PERMUTATION MATRIX TAKES NORTH-WEST-UP TO NORTH-EAST-DOWN, so the output will NOT match TT2CMT.m, which is GCMT (up-south-east).

OUTPUT HERE IS NORTH-EAST-DOWN (AkiRichards convention).

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\begin{split} & \text{In}[1] = & \text{Clear}[\theta, \kappa, \sigma, \rho, \gamma, \beta, M] \\ & \text{V}[\theta_-, \kappa_-, \sigma_-] := \\ & \text{Cos}[\kappa] \cos[\sigma] + \cos[\theta] \sin[\kappa] \sin[\sigma] & \cos[\theta] \sin[\kappa] \cos[\sigma] - \cos[\kappa] \sin[\sigma] & -\sin[\theta] \sin[\kappa] \\ & -\sin[\kappa] \cos[\sigma] + \cos[\theta] \cos[\kappa] \sin[\sigma] & \cos[\theta] \cos[\kappa] \cos[\sigma] + \sin[\kappa] \sin[\sigma] & -\sin[\theta] \cos[\kappa] \\ & \sin[\theta] \sin[\sigma] & \sin[\theta] & \sin[\theta] \cos[\sigma] & \cos[\theta] \\ & \text{Yrot}[\alpha_-] := \begin{pmatrix} \cos[\alpha] & 0 & \sin[\alpha] \\ 0 & 1 & 0 \\ -\sin[\alpha] & 0 & \cos[\alpha] \end{pmatrix}; \\ & \text{R} = \frac{1}{\sqrt{6}} \begin{pmatrix} \sqrt{3} & 0 & -\sqrt{3} \\ -1 & 2 & -1 \\ \sqrt{2} & \sqrt{2} & \sqrt{2} \end{pmatrix}; \\ & \text{f} [\rho_-, \gamma_-, \beta_-] := \begin{pmatrix} \rho\cos[\gamma] \sin[\beta] \\ \rho\sin[\gamma] \sin[\beta] \\ \rho\cos[\beta] \end{pmatrix}; \\ & \text{L}[\rho_-, \gamma_-, \beta_-] := \text{Transpose}[\kappa] \cdot f[\rho, \gamma, \beta]; \\ & \text{U}[\theta_-, \kappa_-, \sigma_-] := \text{V}[\theta, \kappa, \sigma] \cdot \text{Yrot}[-\text{Pi}/4]; \\ & \text{P} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}; \\ & \text{M}[\theta_-, \kappa_-, \sigma_-, \rho_-, \gamma_-, \beta_-] := \\ & \text{P.U}[\theta, \kappa, \sigma] \cdot \text{DiagonalMatrix}[\text{Flatten}[\text{L}[\rho, \gamma, \beta]]] \cdot \text{Transpose}[\text{U}[\theta, \kappa, \sigma]] \cdot \text{Transpose}[\text{P}]; \\ & \text{Clear}[\theta, \kappa, \sigma, \rho, \gamma, \beta] \end{split}
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IN ALL FORMULAS WE WILL set rho = sqrt(2) such that M0 = rho/sqrt(2) = 1.

Double couple.

