HW1 - Sols

1-) a-) $P[A] = \frac{4}{7} \cdot \frac{4}{7} \cdot \frac{4}{7} = 0{,}186$

b-) $P[B] = \frac{4}{7} \cdot \frac{4}{7} \cdot \frac{4}{7} = 0{,}186$

c-) $P[B|A] = \frac{P[B \cap A]}{P[A]} = \frac{P(3<X<5) \cap P(3<Y<5) \cap P(3<Z<5)}{0{,}186}$

$0{,}0156 = \frac{\left(\frac{1}{7}\right)^3}{0{,}186}$

d-) $C = \{X+Y+Z > 5\}$ $\quad C^c = 1 - C = \{X+Y+Z \leq 5\}$

$$C^c = \left\{ \begin{array}{l} (1,1,1), \\ (1,1,2), (1,2,1), (2,1,1), \\ (1,2,2), (2,1,2), (2,2,1) \\ (1,1,3), (1,3,1), (3,1,1) \end{array} \right\}$$

$P[C^c] = \frac{10}{7^3} = 0{,}0291 \quad P(C) = 1 - P(C^c) = 0{,}971$

e-) $P(C|D) = \frac{\overset{\nearrow 0}{P(C \cap D)}}{P(D)} = 0$

2-) a-) $P[S] = P[A] \cdot (P[B] \cdot P[C] + P[D] - P[B] \cdot P[C] \cdot P[D])$

$= 0{,}8 \cdot (0{,}4 \cdot 0{,}7 + 0{,}5 - 0{,}4 \cdot 0{,}7 \cdot 0{,}5) = 0{,}512$

b-) $P[A \mid S'] = \dfrac{P[A \cap S']}{P[S']}$

$P[A \cap S'] = 0{,}8 \cdot 0{,}6 \cdot 0{,}7 \cdot 0{,}5 + 0{,}8 \cdot 0{,}4 \cdot 0{,}3 \cdot 0{,}5 + 0{,}8 \cdot 0{,}6 \cdot 0{,}3 \cdot 0{,}5 = 0{,}29$

$P[S'] = 1 - P[S] = 0{,}488$

$P[A \mid S'] = \dfrac{0{,}29}{0{,}488} = 0{,}59$

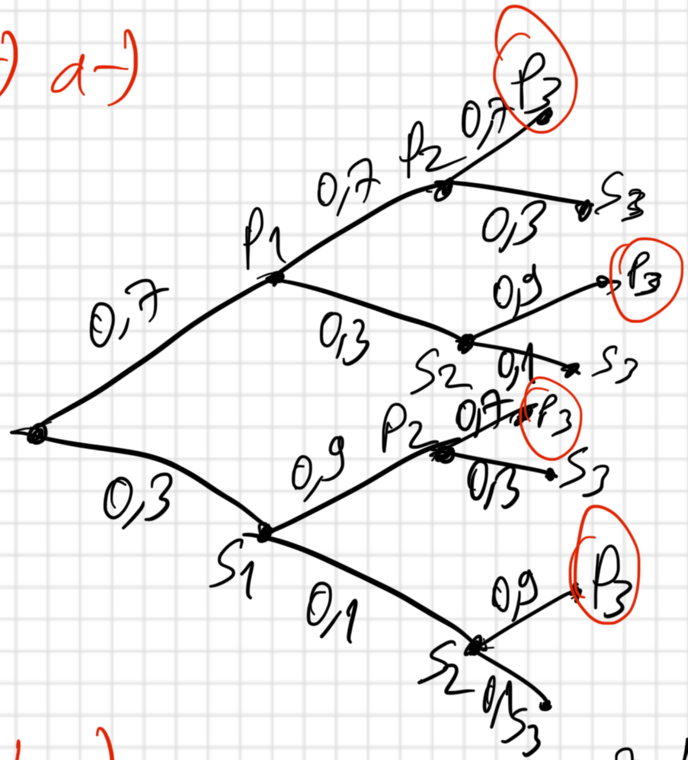
3-) $P[C \mid S] = \dfrac{P[C \cap S]}{P[S]} = \dfrac{0{,}392}{0{,}512} = 0{,}77$

$P[C \cap S] = 0{,}8 \cdot 0{,}4 \cdot 0{,}7 \cdot 0{,}5 + 0{,}8 \cdot 0{,}4 \cdot 0{,}7 \cdot 0{,}5 + 0{,}8 \cdot 0{,}6 \cdot 0{,}7 \cdot 0{,}5 = 0{,}392$

