Foundations of Natural Language Processing - Revision

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Ambiguity and Corpora

The main difficulties of NLP are Variability and Ambiguity, two different problems that arise between the relations between the *meaning* of a sentence and the *language* used in a sentence. Ideally, we'd want one-to-one, but this is rarely the case. Instead, we usually get a one-to-many or many-to-one.

Variability

Variability when one sentence meaning can have multiple language interpretations. For example, the sentence:

> He drew the house

could refer to:

- He made a sketch of the house
- *He showed me* his drawing of the house
- He portrayed the house in his paintings
- He drafted the house *in his sketchbook*

Ambiguity

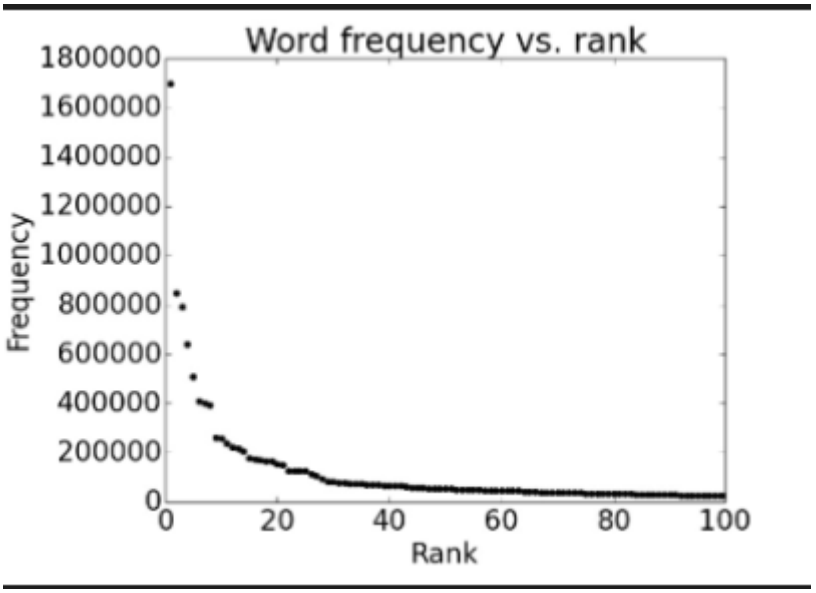
Ambiguity (arguably the more pressing challenge in NLP) is when the language used in a sentence can have *multiple different **valid** meanings*. There are many different types of ambiguity:

- Homophones: *blew* and *blue*
- Word senses: *bank* (finance or river?)
- Part of speech: *chair* (noun or verb?)
- Syntactic structure: *I saw a girl with a telescope*
- Quantifier scope: *Every child loves some movie*
- Multiple: *I saw her duck*
  - (word senses and syntactic)
- Reference: *John dropped the goblet onto the glass table and it broke.*
- Discourse: *The meeting is cancelled. Nicholas isn't coming to the office today*
- Syntactic Ambiguity: *Put the block in the box on the table in the kitchen*
  - If we have an ambiguous sentence like this with n prepositional phrases, the amount of interpretations is Cat_n (*Catalan Numbers*), where

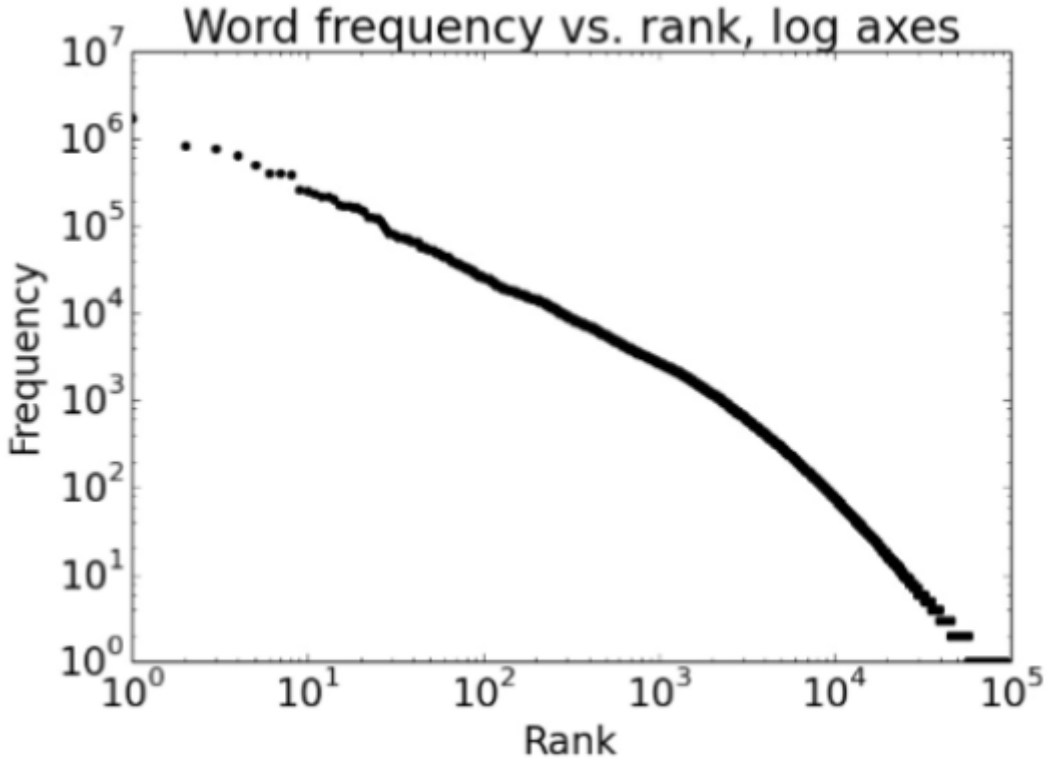
$$Cat_n = \binom{2n}{n} - \binom{2n}{n-1} \sim \frac{4^n}{n^{3/2}\sqrt{\pi}}$$

Zipf's Law

If we count the amount of words in some Corpora to get their *frequency* f, and then *rank them in order of most frequent* (r), we can plot these values against each other. The result looks something like this:



This is quite unreadable, so if we use *logarithmic* axes we can see what's going on:



This phenomenon is known as ***Zipf's Law***, and is formally defined (in the slides) as

$$f \times r \approx k$$

, where k is some constant. Most other online sources seem to fix $k = 1$ and explicitly write that the *frequency of a word is **inversely proportional** to it's rank in our frequency table*.

$$f = \frac{1}{r}$$

The implications of this mean that regardless of how big our corpus is, we will *always* find words we've barely seen before. This proves a problem when we want to find methods of estimating probabilities for *all* words.

Uncertainty in NLP

Most problems we come across are manifestations of uncertainty

- Ambiguity - uncertainty in interpretation
- Variability - uncertainty in a specific realisation
- **Robustness** - uncertainty with potential inputs
  - We may have many different types of input (e.g. formal/informal)
- **Context Dependence** - uncertainty with previous information

All of these uncertainties basically force our hand into using *Probabilistic models*/*Machine Learning*

Probabilistic models

A probabilistic model has a **inputs** (*set of words+context / utterances*) and **outputs** (*set of Part-of-speech (POS) tags / syntactic analyses*)

input	output		input	output
$input_1$	$output_1$		$\langle \text{the}, \underline{\text{list}} \rangle$	
$input_2$	$output_2$		$\langle \text{We}, \underline{\text{list}} \rangle$	
...	...		...	...

Corpora

A *corpus* (plural *corpora*) is a "body of utterances, as words or sentences". Corpora should be naturally occurring to serve as realistic samples of a language. Corpora are also *linguistically* *annotated*; humans have read this text and marked the structures describing syntax/meaning.

Sentiment Analysis

A simple linguistic analysis where we try to tell the sentiment of a given piece of text (good example: a movie review) - is it *positive* or *negative*? This example would be easy to test for accuracy, as we can use the numerical score provided.

A simple sentiment analyser would use a positive and negative "list" (a *sentiment lexicon*) to count the number of positive and negative words. This is a naive approach; words may be ambiguous, may be used in contexts not useful to the overall sentiment

Tokenisation

We want to split a corpus into separate word/punctuation **tokens** (occurrences), not already separated by spaces. These *tokens* are the individual building blocks for any Language Model to act upon.

Annotation methods

Annotating is a potentially long and costly job. We may need to consider

- source data (size? Licensing?)
- annotation scheme (complexity? Guidelines)
- annotators (expertise? training?)
- Quality control procedures

We devise a set of *annotation guidelines* to help annotators produce **consistent** data, and to help users interpret the annotations correctly. The *Penn Treebank* has a >300 page long guideline document!

Inter-annotator agreement

Human annotators are not perfect. Conversely, due to Ambiguity, there may be multiple possible (equally correct) annotations. *IAA* is the process of getting multiple annotators to "agree" on independently annotated samples. The agreement rate can be thought of as an *upper bound* on the accuracy of a system evaluated on that dataset.

Still, some annotation decisions are far more frequent than others. The **Kappa** coefficient K measures agreement between two people making category judgements, correcting for expected chance agreement.
For example, in a scenario where an item is annotated and 4 coding options are equally likely, then the two annotators will agree 25% of the time. Therefore an agreement of 25% will be assigned $K = 0$ and will scale accordingly (e.g. 50% would be $K = 0.333$ since 50 is a third of the way from 25 to 100).

Gold Standard Evaluation

The gold standard is '*the truth*'; what the original writer *actually* meant when producing the individual text. Gold standards are used for both training and evaluation, but **testing must be done on unseen data**.

When designing a system, we often *tune* it by changing configuration options. If we run several experiments on our "test set", we risk **overfitting** it; this set no longer holds as a reliable *proxy* for new data.

K-Fold Cross-validation

We often split our dataset into test/train/dev pieces. We *only training the model on a **training set***. We then can *test the model* on the ***test set***. Devsets are used for evaluating different models, debugging, and optimising.

If our model is *too small* to reasonably create sufficiently sized sets, we can use *k-fold cross validation*. This process breaks the data into k pieces and treats one as a held-out set - the remaining are used to train a model. This held out set is used to test these different *folds*.
We can then combine all learned information through the use of cross-validation.

Evaluation methods

The simplest method of measuring a model's *performance* is the **proportion model**:

$$\frac{\text{right}}{\text{test set}} \times 100$$

This is okay for some tasks, but not all

Advanced evaluation methods

We can use the following methods in a system that can produce true/false positives/negatives (a total of 4 possible answers). We use tp, tn, fp and fn to represent these

Accuracy

The percentage of all observations that were labelled correctly.

$$\frac{\text{tp}+\text{tn}}{\text{tp}+\text{fp}+\text{tn}+\text{fn}}$$

Recall

The percentage of items actually present in the input that were correctly identified by the system.

$$\frac{\text{tp}}{\text{tp}+\text{fn}}$$

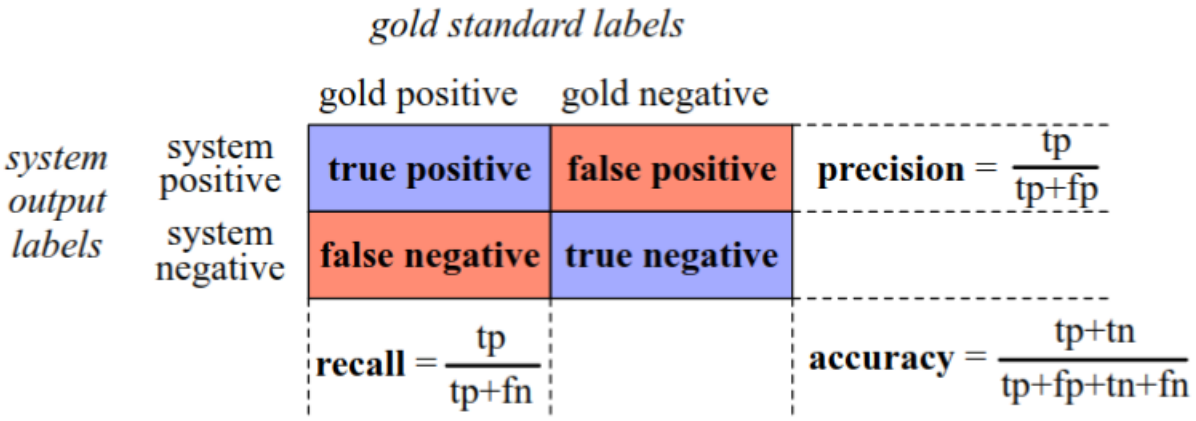
Precision

The percentage of items the system detected *that were actually positive*.

$$\frac{\text{tp}}{\text{tp}+\text{fp}}$$

Confusion Matrices

A way of representing all of these different metrics is through use of a *confusion matrix*.



F-Score

A combination of Precision (P) and Recall (R), the **F-measure** is formally defined as

$$F_\beta = \frac{(\beta^2 + 1)PR}{\beta^2 P + R}$$

Values of $\beta > 1$ favour *recall*, and values of $\beta < 1$ favour precision. When $\beta = 1$ they are balanced; this F_1 metric is the most commonly used and can be more clearly defined as

$$F_1 = \frac{2PR}{P + R}$$

Significance Testing

If we have a model with 95% accuracy, how can we tell if this is good or bad?

We can use an *upper bound* (Inter-annotator agreement rate as discussed earlier) and a lower bound (performance of a "simpler" model) to measure how *significantly* better/worse our model is.

There are two types of significance tests:

- **Parametric**
  - Used when the underlying distribution is *normal/Gaussian*.
- **Non-parametric**
  - Used otherwise

N-gram LMs

> *1.5k words into notes and we're finally talking about an actual language model!!*

Due to the Zipfian curve it's very hard to predict possible sentence structures just based on words alone; we need to determine (ideally, what we actually do is *approximate*) the **plausibility** of a sentence. For example,

$$P(\text{the cat slept peacefully}) > P(\text{slept the peacefully cat})$$
$$P(\text{she studies morphosyntax}) > P(\text{she studies more faux syntax})$$

N-gram models are one such language model that approximates this plausibility for us. They can be used for:

- **Spelling correction**
  - Generate possible "correct" spellings for a certain text and pick the best guess
- **Automatic speech recognition**
  - Generate multiple possible text interpretations of speech and pick the best guess
- **Machine translation**
  - Generate multiple possible translations (in the target language) and pick the be- *you get the idea*

N-grams are most well-known for their use in Prediction

Prediction

We want to compute $P(w|h)$, the probability of a word w given a history h. For example, if our history is "*its water is so transparent that*" and we want to know the probability of our next word being *the*, we would be finding

$$P(\text{the}|\text{its water is so transparent that})$$

A method of calculating this probability would be to use *relative frequency counts* C: $$
\begin{eqnarray}
P(\text{the|its water is so transparent that}) =\
\frac{C(\text{its water is so transparent that the})}{C(\text{its water is so transparent that})}
\end{eqnarray}

$$This is still not sufficient; language is creative and we won't always be able to count entire sentences. We'$$

A **bigram** model ($n = 2$), for example, approximates the probability of a word by *only using* the conditional probability of the preceding word; we would approximate the above transparent water example with

$$P(\text{the}|\text{that})$$

This assumption that we can reasonably estimate the probability of a word based on only the prior is called a **Markov** assumption. N-grams make a Markov assumption that we only need to look $n - 1$ words into the past. For n-gram size N,

$$P(w_n|w_{1:n-1}) \approx P(w_n|w_{n-N+1:n-1})$$

However, given the *bigram assumption for the probability* of an individual word, we can compute the probability of a **complete word sequence** by substituting this into our original

equation to get

$$P(w_{1:n}) \approx \prod_{k=1}^{n} P(w_k|w_{k-1})$$

How do we get the individual *bigram probabilities* $P(w_k|w_{k-1})$? We can estimate probabilities with MLE.

MLE

We can estimate discrete probabilities with

$$P_{RF} = \frac{C(x)}{N}$$

where $C(x)$ is the *count* of x in a large dataset, and N is the total number of items in the dataset.

This is known as **maximum-likelihood estimation**, or more commonly **MLE**.

> *Aside*: this is also known as Relative Frequency estimation, hence why it says P_{RF}.

Estimating bigram probabilities with MLE

To compute a particular bigram probability of a word w_n given previous word w_{n-1}, we compute the count of the bigram $C(w_{n-1}w_n)$ (that is to say, how many times "*previousword word*" appears), and then normalise this by dividing by how many bigrams also share the same first word w_{n-1}:

$$P(w_n|w_{n-1}) = \frac{C(w_{n-1}w_n)}{\sum_w C(w_{n-1}w)}$$

This can be simplified - the amount of times "$w_{n-1}\ w_n$" appears is the same as the *unigram* count for the amount of times "w_{n-1}" appears, so we can instead just write

$$P(w_n|w_{n-1}) = \frac{C(w_{n-1}w_n)}{C(w_{n-1})}$$

Problem with using MLE for sentences

If we have a sentence $\vec{w}$ we could say

$$P_{MLE}(\vec{w}) = \frac{C(\vec{w})}{N}$$

The problem with this is that if we ever get a sentence that *has not appeared* in our corpus, $P = 0$; MLE thinks anything *that **hasn't** occurred will **never** occur*.

Trigram independence assumption

Under a trigram independence assumption, all

- $P(\text{mast}|\text{I spent three years before the})$
- $P(\text{mast}|\text{I went home before the})$
- $P(\text{mast}|\text{I saw the sail before the})$
- $P(\text{mast}|\text{I revised all week before the})$

are estimated as $P(\text{mast}|\text{before the})$. The general rule is

$$P_{MLE}(w_i|w_{i-2},w_{i-1}) = \frac{C(w_{i-2},w_{i-1},w_i)}{C(w_{i-2},w_{i-1})}$$

(The amount of times the three words have appeared over the amount of times the previous two words appeared).

Evaluation of LMs

Types of Evaluation

There are different metrics for which we can evaluate how "good" a LM is.

Extrinsic

Extrinsic evaluations are "plugged into" a functional system, and then the system's performance is measured. This is most reliable, but an evaluation method for the system is also needed and the whole process is time consuming.

Intrinsic

Intrinsic evaluations are inherent to the current task, but choosing the correct measure is difficult; ideally, this correlates with extrinsic measures.

Entropy

Entropy is the *degree of uncertainty* a system has.

The definition of the **entropy** of a random variable X is

$$H(X) = \sum_x -P(x)\log_2 Px$$

It's basically giving the average $\log$, weighted with the probability $P(X)$. Higher values of entropy are present in *more uncertain* systems.

Examples of entropy

One event

$$P(a) = 1$$

$$
\begin{aligned}
H(X) &= -1 \log_2 1 \\
&= 0
\end{aligned}
$$

4 equally likely events

$$
\begin{aligned}
P(a) &= 0.25 \\
P(b) &= 0.25 \\
P(c) &= 0.25 \\
P(d) &= 0.25
\end{aligned}
$$

$$
\begin{aligned}
H(X) &= -0.25 \log_2 0.25 - 0.25 \log_2 0.25 \\
&\quad - 0.25 \log_2 0.25 - 0.25 \log_2 0.25 \\
&= -\log_2 0.5 \\
&= 2
\end{aligned}
$$

3 equally likely events, and one more likely

$$
\begin{aligned}
P(a) &= 0.7 \\
P(b) &= 0.1 \\
P(c) &= 0.1 \\
P(d) &= 0.1
\end{aligned}
$$

$$
\begin{aligned}
H(X) &= -0.7 \log_2 0.7 - 0.1 \log_2 0.1 - 0.1 \log_2 0.1 - 0.1 \log_2 0.1 \\
&= -0.7 \log_2 0.7 - 0.3 \log_2 0.1 \\
&= -(0.7)(-0.5146) - (0.3)(-3.3219) \\
&= 0.36020 + 0.99658 \\
&= 1.35678
\end{aligned}
$$

Encoding entropy

- For a uniform distribution with 2^n outcomes
- When encoding a sequence of events X, we encode each event by a sequence of bits (using as few as possible)
  - The average number of bits needed to encode $X \geq$ entropy of X
- The measured entropy of English is about 1.3. That is to say, on average, a human would need 1.3 yes/no questions (about the context) to predict the next word in a sentence.

Cross-entropy

For most corpora we cannot exactly calculate Entropy as we do not know the *exact* word sequence probability distribution. We can, however, measure how close *an estimated probability distribution* $\hat{P}$ is to the true distribution P via ***cross-entropy***:

$$
H(P, \hat{P}) = \sum_x -P(x) \log_2 \hat{P}(x)
$$

But the problem was that we didn't know $P(x)$!

We can approximate this by using *word sequences* $w_1, \ldots, w_n$ (with large n):

$$H_M(w_1, \ldots, w_n) = -\frac{1}{n} \log_2 P_M(w_1, \ldots, w_n)$$

Lower values of cross-entropy (like entropy) mean the model is *less uncertain (**better**)* at predicting the next word. Cross-entropy is also measured in bits; if we need to guess x words with a cross entropy of y, we will need $x \times y$ yes/no questions.

Perplexity

Perplexity is $2^{\text{cross-entropy}}$, and is the common metric for measuring uncertainty in models.

Smoothing

Smoothing aims to solve the issue of MLE estimating probabilities that *maximise* the training data; making everything else (unseen) *minimal*.

Laplace Smoothing

Add-one smoothing, sometimes Laplace smoothing, basically pretends every possible word was seen one more time than it actually was. We can't simply just add 1 though - the probabilities won't all sum up to 1, so we need to normalise.

We want:

$$\sum_{w_i \in V} \frac{C(w_{i-2}, w_{i-1}, w_i) + 1}{C(w_{i-2}, w_{i-1}) + x} = 1$$

Solve for x:

$$\begin{aligned}
\sum_{w_i \in V} \left(\frac{C(w_{i-2}, w_{i-1}, w_i) + 1}{C(w_{i-2}, w_{i-1}) + x} \right) &= \frac{C(w_{i-2}, w_{i-1}) + x}{C(w_{i-2}, w_{i-1}) + x} \\
\sum_{w_i \in V} C(w_{i-2}, w_{i-1}, w_i) + \sum_{w_i \in V} 1 &= C(w_{i-2}, w_{i-1}) + x \\
C(w_{i-2}, w_{i-1}) + v &= C(w_{i-2}, w_{i-1}) + x \\
v &= x
\end{aligned}$$

where v = vocabulary size.

Unfortunately, this normalisation flips our Zipfian curve and now the super low probabilities happen to the common words such as "*it*", "*the*", and "*and*".

Lidstone Smoothing

Add-α smoothing, sometimes Lidstone smoothing adds α instead, and this formula comes normalised

$$P_{+\alpha}(w_i | w_{i-1}) = \frac{C(w_{i-1}, w_i) + \alpha}{C(w_{i-1} + \alpha v)}$$

This also assumed we know the vocab size in advance, but if we don't we can add a single "unknown" item and use it for all unknown words found during testing. The question on how to choose α is still a bit uncertain; we can use splits and test multiple α values, choosing the one that minimises Cross-entropy on the devset.

