AN APPROACH TO SIMULTANEITY IN TWELVE-TONE MUSIC

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THE OVEREXTENSION and consequent weakening of the traditional tonal functions gave rise, prior to Schoenberg's formulation of his "twelve-tone method," to a number of attempts to base a total musical structure upon a complex of functionally undifferentiated pitch elements. As examples, Debussy's *Voiles*, Scriabin's Seventh Sonata, and Roslavetz' Three Compositions for piano may be cited. Each of these works is based on sets comprising a selection of the notes of the chromatic scale. Scriabin's Seventh Sonata, for instance, is based on the following collection of seven notes and on transpositions of this collection:

Ex. 1

Such a collection cannot be regarded as analogous to a "mode," or a "scale," since (1) its elements are functionally equivalent, (2) transpositions are freely used so that there is a constant circulation of all twelve notes, and (3) there is no special criterion of simultaneity, such as the triad in the major-minor system or the limited consonant intervallic relations of the modal system. The unordered content of the set is the sole general basis of both simultaneity and succession.

A set comprising *all* twelve notes obviously cannot be thus defined in terms of its pitch content. Two different solutions to this problem were devised at about the same time, one by Arnold Schoenberg, the other by Josef Hauer. Hauer's set, or "trope," as he termed it, is defined as the combination of two unordered hexachords of mutually exclusive pitch content. Schoenberg's set is defined as a given permutation of the elements of the chromatic scale, that is, a series, which may be transposed to any pitch level, and which may be transformed by inversion, retrogression, or both. How is this ordered succession compositionally represented? Pitches may be stated in succession or simultaneously. But only succession is defined by the set. How can simultaneously stated pitches be said to represent a succession? To the extent that a consistent principle can be deduced from the practice of Schoenberg and other twelve-tone composers, it is as follows: any group of successive elements in the set may be stated as a simultaneity. This principle, which we will call verticalization, converts the set into something analogous to Hauer's trope, insofar as the set is defined, for harmonic purposes, not by the order of its notes but by the pitch content of its segments. The differences are that there are no precompositional assumptions regarding the size and number of segments into which the set is to be divided, and that the order of the segments themselves is dependent on the original succession.

Verticalization was a basic concept of atonal composition from the beginning. The first three notes of the opening phrase (Ex. 2) of Schoenberg's Opus 11, No. 1, for example,

Ex. 2

are verticalized to form a primary harmonic element of the next phrase.

Ex. 3

The application of this familiar concept to the twelve-tone set is problematical in a number of respects. (1) The relevance of a verticalized set-segment to a given set is ambiguous, the extent of this ambiguity depending on the number of notes in the segment. A succession of 6 dyads of mutually exclusive pitch content, for example, may, in itself, represent any one of 64 different permutations of the 12 notes, while a simultaneity comprising all 12 notes may represent any one of 479,001,600. (2) The possible verticalizations that may be derived from the general set are unsystematic, and largely, from a harmonic point of view, fortuitous, and therefore do not lend themselves to any coherent, over-all control of the harmonic material. (3) For this reason, the verticalization of adjacencies, though it is the only general principle of chord structure that has been deduced from the premise of an ordered succession of the twelve notes, cannot be, and, in fact, has not been, the only criterion of vertical association. Nor do verticalized segments of the set bear any explicit relation to simultaneities that are otherwise derived, as the triad does to nontriadic elements in tonal music.

I am not aware that Schoenberg, Berg, and Webern ever had anything to say about these problems, but it is obvious enough that, as composers, they were aware of them. Consider Schoenberg's Opus 33a: the initial statement of the set

Ex. 4

may represent any one of 13,824 different twelve-tone series. Subsequently, it becomes clear that the actual linear ordering on which the piece is based is as follows:

Ex. 5

The original distribution of the twelve notes into a series of tetrads, however, has far more relevance to the composition as a whole than does this serial arrangement of the notes. The general definition of the Schoenbergian set as an ordered succession of the twelve notes of the chromatic scale is obviously inadequate here, for within each tetrad the order is not specified. Simultaneity cannot represent succession, and for this reason every twelve-tone composition requires a modification of the general definition of the Schoenbergian set.[1] Obviously, this requirement in no way questions the validity of any given twelve-tone work, but it does question the extent to which Schoenberg's method is successful in meeting "the desire for a conscious control of the new means and forms" that led him to formulate this method in the first place.