4-) $P[D' \mid S] = \dfrac{P[D' \cap S]}{P[S]} = \dfrac{0{,}112}{0{,}512} = 0{,}22$

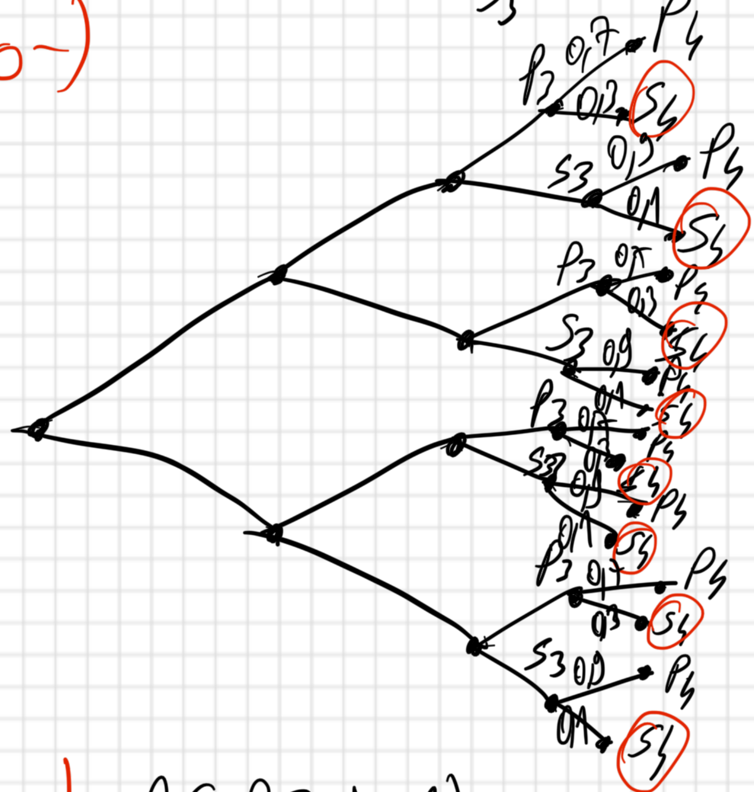
$P[D' \cap S] = 0{,}8 \cdot 0{,}4 \cdot 0{,}7 \cdot 0{,}5 = 0{,}112$

3-) a-)



$$P(P_3) = (0,7)(0,7)(0,7) + (0,7)(0,3)(0,9) + (0,3)(0,9)(0,7) + (0,3)(0,1)(0,9) = 0,748$$

b-)



$$P(S_4) = 0,3 \cdot P(P_3) + 0,1 \cdot P(S_3) = 0,3 \cdot P(P_3) + 0,1 \cdot (1 - P(P_3)) = 0,25$$

c-) $$P(P2 \mid S1) = \frac{P(P2 \cap S1)}{P(S1)} = \frac{(0,3)(0,9)}{0,3} = 0,9$$

d-) $$P(S1 \mid P3) = \frac{P(S1 \cap P3)}{P(P3)} = \frac{(0,3)(0,9)(0,7) + (0,3)(0,1)(0,9)}{0,748} = 0,29$$

4-) a-) $f(p) = \frac{1}{4} \cdot \frac{1}{2^p} \cdot \sum_{q=0}^{\infty} \frac{1}{2^q} = \frac{1}{4} \cdot \left(\frac{1}{1-\frac{1}{2}}\right) \cdot \frac{1}{2^p}$

$$f(p) = \frac{1}{2^{p+1}}$$

b-) $f(q) = \frac{1}{4} \cdot \frac{1}{2^q} \cdot \sum_{q=0}^{\infty} \frac{1}{2^p} = \frac{1}{2^{q+1}}$

$$f(p,q) = f(p) \cdot f(q) = \frac{1}{2^{p+q+2}}$$

independent

c-) $P(P+Q<6 \mid P>2)$

$$= \frac{P[P(P+Q<6) \cap P(P>2)]}{P(P>2)}$$

$$= \frac{\sum_{p=3}^{5} \sum_{q=0}^{5-p} \frac{1}{2^{p+q+2}}}{1 - P(P \leq 2)} = \frac{0,086}{1-\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}\right)}$$