Good Turing Smoothing

Our previous methods changed the denominator which had unintended effects even on the frequent events. Good Turing changes the numerator.

If MLE is $P_{MLE} = \frac{c}{n}$, Good Turing uses *adjusted counts* c^* instead: $P_{GT} = \frac{c^*}{n}$

Each probability gets pushed down to the count class.

c	N_c	P_c	$P_c(\text{total})$	c^*	P_c^*	$P_c^*(\text{total})$
0	N_0	0	0	0	$\frac{\frac{N_1}{N_0}}{N}$	$\frac{N_1}{N}$
1	N_1	$\frac{1}{N}$	$\frac{N_1}{N}$	$2\frac{N_1}{N}$	$\frac{2\frac{N_2}{N_1}}{N}$	$\frac{2N_2}{N}$
2	N_2	$\frac{2}{N}$	$\frac{2N_2}{N}$	$3\frac{2N_2}{N}$	$\frac{3\frac{N_3}{N_2}}{N}$	$\frac{3N_3}{N}$

- c: count
- N_c: number of different items with count c
- P_c: MLE estimate of prob. of that item
- $P_c(\text{total})$: MLE total probability mass for all items with that count.
- c^*: Good-Turing smoothed version of the count
- P_c^* and $P_c^*(\text{total})$: Good-Turing versions of P_c and $P_c(\text{total})$

The basic idea is to arrange the discounts so the amount we *add* to total probability is matched by *all the discounting* in the other rows.

Good-Turing depends on (real) adjacent count

$$
\begin{aligned}
c^* &= (c+1)\frac{N_{c+1}}{N_c} \\
P_c^* &= \frac{c^*}{N} \\
&= \frac{(c+1)\frac{N_{c+1}}{N_c}}{N}
\end{aligned}
$$

Since counts tend to go down as c goes up, the multiplier is < 1.
The sum of all discounts is $\frac{N_1}{N_0}$; we need this to be true given that this is the *Good-Turing* count for row 0 (no count c).

Good Turing: 0-count

$$P(\text{unseen}) = \frac{N_1}{n}$$

This uses MLE, we divide this probability equally amongst all unseen events

$$P_{GT} = \frac{1}{N_0}\frac{N_1}{n} \rightarrow c^* = \frac{N_1}{N_0}$$

Good Turing: 1 count

$$P(\text{once before}) = \frac{2N_2}{n}$$

We estimate the probability that the next observation has count of 2 (add-one). We then divide that probability equally amongst all 1-count events

$$P_{GT} = \frac{1}{N_1}\frac{2N_2}{n} \rightarrow c^* = \frac{2N_2}{N_1}$$

Interpolation

Interpolation solves the problems still remaining with Good Turing Smoothing:

- It assumes we know vocabulary size (no unseen words)
- Doesn't allow "holes" in the counts (if $N_i > 0, N_{i-1} > 0$)
- Applies discounts to high-frequency items

Imagine in a training corpus we saw "*Scottish beer*", but we didn't see "*Scottish beer drinkers*" or "*Scottish beer eaters*". Despite the fact the term "*beer drinkers*" does appear (and "*beer eaters*" does **not**), a trigram model smoothed with add-alpha or Good Turing Smoothing would rate the options equally likely.

The solution is to shorten our window to understand the "*beer drinkers*" information. Interpolation is one method of achieving this (the other being back-off).

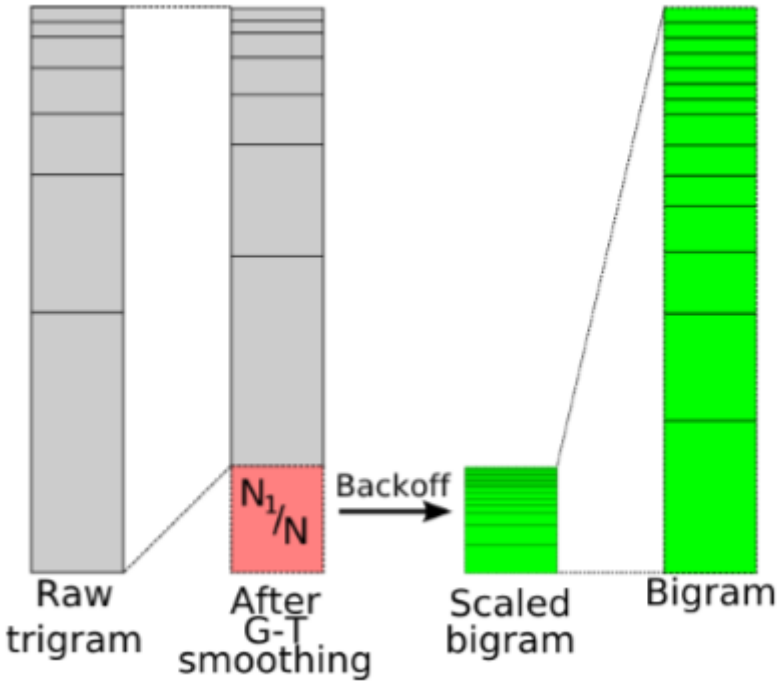
Imagine we are trying to compute a trigram probability $P(w_n|w_{n-2}w_{n-1})$ but have no examples of this particular trigram. We can estimate it with the bigram probability $P(w_n|w_{n-1})$, which we could also estimate (if needed) with the unigram $P(w_n)$. That being said, we need to combine these different estimates - solely using unigrams won't be useful. We also want to weight these probabilities by a λ value such that the final interpolated probability is between 0 and 1. Generally, we can estimate the probability $\hat{P}$ with

$$\begin{aligned}\hat{P}(w_n|w_{n-2}w_{n-1}) = & \lambda_1 P(w_n) \\ & + \lambda_2 P(w_n|w_{n-1}) \\ & + \lambda_3 P(w_n|w_{n-2}w_{n-1})\end{aligned}$$

The values of λ are *hyperparameters*, and are acquired through training. Typically, we hold out a set corpus, and then choose the λ values that **maximise** that corpus, possibly through techniques such as EM.

Katz back-off

Back-off is the process of "backing off" (reverting to, switching) to a $n-1$-gram if we have zero count for a given n-gram. In order for a back-off model to give a correct probability distribution we need to *discount* the higher order n-grams to save the "probability mass" for lower order ones. We do this discounting after Good Turing Smoothing.



Formally, the probability for a back-off n-gram P_{BO} is

$$P_{BO}(w_n|w_{n-N+1:n-1}) = \begin{cases} P^*(w_n|w_{n-N+1:n-1} & \text{if } C(w_{n-N+2:n-1}) > 0 \\ \alpha(w_{n-N+1:n-1})P_{BO}(w_n|w_{n-N+2:n-1}) & \text{otherwise.} \end{cases}$$

Kneser-Ney Smoothing

Kneser-Ney Smoothing solves the issue caused by the **diversity of histories**.

In the *Europarl* corpus, the word "*York*" occurs 477 times - fairly frequent, so unigrams would give it a respectable probability. 473 of those times are preceded by "*New*"; in unseen bigram contexts, "*York*" should have a low probability - but Interpolation/back-off predict this too high due to unigram models.

This can be solved by accounting for the diversity of context. If we have distinct histories for a word $N_{1+}(\bullet w_i) = |\{w_{i-1} : c(w_{i-1}, w_i)\}|$, then using KN smoothing

$$P_{KN}(w_i) = \frac{N_{1+}(\bullet w_i)}{\sum_w N_{1+}(\bullet w_i)}$$

Eagle eyed students will notice this is literally the same things as MLE, just with history counts instead.

How to handle similar words (an early tease of Neural Embeddings)

The above models do not contain any meaning of word *similarity*. If we had

$$w_1 = \text{salmon}; w_2 = \text{swordfish}; C(w_1) >> C(w_2)$$

then $P(\text{salmon}|\text{caught two})$ **would not tell us anything about** $P(\text{swordfish}|\text{caught two})$.

Traditional methods would be to define classes c of words, so we'd be predicting

$$P_{CL} = (w_i|w_{i-1}) = P(c_i|c_{i-1})P(w_i|c_i)$$

Recent versions (covered later) project words into a continuous psace such that words appearing in similar contexts have similar representations (in this space).

Noisy Channel Model

A common problem is NLP is handling spelling errors. We don't want our models to process words that are not real, both in training and generation. We imagine this often with a "*noisy channel*" model, as if the original word has flown through the noisy channel and now spits out the misspelled artefact.

This is a *Bayesian* interface. We observe x (a misspelled word) and our aim is to find the word w that generated this misspelling. Out of all possible words in our vocabulary V, we want to find w such that $P(w|x)$ is highest. Formally, our best estimate $\hat{w}$ is

$$\hat{w} = \underset{w_\in V}{\arg\max} P(w|x)$$

Bayes' rule

The conditional probability $P(a|b)$ can be famously represented as

$$P(a|b) = \frac{P(b|a)P(a)}{P(b)}$$

so if we substitute our previous equation in, we can estimate

$$\hat{w} = \underset{w_\in V}{\arg\max} \frac{P(x|w)P(w)}{P(x)}$$

and since $P(x)$ is static (it's talking about the same error word each time), we can just choose the word that maximises

$$\hat{w} = \underset{w_\in V}{\arg\max} P(x|w)P(w)$$

The *likelihood* of the noisy channel producing a particular observation x is $P(x|w)$, and the *prior probability* of a hidden word is $P(w)$

Edit Distance

We can approach the spelling correction problem from earlier if we assume all corrections can be solved by a series of single character operations (**insertion, deletion**, and **substitution**). Then for each misspelling x:

- Generate a list of all words y_i that differ by 1 character from x_i
- Compute $P(\overrightarrow{x}|\overrightarrow{y})P(\overrightarrow{y})$ for each $\overrightarrow{y}$, return one with the highest value

Imagine we had the mispelling (of "*not*"), "*no*". Our noise model would be $P(\text{no}|\text{not}) = P(n|n)P(o|o)P(-|t)$. Generally, for each *observed sequence* $\overrightarrow{x}$ made up of characters (including spaces) $x_1, \ldots, x_n$, we have

$$P(\overrightarrow{x}|\overrightarrow{y}) = \prod_{i=1}^{n} P(x_i|y_i)$$

We can build a confusion matrix from this:

y/x	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**...**
A	168	1	0	2	5	1	1	0	...
B	0	136	1	0	3	2	5	0	3
C	1	1	111	1	5	11	***36***	0	...
D	1	17	4	157	6	11	5	0	...
E	2	10	0	1	98	27	1	5	...
F	1	0	1	9	73	6	0	6	...
G	1	3	32	1	3	3	127	3	...
H	2	0	0	3	0	4	0	4	...
...	...	...	...	...	...	...	...	...	...

Highlighted, we see that we saw *G* when intention was *C* ***36 times***. A matrix like this can be obtained from manual analysis on a corpus,

The *edit distance* problem now boils "what word produces the fewest character changes" or the **minimum *edit distance* (MED)**. We commonly choose the cost of the 3 operations to be the same (1), but we can have them be whatever they want, even based on characters. We could define

$$\text{cost}(\text{sub}(c, c')) = P(c|c')$$

which means the cost depends on the probability of the proposed character.

The problem

We can now actually compute the alignments using MED, and therefore can make the matrix from before. **However**! we want to compute the alignments using the costs from our

noise model - but until we have the alignments, we can't estimate the noise model. But now we can't estimate the alignments from the noise model...

Using *parameter estimation*, we can use expectation maximisation to solve this.

EM

Expectation Maximisation is the process when we estimate these parameters by making what we observe *maximally* likely.

1. Set all parameters (in our previous cases, matrix values) to arbitrary values (e.g. all costs = 1)
2. Using these parameters, compute optimal values for the variables (in our case, run Edit Distance)
3. Using these alignments, **recompute** the parameters

We repeat steps 2 and 3 until the parameters stop changing.

Hard EM vs True EM

The previous explanation of EM is *hard EM* - there are no "soft/fuzzy" decisions. In *True EM*, we compute the *expected* values of the variables; this is guaranteed to converge to a local optimum of the likelihood function.

Likelihood functions

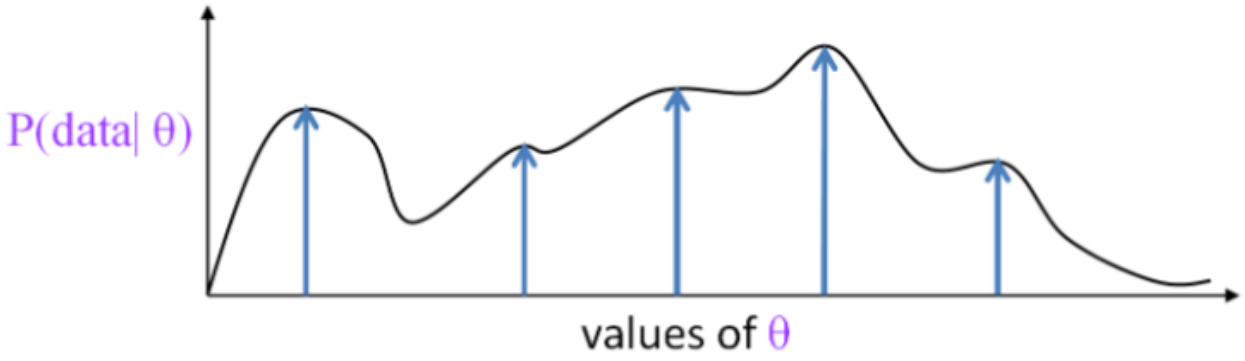
If we call the parameters of our model θ (in our case θ is the set of all character rewrite possibilities $P(x_i|y_i)$, we can compute $P(\text{data}|\theta)$.
If our data contains hand-annotated character alignments, then

$$P(\text{data}|\theta) = \prod_{i=1}^{n} P(x_i|y_i)$$

- If the alignments α are latent, we instead sum of possible alignments

$$P(\text{data}|\theta) = \sum_{a} \prod_{i=1}^{n} P(x_i|y_i, a)$$

The likelihood $P(\text{data}|\theta)$ can have multiple local optima.



True EM is guaranteed to converge to one of these, but not guaranteed to find the **global** optimum.

Naive Bayes

If we have a document d and a set of categories C (e.g. spam/not spam, for an email app), we want to assign d to the *most probable* category $\hat{c}$:

$$\hat{c} = \text{argmax}_{c \in C} P(c|d)$$
$$= \text{argmax}_{c \in C} P(d|c)P(c)$$

So we need to define $P(d|c)$ and $P(c)$.

Naive Bayes Assumption and modelling $P(d|c)$

We can define a set of *features* (prescence of certain words/sequences, part of speech, etc) to help us classify the documents, and represent each document d as a set of features $f_1, f_2, \ldots, f_3$. We can then model $P(d|c)$ as

$$P(d|c) = P(f_1, f_2, \ldots, f_n|c)$$

Using a ***naive Bayes assumption*** (features are conditionally independent) we can represent $P(d|c)$ further as

$$P(d|c) \approx P(f_1|c)P(f_2|c)\ldots P(f_n|c)$$

Naive Bayes Classifier

Combining everything, given a document d with features $f_1, f_2, \ldots, f_n$ and a set of categories C, choose the class $\hat{c}$ where

$$\hat{c} = \text{argmax}_{c \in C} P(c) \prod_{i=1}^{n} P(f_i|c)$$

where:

- $P(c)$ is the **prior probability** of class c before observing any data.
- $P(f_i|c)$ is the probability of **seeing** f_i in class c.

Estimating priors and feature probabilities

$\hat{P}(c)$

We estimate $P(c)$ by dividing the number of documents N_c in our training data with class c by the total number of documents N_{doc}.

$$\hat{P}(c) = \frac{N_c}{N_{\text{doc}}}$$

$\hat{P}(f_i|c)$

We normally estimate

$$\hat{P}(f_i|c) = \frac{\text{count}(f_i, c) + \alpha}{\sum_{f \in F} \text{count}(f, c), + \alpha}$$

where:

- $\text{count}(f_i, c)$ is the number of times f_i occurs in class c.
- F is the set of possible features.
- α is the smoothing parameter. This is included as due to our assumption, any "*zero-features*" will result in a flat zero probability for that class (due to multiplying them all together).
    - J&M claims this is often just add-one.

Costs

If we use Naive Bayes with small probabilities we run into *very* small eventual final probabilities. Many implementations solve this by using *costs* (negative log probabilities, summed, solved for the *lowest* cost overall). With this, Naive Bayes often looks like

$$\hat{c} = \text{argmin}_{c \in C} + ((-\log P(c) + \sum_{i=1}^{n} - \log P(f_i|c))$$

Naive Bayes is now a *linear classifier*, because it uses a linear function (in log space) over the input features.

Semi-supervised learning (with EM)

Semi-supervised learning is a method of training the Naive Bayes model on *unlabelled data*, of which there is **much** more of.

We give a model a set of documents with features, classified to specific classes. Using EM, we then get our model to estimate a class and say *how confident* it is of that decision. This then affects *retraining* the model.

Logistic Regression

Logistic Regression is the process of solving the task of classifying outputs based on features. Each weight w_i is associated with an input feature x_i, representing how important that feature is to the classification decision.

$\sigma(z)$

$$z = (\sum_{i=1}^{n} w_i x_i) + b$$

where b is a bias term. z is also $w \cdot x + b$ but both mean the same thing.

The *sigmoid function* forces this value to be a probability - forced between 0 and 1.

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

We now need to ensure the cases sum to 1. In the case of two cases ($P(y = 1)$ and $P(y = 0)$)

$$\begin{aligned} P(y = 1) &= \sigma(w \cdot x + b) \\ &= \frac{1}{1 + e^{-w \cdot x + b}} \\ P(y = 0) &= 1 - \sigma(w \cdot x + b) \\ &= 1 - \frac{1}{1 + e^{-w \cdot x + b}} \\ &= \frac{e^{-w \cdot x + b}}{1 + e^{-w \cdot x + b}} \end{aligned}$$

Side note: it is often more convenient to use $\exp(x)$ than e^x, and this document will do so often from here forth.

Softmax

The classifier discussed above would only work with two classifiers, as we could set a *boundary* at 0.5. If we wanted more possible classes, we need a *generalisation* of the sigmoid; we want to compute $P(y_k = 1|x)$.

Softmax takes a vector $z = [z_1, z_2, \ldots, z_K]$ of K values and maps them to a probability function.

$$\text{softmax}(z_i) = \frac{\exp(z_i)}{\sum_{j=1}^{K} \exp(z_j)}, 1 \leq i \leq K$$

Softmax of z is then a vector itself

$$\text{softmax}(\mathbf{z}) = [\frac{\exp(z_1)}{\sum_{i=1}^{K}\exp(z_i)}, \frac{\exp(z_2)}{\sum_{i=1}^{K}\exp(z_i)}, \ldots, \frac{\exp(z_K)}{\sum_{i=1}^{K}\exp(z_i)}]$$

Logistic Regression Learning

We require two components to learn the parameters of the model (weights $\mathbf{w}$ and bias b)

- A cost function; a function that measures how close the system output and gold standard output are.
- An optimisation algorithm to iteratively update the weights

Cross-entropy loss

We define a loss function L as

$$L(\hat{y}, y) = \text{How much } \hat{y} \text{ differs from the true } y$$

We first get a loss function that prefers the correct class labels to be *more likely*, called **conditional maximum likelihood esimation**, or CMLE. Given items $x^{(1)}, \ldots, x^{(N)}$ with labels $c^{(1)}, \ldots, c^{(N)}$, choose

$$\hat{w} = \underset{\vec{w}}{\text{argmax}} \sum_j \log P(c^{(j)}|x^{(j)})$$

If we invert this to obtain the *lowest negative log likelihood loss*, this is called the **cross-entropy loss**. Say we want to maximise the probability of $p(y|x)$; there are two outcomes

$$p(y|x) = \hat{y}^y(1-\hat{y})^{1-\hat{y}}$$

If $y = 1$, this simplifies to $\hat{y}$, and if $y = 0$, to $1 - \hat{y}$.

If we take the log of both sides

$$\log p(y|x) = y \log \hat{y} + (1-y)\log(1-\hat{y})$$

To obtain cross-entropy loss L_{CE}, we just flip the sign:

$$L_{CE} = -[\log p(y|x) = y \log \hat{y} + (1-y)\log(1-\hat{y})]$$

and knowing $\hat{y} = \sigma(w \cdot x + b)$:

$$L_{CE} = -[y \log \sigma(w \cdot x + b) + (1-y)\log(1-\sigma(w \cdot x + b))]$$

Gradient Descent

In general, gradient descent minimises the loss function. L is parameterised by weights ((w, b), but in this example we're gonna denote them with θ). We want to find a set of weights

that minimises the loss function

$$\hat{\theta} = \arg\min_{\theta} \frac{1}{m} \sum_{i=1}^{m} L_{CE}(f(x^{(i)}; \theta), y^{(i)})$$

Gradient descent finds the *gradient of the loss function* at a current point and moves in the *opposite direction*. We can take the derivative to find the value of the slope $\frac{d}{dw} L(f(x; w), y)$.

The Gradient Descent formula

I don't think the exam needs us to go into further detail on the cost-function-specifics formula, so in the essence of time, I will steal a final formula from *Chris'* excellent notes

$$\theta^{\text{new}} = \theta - \alpha \nabla J(\theta)$$

where

- θ^{new} is our new parameters
- θ is our old parameters
- α is some small weighting constant applied to the next bit
- $\nabla J(\theta)$ is the *vector derivative* of our cost function, when applied to the *old parameters* θ

MaxEnt

Maximum Entropy models are best suited if we have a *lot* of feautres, and the features do not follow the assumption of conditional independence. It is similar to Naive Bayes in that

$$\hat{c} = \arg\max_{c \in C} P(c|x)$$

but we model $P(c|x)$ *directly*. This is done by, given observations $\overrightarrow{x}$ and a class c,

$$P(c|\overrightarrow{x}) = \frac{1}{Z} \exp(\sum_i w_i f_i(\overrightarrow{x}, c))$$

where the normalisation constant $Z = \sum_{c'} \exp(\sum_i w_i f_i(\overrightarrow{x}, c'))$

We train MaxEnt like any other form of Logistic Regression Learning.

Morphology Parsing

Morphology is the structure of words. Words aren't the smallest unit of language, and in many languages the morphology is rich.

Words are often constructed with **stems** ("*the dictionary bit*", house, combine, eat, walk) and **affixes** ("*the grammar parts*", prefixes, suffixes, infixes, circumfixes).