In 1939, in an attempt to arrive at some general principles for the government of simultaneity in twelve-tone music, I considered the possibility of constructing special sets whose linear adjacencies would present a coherent pattern, sets more likely to suggest consistent harmonic

[1] Another conclusion, that "the effect of serial activity exists merely in the composer's imagination," was reached by Peter Stadlen in his article, "Serialism Reconsidered," *The Score*, February, 1958. Replies by Walter Piston, Roberto Gerhard, and Roger Sessions appeared in the following issue of *The Score*, July, 1958, and a rebuttal by Mr. Stadlen in the November 1958 issue, and my article, "Theory and Practice in Twelve-Tone Music (Stadlen Reconsidered)," in the issue of June, 1959.

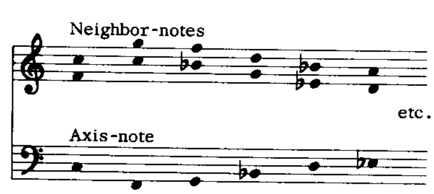
procedures than does the general set. It occurred to me that the semitonal scale and the circle of fifths each comprise linear adjacencies of a single type. In each case, verticalized segments of identical extent generate the twelve transpositions of a single collection of pitch elements. Obviously, neither the semitonal scale nor the circle of fifths is useful as an explicit twelve-tone set. However, each of these formations can generate not only a "one-interval" set, but also an "all-interval" set, as illustrated in Exx. 6a and 6b. In Ex. 6a the successive notes of the set proceed alternately along diverging series of fifths, in Ex. 6b they proceed alternately along diverging series of semitones. Ex. 6c illustrates a set that is similarly derived from two diverging "whole-tone scales."

Ex. 6a

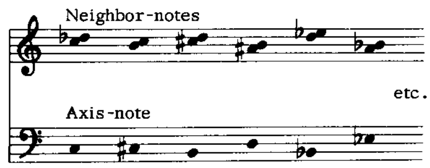
Ex. 6b

Ex. 6c

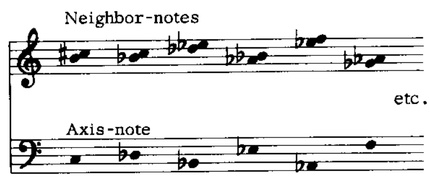
It will be seen that the notes immediately adjacent to any given element (we will call the former "neighbor-notes" and the latter "axis-note") always form a perfect fifth (or its inversion) in Ex. 6a, a semitone in Ex. 6b, and a whole tone in Ex. 6c. The three-note segments within each set are not identical in structure, however, as they are in the case of the two "one-interval" sets—the series of fifths and the series of semitones. Let us regard each element of the set in turn as an axis-note. The series of intervals formed by the succession of adjacency pairs will be seen to invert that formed by the succession of axis-notes.



Ex. 7a,



Ex. 7b



Ex. 7c

Of the three sets illustrated in Ex. 6, I have found only the first to be of compositional interest. I will therefore ignore the other two sets in the remainder of this discussion. Anyone interested in doing so can easily apply the precompositional operations described below to these two sets as well.

Thus far I have considered the axis- and neighbor-note combinations that may be derived from the set in only one of its four transformations. The axis- and neighbor-note combinations that may be derived from a given transposition of the inversion are illustrated below (Ex. 8). I will disregard the retrograde and the retrograde-inversion, since these cannot alter the unordered pitch content of segments of the set, which is all that concerns us here.

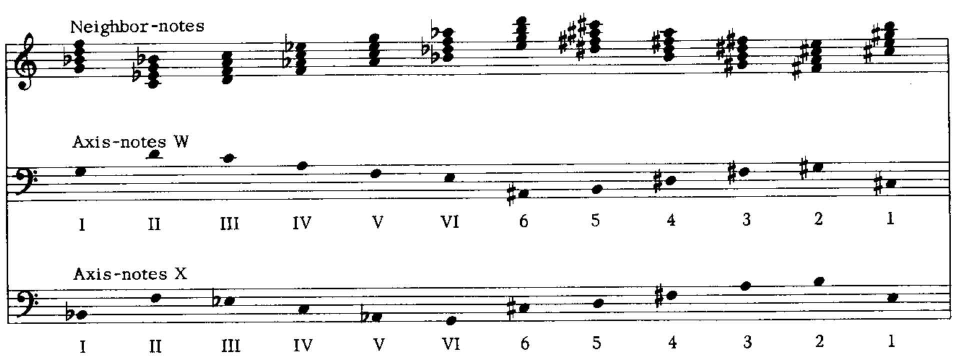
Ex. 8

Suppose now that a given transposition of the prime is assumed to be paired with a given transposition of the inversion. Just such an assumption is basic to Schoenberg's twelve-tone practice during the last twenty years of his life. The prime form of the characteristic Schoenbergian set is normally paired with the inversion, and the retrograde with the retrograde-inversion, with the relative transposition of each predetermined by the requirement that there be no duplication of pitch elements among corresponding hexachords of paired forms of the set. Our purpose here is to examine the complex of neighbor-notes associated with each axis-note where the latter is regarded as a given pitch element common to the prime and the inversion, with all transpositional relations between the paired forms of the set considered in turn.