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AS IS

$$\begin{aligned} &\text{Model} &= & \mathbf{M} \left[ \boldsymbol{\theta}, \boldsymbol{\kappa}, \boldsymbol{\sigma}, \sqrt{2}, \boldsymbol{\gamma}, \boldsymbol{\beta} \right] [\mathbf{I}, \mathbf{I}] \mathbf{I} \\ &\text{Out(N)} = \left[ \sqrt{\frac{2}{3}} \cos[\beta] + \frac{2 \sin[\beta] \sin[\gamma]}{\sqrt{3}} \right] \cos[\beta] \cos[\beta] \sin[\kappa] - \cos[\kappa] \sin[\kappa] - \cos[\kappa] \sin[\alpha])^2 + \\ & \left[ \sqrt{\frac{2}{3}} \cos[\beta] - \cos[\gamma] \sin[\beta] - \frac{\sin[\beta] \sin[\gamma]}{\sqrt{3}} \right] \\ & \left[ \frac{\sin[\beta] \sin[\kappa]}{\sqrt{2}} - \frac{\cos[\kappa] \cos[\beta] + \cos[\beta] \sin[\kappa] \sin[\alpha]}{\sqrt{2}} \right]^2 \\ & \left[ \sqrt{\frac{2}{3}} \cos[\beta] + \cos[\gamma] \sin[\beta] - \frac{\sin[\beta] \sin[\gamma]}{\sqrt{3}} \right] \\ & \left[ -\frac{\sin[\beta] \sin[\kappa]}{\sqrt{2}} - \frac{\cos[\kappa] \cos[\alpha] + \cos[\beta] \sin[\kappa] \sin[\alpha]}{\sqrt{2}} \right]^2 \\ & & \left[ -\frac{\sin[\beta] \sin[\kappa]}{\sqrt{2}} - \frac{\cos[\kappa] \cos[\alpha] + \cos[\beta] \sin[\kappa] \sin[\alpha]}{\sqrt{2}} \right]^2 \\ & & \left[ -\frac{\sin[\beta] \sin[\kappa]}{\sqrt{2}} - \frac{\cos[\kappa] \sin[\gamma]}{\sqrt{3}} \right] \left( -\cos[\beta] \cos[\kappa] \cos[\alpha] - \sin[\kappa] \sin[\alpha] \right) \\ & & \left[ \cos[\beta] \cos[\kappa] \cos[\beta] + \sin[\kappa] \sin[\alpha] \right] - \left[ -\frac{\sqrt{\frac{2}{3}}}{\sqrt{2}} \cos[\beta] + \cos[\gamma] \sin[\alpha]} \right] \\ & & \left[ -\frac{\cos[\kappa] \sin[\beta]}{\sqrt{2}} - \frac{-\cos[\beta] \sin[\kappa] + \cos[\beta] \cos[\kappa] \sin[\alpha]}{\sqrt{2}} \right] \\ & & \left[ -\frac{\cos[\kappa] \sin[\beta]}{\sqrt{2}} + \frac{-\cos[\beta] \sin[\kappa] + \cos[\beta] \cos[\kappa] \sin[\alpha]}{\sqrt{3}} \right] \\ & & \left[ -\frac{\cos[\kappa] \sin[\beta]}{\sqrt{2}} - \frac{-\cos[\beta] \sin[\kappa] + \cos[\beta] \cos[\kappa] \sin[\alpha]}{\sqrt{2}} \right] \\ & & \left[ \frac{\cos[\kappa] \sin[\beta]}{\sqrt{2}} - \frac{-\cos[\beta] \sin[\kappa] + \cos[\beta] \cos[\kappa] \sin[\alpha]}{\sqrt{2}} \right] \\ & & \left[ \frac{\cos[\kappa] \sin[\beta]}{\sqrt{2}} - \frac{-\cos[\beta] \sin[\kappa] + \cos[\beta] \cos[\kappa] \sin[\alpha]}{\sqrt{2}} \right] \\ & & \left[ \frac{\cos[\alpha] \sin[\beta]}{\sqrt{2}} - \frac{\sin[\beta] \sin[\gamma]}{\sqrt{3}} \right] \left( \frac{\cos[\beta]}{\sqrt{2}} - \frac{\sin[\beta] \sin[\alpha]}{\sqrt{2}} \right) \\ & & \left[ \frac{\sqrt{\frac{2}{3}} \cos[\beta] - \cos[\gamma] \sin[\beta]}{\sqrt{3}} - \frac{\sin[\beta] \sin[\gamma]}{\sqrt{3}} \right] \left( \frac{\cos[\beta]}{\sqrt{2}} - \frac{\sin[\beta] \sin[\alpha]}{\sqrt{2}} \right) \left( \frac{\cos[\beta]}{\sqrt{2}} + \frac{\sin[\beta] \sin[\alpha]}{\sqrt{2}} \right) \\ & & \left[ \frac{\sqrt{\frac{2}{3}}}{\sqrt{3}} \cos[\beta] + \cos[\beta] - \frac{\sin[\beta] \sin[\gamma]}{\sqrt{3}} \right] \left( \frac{\cos[\beta]}{\sqrt{2}} - \frac{\sin[\beta] \sin[\alpha]}{\sqrt{2}} \right) \left( \frac{\cos[\beta]}{\sqrt{2}} + \frac{\sin[\beta] \sin[\alpha]}{\sqrt{2}} \right) \\ & & \left[ \frac{\sqrt{\frac{2}{3}}}{\sqrt{3}} \cos[\beta] + \cos[\beta] \sin[\beta] \sin[\gamma]}{\sqrt{3}} \right] \left( \frac{\cos[\beta]}{\sqrt{2}} - \frac{\sin[\beta] \sin[\alpha]}{\sqrt{2}} \right) \left( \frac{\cos[\beta]}{\sqrt{2}} + \frac{\sin[\beta] \sin[\alpha]}{\sqrt{2}} \right) \\ & & \left[ \frac{\sqrt{\frac{2}{3}}}{\sqrt{3}} \cos[\beta] + \cos[\beta] \sin[\beta] \sin[\gamma]} \right] \left( \frac{\cos[\beta]}{\sqrt{2}} - \frac{\sin[\beta] \sin[\alpha]}{\sqrt{2}} \right) \left( \frac{\cos[\beta]}{\sqrt{2}} + \frac{\sin[\beta] \sin[\alpha]}{\sqrt{2}} \right) \\ & & \left[ \frac{\sqrt{\frac{2}{3}}}{\sqrt{3}} \cos[\beta] + \cos[\beta] \sin[\beta] \sin[\gamma]} \right] \left( \frac{\cos[\beta]}{\sqrt{2}} - \frac{\sin[\beta] \sin[\alpha]}{\sqrt{2}} \right) \\ & & \left[ \frac{\sqrt{\frac{2}{3}}}{\sqrt{3}} \cos[\beta] + \frac{\sin[\beta] \sin[\alpha]}{\sqrt{3}} \right] \left( \frac{\cos[\beta]}{\sqrt{2}} - \frac{\sin[\beta] \sin[\alpha]}{\sqrt{2}} \right) \\ & & \left[ \frac{\sqrt{\frac{2}{3}}}{\sqrt{2}} \cos[\beta] + \frac{\sin[\beta] \sin[\alpha]}{\sqrt{2}} \right] \\ & & \left[ \frac{\cos[\beta]}{\sqrt{2}} - \frac{\sin[\beta] \sin[\alpha]}{\sqrt{2}} \right] \\ & & \left[ \frac{\cos[\beta]}{\sqrt{2}} - \frac{\sin[\beta]$$