Methods of combining stems and affixes

There are four methods to combine **stems** and **affixes**:

- **Inflection**
  - stem + grammar affix (**no change** to grammatical category)
  - *walk -> walking*
- **Derivation**
  - stem + grammar affix (**change** to grammatical category)
  - *combine -> combination*
- **Compounding**
  - stems together
  - *doghouse*
- **Cliticization**
  - *I've, we've, he's*

Parsing and generation

When we discuss *parsing*, we're going from the surface to lexical form e.g. *foxes -> fox* + ***N*** + ***PL***.

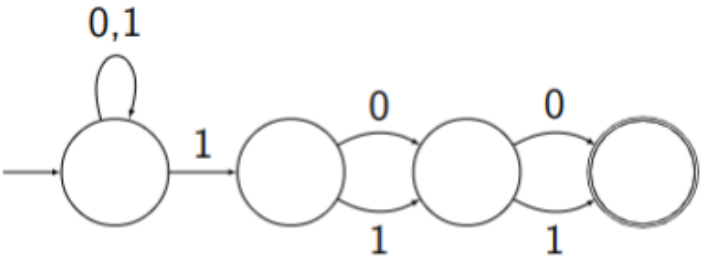
Generation goes the opposite way; *fox* + ***N*** + ***PL***-> *foxes*.

We often want to produce an intermediate form corresponding to analysis in terms of *morphemes* (minimal meaningful units) before applying *orthological rules*. For our *foxes* example, we may produce "*fox* ^ *s* #", where morpheme boundary = ^ and # = word boundary.

FSAs

A *nondeterministic finite state automaton (NFA)* is a finite state machine where a state can have more than one outgoing arc.

E.g., $(0|1)^*1(0|1)^2$ is captured by the following:



An *epsilon NFA* (ϵ-NFA) allows an input (in parsing) or output (in generation) defined by an arc to be the empty string.

Finite State Transducers

FSTs are ϵ-FSAs over an input alphabet $\sum$ can capture transitions that (optionally) produce *output* symbols (possibly over a different alphabet $\prod$).

Formally, a *finite state transducer* T with inputs from $\sum$ and outputs from $\prod$ consists of:

- states Q, S (for start), F (for finish)
- a transition relation $\Delta \subseteq Q \times (\sum \cup \{\epsilon\}) \times (\prod \cup \{\epsilon\}) \times Q$
  - This defines a **many-step transition relation** $\hat{\Delta} \subseteq Q \times \sum^* \times \prod^* \times Q$

Porter Stemmer

A lexicon-free method for getting the stem from a given word

> ATIONAL -> ATE (relational -> relate)
> ING -> ϵ if stem contains a vowel (motoring -> motor)
> SSES -> SS (grasses -> grass)

POS Tagging and HMMs

A quick aside on POS taggins

POS tagging is hard for several reasons:

- Ambiguity
- Sparse data
- the model needs to decide tags based on the word itself, but also surrounding words
- diverse morphologies

This also results in *many* of them.

From FSAs to HMMs

We can generalise our ideas from FSAs and say to generate a sentence of length m,

- let $t_0 =$ `<s>`
- for $i = 1$ to n
  - Choose a tag conditioned on previous tag $P(t_i|t_{i-1})$
  - Choose a word conditioned on its tag $P(w_i|t_i)$

A way of viewing this model would be to "walk" through *states* in a graph, where each state represents a tag. The probability of moving from state s to s' is $P(t_i = s'|t_{i-1} = s)$.

Formalising the tagging problem

To find the best tag sequence T for *untagged sentence* S

$$\underset{T}{\operatorname{argmax}}\, p(T|S)$$

We can use Bayes' rule to give us

$$P(T|S) = \frac{p(S|T)p(T)}{p(S)}$$

But if we only care about argmax_T we can drop $p(S)$:

$$\underset{T}{\text{argmax}}\, p(T|S) = \underset{T}{\text{argmax}}\, p(S|T)p(T)$$

$P(T)$ is the state transition sequence:

$$P(T) = \prod_i P(t_i|t_{i-1})$$

$P(S|T)$ are the emission probabilities:

$$P(S|T) = \prod_i P(w_i|t_i)$$

For any specific tag sequence T we can compute $P(S,T) = P(S|T)P(T)$

$$P(S|T)P(T) = \prod_i P(w_i|t_i)P(t_i|t_{i-1})$$

Markov Assumption

The **Markov Assumption** we're making here is that each tag is only dependent on the previous one (*bigram*) and that words are independent given tags.

Other previous models have had *markov assumptions* such as N-grams.

HMMs

For POS tagging, our "*generative*" **Hidden Markov Model** will do three things:

- **Model**: parameterised model of how both words and tags are generated $P(x, y|\theta)$ (*the transition and emission probabilities*)
- **Learning**: use a labelled training set to estimate most likely parameters of the model $\hat{\theta}$
- **Decoding**: $\hat{y} = \text{argmax}_y\, P(x, y|\theta)$ (Viterbi)

Viterbi

Like Edit Distance, *Viterbi* is a **dynamic programming** approach to solve the problem. It saves us having to enumerate over all possible tag sequences to find the most probable one (with c possible tags for each of the n words in a sentence, we'd have to enumerate all c^n possible tag sequences).

The algorithm goes as follows (thanks again to *Chris*, these are his notes I'm shamelessly stealing here):

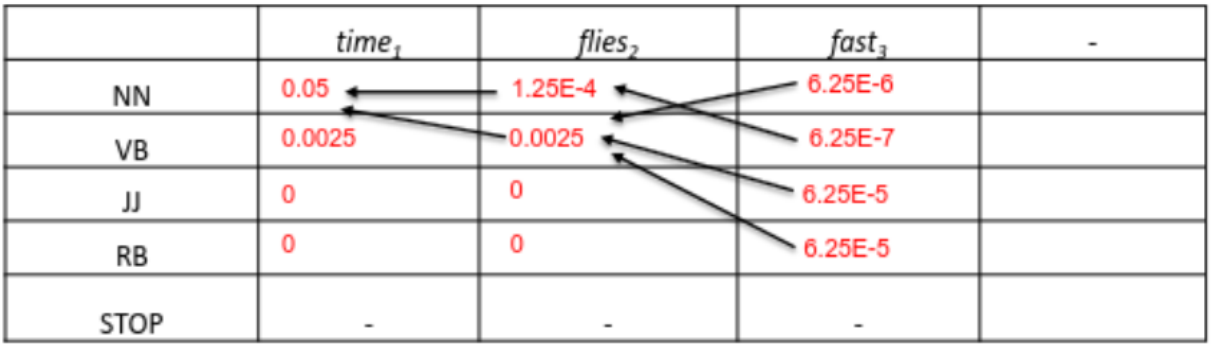
1. We initialise a new matrix, filling the first column as follows: $v_i^1 = a_{(\text{START},i)} b_{(i,x^1)}$ where a is the *transition probabilities* matrix and b is the *emission probabilities* matrix

a_{ij}	STOP	NN	VB	JJ	RB
START	0	0.5	0.25	0.25	0
NN	0.25	0.25	0.5	0	0
VB	0.25	0.25	0	0.25	0.25
JJ	0	0.75	0	0.25	0
RB	0.5	0.25	0	0.25	0

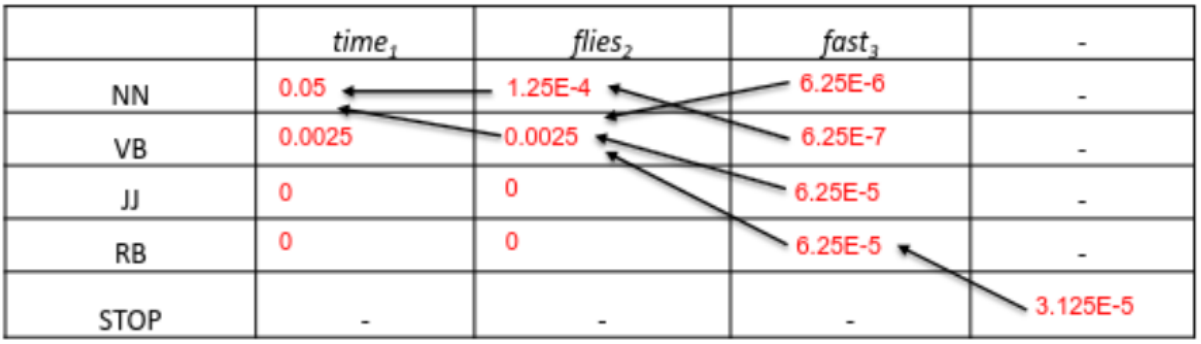
b_{ik}	time	flies	fast	...	...	...
NN	0.1	0.01	0.01	...	...	...
VB	0.01	0.1	0.01	...	...	...
JJ	0	0	0.1	...	...	...
RB	0	0	0.1	...	...	...

	$time_1$	$flies_2$	$fast_3$	-
NN	0.5x0.1=0.05			
VB	0.25x0.01=0.0025			
JJ	0			
RB	0			
STOP	-	-	-	

2. The next column can now be filled in, according to the following: $v_j^t = \left(\max_i \left(v_i^{t-1} a(i,j)\right)\right) b(j, x_t)$, where $j \in [1, 2, \ldots, N]$ and $t \in [2, \ldots, |x|]$. Back-pointers should be tracked.
3. Repeat step 2 for each subsequent value of t, unless you've reached the end of the sentence ($t = |x|$) in which case proceed to step 4.



4. Fill the final cell according to the following: $v_{\text{STOP}}^{|x|+1} = \max_i \left(v_i^{|x|} a_{i,\text{STOP}}\right)$



5. Retrace from the final cell using the backpointers to get the most likely sequence of tags.

Syntax and Parsing

Constituency tests

We need methods of testing whether a group of words form a *constituent*: that is to say, they should all be considered one object(/token?) in the scope of the sentence as a whole

Coordination

Only constituents (of the same type) can be **coordinated** using conjunction words like *and*, *or*, and *but*.

The following sentences would **pass** the test:

> Her friends from Peru **went to the show**.
> Mary *and* her friends from Peru **went to the show**.

> **Should I go** through the tunnel*?*
> **Should I go** through the tunnel *and* over the bridge*?*

The following sentences would **fail** the test:

> **We** peeled the **potatoes**.
> **We** peeled the *and* washed the **potatoes**.

Clefting

Only a constituent can appear in the frame "____ *is/are/who/what/where/when/why/how ...*"

The following sentences would **pass** the test:

> **They put the boxes** in the basement.
> In the basement *is where* **they put the boxes**.

The following sentences would **fail** the test:

> **They** put the boxes **in the basement**.
> Put the boxes *is what they did* **in the basement**

Context Free Grammars

CFGs, or **Context Free Grammars** are a way to represent grammar. Formally, a **CFG** is a tuple of 4 elements $G = (V, \sum, R, S)$:

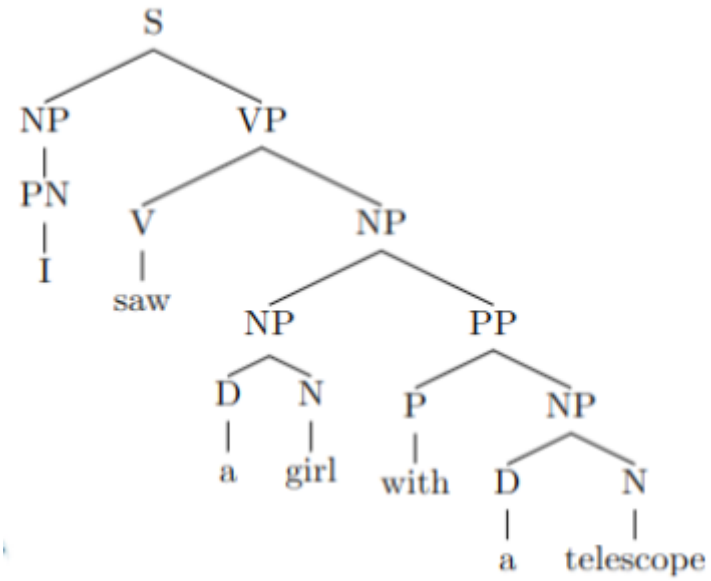
- V - the set of *non-terminals*.
- $\sum$ - the set of *terminals*
- R - the *set of rules* in the form $X \to Y_1, Y_2, \ldots, Y_n$ where $n \geq 0$
  - $X \in Y, Y_i \in V \cup \sum$
- S is a dedicated start symbol

The term "context-free" is due to a subtree only being affected by what the parent's other child is, but **not the context**.

Constituency trees

CFGs can be represented in trees for each consituent. Here's the tree for

> I saw a girl with a telescope



As evident, this can be used to "solve" (read: set in stone one single answer to) Structural ambiguity.

Parsers

All parsers have two fundamental properties:

- **Directionality**: the sequence in which the structures are constructed (*top-down, bottom-up, mixed*)
- **Search strategy**: the order in which the search space of possible outcomes is explored.
  - **Depth-first search**: explore one branch of the search space at a time as far as possible; if dead-end, *backtrack*
  - **Breadth-first search**: explore all possible branches in parallel (simulated/the real thing); requires storing many incomplete parses in memory at once
  - **Best-first search**: score each partial parse and pursue the options *sorted by highest-scoring first*.

The commonly used CYK Algorithm is a *bottom-up* parser for CFGs, but only supports rules in Chomsky Normal Form.

CYK Algorithm

The **Cocke-Kasami-Younger** (CYK) algorithm is a parsing algorithm for Context Free Grammars. It is generalisable to Probabilistic Context-Free Grammars, but the CFG must be written in CNF.

Chomsky Normal Form

A **CNF** grammar only contains rules in one of two forms:

$$C \to x$$
$$C \to C_1 C_2$$

That is to say, each rule expands to either a POS tag or *two inner rules*.

Converting to CNF

Any CFG can be converted to an equivalent CNF - the syntactic tree will look different, but the language remains the same. There are three steps to get a valid CNF grammar:

1. Remove any empty (*epsilon*) productions ($C \to \epsilon$).
2. Get rid of any *unary* rules ($C \to C_1$).
3. Split rules so we get binary rules ($C \to C_1\ C_2 \ldots C_n\ (n > 2)$).

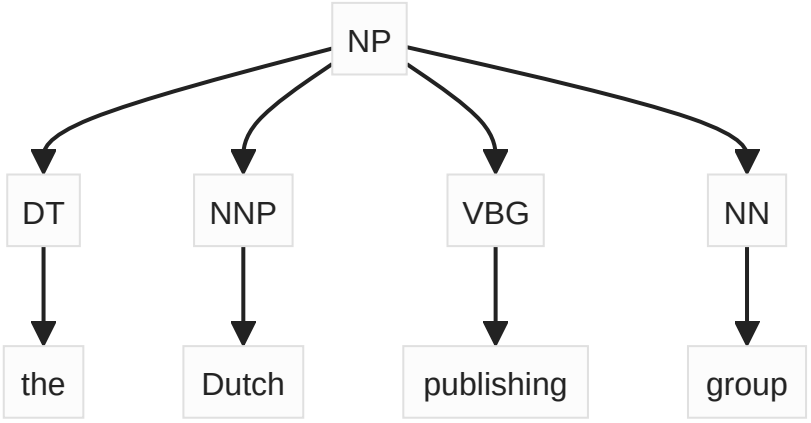
The Actual CYK Algorithm

The CYK algorithm can *automate* the process of Converting to CNF.

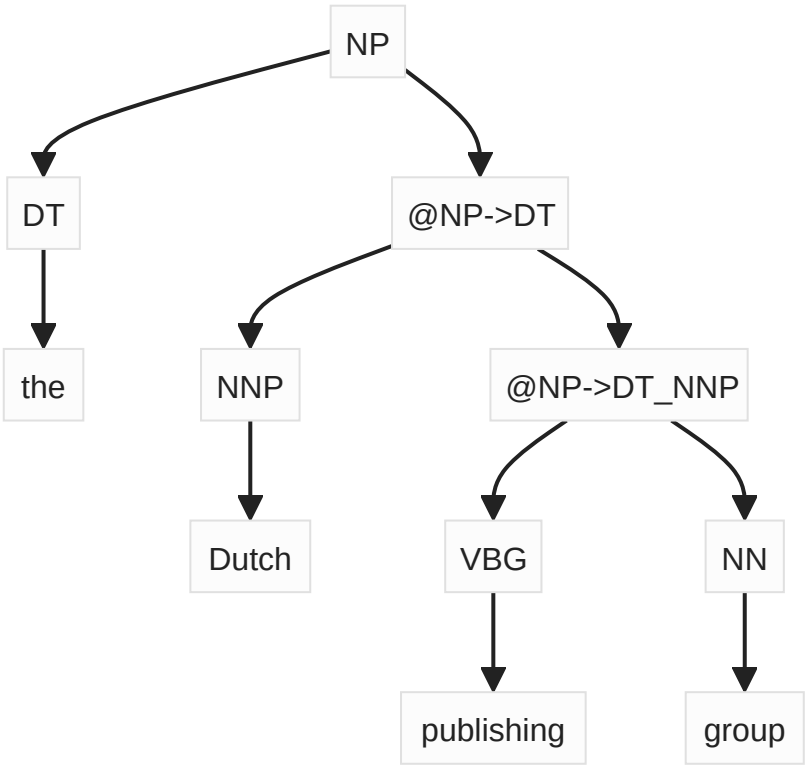
Consider the following rule in a non-CNF grammar: $NP \to DT\ NNP\ VBG\ NN$.
To get a set of *binary rules* ***which are equivalent***, we can create the following:

$$NP \to DT\ X$$
$$X \to NNP\ Y$$
$$Y \to VBG\ NN$$

Instead of binarising the rules, we can binarise trees on preprocessing.



This is known as **lossless Markovization** in PCFG circles.



CYK for parsing

We can use CYK to parse a sentence.

Given a:

- grammar $G = (V, \sum, R, S)$
- sequence S of words $(w) = (w_1, w_2, \ldots, w_n)$
  produce a *parse tree* for w.

If we imagine "fence posts" between each word and one on each end, labelled starting from 0, we can use `span(i, k)` to refer to words between fence posts *i* and *j*.

The general idea is:

1. We compute for every `span` a set of admissible labels
   - This starts from small trees (i.e single words) and proceeds to larger ones
2. When we're done, we check if S is among the admissible labels for the whole sentence
   - That is to say, if a tree with the signature $[0, n, S]$ exists
3. If yes, *the sentence belongs to the language*.

This does not always give us direct clear meaning. If we don't have absolute values (one possible label for each word), then we have an ambiguous sentence

Probabilistic Context-Free Grammars

PCFGs extend the regular Context Free Grammars by adding a probability that a rule will occur. Rules instead follow the format

$$A \to \beta[p]$$

where p is the probability of the expansion. All possible expansion probabilities must sum to 1. This solves the ambiguity problem in CFGs, as we can now choose the *most* likely outcome.

CYK for PCFGS

CYK can now be used to produce the *most-likely* parse $\hat{T}$ for a given sentence S. Instead of putting down all possible tags for a given word, we put the most probable one, and continue from there. Alternatively, we can iterate through all possible parse trees and choose the most likely from that one. Such **treebanks** exist to use with a PCFG parser.

MLE and Smoothing for PCFG Parsing

We can estimate the rule probabilities to assist with parsing by using MLE (and applying any Smoothing if this causes problems).

$$P(x \to \alpha) = \frac{C(X \to \alpha)}{C(X)}$$

Unary Closure

If we had a loop (e.g. *A -> B -> A -> C*), the probability of this rule would be $\leq$ the probability of A->C. If we need to recover the tree, we store *backpointers* to the elements which the rule was built from. We can use these to maintain the probability distribution.

CYK Pruning

We can speed up the algorithm by *pruning* the data parsed.

- **Basic pruning**
  - For every span (i, j), we store only the labels which have the probability at most N times smaller than the prob of the most probably label for this span. This *does not* check all rules, but only ignores subtrees with negligible probabilities.
- **Coarse-to-fine pruning**
  - We parse with a smaller simpler grammar, and pre-compute probabilities for each span. We only use the ones with non-negligible probability from the previous grammar.

Parser Evaluation

Intrinsic and Extrinsic

Intrinsically evaluating parsers are either *manual* (human judgement) or *automatic* (evaluating against human expert **gold standard** annotations).

Extrinsically evaluating parsers involves scoring syntactic representations by comparing how well a system *using this representation* performs on some task.

Bracket Scores

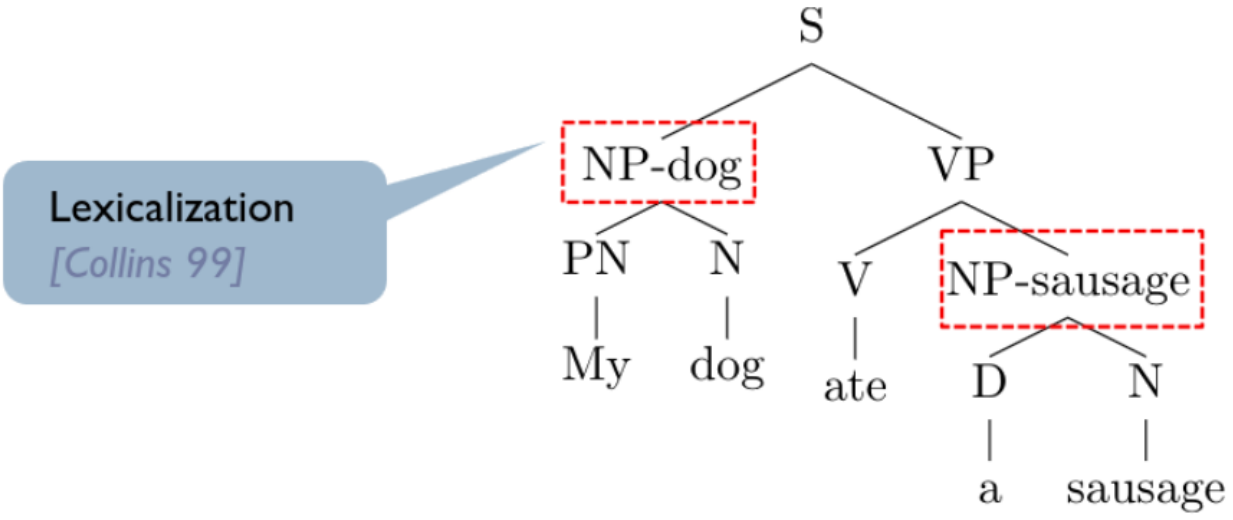
A common measure of parser evaluation, this method regards a tree as a collection of brackets $[\min, \max, C]$. The set of brackets is predicted by a parser, and compared against the gold standard. We use Precision, Recall, and F1 to score this. F_1 remains a composition of *precision* and *recall* but their definitions here are slightly adjusted to better fit to PCFGs:

$$\text{labelled recall} = \frac{\#\text{ of correct consituents in hypothesis parse of S}}{\#\text{ of correct consituents in reference parse of S}}$$

$$\text{labelled precision} = \frac{\#\text{ of correct consituents in hypothesis parse of S}}{\#\text{ of total consituents in hypothesis parse of S}}$$

Structural Annotation

Regular PCFGs (*treebank*) do not produce the best parsers because *they do not encode anything more beyond single rules*. To extend this, we (ironically, for Context Free Grammars) need to incorporate some form of context, usually into the parents. This is known as *lexicalisation*.



