

1-) [20 Points] Seven Sided Dices

Three fair **seven-sided** dice are thrown independently and the number of dots on the top face of each is observed. Let the random variable X be the number of dots on the top face of the first die, Y be the number of dots on the top face of the second die, and Z be the number of dots on the top face of the third die.

a-) Event A is defined as “the maximum of the thrown dice being less than 5”.

Find the probability $P[A]$.

b-) Event B is defined as “the minimum of the thrown dice being greater than 3”.

Find the probability $P[B]$.

c-) Find the probability $P[B | A]$.

d-) Event C is defined as “the sum of the thrown dice being greater than 5”.

Find the probability $P[C]$.

e-) Event D is defined as “the maximum of the thrown dice being less than 2”.

Find the probability $P[C | D]$.

2-) [20 Points]

A circuit system between X and Y lines is given in Figure 1. Additionally, the probabilities of each component functioning correctly are provided. Assume that;

- The circuit components A , B , C and D fails independently.
- If component A is functional, then the operation of either line 1 or line 2 is sufficient for the circuit system between X and Y to function.

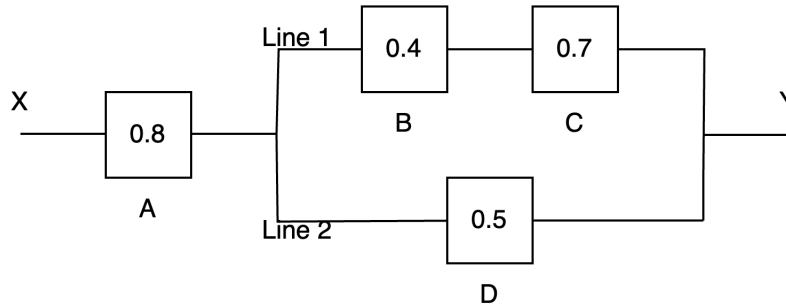


Figure 1: Circuit System

a-) What is the probability of the circuit system works?

b-) Given that the system is not working, what is the probability that the component A is working?

c-) Given that the system is working, what is the probability that the component C is working?

d-) Given that the system is working, what is the probability that the component D is not working?



3-) [20 Points] Software Practice

Mehmet is a software engineering student at TOBB-ETÜ. He has a habit of practicing his coding skills every day. If he practices coding on a given day, the probability that he will practice (or skip) on the next day is 0.7 (or 0.3, respectively). If he skips practicing on a given day, the probability that he will practice (or skip) on the next day is 0.9 (or 0.1, respectively). Mehmet (by default) practices when he starts his routine.

Let's denote:

- P_k : the event that Ali practices at the end of the k th day, for $k=1, 2, \dots$ respectively.
- S_k : the event that Ali skips practice at the end of the k th day, for $k=1, 2, \dots$ respectively.

a-) What is the probability of $P(P_3)$?

b-) What is the probability of $P(S_4)$?

c-) What is the likelihood of him practicing after two days, considering that he skipped practice at the end of the first day?

d-) What is the likelihood of him having skipped practice at the end of the first day, given that he is practicing after three days?

4-) [20 Points] Shopping Center

Random variables P and Q represent the number of customers that enter 2 separate stores during a certain 10-minute period. These stores are located in the same shopping center so it is important that store managers coordinate their operations if necessary.

The joint distribution of P and Q is known to be

$$f(p, q) = \frac{1}{2^{(p+q+2)}}; p \in \{\mathbb{Z}^+ \cup \{0\}\} \text{ and } q \in \{\mathbb{Z}^+ \cup \{0\}\}$$

a-) Find the marginal distribution of P .

b-) Are the two random variables P and Q independent or not independent? Explain the reason mathematically.

c-) What is the probability that, during the time period in question, less than 6 customers enter the two stores given that P is more than 2?

d-) What is the expected value of customers that enter store Q during the 10-minute period?

5-) [20 Points] Growing Bacteria Species

Consider two different species of bacteria, Species X and Species Y, growing in a petri dish. The lifetime of each bacterium is a random variable. The joint density functions of these variables is given by:

$$f(x, y) = \begin{cases} cxy^2 & \text{if } x \geq 0, y \geq 0, x + y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

And, the random variable Z that corresponds the evolution probability of the species denoted as:

$$Z = \begin{cases} \max\{X, Y\} & 0 \leq x \leq \frac{1}{2}, 0 \leq y \leq \frac{1}{2}, x + y \leq 1 \\ 1 & \{x \geq \frac{1}{2}, y \geq 0, x + y \leq 1\} \cup \{x \geq 0, y \geq \frac{1}{2}, x + y \leq 1\} \\ 0 & \text{otherwise} \end{cases}$$

- a-) Find **c**.
- b-) Find **CDF** of Z and draw it.
- c-) Find **PDF** of Z and draw it.
- d-) Find **E[Z]**.

Submission (Deadline: 16.02.2024 Friday 15.00)

- Bring your paper at Teknoloji-Merkezi-217 (can be a box in front of the door, if you can't find anyone or there is no box at the door, throw it under the door.)
- Submit a pdf of your answers to the <https://uzak.etu.edu.tr> site.

Both are required. Be aware that your answers should be in-order.