In[35]:= 
$$M \left[ \theta, \kappa, \sigma, \sqrt{2}, \gamma, \beta \right] [[1, 2]]$$

$$\begin{aligned} & \text{Out}[35] = & - \left( \sqrt{\frac{2}{3}} \; \cos[\beta] + \frac{2 \sin[\beta] \sin[\gamma]}{\sqrt{3}} \right) \left( \cos[\theta] \cos[\sigma] \sin[\kappa] - \cos[\kappa] \sin[\sigma] \right) \\ & \quad \left( \cos[\theta] \cos[\kappa] \cos[\sigma] + \sin[\kappa] \sin[\sigma] \right) - \left( \sqrt{\frac{2}{3}} \; \cos[\beta] - \cos[\gamma] \sin[\beta] - \frac{\sin[\beta] \sin[\gamma]}{\sqrt{3}} \right) \\ & \quad \left( -\frac{\cos[\kappa] \sin[\theta]}{\sqrt{2}} - \frac{-\cos[\sigma] \sin[\kappa] + \cos[\theta] \cos[\kappa] \sin[\sigma]}{\sqrt{2}} \right) \\ & \quad \left( -\frac{\sin[\theta] \sin[\kappa]}{\sqrt{2}} - \frac{\cos[\kappa] \cos[\sigma] + \cos[\theta] \sin[\kappa] \sin[\sigma]}{\sqrt{2}} \right) - \\ & \quad \left( \sqrt{\frac{2}{3}} \; \cos[\beta] + \cos[\gamma] \sin[\beta] - \frac{\sin[\beta] \sin[\gamma]}{\sqrt{3}} \right) \\ & \quad \left( -\frac{\cos[\kappa] \sin[\theta]}{\sqrt{2}} + \frac{-\cos[\sigma] \sin[\kappa] + \cos[\theta] \cos[\kappa] \sin[\sigma]}{\sqrt{2}} \right) \\ & \quad \left( -\frac{\sin[\theta] \sin[\kappa]}{\sqrt{2}} + \frac{\cos[\kappa] \cos[\sigma] + \cos[\theta] \sin[\kappa] \sin[\sigma]}{\sqrt{2}} \right) \end{aligned}$$