Vertical & Horizontal Markovisation

A form of lexicalisation, in which each non-terminal in the tree is annotated with its lexical head. It also solves the problem of *close attachment*, which from what I can tell is when `PP` attaches to the closest preceding `NP` (it solves it by distinguishing `NP`s).

Vertical Markovisation increases context, whereas Horizontal Markovisation (a form of *binarisation*) tries to reduce context. We can combine different orders of **both** vertical and horizontal markovisation to best optimise the model's performance

EM for splitting

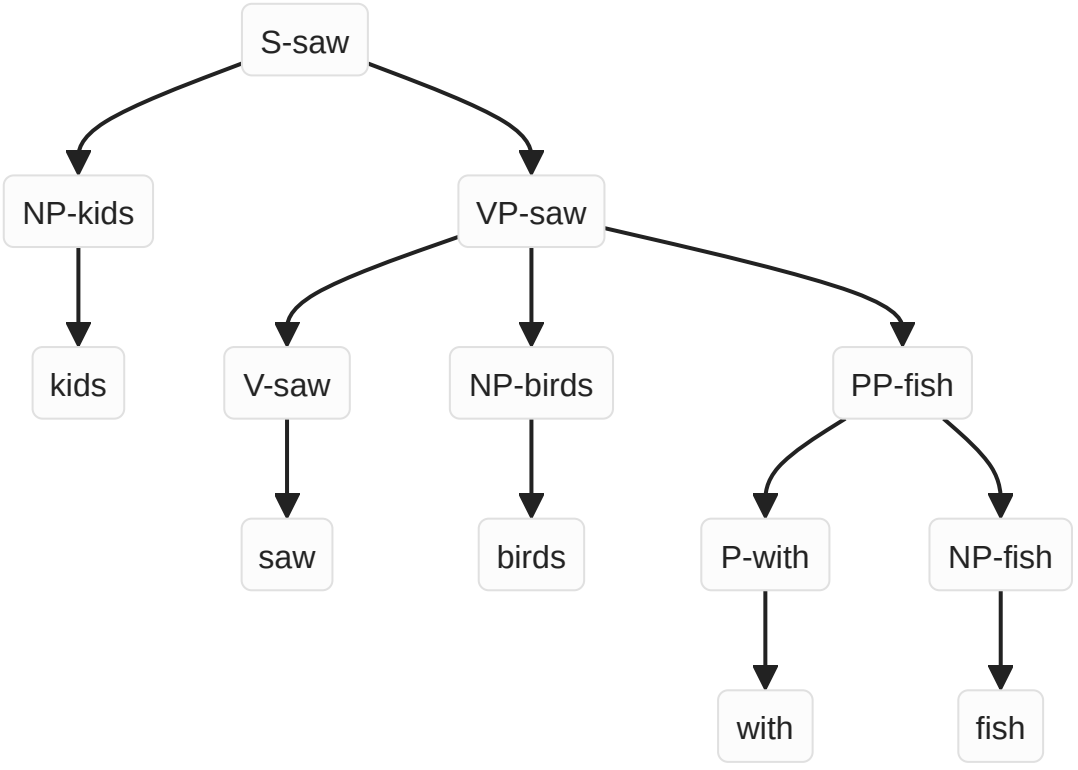
If tags in a treebank are too course, we can *split* them which leads to a boost in performance. We can "learn" these splits from data (aswell as do them manually) using EM.

Anchored Rules

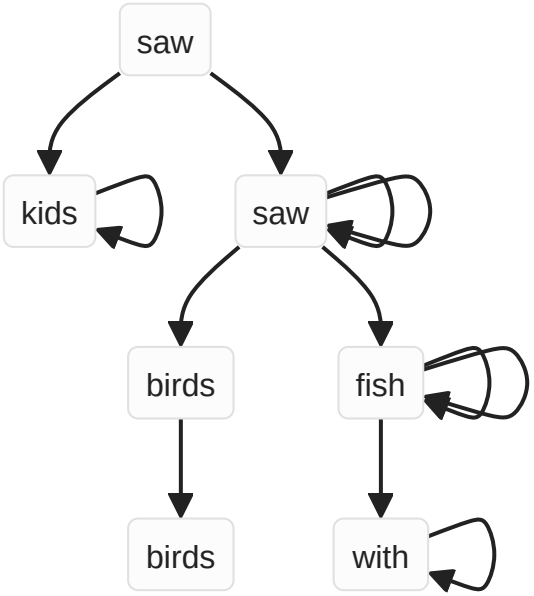
Instead of a rule probability being constant, we have it predicting for a *given span* in the chart. Instead of $P(C_1, C_2, C_3)$ we augment this with $\min, \max$, and mid.

Dependency Parsing

If we have our *lexicalised* tree (annotated),



we can first *remove the phrasal categories* and start *collapsing chains of duplicates*



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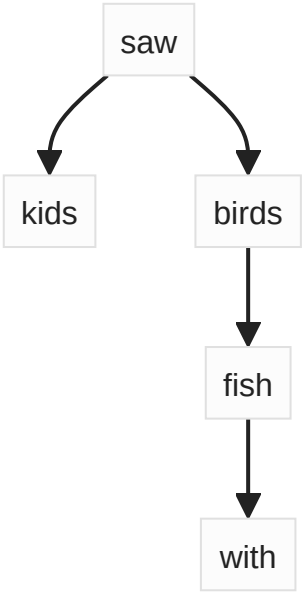
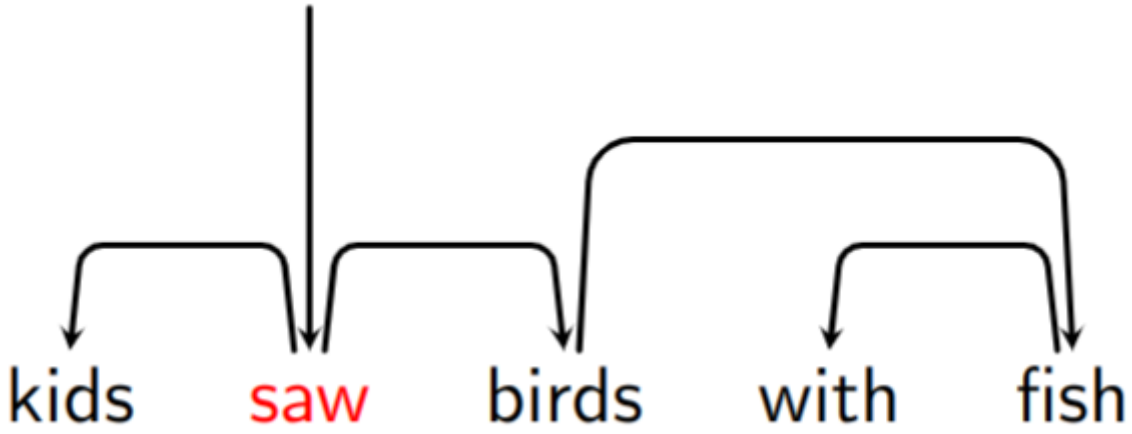
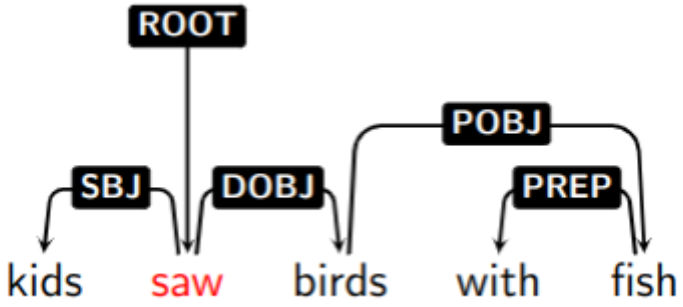


Now we have a tree where in this format we can represent it in a format best suited to *parsing dependencies* (parents)



Edge Labels

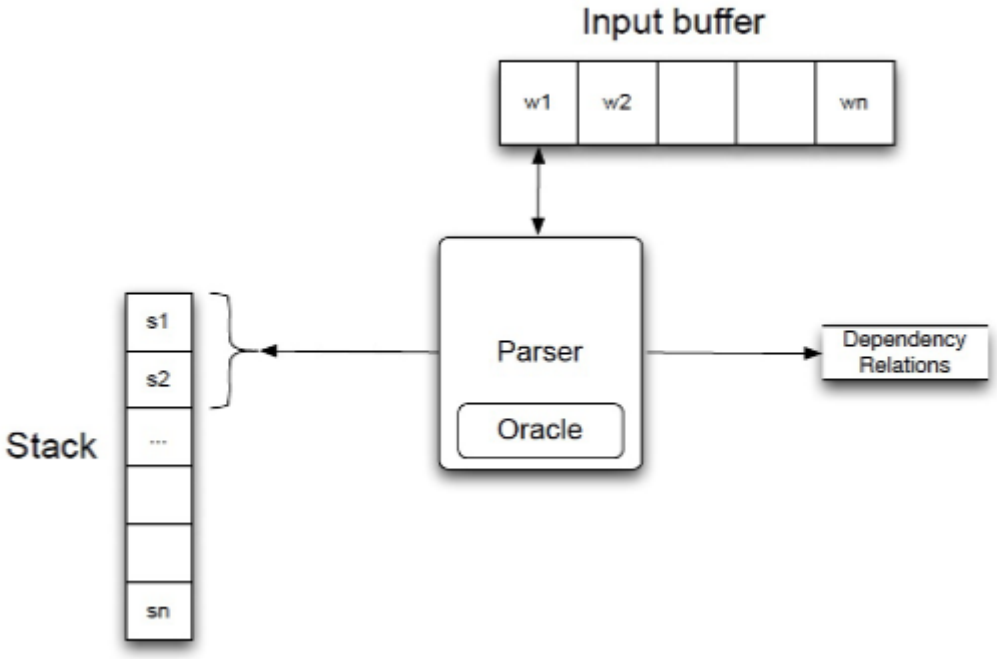
We can further annotate these **relations** (*edges*) with *labels*



Projectivity

A dependency parse is said to be **projective** if every subtree occupies a *contiguous* span of the sentence (that is to say, there is no crossing of edges)

Transition-based dependency parsing

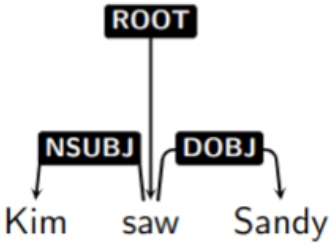


There are three actions we can do with each word:

- **LeftArc**: Assign head-dependent relation between $s1$ and $s2$, pop $s2$
- **RightArc**: Assign head-dependent relation between $s2$ and $s1$, pop $s1$
- **Shift**: Put $w1$ at the top of the stack

This is sometimes called the *shift-reduce* algorithm; an alternative to CYK that doesn't even require a grammar.

Step	Stack	Word List	Action	Relations
0	[root]	[Kim,saw,Sandy]	Shift	
1	[root,Kim]	[saw,Sandy]	Shift	
2	[root,Kim,saw]	[Sandy]	LeftArc	nsubj(saw,Kim)
3	[root,saw]	[Sandy]	Shift	
4	[root,saw,Sandy]	[]	RightArc	dobj(saw,Sandy)
5	[root,saw]	[]	RightArc	root→saw
6	[root]	[]		



Graph-based dependency parsing

We create a fully connected directed graph of all possible edges and choose the *best ones that **form a tree**.* We do this by assigning a non-negative score to each possible edge, and using the **maximum spanning tree** algorithm to find the tree with the highest total score.

Conversion-based dependency parsing

It is sometimes better to constituency parse first, then *convert* to dependencies. Some treebanks are only available in dependency form.

Compositional Semantics

A goal of NLP is to understand what people *mean* when they talk. But how do we know if we've succeeded?

We can use **semantics** to relate expressions to "the world", including both their *truth conditions and denotation* (literal meaning) and *connotation* (other associations).

We can exploit **compositionality** to augment a grammar with a *semantic component* which will deterministically derive the *logical* form of a sentence from its syntax tree.

This representation should be **unambiguous**, support **automated inference**, and be **verifiable, somehow**.

First-order logic for semantics

We use first order logic to represent sentences. This is better than propositional logic as all meaning would be embedded into the individual clauses, entirely defeating the point of representing language.

1. *Fred ate rice*: $\text{eat}(\text{fred}, \text{rice})$
2. *Everyone ate rice*: $\forall x.\, \text{eat}(x, \text{rice})$
3. *Someone ate rice*: $\exists x.\, \text{eat}(x, \text{rice})$
4. *Every dog had a bone*: $\forall x(\text{dog}(x)) \rightarrow \exists y(\text{bone}(y) \wedge \text{have}(x, y)))$
5. *Every dog had a bone*: $\exists y(\text{bone}(y) \wedge \forall x.\, (\text{dog}(x) \rightarrow \text{have}(x, y)))$

2 entails 1 and 3; 1 entails 3, 5 entails 4!

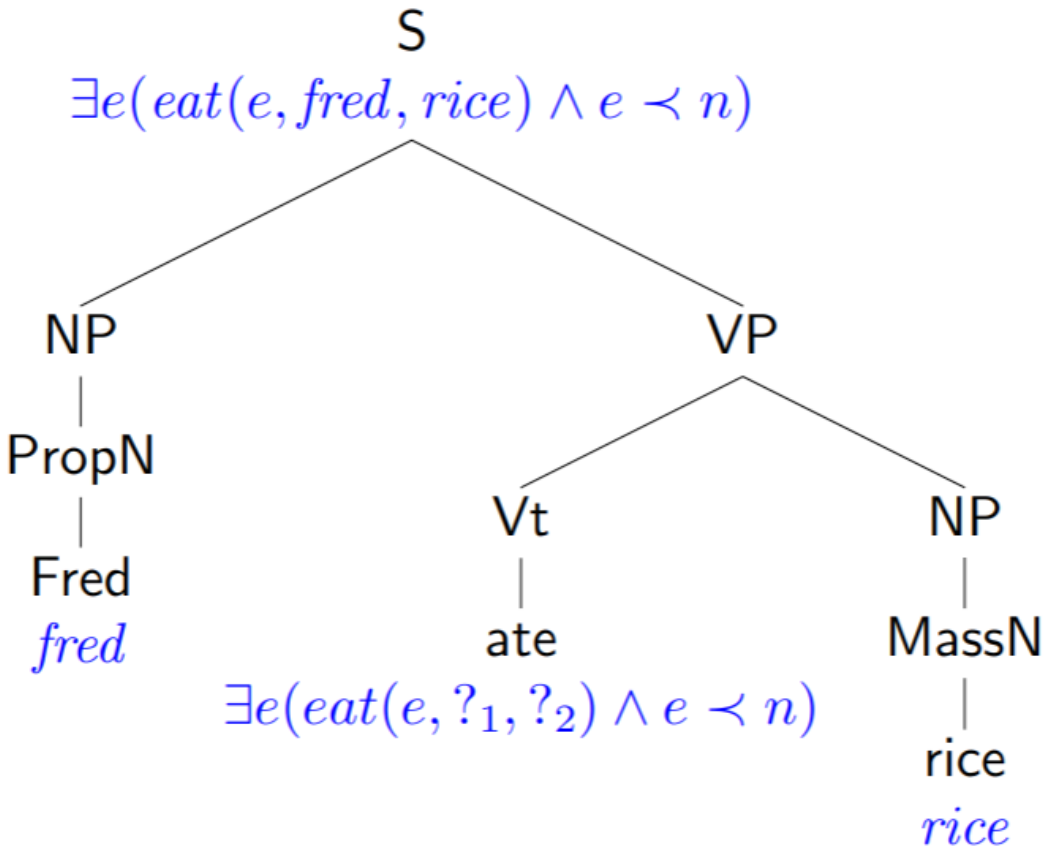
Davidsonian Semantics

We can introduce an event argument e to an 'action'

Tense: *Fred ate rice*: $\exists e(\text{eat}(e, \text{fred}, \text{rice}) \wedge e \prec n)$
Modifiers: *Fred ate rice with a fork at midnight*:

$$\exists e(\text{eat}(e, \text{fred}, \text{rice}) \wedge e \prec n \wedge \exists x(\text{with}(e, x) \wedge \text{fork}(x)) \wedge \text{at}(e, \text{midnight}))$$

The second sentence now entails the first via $\wedge - \text{elimination}$. We now can exploit **compositionality** to augment a grammar with a semantic component which deterministically derives the *logical form* of a sentence from its syntax tree. We're aiming for something like this:



Lambda-calculus and Beta reduction

Using lambda calculus allows us to work with "partially constructed formulae". If φ is a *well-formed* FoL expression and x is a variable, then $\lambda x \varphi$ is a well formed FoL expression (a function known as the λ-*term*).

$$\lambda x \varphi(a) = \varphi[x/a]$$

When we apply this function this is known as ***Beta (β) reduction***. For example:

$$\lambda y \lambda x (\exists e(\text{eat}(e, x, y) \wedge e \prec n))(rice) \text{ becomes}$$

$$\lambda y \lambda x (\exists e(\text{eat}(e, x, rice) \wedge e \prec n))$$

In short, if we have $\lambda x.\,(x(a\,b\,x))\,n$, beta reducing this replaces all instances of x in $x(a\,b\,x)$ with n.

Scope Ambiguities

A typical parse does not uniquely determine the scope of snippets containing both terms *every* and *a*. We can possible enumerate all interpretations (bad and stupid), or build lambda functions via syntax that *underspecify* the realtive semantics of the quantifiers.

- So the syntax tree:LF is 1:1, but the LF describes several FoL formulae

Discourse Coherence

Coherence

Adding context to events can change our minds on how we interpret then, even if the original context is still present.

Coherence is important (to associating otherwise unrelated sentences together), but it can be challenging to represent it computationally.

The following examples showcase some challenging problems a computer may be asked to solve

Time

> John hit Max on the back of his neck.
> Max fell. John pushed him.
> Max rolled over the edge of the cliff.

Word meanings

> A: Did you buy the apartment?
> B: Yes, but we rented it./ No, but we rented it.

Bridging

> John took an engine from Avon to Dansville.
> He picked up a boxcar./He also took a boxcar.

Dishonesty

> a. M (to K and S): Karen ’n’ I’re having a fight,
> b. M (to K and S): after she went out with Keith and not me.
> c. K (to M and S): Wul Mark, you never asked me out.

> a. P: Do you have any bank accounts in Swiss banks, Mr. Bronston?
> b. B: No, sir.
> c. P: Have you ever?
> d. B: The company had an account there for about six months, in Zurich.

Gestures

We require multi-model models to connect speech and gestures. Obviously, this is quite difficult to represent an example on in my notes, so as you read this wave your arms around to kinda, sort of, get the point across?

If you spot anyone in the library doing this, please email me because it sounds hilarious.

SDRT and Logical Form

Segmented Discourse Representation theory is the **logical form** of monologue. The logical form consists of:

- Set A of labels $\pi_1, \pi_2, \ldots$
  - each label stands for a discourse segment
- A mapping $\mathcal{F}$ from each label to a formula representing its content
- Vocabulary contains coherence relations e.g. *Elaboration(π_1, π_2)*

Example, using John's safe

π_1: John can open Bill's safe
π_2: He knows the combination

π_0: *Explanation*(π_1, π_2)
π_1: ιx(*safe*(x) & *possess*(x, *bill*) & *can*(*open*(e_1, *john*, x)))
π_2: ιy(*combination*(y) & ***of***($y, \mathbf{x}$) & *knows*(***john***, y))

Supervised learning for SDRT

We can use corpora annotated with discourse structure, but these are scarce.

Training on 100 dialogues

Parser based on Collins' parsing model:

- 72% f-score on segmentation (baseline: 53.3%)
- 48% f-score on segmentation and coherence relations (baseline: 7.4%)
- Doesn't attempt to estimate LFs of clauses.

Training on Groningen Meaning Bank

Neural semantic parser, RNN computes structure first, fills in arguments later:

- 77% f-score on segmentation, coherence relations *and* LFs of clauses
- State of the Art on extended text!

Training on STAC

BERT to encode pairs of discourse units, simple NN to predict attachments, multi-task learning for labelling.

- 80% f-score on segmentation; 71% f-score on coherence relations.
- State of the Art on multi-party dialogue!

Lexical Semantics

Hyponyms and Hypernyms

Words can be *hyponyms* or hypernyms

- A **hyponym** is a subset of another word
- A **hypernym** is a *superset* of another word
  An **A-B** relationship of these words is called an *ontology*.

Regular Polysemy

Polysemy is when two separate word senses are in some way related to each other, we can capture patterns. **Regular** polysemy is when two words exhibit polysemy for the same reason (e.g. *Cherry* and *Orange* both meaning a **colour** and **fruit**).

Word Sense disambiguity

A word is ambiguous if, for a given POS for that word, there is more than one sense. Words can have multiple (related/unrelated) senses.

