch-class set of three elements can be permuted into only six different orderings. Regardless of how we permute or combine the pitch representations of three pitch classes, we will recognize transpositions and inversions as equivalents of the same collection. The pitch-class set of nine elements that completes the 12-tone aggregate may be permuted in 362,880 different ways. Wouldn't this rather unduly complicate the problem of recognizing its transpositions and inversions as representations of the same set?

In the second scene of Berg's *Lulu* Alwa suddenly enters with an urgent message for his father, editor-in-chief of a newspaper: "A revolution has broken out in Paris!" Dr. Schön, however, is momentarily preoccupied in trying to force open the door behind which Lulu's husband has just cut his throat. Alwa continues, "No one at the editor's desk knows what to write," to an ordering of the twelve notes that seems to have no relevance whatever to his characteristic tone row, or to any of the other 12-tone sets on which the opera is based. A haphazard arrangement of the twelve notes would seem to be an appropriate musical metaphor for the sense of these words, but, with Berg, the more obvious the metaphor the more likely it is that the composer will have hidden an even better metaphor beneath it. The persistent analyst will eventually discover that this seemingly disordered 12-tone set simultaneously unfolds Alwa's tone row in a strictly ordered cyclically permuted inversion and a strictly ordered cyclically permuted transposed retrograde inversion, a serial relation which only a 12-tone composer who does not know what to write could contrive. Only composers who did not know what to write could possibly have contrived the musical relations described in Allen Forte's book, *The Structure of Atonal Music.*

San Francisco Symphony