It was pointed out at the beginning of this paper that the verticalization of segments of the set as a basis for simultaneity in twelve-tone music is problematical in three important respects, one of which was that the relation among the possible resultant verticalizations was likely to be unsystematic and fortuitous, resistant to compositionally coherent practice. Obviously, this objection cannot be raised against the principle of verticalization as applied to the specially constructed set under consideration here—a set which permits all possible verticalizations to be precompositionally formulated in a totally systematic fashion, regardless of which of its forms are employed or how extensive the verticalized segments. Ex. 9a presents such a precompositional statement of verticalized three-note segments of a fixed transposition (beginning on C) of the prime combined with three-note segments derived in turn from each of six nonequivalent transpositions of the inversion. Thus, the simultaneities resulting from the conjunction of the "neighbor-notes" in the first stave of Ex. 9a with "axis-notes X" combine the segments of the prime on C with those of the inversion on C around their common axis-notes. Since tritone transpositions of each set-form are equivalent and are found by retrograding the given set-form, Ex. 9a actually comprises all twelve transpositions of the inversion. In Ex. 9b the inversion (beginning on C) is similarly related to each transposition of the prime. In Ex. 9a the axis-notes are arranged in the order in which they appear in the inversional forms of the set, and in Ex. 9b they are arranged in the order in which they appear in the prime forms. The combined adjacencies around a given axis-note fall into three different intervallic structures, which I call "modes." A given prime will form an identical set of combined adjacencies with each of two different transpositions of the inversion, respectively designated as "axis-notes W" and "axis-notes X." Thus the neighbor-notes around each axis-note of the prime on C combined with the inversion on C are identical with those around the axis-notes a fourth higher resulting from the combination of the prime on C with the inversion on F. And a given inversion will form an identical set of combined adjacencies with each of two different transpositions of the prime, respectively designated as "axis-notes Y" and "axis-notes Z." The order numbers of the axis- and neighbor-note combinations (referred to as "chords" in the remainder of this discussion) within a given form (W, X, Y, or Z) of a given mode are identified by roman and arabic numerals, as shown. Equivalent arabic and roman numerals represent tritone transpositions of the same chord.