Pattern	Participating Senses	Example Sentences
Animal for fur	Mink, chinchilla, rabbit, beaver,	The mink drank some water /

Pattern	Participating Senses	Example Sentences
	raccoon*, alpaca*, crocodile*	She likes to wear mink
Animal for meat	Chicken, lamb, fish, shrimp, salmon*, rabbit*, lobster*	The chicken drank some water / The chicken is tasty
Animal/Object for personality	Chicken, sheep, pig, snake, star*, rat*, doll*	The chicken drank some water / He is a chicken
Artifact for activity	Shower, bath, sauna, baseball	The shower was leaking / The shower was relaxing
Artist for product	Writer, artist, composer, Shakespeare, Dickens*, Mozart*, Picasso*	The writer drank a lot of wine / The writer is hard to understand
Body part for object part	Arm, leg, hand, face, back*, head*, foot*, shoulder*, lip*	John's arm was tired / The arm was upholstered
Building for people	Church, factory, school, airplane	The church was built 20 years ago / The church sang a song
Coercion		The bottle is made of steel / He drank half of the bottle
Complement	Begin, start, finish, try	John began reading the book / John began the book
Container for contents	Bottle, can, pot, pan, bowl*, plate*, box*, bucket*	The price of the coffee was low / John asked the price of the coffee
Figure for Ground	Window, door, gate, goal	The window is broken / The cat walked through the window
Grinding	Apple, chair, fly	The apple was tasty / There is apple all over the table
Instrument for action	Hammer, brush, shovel, tape, lock*, bicycle*, comb*, saw*	The hammer is heavy / She hammered the nail into the wall
Instance of an entity for kind	Tennis, soccer, cat, dog, class*, dinner*, chair*, table*	Tennis was invented in England / Tennis was fun today
Location / Place at location	Bench, land, floor, ground, box*, bottle*, jail*	The bench was made of pine / The coach benched the player
Material for artifact	Tin, iron, china, glass, linen*, rubber*, nickel*, fur*	Watch out for the broken glass / He filled the glass with water
Object for contents	Book, CD, DVD, TV*, magazine*, newspaper*	The heavy, leather-bound book / The book is funny
Object for placing at goal	Water, paint, salt, butter, frame*, dress*, oil*	The water is cold / He watered the plant.
Object for taking from source	Milk, dust, weed, peel, pit*, skin*, juice*	The milk tastes good / He milked the cow

Pattern	Participating Senses	Example Sentences
Occupation for role in action	Boss, nurse, guard, tutor	My boss is nice / He bossed me around
Place for an event	Vietnam, Korea, Waterloo, Iraq	It is raining in Vietnam / John was shot during Vietnam
Place for an institution	White House, Washington, Hollywood, Pentagon, Wall Street*, Supreme Court	The White House is being repainted / The White House made an announcement
Plant for food or material	Corn, broccoli, coffee, cotton, lettuce*, eggs*, oak*, pine*	The large field of corn / The corn is delicious
Portioning	Water, beer, jam	She drank some water / She bought three waters
Publisher for product	Newspaper, magazine, encyclopedia, Wall Street Journal*, New York Times*	The newspaper is badly printed / The newspaper fired three employees
Visual Metaphor	Beam, belt, column, stick, bug*, leaf*	Most of the weight rests on the beam / There was a beam of light

Altering Naive Bayes for WSD

We can alter *Naive Bayes* to include word senses as a feature (the question on *what features to use?* still remains).

$$\hat{s} = \arg\max_{s \in S} P(s) \prod_{j=1}^{n} P(f_j|s)$$

WordNet

We can solve the problems above by mapping the *ontologies* to a graph. Tools like WordNet exist for this reason.

Named Entity Recognition

WordNet has many different types of *named entity* tags to incorporate complex proper nouns. NER systems typically use some form of feature-based tagging, with features such as capitalisation being often considered. A list of known names is a **gazetteer**, and is sometimes used.

Supersense tagging

In the sense that Named Entity Recognition breaks down entity tags, this is a form of *supersense* tagging. WordNet does this often:

N:TOPS	N:OBJECT	V:COGNITION
N:ACT	N:PERSON	V:COMMUNICATION
N:ANIMAL	N:PHENOMENON	V:COMPETITION
N:ARTIFACT	N:PLANT	V:CONSUMPTION
N:ATTRIBUTE	N:POSSESSION	V:CONTACT
N:BODY	N:PROCESS	V:CREATION
N:COGNITION	N:QUANTITY	V:EMOTION
N:COMMUNICATION	N:RELATION	V:MOTION
N:EVENT	N:SHAPE	V:PERCEPTION
N:FEELING	N:STATE	V:POSSESSION
N:FOOD	N:SUBSTANCE	V:SOCIAL
N:GROUP	N:TIME	V:STATIVE
N:LOCATION	V:BODY	V:WEATHER
N:MOTIVE	V:CHANGE	

Distributional Semantics

Distributional hypothesis

The idea that words in *similar contexts* imply *similar meanings*. For example, take the *unknown* word

> ongchoi

This word is borrowed from Cantonese; it's quite likely you've never seen it before. It would be unfair to ask you what it meant, but if I said "here are some snippets containing the word"

> *Ongchoi is delicious sauteed with garlic*
> *Ongchoi is superb over rice*
> *...ongchoi leaves with salty sauces*

You could probably reasonably infer the meaning of **ongchoi** - *a leafy green*

First and second order co-occurence

There are two types of *co-occurence* between words:

First-order co-occurence

- *Syntagmatic association*
- Typically near each other; *wrote* is first-order associate of *book*

Second-order co-occurence

- *Paradigmatic association*
- Words that have **similar neighbours**; *wrote* is a second-order associate of *said* and *remarked*

PMI

Pointwise mutual information tells us how likely the co-occurence is than if the words were independent.

$$\text{PMI}(x, y) = \log_2 \frac{P(x, y)}{P(x)P(y)}$$

$P(x, y)$ is the probability of seeing the words x and y in the same context.
$P(x)P(y)$ is the *predicted* probability of the same **if x and y are independent**.

Vector space representation of context vectors

This idea that context represents a word's meaning and that similar contexts imply similar meanings lends itself to representing these words in a *vector space*. Such a space would require many dimensions, so any 2d or 3d representations are due to the limitations of note-taking.

Evaluation: word association

Despite these vectors being abstract, they are still *vectors* - we can measure the association strength of words via typical vector means.

The first intuitive idea is to use *Euclidean Distance*

$$\sqrt{(\sum_i (v_i - w_i)^2)}$$

This idea won't work well if *one or more dimension has an extreme value*. Vectors become longer when they have higher values in each dimension so more frequent words have higher dot products, so this similarity metric is very sensitive to word frequency.

We could also take the *dot product* of two vectors $\vec{v}$ and $\vec{y}$.

$$\vec{v} \cdot \vec{w} = \sum_i v_i w_i$$

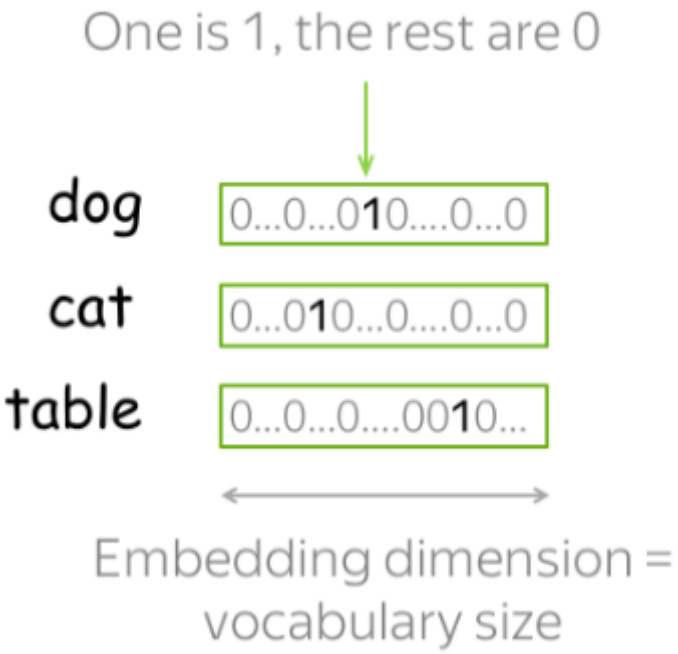
This also falls prey to the frequency issue mentioned above, but if we *normalise the dot product* we solve this. Normalising the dot product like this is the same as finding the $cosine$ of the angle between the vectors

Neural Embeddings

The previous discussion on Vector space representation of context vectors has another, more common name - **embeddings**.

One-hot Vectors

Solving the problem of "*how do we get these word vectors?*", **one-hot vectors** are a means to represent words. For the i-th word in the vocabulary, a word vector has 1 in the i-th dimension and 0 in all the rest.



This grows with the size of the vocabulary, which isn't very ideal, but also this method doesn't actually *capture* any semantic meaning! We need better measures, as our model will be useless at predicting words if it doesn't know what they mean.

Latent Semantic Analysis

LSA is a technique which analyses a document to infer meaning *from contexts*. We apply a *sliding context window* to the document. For sake of example, let's say we use a window of $L = 2$ on the following sentence:

> I saw a cute grey cat playing in the garden

If our **central** word was "cat", our **context** words would be "cute", "grey", "playing", and "in"

> I saw a *cute grey* ***cat*** *playing in* the garden

LSA then forms a $N(w, c)$ matrix, where each cell tells us the number of times *word* w appeared in *context* c. We calculate this value by with two sets of vectors: *word (central) vectors* and *context vectors*; when a word is the central one, we use the value from that, and when it is a context word, we use that. Formally, the value of each element is

$$\mathrm{tf}(w, d) \cdot \mathrm{idf}(w, D)$$

where

- we are focusing on word w
- we are focusing on document d, from a collection D
- tf is *term frequency* ($N(w, d)$); the amount of times the word w appears in the document d
- idf is the *inter-document frequency*

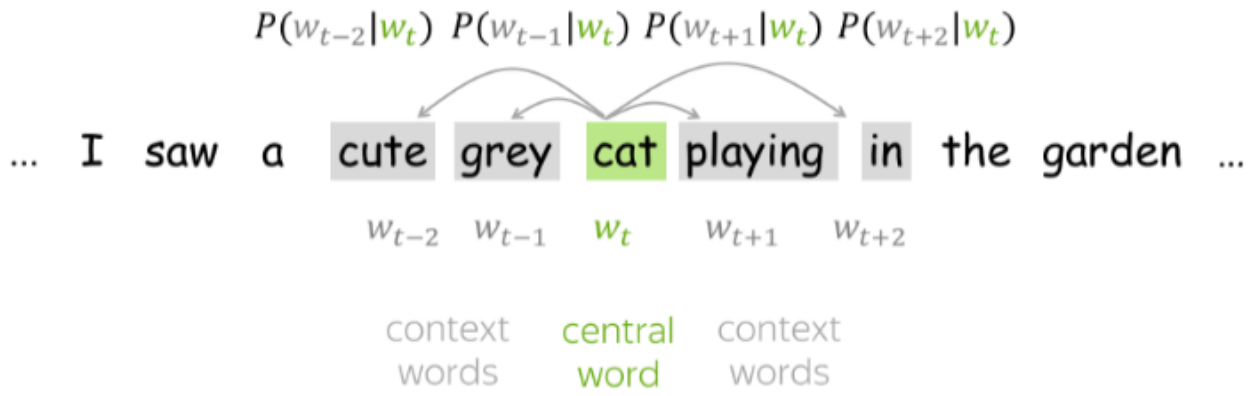
$$\log \frac{|D|}{|d \in D : w \in d|}$$

  This isn't very good unless we use PMI to calculate our co-occurences.

Word2Vec

Following from this research, *Word2Vec* is an iterative model that learns word vectors by teaching to **predict contexts**. It does this by predicting and optimising our *context* vectors from earlier. These vectors are usually initialised to a normal or Gaussian distribution.

Skip-gram model



Mikolov's skipgram is a way to formally represent our *sliding window* example from earlier. The highlighted w_t probability is calculated from the *central word* vectors, whereas the other W_{t-n} vectors are calculated from the *context word* vectors.

Negative-Log-Likelihood Cost Function

For each position $t = 1, \ldots, T$ in a corpus, *Word2Vec* predicts the likelihood (the context words) within an m sized window given a central word w_t:

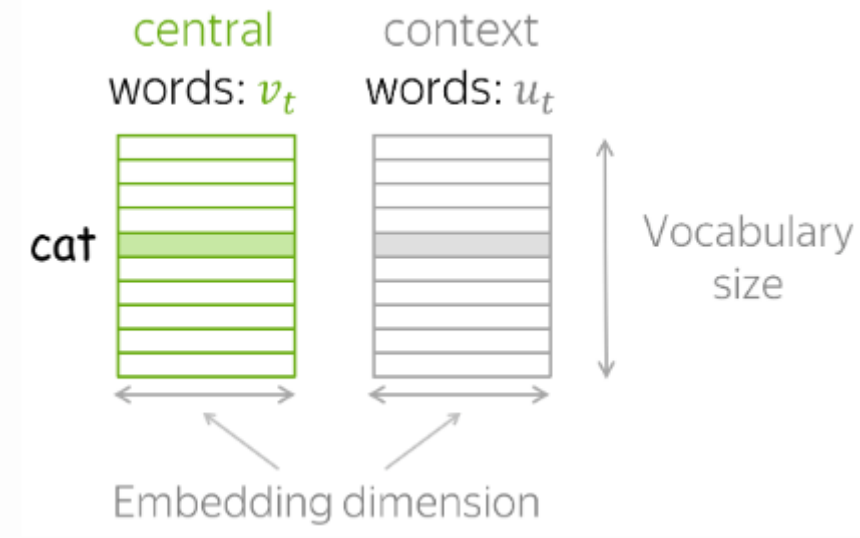
$$\text{Likelihood} = L(\theta) = \prod_{t=1}^{T} \prod_{-m \leq j \leq m, j \neq 0} P(w_{t+j} | w_t, \theta)$$

θ represents all variables to be optimised. The **actual cost function** (objective function) $J(\theta)$ is the *average* of this negative-log-likelihood

$$\textbf{Loss} = J(\theta) = -\frac{1}{T} \log L(\theta) = -\frac{1}{T} \sum_{t=1}^{T} \sum_{-m \leq j \leq m, j \neq 0} \log P(w_{t+j} | w_t, \theta)$$

Computing $P(w_{t+j} | w_t, \theta)$

Recall we have *two* vectors for each word:



We can bring back our old friend Softmax to compute the probability $P(o|c)$ for a *central* word c and a *context* (outside) word o:

$$P(o|c) = \frac{\exp(u_o^T v_c)}{\sum_{w \in V} \exp(u_w^T v_c)}$$

The u^T here just means the transpose of the vector, turning it into a row vector so it can be *dot producted* with v_c

Optimisation with Gradient Descent

We want to train the parameters (θ) v_w and u_w for all w words in the vocabulary.

> ### The Gradient Descent formula
>
> I don't think the exam needs us to go into further detail on the cost-function-specifics formula, so in the essence of time, I will steal a final formula from *Chris'* excellent notes

$$\theta^{\text{new}} = \theta - \alpha \nabla J(\theta)$$

where

- θ^{new} is our new parameters
- θ is our old parameters
- α is some small weighting constant applied to the next bit
- $\nabla J(\theta)$ is the *vector derivative* of our cost function, when applied to the *old parameters* θ

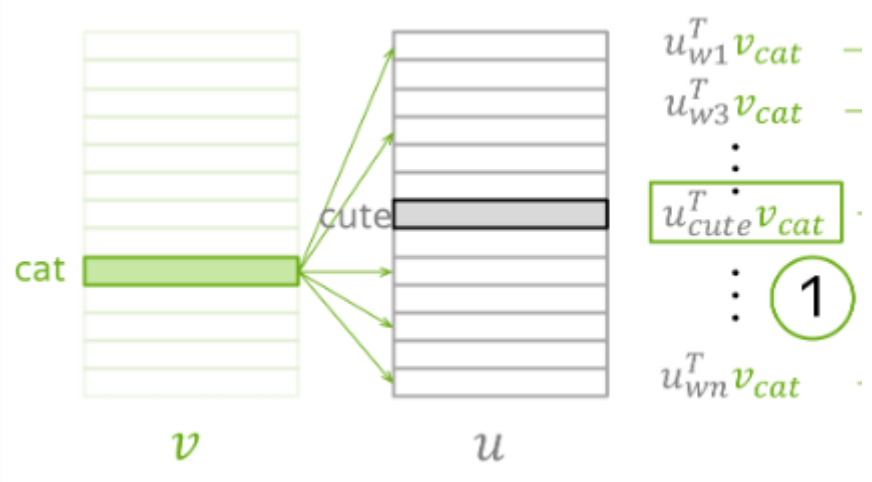
Each update is for a single pair of words - one *center word* and one of its *context words*. Say with our previous example, our *central word* is "**cat**", and our *context word* is "**cute**", our loss term becomes

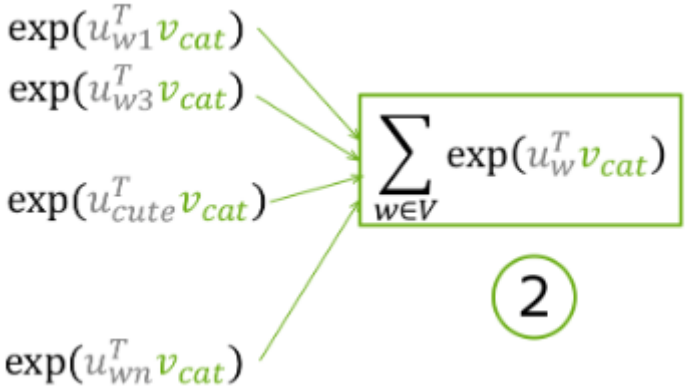
$$\begin{aligned} J_{t,j}(\theta) &= -\log P(\text{cute}|\text{cat}) \\ &= -\log \frac{\exp(u_{\text{cute}}^T v_{\text{cat}})}{\sum_{w \in Voc} \exp u_w^T v_{\text{cat}}} \\ &= -u_{\text{cute}}^T + \log \sum_{w \in Voc} \exp u_w^T v_{\text{cat}} \end{aligned}$$

- We only use
  - from vectors for **central words**
  - v_{cat}
  - from vectors for **context words**
  - all u_w (for all words in Vocab)

  The full gradient descent process is:

1. Take the *dot product* of v_{cat} with all u.





2. $\exp$ and $\sum$ (sum) all.
3. Get the loss for this step.

$$J_{t,j}(\theta) = -u_{\text{cute}}^T v_{\text{cat}} + \log \sum_{w \in V} \exp(u_W^T v_{\text{cat}})$$

4. Evaluate the gradient and make an update

$$v_{\text{cat}} := v_{\text{cat}} - \alpha \frac{\partial J_{t,j}(\theta)}{\partial v_{\text{cat}}}$$
$$u_w := u_w - \alpha \frac{\partial J_{t,j}(\theta)}{\partial v_{u_w}} \forall w \in V$$

   (∂ is talking about *partial derivatives* here. It's so we know "which way to go" to minimise the cost of J)

This has the effect of *increasing* the similarity (recall: dot product) of v_{cat} and u_{cute}, and *decreasing* the similarity between v_{cat} and all other words w in the vocabulary. The face we iterate over *all context words* means this will not negatively impact the other context words on average.

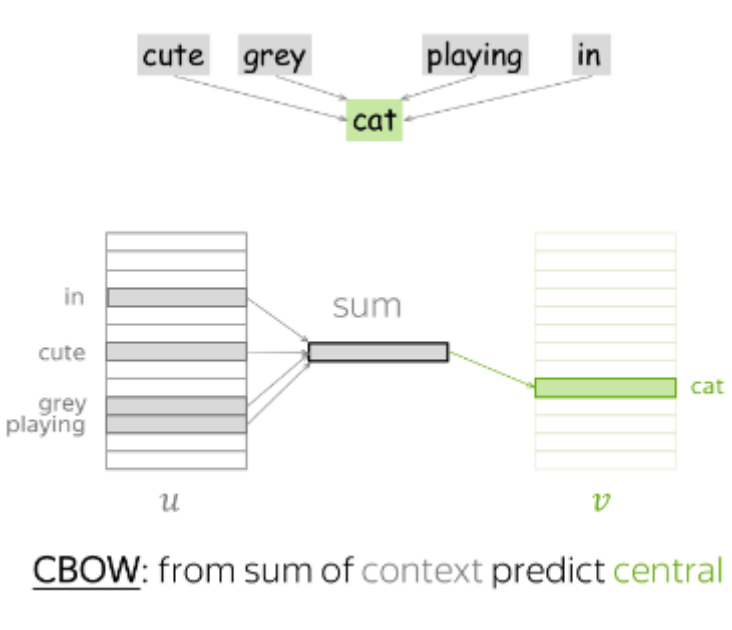
Negative Sampling

Normally, we update (*majoratively decrease*) **all** context words when making an update (to u_w). *Negative Sampling* is the process of only randomly decreasing K selected words in u_w. This trains *Word2Vec* efficiently, and as long as these samples are uniformly distributed, it will sample less frequent words more often ($U(w)$->$U^{3/4}(w)$)

Continuous Bag of Words model

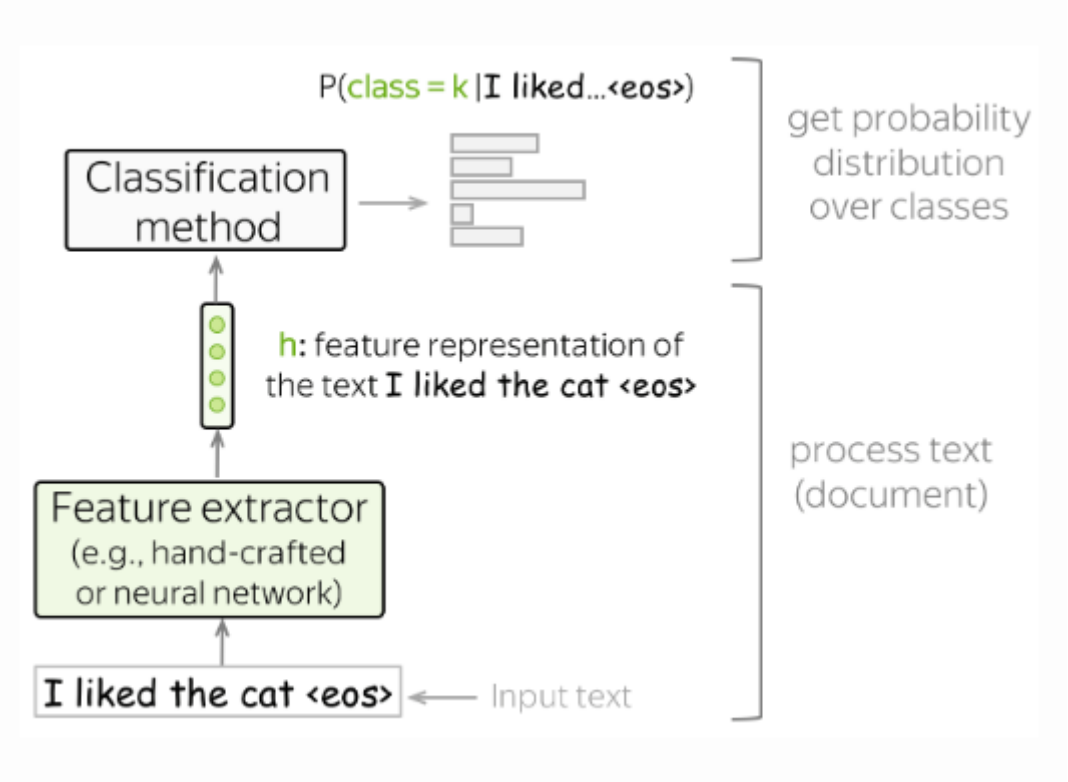
The previously mentioned Skip-gram model has been the basis for Word2Vec; it predicts *context words* given a *central word*. The *Continuous Bag of Words (**CBOW**)* model predicts

a central word given *context words*, by summing the context vectors.



Neural Classifiers

We can use our networks to *classify* a document of text. We first use a feature extractor (which can be hand crafted, or also learned with a *Neural Network*) to obtain a *feature representation* of the text, and then can use a classification method to get a probability distribution over possible classes $P(\text{class} = k|\text{input text})$.