## In[36]:= $\mathbf{M} \left[ \boldsymbol{\theta}, \kappa, \sigma, \sqrt{2}, \gamma, \beta \right] [[1, 3]]$

$$\begin{aligned} & \text{Out}[36] = & -\text{Cos}\left[\sigma\right] \left(\sqrt{\frac{2}{3}} \; \text{Cos}\left[\beta\right] + \frac{2 \sin\left[\beta\right] \sin\left[\gamma\right]}{\sqrt{3}} \right) \sin\left[\theta\right] \; & (\text{Cos}\left[\theta\right] \; \text{Cos}\left[\sigma\right] \sin\left[\kappa\right] - \text{Cos}\left[\kappa\right] \sin\left[\sigma\right]) \; - \\ & \left(\sqrt{\frac{2}{3}} \; \text{Cos}\left[\beta\right] - \text{Cos}\left[\gamma\right] \sin\left[\beta\right] - \frac{\sin\left[\beta\right] \sin\left[\gamma\right]}{\sqrt{3}} \right) \left(\frac{\text{Cos}\left[\theta\right]}{\sqrt{2}} - \frac{\sin\left[\theta\right] \sin\left[\sigma\right]}{\sqrt{2}} \right) \\ & \left(-\frac{\sin\left[\theta\right] \sin\left[\kappa\right]}{\sqrt{2}} - \frac{\text{Cos}\left[\kappa\right] \cos\left[\sigma\right] + \text{Cos}\left[\theta\right] \sin\left[\kappa\right] \sin\left[\sigma\right]}{\sqrt{2}} \right) - \\ & \left(\sqrt{\frac{2}{3}} \; \text{Cos}\left[\beta\right] + \text{Cos}\left[\gamma\right] \sin\left[\beta\right] - \frac{\sin\left[\beta\right] \sin\left[\gamma\right]}{\sqrt{3}} \right) \left(\frac{\text{Cos}\left[\theta\right]}{\sqrt{2}} + \frac{\sin\left[\theta\right] \sin\left[\sigma\right]}{\sqrt{2}} \right) \\ & \left(-\frac{\sin\left[\theta\right] \sin\left[\kappa\right]}{\sqrt{2}} + \frac{\text{Cos}\left[\kappa\right] \cos\left[\sigma\right] + \text{Cos}\left[\theta\right] \sin\left[\kappa\right] \sin\left[\sigma\right]}{\sqrt{2}} \right) \end{aligned}$$

