

## 1-) [20 Points] Seven Sided Dices

Three fair **seven-sided dice**, each numbered from 1 to 7, are simultaneously rolled, and the numbers on the top face of each die are observed. Let random variable  $X$  represent the number observed on the top face of the first die,  $Y$  represent the number observed on the top face of the second die, and  $Z$  represent the number observed on the top face of the third die.

a-) Event  $A$  is defined as “the maximum of the numbers observed at the end of the process is less than 5”. Find the probability  $P[A]$ .

b-) Event  $B$  is defined as “the minimum of the numbers observed at the end of the process is greater than 3”. Find the probability  $P[B]$ .

c-) Find the probability  $P[B | A]$ .

d-) Event  $C$  is defined as “the sum of numbers being greater than 5”.

Find the probability  $P[C]$ .

e-) Event  $D$  is defined as “the maximum of the numbers observed at the end of the process is less than 2”. Find the probability  $P[C | D]$ .

## 2-) [20 Points] Circuit System

A circuit system between  $X$  and  $Y$  lines is given in Figure 1. Additionally, the probabilities of each component functioning correctly are provided. It is known that;

- The circuit components  $A$ ,  $B$ ,  $C$  and  $D$  fails independently.
- If component  $A$  is functional, then the operation of either line 1 or line 2 is sufficient for the circuit system between  $X$  and  $Y$  to function.

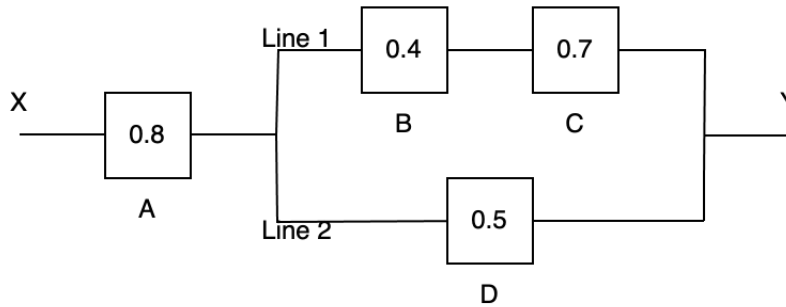


Figure 1: The Circuit System

a-) What is the probability of the circuit system works?

b-) Given that the system is not working, what is the probability that the component  $A$  is working?

c-) Given that the system is working, what is the probability that the component  $C$  is working?

d-) Given that the system is working, what is the probability that the component  $D$  is not working?



### 3-) [20 Points] Software Practice

Mehmet is a computer engineering student at TOBB-ETÜ. He has a habit of practicing his coding skills every day. If he practices coding on a given day, the probability that he will practice (or skip) on the next day is 0.7 (or 0.3, respectively). If he skips practicing on a given day, the probability that he will practice (or skip) on the next day is 0.9 (or 0.1, respectively). Mehmet (by default) practices when he starts his routine.

Let's denote;

- $P_k$ : the event that Mehmet practices at the end of the  $k$ th day, for  $k=1, 2, \dots$  respectively.
- $S_k$ : the event that Mehmet skips practice at the end of the  $k$ th day, for  $k=1, 2, \dots$  respectively.

a-) What is the probability of  $P(P_3)$ ?

b-) What is the probability of  $P(S_4)$ ?

c-) What is the likelihood of him practicing after two days, considering that he skipped practice at the end of the first day?

d-) What is the likelihood of him having skipped practice at the end of the first day, given that he is practicing at the end of the third day?

### 4-) [20 Points] Shopping Center

Random variables  $P$  and  $Q$  represent the number of customers that enter 2 separate stores during a certain 10-minute period. These stores are located in the same shopping center so it is important that store managers coordinate their operations if necessary.

The joint distribution of  $P$  and  $Q$  is known to be

$$f(p, q) = \frac{1}{2^{(p+q+2)}}; p \in \{\mathbb{Z}^+ \cup \{0\}\} \text{ and } q \in \{\mathbb{Z}^+ \cup \{0\}\}$$

a-) Find the marginal distribution of  $P$ .

b-) Are the two random variables  $P$  and  $Q$  independent or not independent? Explain the reason mathematically.

c-) What is the probability that, during the time period in question, less than a total of 6 customers enter the two stores given that  $P$  is more than 2?

d-) What is the expected value of customers that enter store  $Q$  during the 10-minute period?



## 5-) [20 Points] Lifetime of Components at a Computer System

Let  $X$  and  $Y$  be the lifetime of two components in a computer system. If the joint density functions of these variables is

$$f(x, y) = \begin{cases} cxy^2 & \text{if } x \geq 0, y \geq 0, x + y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- a-) Find  $c$ .
- b-) Find **CDF** of  $X$  and draw it.
- c-) Find **PDF** of  $X$  and draw it.
- d-) Find  $P(Y \leq 1/3 \mid X = 0.4)$ .
- e-) Find  $P(X + Y \geq 1/3)$ .
- f-) Find  $E[X]$ .

## Submission (Deadline: 16.02.2024 Friday 15.00)

- Bring your stapled paper at Teknoloji-Merkezi-217 (can be a box in front of the door, if you can't find anyone or there is no box at the door, throw it under the door.)
- Submit a scanned pdf of your answers to the <https://uzak.etu.edu.tr> site.

***Important Note:** Both are required. Be aware that your answers should be in-order.*