Classification techniques

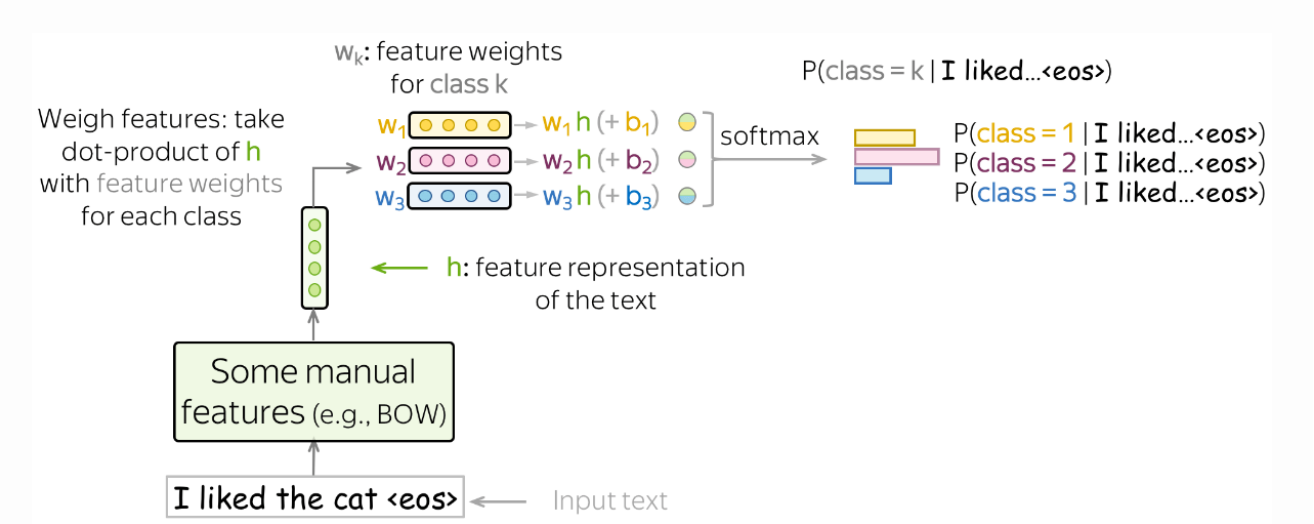
A typical classification technique for this involves Logistic Regression. Remember we are interested in finding out $P(y = k|x)$. The pipeline here goes as follows:

1. Get the feature representation $h = (f_1, f_2, \ldots, f_n)$
2. Take $w^i = w_1^i, \ldots, w_n^i$ - vectors with feature weights for each of the classes
3. For each class, take the dot product of the feature representation h with feature weights $w^{(k)}$:

$$w^{(k)}h = w_1^{(k)} \cdot f_1 + \ldots + w_n^{(k)} \cdot f_n$$

4. Use Softmax to get class probabilities

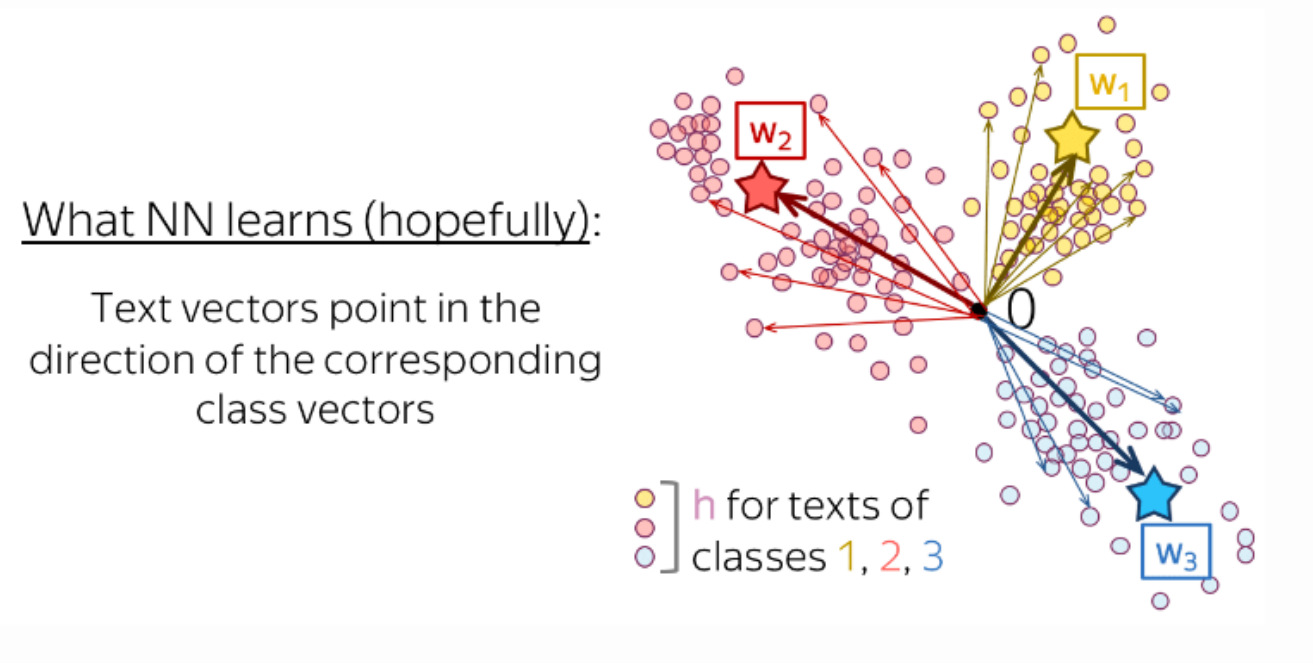
$$P(\text{class} = k|h) = \frac{\exp(w^{(k)}h)}{\sum_{i=1}^{K} \exp(w^{(i)}h)}$$



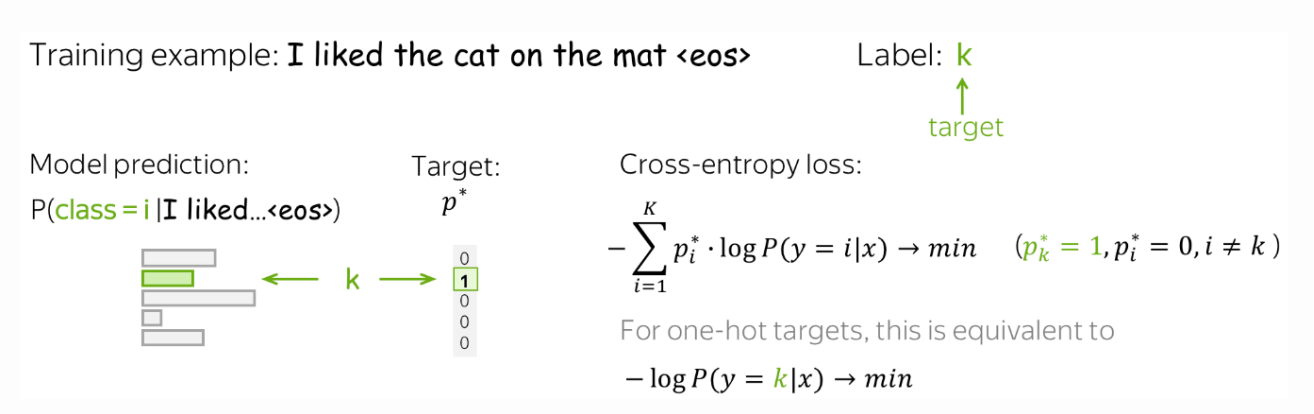
We also use MLE to train the model, maximising the probability of the training data.

Representing documents as vectors

We can continously train a Neural Network to *create **it's own** classes*, and obtain it's *own* feature representation. Ideally, we'd want a NN to be able to show if two documents are in the same class, due to their document vectors being close to each other.



The standard loss function for training such a model is Cross-entropy loss.



We then evaluate the loss, and alter the probability distribution (by *decreasing* all other **wrong** classes and *increasing* the **right** ones).

Bag of Embeddings

But what actually goes on to get our neurally-learned representation of the text? We can either sum all token embeddings (*Bag of Embeddings*) or use a weighted sum (possibly `tf-idf` from earlier).

CNNs

Convolutional Neural Networks have **translational invariance**; if we wanted to find **if an image contains a cat**, and didn't care **where the cat was**, we could use a CNN and process all images **containing cats somewhere** *equally*. We can use a CNN for text in certain contexts. For example, if a feature is *very informative*, sometimes we don't care **where** in a text it appears - just as long as it **does appear**.

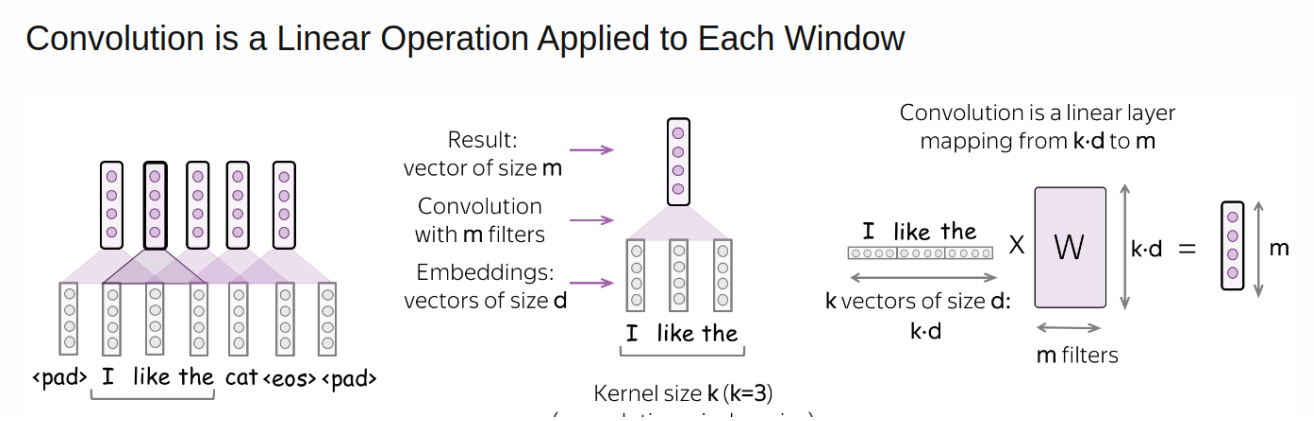
> `<pad>` ***I like the*** cat on a mat `<eos>` `<pad>`

> `<pad>` I ***like the cat*** on a mat `<eos>` `<pad>`

> `<pad>` I like ***the cat on*** a mat `<eos>` `<pad>`

> `<pad>` I like the ***cat on a mat*** `<eos>` `<pad>`

We can also use pooling to aggregate matches of patterns.
A convolution is a *linear operation* applied to each window.



Formally,

- $(x_1, \ldots, x_n)$ are representations of the input words $x_i \in \mathbb{R}^d$
- d is the size of an input embedding
- k is the *kernel size* - the length of a convolution window
- m is the output channels, the number of convolution filters (produced by the CNN). A convolution is a linear layer $W \in \mathbb{R}^{(k \cdot d) \times m}$. For a k sized window, it takes the concatenation of the vectors

  $$u_i = [x_i, \ldots, x_{i+k-1}] \in \mathbb{R}^{k \cdot d}$$

  and multiplies by the convolution matrix

  $$F_i = u_i \times W$$

We use F because using the filter (f) we have extracted a feature! We'll extract m of these.

Pooling

There are two types of pooling used:

- **Max** pooling takes the maximum over each dimension, so we get the maximum values for each feature from all the filters
- **Mean** pooling works exactly the same but computes the mean of each feature from all the filters.

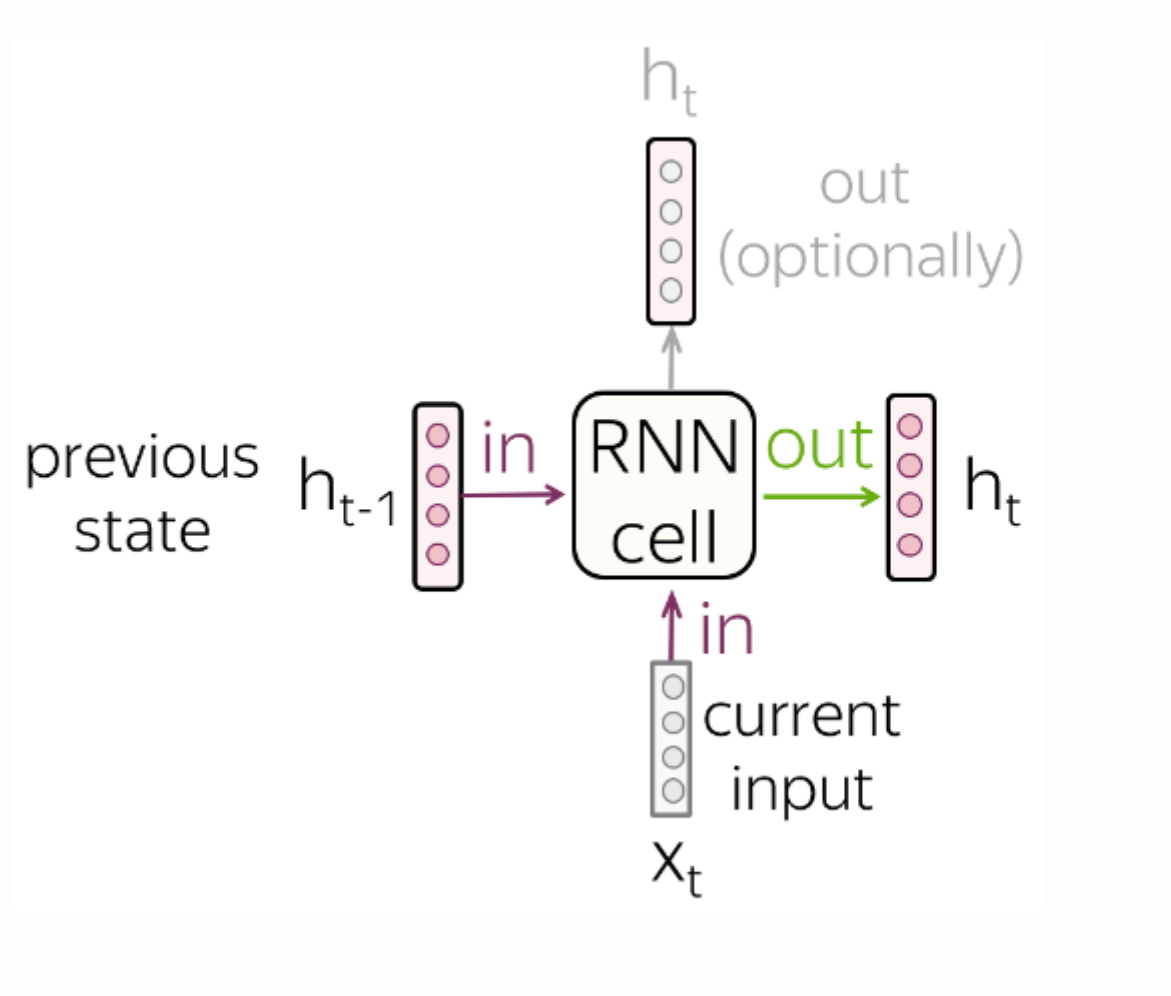
Pooling goes in *strides*, so we have multiple pools for different windows of text. We apply a *convolution* to these individual features and then can use *global pooling* (max) to average a final vector of feature strengths for the whole network.

Recurrent Neural Networks

In contrast to CNNs, *RNNs* "read" a sequence of tokens one by one and process the information. The main feature of RNNs is that it **remembers** information (more formally, "*the value of some unit is directly, or indirectly, dependent on its own earlier outputs as an input*"). This one-by-one nature makes these models particularly easy to adapt for generation.

Anatomy of an RNN cell

Each individual cell of an RNN receives a new input vector (such as a token embedding) and the *previous network state* (which hopefully encodes all the previous information). It then produces an output from this:



(It can also spit out a copy of this state, to be used in augments such as Multi-layer RNN).

Vanilla RNN model

By combining these cells alongside token embeddings, we can iterate through all tokens and produce a final result. To get from previous state h_{t-1} to h_t with input x_t, we perform

$$h_t = \tanh(h_{t-1}W_h + x_tW_x)$$

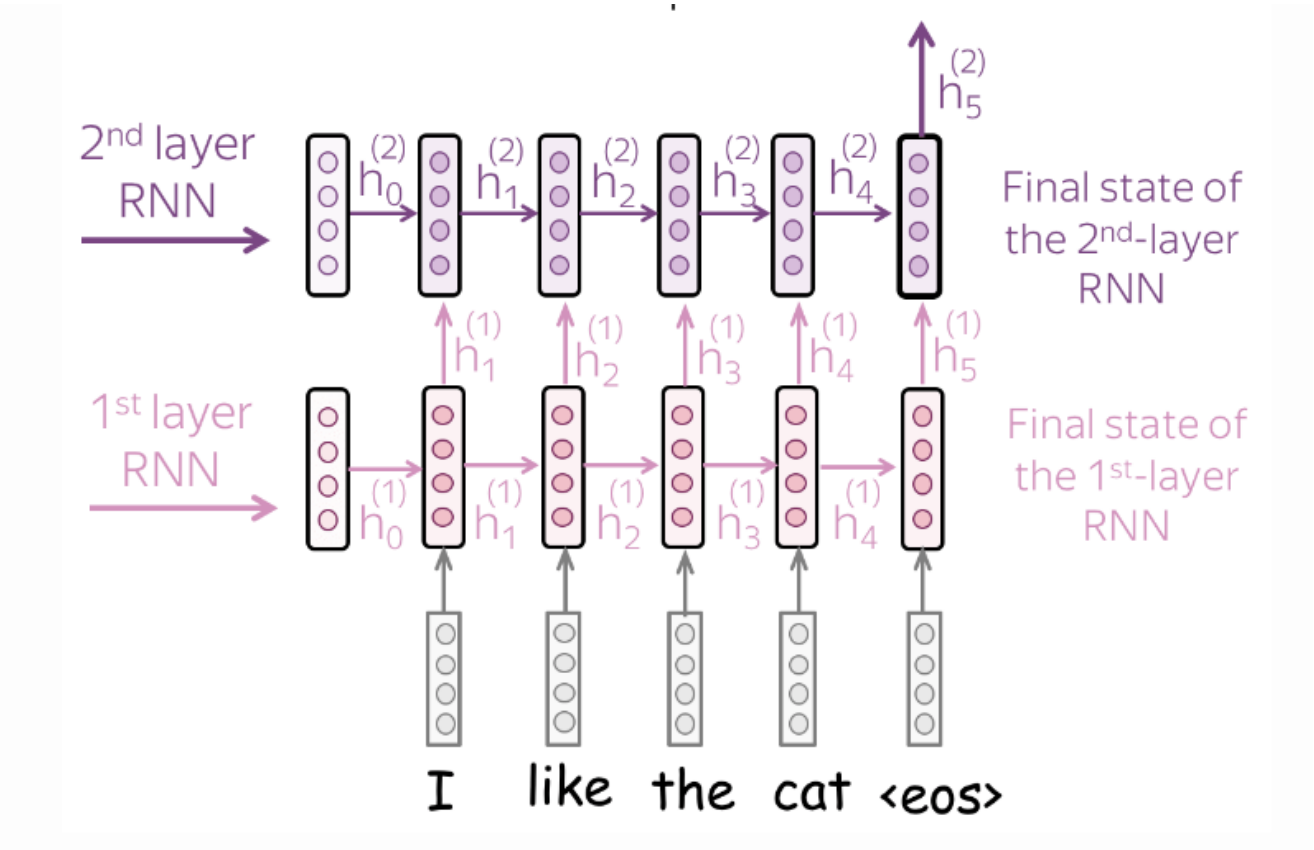
Here, $\tanh()$ is our *activation function* ($\mathbf{ReLU}$ is often used instead too), and the W represent *hyperparameters* (can be optimised through training).

However, if we just "read the last state", it might not be very good for classifications that require a bit more thinking

Improving the Vanilla RNN model

Multi-layer RNN

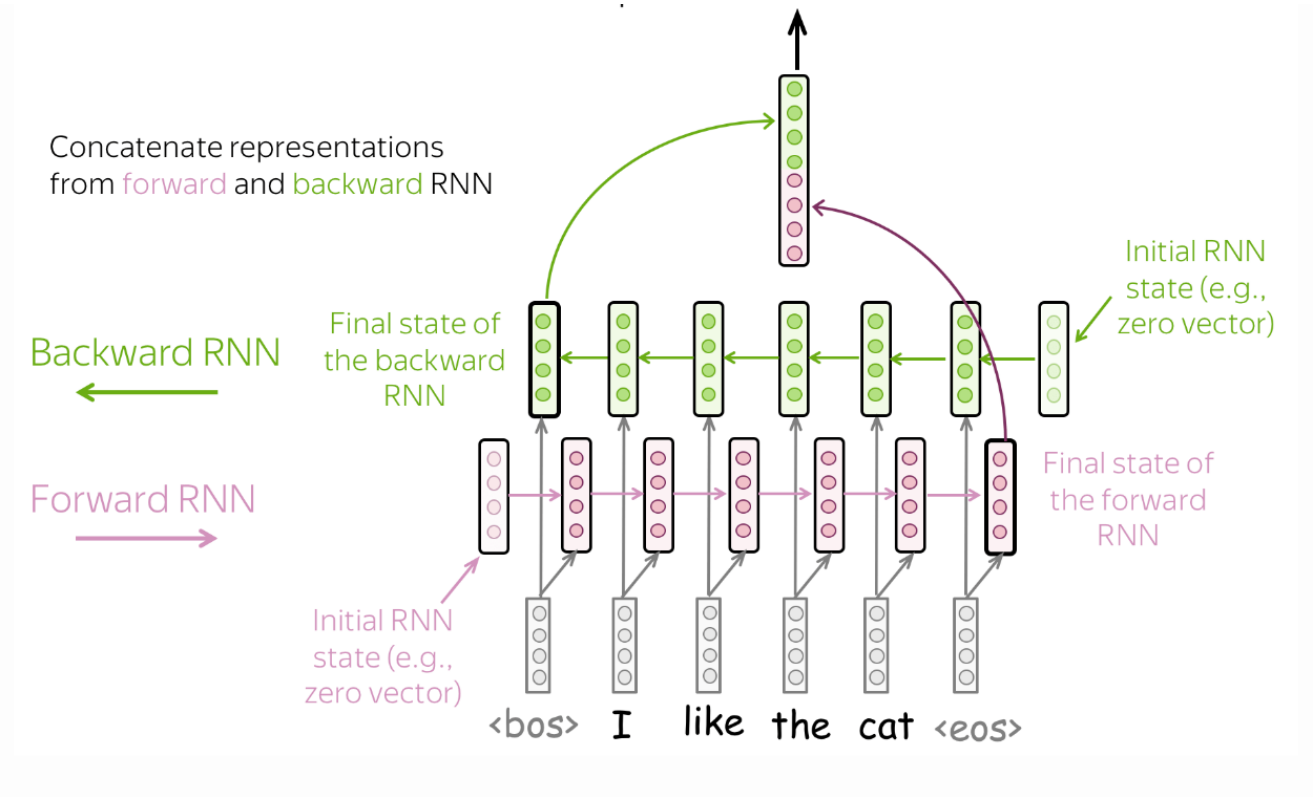
If we stack more layers, piping the final result of layer $n-1$ as the *input* (x_t) to the final cell in layer n, then inputs for the higher RNN are representations coming from previous layers (we also do this using the copy for each cell in the layer, said earlier).