First Mode

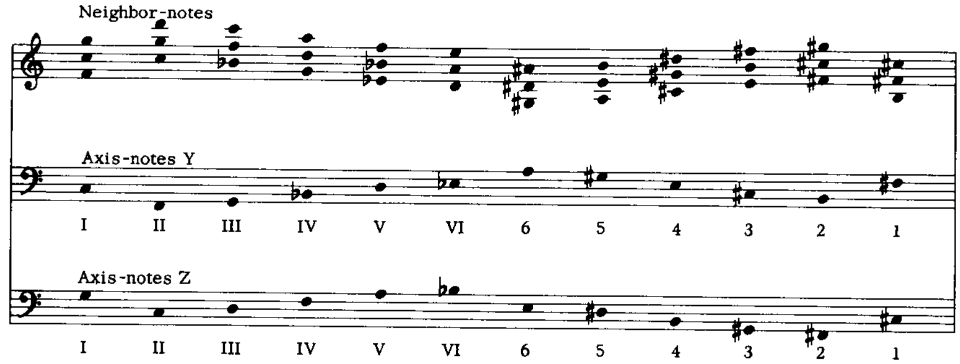
Second Mode



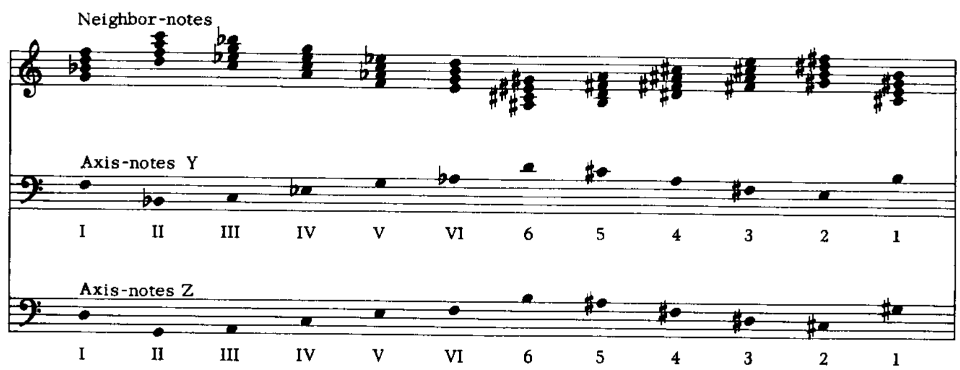
Third Mode

Ex. 9a

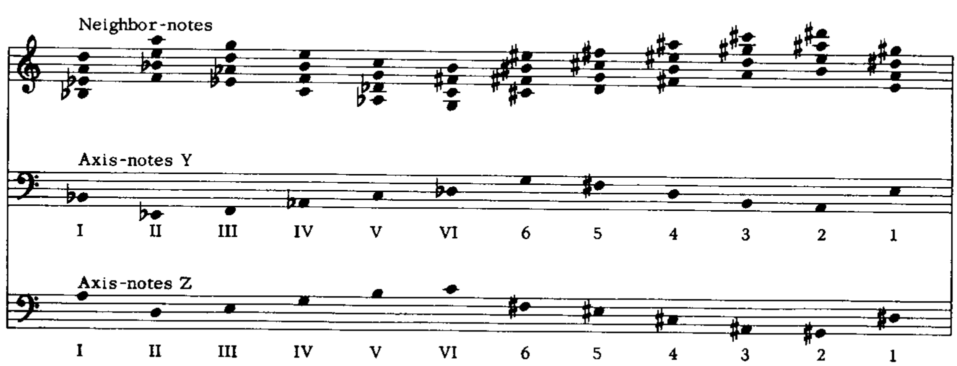
First Mode



Second Mode



Third Mode



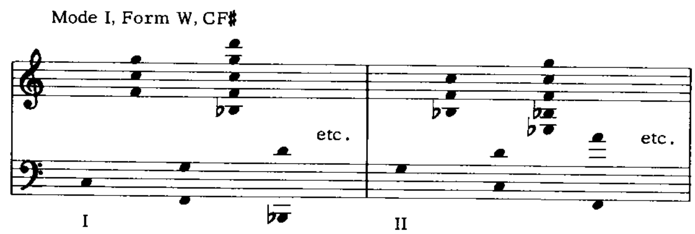
Ex. 9b

Exx. 9a and 9b may of course be transposed to any other pitch level. Since both the fixed prime on which Ex. 9a is based and the fixed inversion on which Ex. 9b is based begin on C and end on F♯, I refer to them as being in "the key of CF♯" (equivalent to F♯C). It will be noted that within each form of each mode there are certain chords whose axis-notes duplicate components of their neighbor-note combinations. Because of these duplications, certain chords within each mode will be found to be common to various forms and keys. Within each mode in any given key, Chords I and 1 are common to all four forms. The remaining pivotal chords can be deduced from the following table of such chords in the key of CF♯.

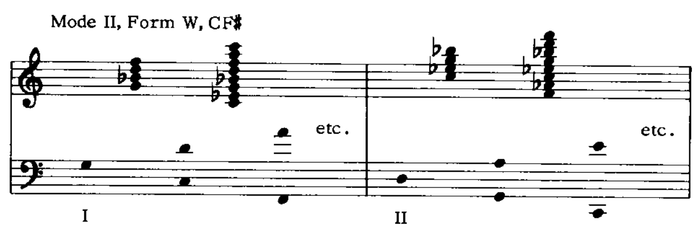
Chord	Key	Mode form	Chord	Key	Mode	Forms
I, 1	C F♯	First Mode	W = I, 1	C F♯	First Mode	W,X,Y,Z
II, 2	C F♯	First Mode	X = I, 1	F B	First Mode	W,X,Y,Z
III, 3	C F♯	Second Mode	W = I, 1	G C♯	Second Mode	W,X,Y,Z
IV, 4	C F♯	Second Mode	X = I, 1	B♭ E	Second Mode	W,X,Y,Z
V, 5	C F♯	Third Mode	W = I, 1	D G♯	Third Mode	W,X,Y,Z
VI, 6	C F♯	Third Mode	X = I, 1	E♭ A	Third Mode	W,X,Y,Z
I, 1	C F♯	First Mode	Y = I, 1	C F♯	First Mode	W,X,Y,Z
II, 2	C F♯	First Mode	Z = I, 1	G C♯	First Mode	W,X,Y,Z
III, 3	C F♯	Second Mode	Y = I, 1	F B	Second Mode	W,X,Y,Z
IV, 4	C F♯	Second Mode	Z = I, 1	D G♯	Second Mode	W,X,Y,Z
V, 5	C F♯	Third Mode	Y = I, 1	B♭ E	Third Mode	W,X,Y,Z
VI, 6	C F♯	Third Mode	Z = I, 1	A D♯	Third Mode	W,X,Y,Z