$$= 0,688$$

d-) $E[Q] = \sum_{q=0}^{\infty} q \cdot \frac{1}{2^{q+1}} = \frac{1}{2} \cdot \sum_{q=0}^{\infty} \frac{q}{2^q} = 1$

$$\sum_{q=0}^{\infty} x^q = \frac{1}{1-x}$$

$$\sum_{q=0}^{\infty} q x^{q-1} = \frac{1}{(1-x)^2} \xrightarrow{x=\frac{1}{2}} \sum_{q=0}^{\infty} \frac{2q}{2^q} = \frac{1}{\left(\frac{1}{2}\right)^2} = 4$$

$$\sum_{q=0}^{\infty} \frac{q}{2^q} = 2$$

5-) a-) $\int_{x=0}^{1}\int_{y=0}^{1-x} cxy^2\,dy\,dx = 1$

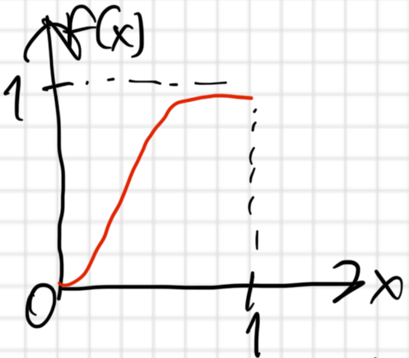
$$= \int_0^1 \left(\frac{cxy^3}{3}\Big|_0^{1-x} \right) = \int_0^1 c\cdot x\cdot \frac{(1-x)^3}{3}$$

$$= c\left(\frac{x^2}{6} - \frac{1}{3}x^3 + \frac{x^4}{4} - \frac{x^5}{15} \right)\Big|_0^1 = \frac{c}{60} = 1$$

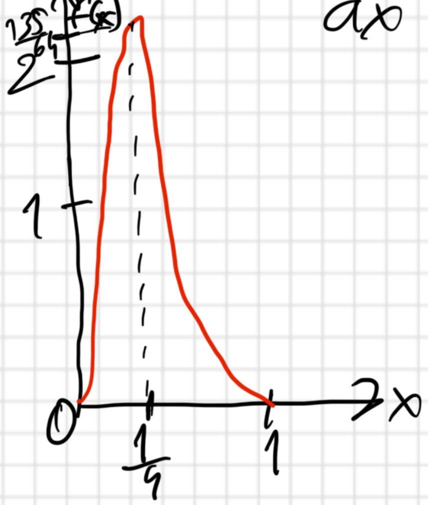
$c = 60$

b-) $F(x) = \int_{x=0}^{x}\int_{y=0}^{1-x} 60xy^2\,dy\,dx = \int_0^x 20x\cdot(1-x)^3$

$$= 10x^2 - 20x^3 + 15x^4 - 4x^5$$



c-) $f(x) = \frac{d}{dx}F(x) = 20x - 60x^2 + 60x^3 - 20x^4$



d-) $P(Y \leq \frac{1}{3} \mid X = 0{,}4) = \dfrac{\int_{y=0}^{\frac{1}{3}} 60 \cdot (0{,}4) y^2 \cdot dy}{\int_{y=0}^{0{,}6} 60 \cdot (0{,}4) \cdot y^2 \cdot dy}$

$= \dfrac{8y^3 \Big|_0^{\frac{1}{3}}}{8y^3 \Big|_0^{0{,}6}} = \dfrac{0{,}3}{1{,}73} = 0{,}171$

e-) $P(X+Y \geq \frac{1}{3})$

$x+y \leq 1 \rightarrow 0 \leq x \leq \frac{1}{3} \rightarrow \frac{1}{3} - x \leq y \leq 1-x$

$\rightarrow \frac{1}{3} \leq x \leq 1 \rightarrow 0 \leq y \leq 1-x$

$\int_{x=0}^{\frac{1}{3}} \int_{y=\frac{1}{3}-x}^{1-x} 60xy^2 \, dy\, dx + \int_{\frac{1}{3}}^{1} \int_{0}^{1-x} 60xy^2 \, dy\, dx = \frac{130}{243} + \frac{112}{243}$

$= 0{,}99$

f-) $E[X] = \int_0^1 x(20x - 60x^2 + 60x^3 - 20x^4) = \frac{1}{3}$