The idea behind this is lower layers will capture *lower-level phenomena* (i.e **phrases**) and the higher levels will capture *high level things* (such as **topics**).

Bidirectional RNN

If we run two RNNs going in opposite directions as one layer, we can train it equally.



This can make it forget the *structure* of the document, and it can be hard to parse in retrospect.

Residual connections

When stacking multiple layers, the gradients don't propagate as well. If we add a block's input to it's output, this solves the problem. This is known as a *residual connection*. If we apply a gated sum (the gate $g = \sigma(Wx + b)$) to the input x and output h, and then combine, this is known as a *highway connection*.

Neural Language Modelling

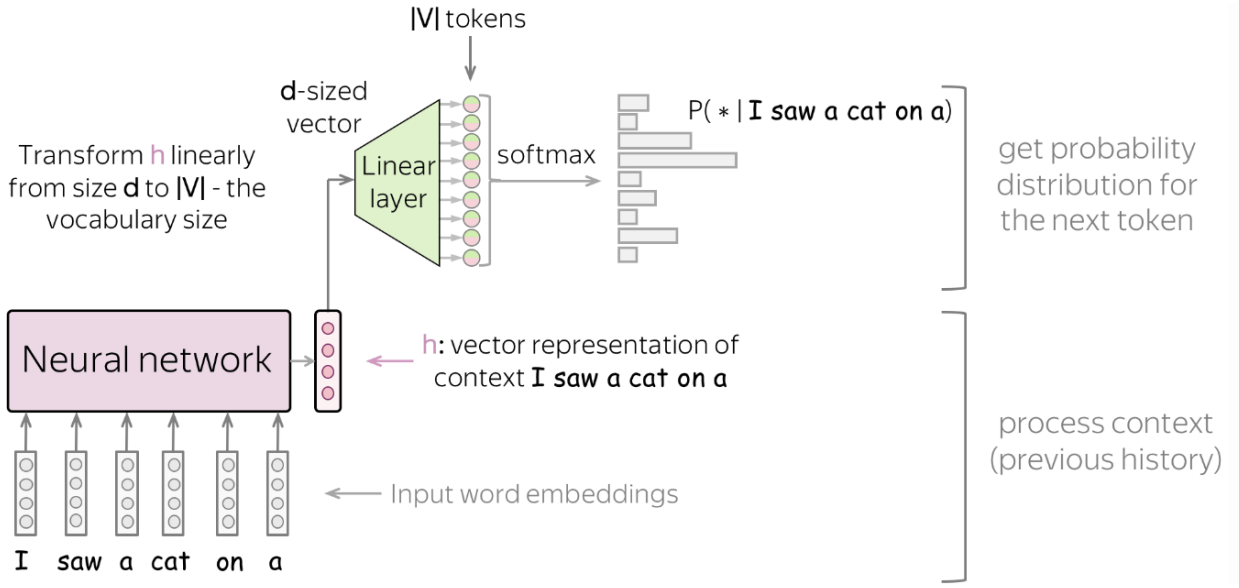
A *language model* assigns the probability to a sequence of words $y_1, y_2, \ldots, y_n$ relying on the chain rule

$$P(y_1, y_2, \ldots, y_n) = \prod_{t=1}^{n} P(y_t | y_{<t})$$

Anatomy of a neural language model

NLMs

1. Produce a representation of the prefix (currently selected token, in sentence context)
2. Generate a probability distribution $P(*|(\text{previous tokens}))$ over the next token. Predicting the next token is a classification problem, as earlier discussed!



Training a neural language model (loss)

We define our loss function as

$$\textbf{Loss} = -\log(p(y_t|y_{<t}))$$

We apply this function to

Comparing N-gram LMs and RNN LMs

Ngram language models

- relies on a short prefix, to get a distribution over next tokens
- explicit independence assumption (can't use context outside of the ngram window)
- smoothing is necessary

RNN language models

- 'compresses' the past into a state, used to compute the distribution over next tokens
- no independence assumptions; the gradient descent learns to compress the past
- all the information is carried through hidden states (hard to carry it across long distances)

Parallels between RNN state and HMM state learnt in an unsupervised way through EM

Comment: we could also define CNN language models, which would make the explicit independence assumptions but would not require smoothing and has some nice properties.

Text Generation and Encoder-Decoder Models

While the immediate thought might be straight to *ChatGPT* (and we will get onto Transformers), most topics in text-generation cover the field of *machine translation*, and this is where most research and write-up has been.

Greedy Decoding

We could theoretically just sample the most probable word generated by our Neural Language Model (called *greedy decoding*). This can result in *boring, common* words **always** getting chosen.

We can slightly control this with *temperature*.

$$\frac{\exp(h^T w)}{\sum_{w_i \in V} \exp(h^T w)} \rightarrow \frac{\exp(\frac{h^T w}{\mathcal{T}})}{\sum_{w_i \in V} \exp(\frac{h^T w}{\mathcal{T}})}$$

The higher the temperature, the *flatter* the probability distribution. When sampling with Softmax, this results in more *diverse* samples

Encoder-decoder framework

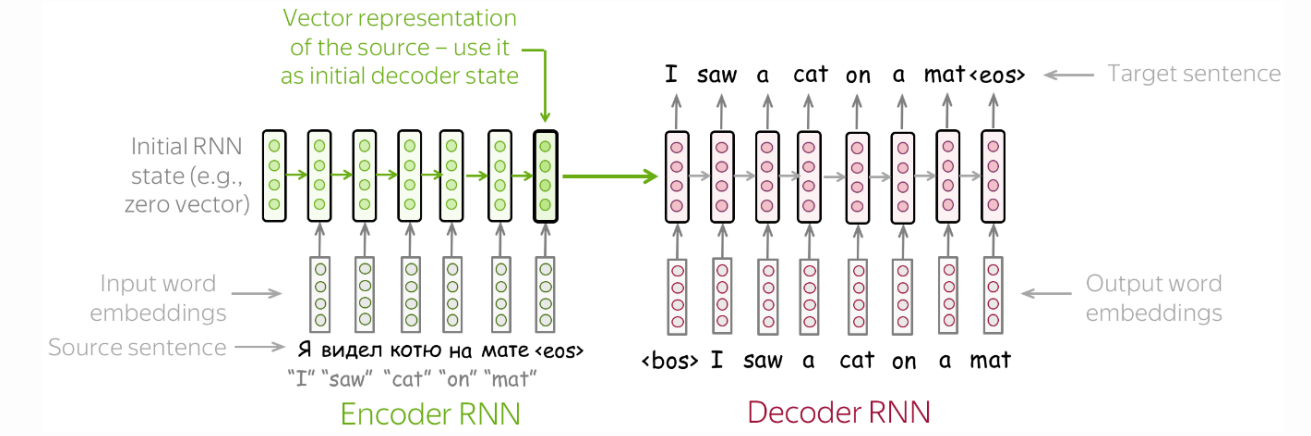
A form of *sequence-to-sequence modelling*, encoder-decoders consists of two (obvious) parts. The **encoder** takes in a *source sentence* and converts it to a *representation of that sentence* to be *fed into the **decoder** part*. The **decoder** then takes this *representation* and produces a *target sentence*.

Conditional Language Modelling

Seq2Seq tasks can be modelled as *Conditional Language Models*; the difference from a regular *language model* now being a *condition on source* x.

$$P(y_1, y_2, \ldots, y_n | x) = \prod_{t=1}^{n} p(y_t | y_{<t}, x)$$

RNNs for encoder-decoder



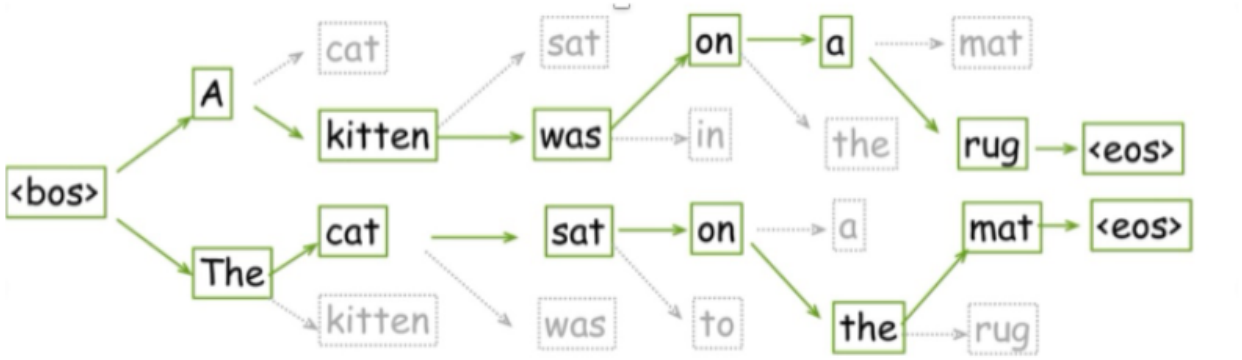
We go through each token in the *source sentence* and then also *the representation*.

Beam search

The probability of a sentence using the Recurrent Neural Networks RNNs for encoder-decoder framework is now

$$y' = \arg\max_y p(y|x) = \arg\max_y \prod_{t=1}^{n} p(y_t|y_{<t}, x)$$

We now need to find $\arg\max$. We can use Greedy Decoding from earlier, but another idea is to keep track of *possible decodings*, and then pick the likely ones as we iterate through the tree of possibilities generated from the decoder.



Usually, the beam size is 4-10. Increasing beam size is kinda against the point - it'll end up close to greedy decoding while being slower.

Evaluation: BLEU metric

Idea: count overlapping *n-grams*.

Actual BLEU is more complicated because it needs to:

- aggregate over the *entire test set*
- aggregate over n-grams *of different order*
- penalise short translation

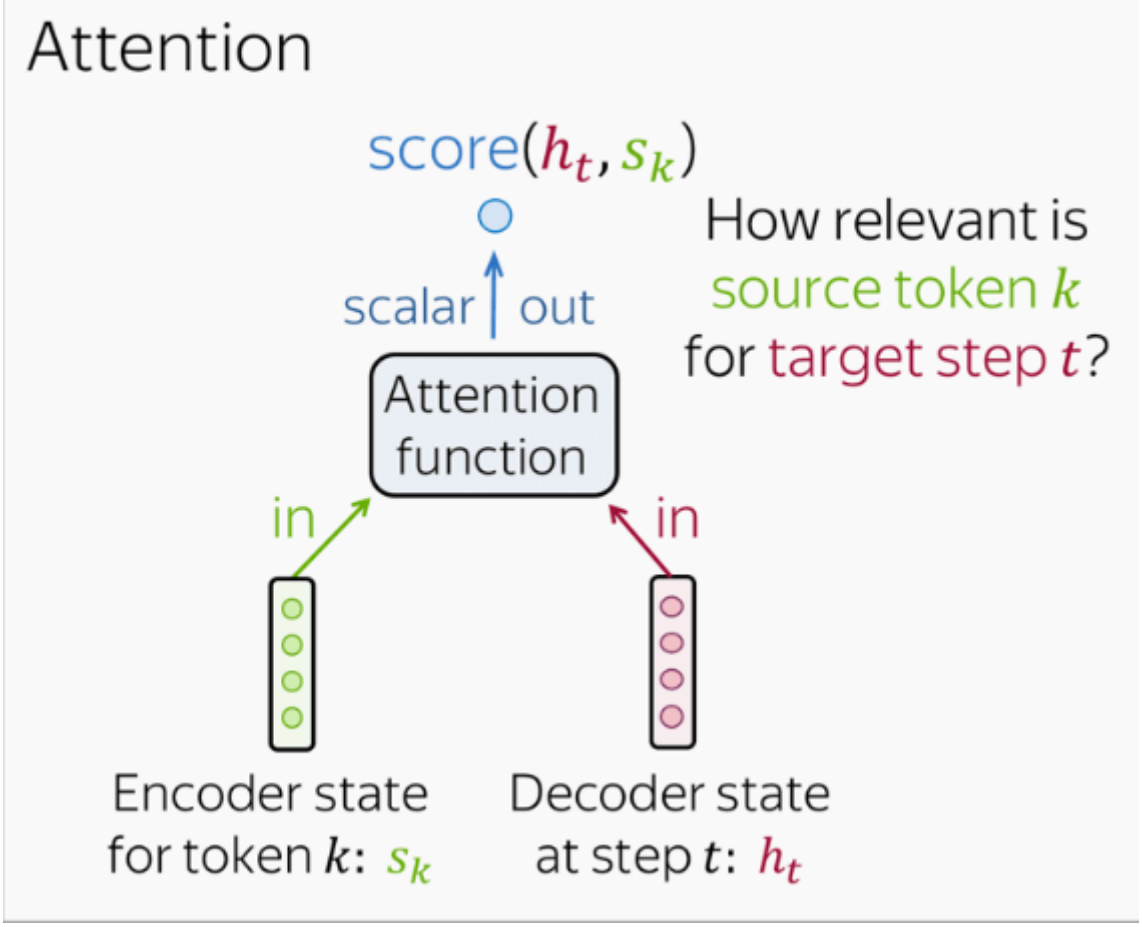
Subword tokenisation

Usually, tokens have referred to *words*. If we focus on *subwords* (e.g. "unrelated" -> "un@@" "related) we can expand our vocabulary. This comes with extra computation costs but it's really good, and used in virtually any modern neural model.

Attention

If we are forced to encode rich information into a limited size vector, we will inevitably miss something - and it could be important!

Neural Attention Model



At each decoder step, attention

- receives attention input
  - a decoder state h_t and *all encoder states* $s_1, s_2, \ldots, s_m$
- Computes the attention scores
  - the "relevance" of encoder state s_k for decoder state h_t
- Computes the attention weights
  - this is a probability distribution of Softmax applied to the attention scores
- Computes the attention output
  - The weighted sum of encoder states (per state???????!?!?!?!?!?!?!?!?!?) with attention weights

Formally, we go:

- **Attention input**
  - "all encoder states, one decoder state"
    - $s_1, s_2, \ldots, s_m \quad h_t$
- **Attention scores**
  - "How relevant is *source token* k for *target step* t?"
    - $\text{score}(h_t, s_k), k = 1..m$
- **Attention weights**

- "attention weight for *source token* k at *decoder step* t"
  - $$a_k^{(t)} = \frac{\exp(\text{score}(h_t, s_k))}{\sum_{i=1}^{m} \exp(\text{score}(h_t, s_i))}, k = 1..m$$

- **Attention output**
  - "source context for *decoder step* t"
    - $$c^{(t)} = a_1^{(t)} s_1 + a_2^{(t)} s_2 + \ldots + a_m^{(t)} s_m = \sum_{k=1}^{m} a_k^{(t)} s_k$$

Attention models can also be trained *end-to-end*, since everything is *differentiable* (recall Gradient Descent).

Attention scores

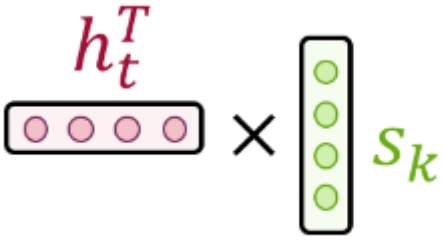
What actually goes on in

$$\text{score}(h_t, s_k)$$

There are different metrics we can use, given h_t, s_k, and a set of weights $[W]$

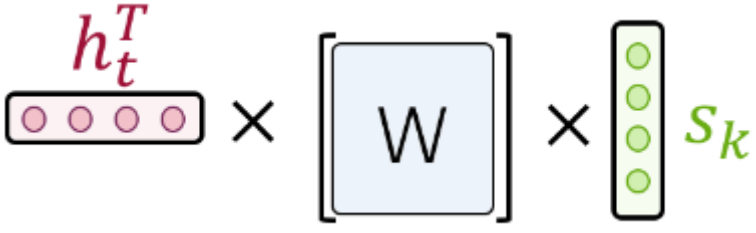
Dot product

$$\text{score}(h_t, s_k) = h_t^T s_k$$



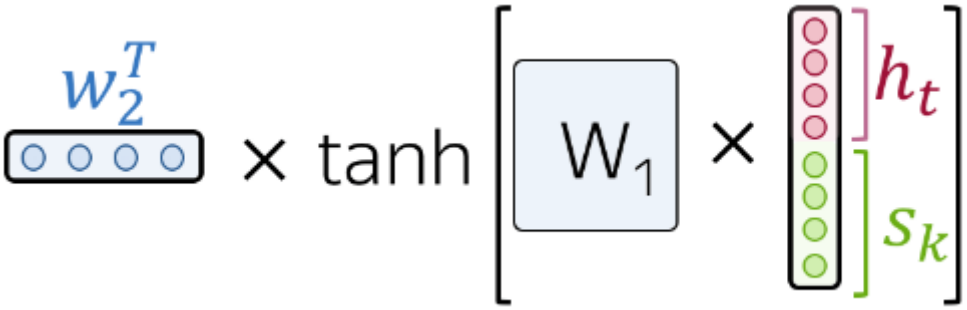
Bilinear

$$\text{score}(h_t, s_k) = h_t^T [W] s_k$$



Multilayer perceptron

$$\text{score}(h_t, s_k) = w_2^T \cdot \tanh([W_1] \times [h_t, s_k])$$



Bahdanau attention

A Bidirectional RNN encoder.
Uses a Multilayer perceptron attention score.
The attention is applied between decoder steps: state h_{t-1} is used to compute attention $c^{(t)}$, and both are passed to the decoder at step t.

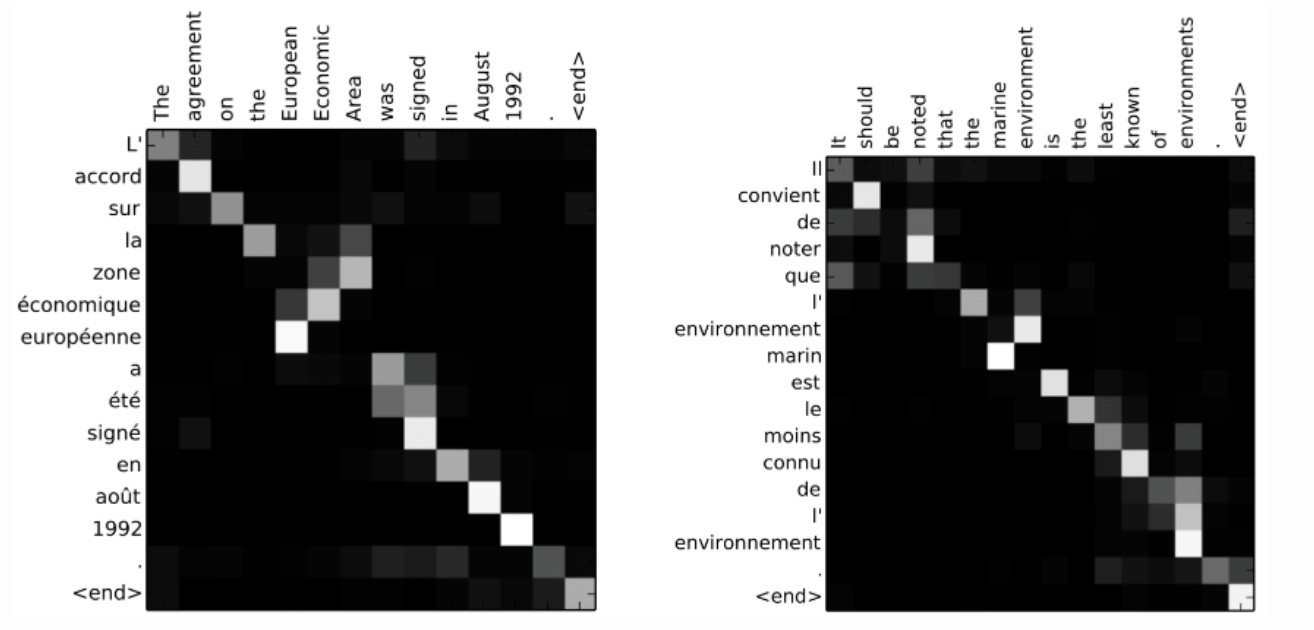
Luong Attention

A unidirectional RNN encoder.
Uses a Bilinear function.
The attention is applied after state h_t has finished computing. The output is not immediately sent as final: an updated representation ($\tilde{h}_t$) using $c^{(t)}$ and h_t is sent to make the actual prediction.

$$\tilde{h}_t = \tanh(W_c[h_t, c^{(t)}])$$

Attention and alignment

Going back to our machine translation assignment, we can see the *alignment* of all source tokens, and see which ones are actually counting.



Transformers

Transformers ask the question "why can't we do *everything* with Attention?"