$$\begin{split} & \ln[37] = & \mathbf{M} \bigg[ \boldsymbol{\theta}, \boldsymbol{\kappa}, \boldsymbol{\sigma}, \sqrt{2}, \boldsymbol{\gamma}, \boldsymbol{\beta} \bigg] \big[ [\mathbf{2}, \mathbf{3}] \big] \\ & \text{Out}[37] = & -\text{Cos}[\sigma] \left( \sqrt{\frac{2}{3}} \, \text{Cos}[\beta] + \frac{2 \, \text{Sin}[\beta] \, \text{Sin}[\gamma]}{\sqrt{3}} \right) \, \text{Sin}[\theta] \, (-\text{Cos}[\theta] \, \text{Cos}[\kappa] \, \text{Cos}[\sigma] - \text{Sin}[\kappa] \, \text{Sin}[\sigma]) - \\ & \left( \sqrt{\frac{2}{3}} \, \text{Cos}[\beta] + \text{Cos}[\gamma] \, \text{Sin}[\beta] - \frac{\text{Sin}[\beta] \, \text{Sin}[\gamma]}{\sqrt{3}} \right) \left( \frac{\text{Cos}[\theta]}{\sqrt{2}} + \frac{\text{Sin}[\theta] \, \text{Sin}[\sigma]}{\sqrt{2}} \right) \\ & \left( \frac{\text{Cos}[\kappa] \, \text{Sin}[\theta]}{\sqrt{2}} - \frac{-\text{Cos}[\sigma] \, \text{Sin}[\kappa] + \text{Cos}[\theta] \, \text{Cos}[\kappa] \, \text{Sin}[\sigma]}{\sqrt{2}} \right) - \\ & \left( \sqrt{\frac{2}{3}} \, \text{Cos}[\beta] - \text{Cos}[\gamma] \, \text{Sin}[\beta] - \frac{\text{Sin}[\beta] \, \text{Sin}[\gamma]}{\sqrt{3}} \right) \left( \frac{\text{Cos}[\theta]}{\sqrt{2}} - \frac{\text{Sin}[\theta] \, \text{Sin}[\sigma]}{\sqrt{2}} \right) \\ & \left( \frac{\text{Cos}[\kappa] \, \text{Sin}[\theta]}{\sqrt{2}} + \frac{-\text{Cos}[\sigma] \, \text{Sin}[\kappa] + \text{Cos}[\theta] \, \text{Cos}[\kappa] \, \text{Sin}[\sigma]}{\sqrt{2}} \right) \\ & \text{SIMPLIFY} \\ & \text{In}[20] := \, \, \, \frac{1}{3} \left( \sqrt{6} \, \, \text{Cos}[\beta] - \text{Sin}[\beta] \, \left( \frac{1}{2} \, \sqrt{3} \, \, \text{Cos}[\kappa]^2 \, (-1 + 3 \, \text{Cos}[2\sigma]) \, \text{Sin}[\gamma] + \frac{6 \, \text{Cos}[\alpha] \, \text{Sin}[\alpha]}{\sqrt{6}} \, \text{Sin}[\alpha] \, \text{Sin}[\alpha]} \, \text{Sin}[\alpha] \right) \cdot \text{Sin}[\alpha]^2 \right) \\ & \text{Sin}[\alpha] \cdot \left( \frac{1}{3} \, \text{Cos}[\alpha] \, \text{Sin}[\alpha] \, \text{Sin}[\alpha]} \right) \cdot \text{Sin}[\alpha]^2 \right) \\ & \text{Sin}[\alpha] \cdot \left( \frac{1}{3} \, \text{Cos}[\alpha] \, \text{Sin}[\alpha] \,$$

