[100 Points] University Exam Simulation

A student is going to take a university exam.

You can find information about the exam below:

• There are a total of **100 questions**.

• There are a total of **200 minutes** and it is known that the student will stay until end of the exam.

• One can get maximum **500 points** and minimum **0 points** from the exam. Each net point worth **5 points**. Every **4 incorrect** answers cancel out **1 correct** answer. For example, if the student marked 70 correct, 12 incorrect and 18 blank answers, that correponds to 67 net points, and the student will get 67 * 5 = 335 points in total. (Points can be floating numbers, so do not round the numbers. If net points is negative or zero the student will get 0 points.)

• It is required that **200 points** in total for passing the exam.

• The student moves on to the next question once they have answered or left a question blank. **Once answered or left blank, one cannot move back to the questions earlier.** The decision to answer or leave the next question blank is independent of what was done on the previous question.

• As the remaining time decreases, the student becomes more stressed and the probability of making mistakes increases. The probability of giving the correct answer, the wrong answer, and leaving the question blank changes depending on the given remaining time.

For each question in the exam;

The probability of giving a correct answer:

$$p_c = 0.6 + \frac{(2t-40)}{1000}$$

The probability of giving a wrong answer:

$$p_w = 0.2 + \frac{(30-t)}{1000}$$

The probability of leaving a question blank:

$$p_b = 1 - (p_c + p_w)$$

Here, t represents the remaining time in minutes. ($1 \leq t \leq 200$)

a-) Knowing that the student has full time at the beginning of the exam, let X be the number of questions the student answered correctly in the exam, Y be the number of questions the student answered incorrectly, and Z be the number of questions the student left blank. Find $P_x(x)$, $P_y(y)$, and $P_z(z)$, and plot their PMFs using matplotlib in python. Save the graphs as **pxpmf.png**, **pypmf.png** and **pzpmf.png** correspondingly.

b-) Find the probability of getting points **more than an M number** at the beginning of the exam knowing that the student has full time? Print the value. (If you cannot do it for variable M, do it for M=400. There will be a point deduction in this case.)

Note: The program will stall here and wait for user input to continue, you can assume that there will be always a valid input and enter key is pressed after the input. Take **M** as integer input between 0 and 500.

c-) Let **s** be the probability of the student passing the exam **successfully** when the exam is over while knowing that the remaining time is **120 minutes**, and **given 20 correct, 7 wrong, 4 empty answers** until this time. Find **s** and print the value.

d-) If it is known that this student leaves **at least 8 out of 100 questions in the exam blank** in total, does the probability value in **part b** change? If it changes, find the new value, if not print the old one.

e-) Let's assume the student's problem-solving process is divided into **three phases**. After completing phase 1, the student takes a **4-minute** break to rest their eyes. Following phase 2, the student takes a longer, **6-minute** break before proceeding to phase 3. This schedule allows the student to maintain focus and efficiency throughout the problem-solving process. However, it's important to note that all these break times are included in the total exam duration.

Suppose that **the remaining time t** depends on the corresponding **expected times** of **exponential random variables** for each phases;

• In the first phase, the student will solve the first 20 questions of the exam and the expected time to complete this phase is 40 minutes.

• In the second phase, the student will solve the next 30 questions of the exam and the expected time to complete this phase is 40 minutes.

• In the third phase, the student will solve the last 50 questions of the exam and the expected time to complete this phase is 110 minutes.

At the end of the exam what is the probability of getting **less than 300 points** by considering the start of each phase according to the given expected times? Print the value.

Note: *Realize that you need to calculate remaining times at the beginning of each phase by using given expected times while considering the breaks and break times and then calculate the probabilities. You should also consider the number of questions answered in each phase.* ***You will not use exponential random variables for this (e) part actually.***

f-) According to the given the information in **part e**, what is the probability that the first phase of the exam lasts **longer than 25 minutes**? Print the value and then plot the count-probability graph by repeating it **1000 times** using matplotlib in python. Save the graph as **countprob.png**.

Note: *It is possible to not pass to other phases, so phase 1 can last until the end of the exam.*

g-) According to the given the information in **part e** and also when you consider if the exam's total duration can be **extended as you wish as possible**, what is the probability that the solution time of the **second 30 questions** will be **between 60 and 80** minutes? Print the value. Also, plot the PDF of the Erlang distribution using matplotlib in python. Save the graph as **erlangpdf.png**.

Note: *Note that sum of two exponential random variables becomes an Erlang random variable. Erlang distribution's PDF is defined as;*

$$f\,(n,\,\lambda,\,x)\;=\;\frac{\lambda^{n}\cdot x^{n-1}\cdot e^{-\lambda\,x}}{(n-1)!}$$

where n equals to 2 (numbers of exponential random variable) here and λ is the same as λ of the exponential random variable.

You will write a python code named ***examsim.py***. All questions should be solved in that one script. You should indicate that which part of the code is solving which part of the question in your comments. (such as *# part a*, *# part b* etc.)

Important Note: *You are just allowed to use these ready-made libraries;*

- ***random*** *for generating random numbers*
- ***numpy*** *for holding the data*
- ***matplotlib*** *for plotting*
- ***scipy*** *just for integration and derivation functions*

if it is needed.

You are ***not allowed*** *to use predefined random variable functions -even from* ***scipy****- (such as* ***erlang.pdf()****) and need to* ***write them from scratch****. You can just use them to control your results on by own if you want, but do not place in the code. And you are* ***not allowed*** *to use libraries any other than* ***random****,* ***numpy****,* ***matplotlib*** *and* ***scipy****.*

Important Note: *p_c, p_w and p_b probabilities of each question's are decided at the start of remaining time, and after, stays same for each new question to answer, from this point of time.*

Important Note: *If needed in a given question, while generating random numbers use* ***seed number*** *as* ***1000*** *for each different random variable to get same results in different times even it is random, and again, if needed in a given question, use* ***1000*** *for iterations.*

Your code must be run via:

```
unzip <Student Number>.zip
cd <Student Number>
python3 examsim.py
```

and must give the outputs in the terminal formatted exactly as:

```
b-) mtMor400: <the_number_you_find>
c-) s: <the_number_you_find>
d-) news: <the_number_you_find>
e-) lt300: <the_number_you_find>
f-) p1m25: <the_number_you_find>
g-) b6080: <the_number_you_find>
```

and must give plotted outputs as png **without showing** them. Name axis and titles in the plots properly.

Submission (Deadline: 22.03.2024 23.59)

1-) examsim.py
2-) pxpmf.png
3-) pypmf.png
4-) pzpmf.png
5-) countprob.png
6-) erlangpdf.png
Compress the files and save them as <Student Number>.zip and upload them to `https://uzak.etu.edu.tr`.