The pair of axis-notes on which each pair of chords designated in the column on the left is based is given in the two letter-names that designate the key at the left. Within each mode, among the various keys, equivalent W and Y forms and equivalent X and Z forms are found, the chords in the one merely being restated in a different sequence in the other. These equivalent modal forms can be deduced from the above table: Form W in the mode and key given at the left will be equivalent to Form Y in the mode and key given in the same line at the right, X will correspond similarly to Z, Y to W, and Z to X. The total harmonic material that may be derived from the verticalization of three-note segments of the basic set may thus be reduced to the series of chords found in six nonequivalent modal forms, each of which is statable in six nonequivalent keys.

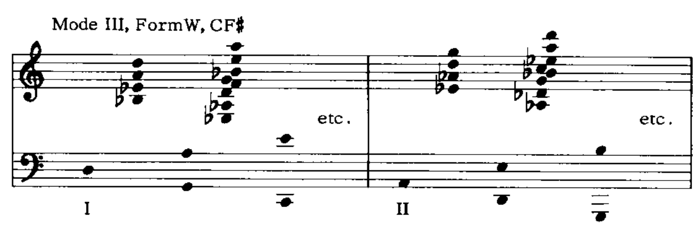
So far, we have considered the verticalization only of three-note segments of the set. The verticalized segment may be extended by the addition of its next adjacent elements in the set. Regardless of the extent of the segment, total symmetry of the same type that we have discovered in our study of three-note segments will be found. In Ex. 10 axis- and neighbor-note combinations derived by pairing extended segments of the prime and the inversion are illustrated. As before, segments sharing a common axis-note are combined. Successive additions to this common axis-note are shown.



Ex. 10a



Ex. 10b



Ex. 10c

The precompositional formulation of this material as given above was completed in 1939 within a very short span of time, and was entirely motivated by personal compositional requirements that the traditional twelve-tone system, as I then understood it, failed to fulfill. The anticipated advantages of my twelve-tone harmonic modes were not immediately realized compositionally, however. It was not until some time later, after I had come to understand that the precompositional reduction of harmonic possibilities eliminated only one of several problematical aspects of verticalization, that I was able to employ the new material compositionally. These additional respects in which verticalization remains problematical, briefly mentioned at the beginning of this discussion, have to do with the relevance of the linear structure of the set to simultaneities that can only be explained as deriving from unordered segments of the set, and with the relation, in any given twelve-tone work, of simultaneities so derived to simultaneities derived in other ways, or to simultaneities not, strictly speaking, "derived" at all—what one might call "circumstantial simultaneities." It was not until I eliminated these problems by making verticalization the *sole* basis for simultaneity and by discarding the traditional function of the set as a criterion of linear relationships, that I was able to exploit the harmonic modes compositionally.[2] The impossibility of retaining the linear function of the set should have been self-evident for several reasons: the restriction to a single type of set-structure, the elimination of the retrograde and retrograde-inversion as independent aspects of the set, and the derivation of each simultaneity from combined segments of the prime and inversion.

It may be objected that this revision of the primary assumptions of the twelve-tone system solves one problem by creating another, for it precompositionally defines simultaneity but not succession, whereas the Schoenbergian "tone-row" defines succession but not simultaneity. The point is, however, that the compositional implications of a system that defines simultaneity but not succession are quite different from those of a system that does the opposite. There are significant respects in which composition in the "Twelve-Tone Modal System" suggests analogies with traditional tonal rather than with traditional twelve-tone music. The function of rhythm in a musical language that is characterized by predefined simultaneities is not the same as in a musical language that is characterized by the *absence* of predefined simultaneities. I do not hesitate to represent a verticalized set-segment by a partial statement of its content, a practice that is not found in most twelve-tone music.[3] Finally, according to Schoenberg, his row was "invented to substitute for some of the unifying and formative advantages of scale and tonality," but at the same time, it "functions in the manner of a motive." In the "Twelve-Tone Modal System" these two functions are separate, just as they are in the diatonic system. This, it seems to me, is a decided advantage.

[2] See my Sonata for Piano, Southern Music; Three Movements for Orchestra, T. Presser Co.; Six Preludes for Piano, in *New Music for the Piano,* Lawson-Gould.

[3] The consequences of the traditional "rule" that no note may be omitted from any statement of the set are usually neglected. See Perle, *op. cit., The Score,* June, 1959 and "Schönberg's Late Style," *The Music Review,* November, 1952.