$$\begin{aligned} & \text{In}[20] = & \text{Simplify} \Big[ \mathbf{M} \Big[ \boldsymbol{\theta}, \boldsymbol{\kappa}, \boldsymbol{\sigma}, \sqrt{2}, \boldsymbol{\gamma}, \boldsymbol{\beta} \Big] \big[ [\mathbf{1}, \mathbf{1}] \big] \Big] \\ & \text{Out}[20] = & \frac{1}{3} \left( \sqrt{6} \, \cos[\beta] - \sin[\beta] \, \left( \frac{1}{2} \, \sqrt{3} \, \cos[\kappa]^2 \, (-1 + 3 \cos[2\sigma]) \, \sin[\gamma] + \\ & & 6 \cos[\kappa] \, \cos[\sigma] \, \sin[\kappa] \, \left( \cos[\gamma] \, \sin[\theta] + \sqrt{3} \, \cos[\theta] \, \sin[\gamma] \, \sin[\sigma] \right) + \sin[\kappa]^2 \\ & & \left( -\frac{1}{2} \, \sqrt{3} \, \cos[\theta]^2 \, (1 + 3 \cos[2\sigma]) \, \sin[\gamma] + \sqrt{3} \, \sin[\gamma] \, \sin[\theta]^2 + 6 \cos[\gamma] \, \cos[\theta] \, \sin[\theta] \, \sin[\sigma] \right) \right) \Big) \\ & & \text{In}[21] = & \textbf{Simplify} \Big[ \mathbf{M} \Big[ \boldsymbol{\theta}, \boldsymbol{\kappa}, \boldsymbol{\sigma}, \sqrt{2}, \boldsymbol{\gamma}, \boldsymbol{\beta} \Big] \big[ [\mathbf{2}, \mathbf{2}] \big] \Big] \\ & \text{Out}[22] = & \frac{1}{6} \left( 2\sqrt{6} \, \cos[\beta] + \\ & \text{Sin}[\beta] \, \left( \sqrt{3} \, \cos[\theta]^2 \, \cos[\kappa]^2 \, (1 + 3 \cos[2\sigma]) \, \sin[\gamma] - 2 \left( \sqrt{3} \, \cos[\sigma]^2 \, \sin[\gamma] \, \sin[\kappa]^2 - 3 \cos[\gamma] \right) \\ & \text{Cos}[\sigma] \, \sin[\theta] \, \sin[2\kappa] - 2\sqrt{3} \, \sin[\gamma] \, \sin[\kappa]^2 \, \sin[\sigma]^2 + \cos[\kappa]^2 \, \sin[\theta] \\ & \left( \sqrt{3} \, \sin[\gamma] \, \sin[\theta] + 6 \cos[\gamma] \, \cos[\theta] \, \sin[\gamma] \, \sin[\sigma]^2 + \cos[\theta] \, \sin[\gamma] \, \sin[2\kappa] \, \sin[2\sigma] \right) \Big) \\ & \text{In}[22] = & \textbf{Simplify} \Big[ \mathbf{M} \Big[ \boldsymbol{\theta}, \boldsymbol{\kappa}, \boldsymbol{\sigma}, \sqrt{2}, \boldsymbol{\gamma}, \boldsymbol{\beta} \Big] \big[ [\mathbf{3}, \mathbf{3}] \big] \Big] \\ & \text{Out}[22] = & \frac{1}{6} \left( 2\sqrt{6} \, \cos[\beta] + \\ & \text{Sin}[\beta] \, \left( -2\sqrt{3} \, \cos[\theta]^2 \, \sin[\gamma] + \sqrt{3} \, (1 + 3 \cos[2\sigma]) \, \sin[\gamma] \, \sin[\theta]^2 + 6 \cos[\gamma] \, \sin[2\theta] \, \sin[\sigma] \right) \Big) \\ & \text{In}[23] = & \textbf{Simplify} \Big[ \mathbf{M} \Big[ \boldsymbol{\theta}, \boldsymbol{\kappa}, \boldsymbol{\sigma}, \sqrt{2}, \boldsymbol{\gamma}, \boldsymbol{\beta} \Big] \big[ [\mathbf{1}, \mathbf{2}] \big] \Big] \\ & \text{Out}[23] = & \frac{1}{3} \left( 3 \cos[\gamma] \, \sin[\beta] \, \sin[\theta] \, \left( \cos[\kappa]^2 \, \cos[\sigma] - \cos[\sigma] \, \sin[\kappa]^2 + \cos[\theta] \, \sin[2\kappa] \, \sin[\sigma] \right) - \\ & \frac{3}{16} \, \sqrt{3} \, \sin[\beta] \, \sin[\theta] \, \left( \cos[\kappa]^2 \, \cos[\sigma] - \cos[\sigma] \, \sin[\kappa]^2 + \cos[\theta] \, \sin[2\kappa] \, \sin[\sigma] \right) - \\ & \frac{3}{16} \, \sqrt{3} \, \sin[\beta] \, \sin[\theta] \, \left( \cos[\kappa]^2 \, \cos[\sigma] - \cos[\sigma] \, \sin[\kappa]^2 + \cos[\theta] \, \sin[2\kappa] \, \sin[\sigma] \right) - \\ & \frac{3}{16} \, \sqrt{3} \, \sin[\beta] \, \sin[\beta] \, \sin[\theta] \, \left( \cos[\kappa]^2 \, \cos[\sigma] - \cos[\sigma] \, \sin[\kappa]^2 + \cos[\theta] \, \sin[2\kappa] \, \sin[\sigma] \right) - \\ & \frac{3}{16} \, \sqrt{3} \, \sin[\beta] \, \sin[\theta] \, \left( \cos[\kappa]^2 \, \cos[\sigma] - \cos[\sigma] \, \sin[\kappa]^2 + \cos[\theta] \, \sin[2\kappa] \, \sin[\sigma] \right) - \\ & \frac{3}{16} \, \sqrt{3} \, \sin[\beta] \, \sin[\theta] \, \left( \cos[\kappa]^2 \, \cos[\sigma] - \cos[\sigma] \, \sin[\kappa]^2 + \cos[\theta] \, \sin[\sigma] \right) - \\ & \frac{3}{16} \, \sqrt{3} \, \sin[\theta] \, \sin[\theta] \, \left( \cos[\kappa]^2 \, \cos[\sigma] - \cos[\sigma] \, \sin[\kappa]^2 + \cos[\theta] \, \cos[\theta] \,$$

$$\begin{aligned} & \sup_{i \in \mathbb{N}^{2}} & \sup_{i \in \mathbb{N}^{2}} \{ \mathbf{N}_{i} \mathbf{N}_{i}, \mathbf{N}_{i}, \mathbf{N}_{i}, \mathbf{N}_{i}, \mathbf{N}_{i} \mathbf{N}_{i}, \mathbf{N}_{i} \} [[1, 3]] \} \\ & \sup_{i \in \mathbb{N}^{2}} & \sup_{i \in \mathbb{N}^{2}} \{ 2\sqrt{3} \cos[\alpha] \sin[\gamma] \sin[\gamma] \cos[\alpha] \cos[\alpha] \sin[\kappa] - \cos[\kappa] \sin[\alpha] + \\ & 2\cos[\gamma] (\cos[\beta] \cos[\kappa] \cos[\alpha] + \cos[\alpha]^{2} \sin[\kappa] \sin[\alpha] - \sin[\beta]^{2} \sin[\kappa] \sin[\alpha] ) \} \\ & \sup_{i \in \mathbb{N}^{2}} & \sup_{i \in \mathbb{N}^{2}} \{ \mathbf{N}_{i} \mathbf{N}_{i}, \mathbf{N}_{i$$

Test cases:

Clear  $[\theta, \kappa, \sigma, \rho, \gamma, \beta]$  $\theta = Pi/4;$  $\kappa = \text{Pi}/3;$  $\sigma = -Pi/3$ ;  $\rho = 1;$  $\gamma = Pi/12;$  $\beta = Pi/3;$ MatrixForm [Simplify[M[ $\theta$ ,  $\kappa$ ,  $\sigma$ ,  $\rho$ ,  $\gamma$ ,  $\beta$ ]]]

MatrixForm [N[M[ $\theta$ ,  $\kappa$ ,  $\sigma$ ,  $\rho$ ,  $\gamma$ ,  $\beta$ ]]]

$$\begin{pmatrix} \frac{1}{768} \left(123 - 90\sqrt{2} + 221\sqrt{3} + 18\sqrt{6}\right) & \frac{1}{256} \left(9 + 6\sqrt{2} - 27\sqrt{3} - 10\sqrt{6}\right) & \frac{1}{128} \left(-9 - 15\sqrt{2} + 3\sqrt{3} + \sqrt{6}\right) \\ \frac{1}{256} \left(9 + 6\sqrt{2} - 27\sqrt{3} - 10\sqrt{6}\right) & \frac{1}{768} \left(-39 + 90\sqrt{2} + 239\sqrt{3} - 18\sqrt{6}\right) & -\frac{3}{128} \left(1 - \sqrt{2} - \sqrt{3} + 5\sqrt{6}\right) \\ \frac{1}{128} \left(-9 - 15\sqrt{2} + 3\sqrt{3} + \sqrt{6}\right) & -\frac{3}{128} \left(1 - \sqrt{2} - \sqrt{3} + 5\sqrt{6}\right) & \frac{1}{192} \left(-21 - 19\sqrt{3}\right) \\ \begin{pmatrix} 0.550254 & -0.210059 & 0.596548 & -0.236747 \\ -0.176309 & -0.236747 & -0.280776 \end{pmatrix}$$