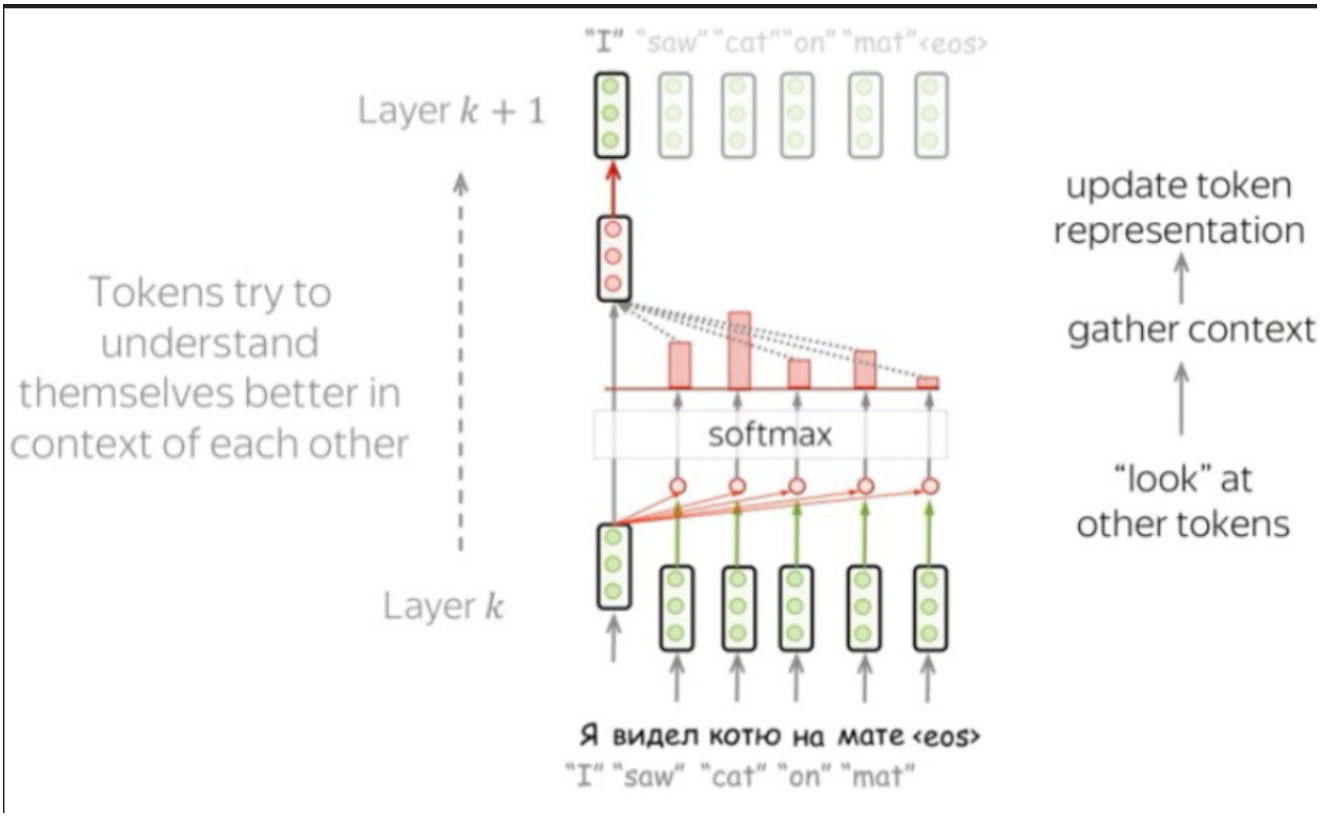
Any by everything, they mean *processing in **both** encoder and decoder*, and also *encoder/decoder interaction*.

Using this, transformers can floor Recurrent Neural Networks because each token literally knows the whole sentence; all of a transformer's encoder token can interact with each other.

Transformers come in "blocks", each a multi-layer network that maps sequences of input vectors $(x_1, \ldots, x_n)$ to sequences of output vectors $(z_1, \ldots, z_2)$

Self-attention

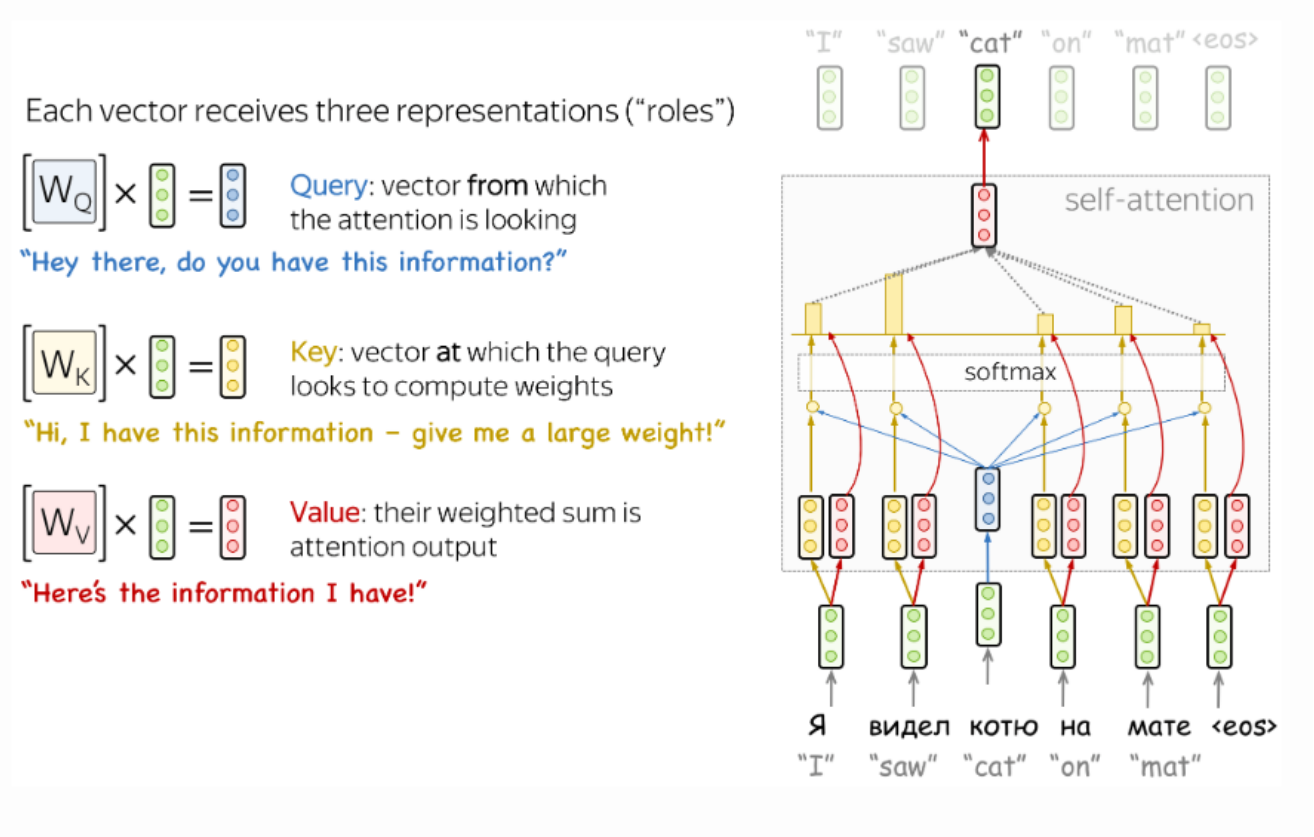
For each state in a set of states (say, in the *encoder*), we will use attention for **all other states** in the *same set*.



This computation happens in parallel for each state

Query-key-value attention

To keep this more structured, each input token receives three representations depending on what purpose it's serving.

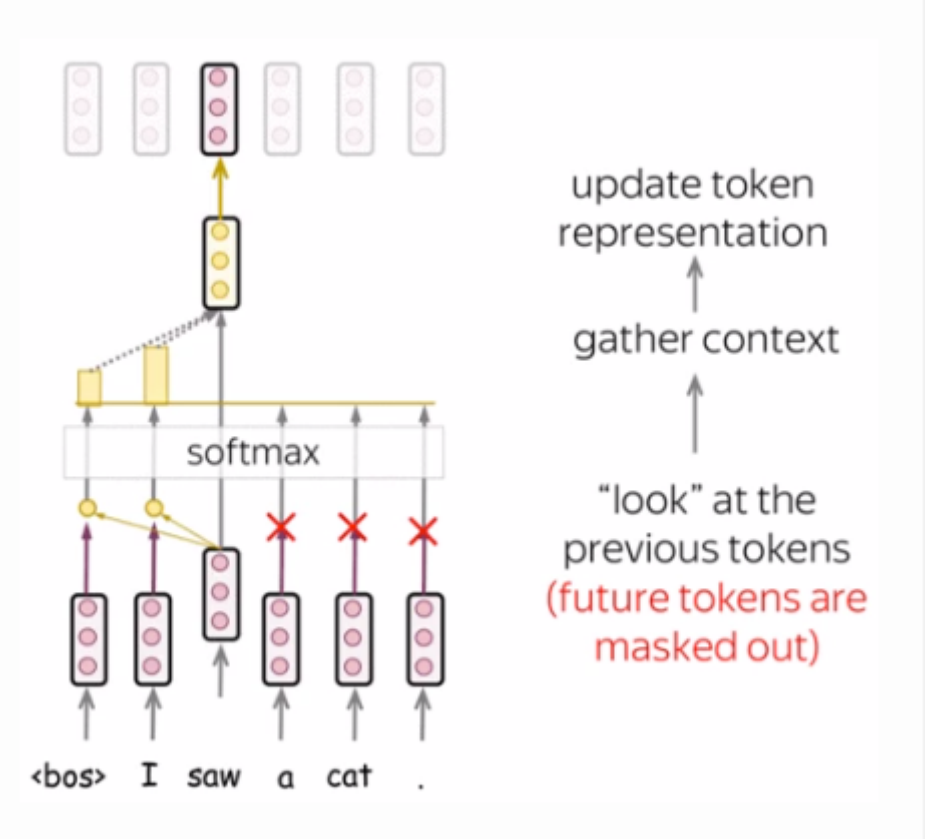


Attention Formula

$$\text{Attention}(q, k, v) = \text{softmax}(\frac{qk^T}{\sqrt{d_k}})v$$

Masked self-attention

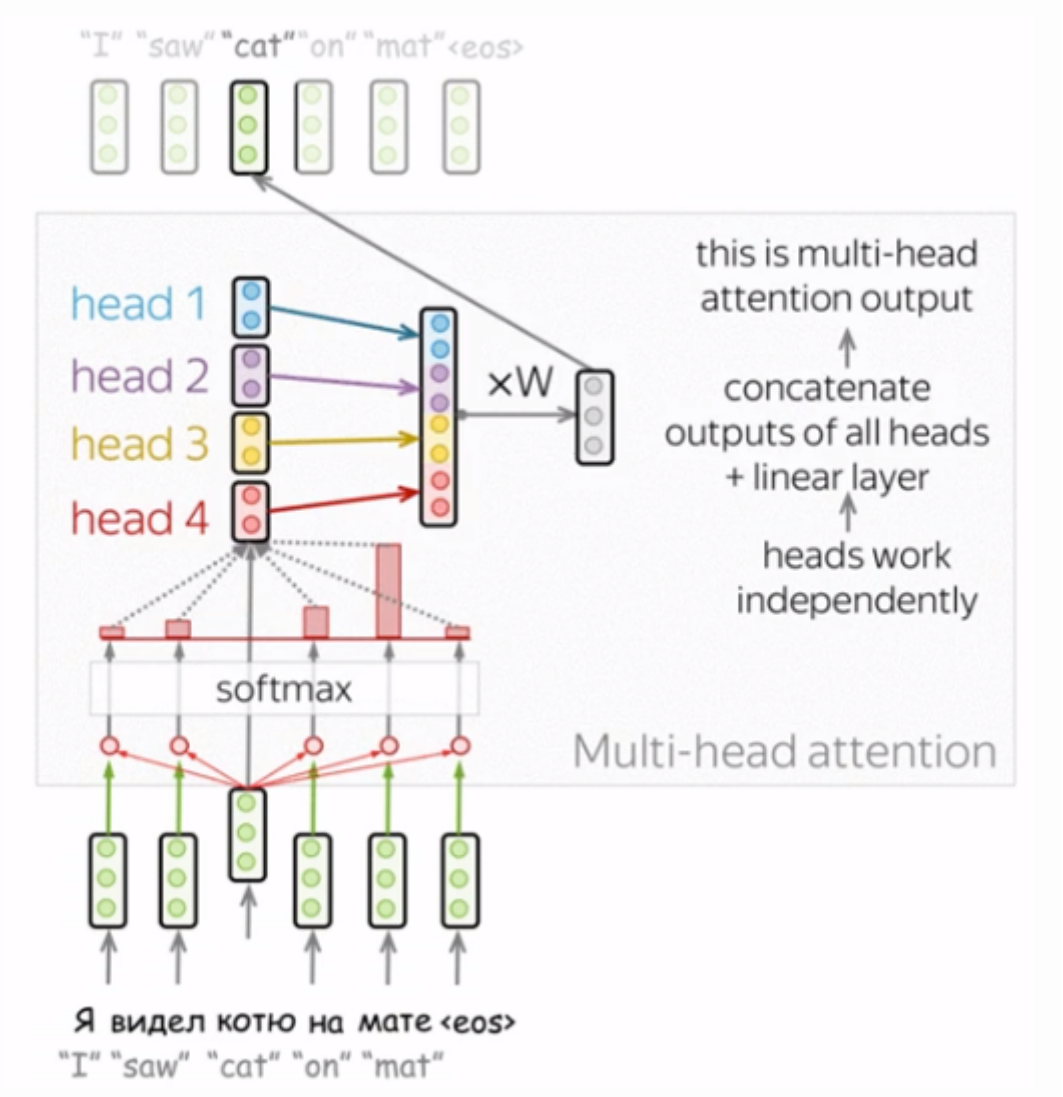
We will run into problems when using the *decoder* for generating text - it can't look into the future! We perform attention with a mask to hide the future tokens.



Even during training, we can't get the decoder to "see future". This masking also makes training transformers $O(1)$.

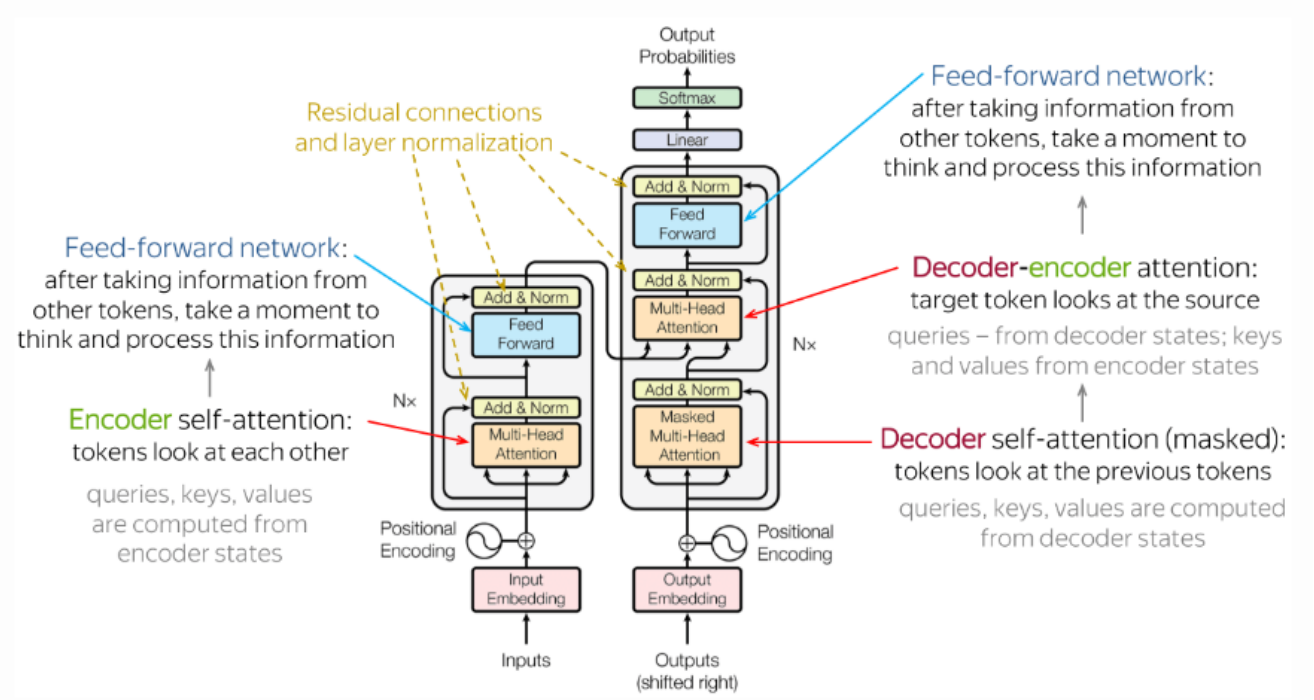
Multi-head attention

Each word can be part of many relations (verb inflection, gender agreement, etc)



This is the motivation behind *multi-head* attention - we capture *different ideas* (recall multi-layer only *refines* these ideas).

Transformer Architecture



Feed-forward blocks

Between layers, we include a *feed-forward* block - two linear layers with $\mathbf{ReLU}$ between them

$$FFN(x) = \max(0, xW_1 + b_1)W_2 + b_2$$

This is used to process the new information gained from attention.

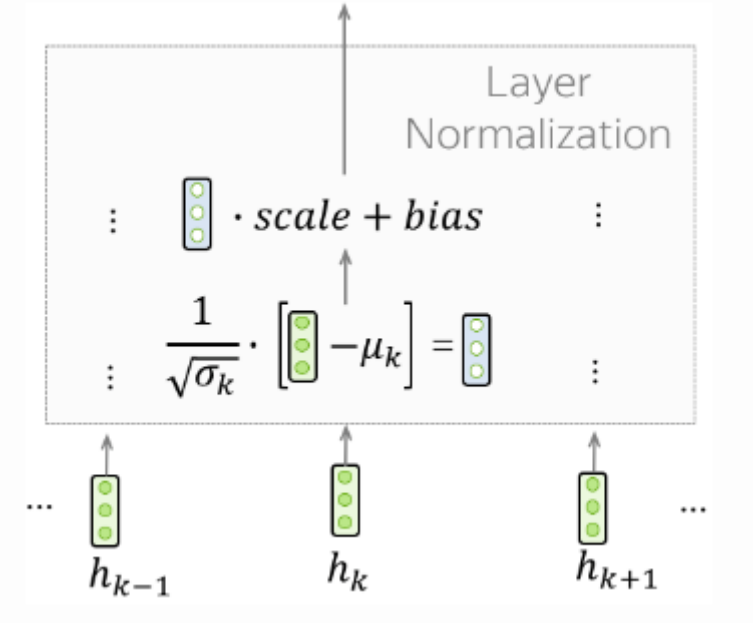
We also use

> ## Residual connections
>
> When stacking multiple layers, the gradients don't propagate as well. If we add a block's input to it's output, this solves the problem. This is known as a *residual connection*. If we apply a gated sum (the gate $g = \sigma(Wx + b)$) to the input x and output h, and then combine, this is known as a *highway connection*.

Layer normalisation

We normalise the vectors in each layer to make sure they don't get too **crazy!!!!**

LayerNorm has trainable parameters scale and bias (trainable for each layer).



Positional encoding

Transformers don't inherently know the order of an input. These can be learned, but fixing *positional encodings* doesn't hurt quality. Transformers use

$$PE_{pos,2i} = \sin(pos/10000^{2i/d_{\text{model}}})$$
$$PE_{pos,2i+1} = \cos(pos/10000^{2i/d_{\text{model}}})$$

and each input is a **sum of the two embeddings** *token* and *position*.

Transfer Learning

The general idea of *transfer learning* is to "transfer" knowledge from one model/task to another.

Using pre-trained embeddings and fine-tuning

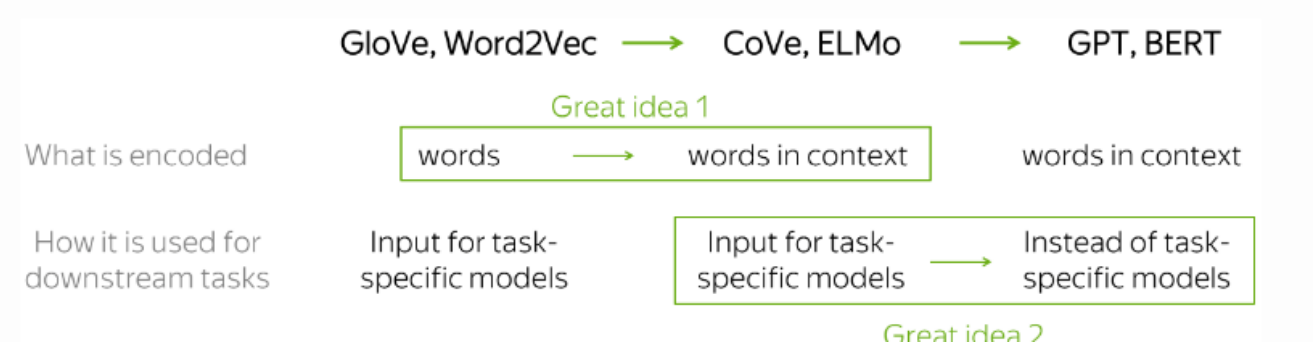
We usually have *pre-trained* embeddings, but can also start from *scratch*

- **From scratch**
  - might not even *learn relationships*
- **Pre-trained**
  - Using pre-trained embeddings from stuff such as Word2Vec will lead to embeddings that **know relationships** between words but are *not specific*
- **pre-training, and *then* fine-tuning**
  - *now specific to our task*

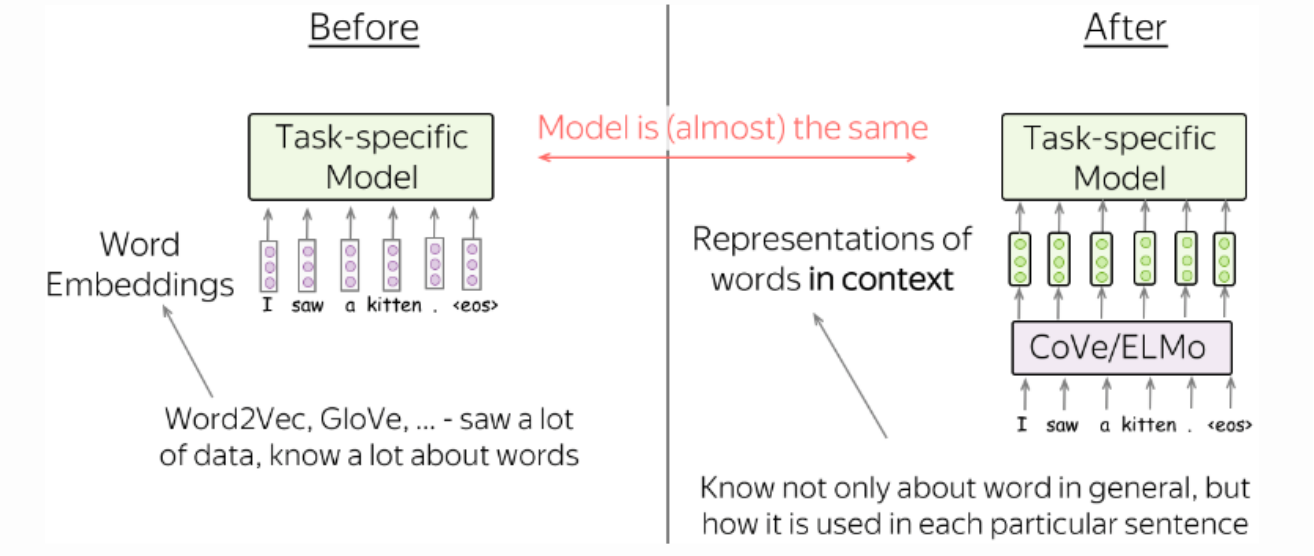
Limitations of transfer with word embeddings

- They encode word meanings without *considering context* (merging all senses for a word)
- They also lack *larger linguistic unit* representation (phrases, sentences, paragraphs).

Word-in-context embeddings

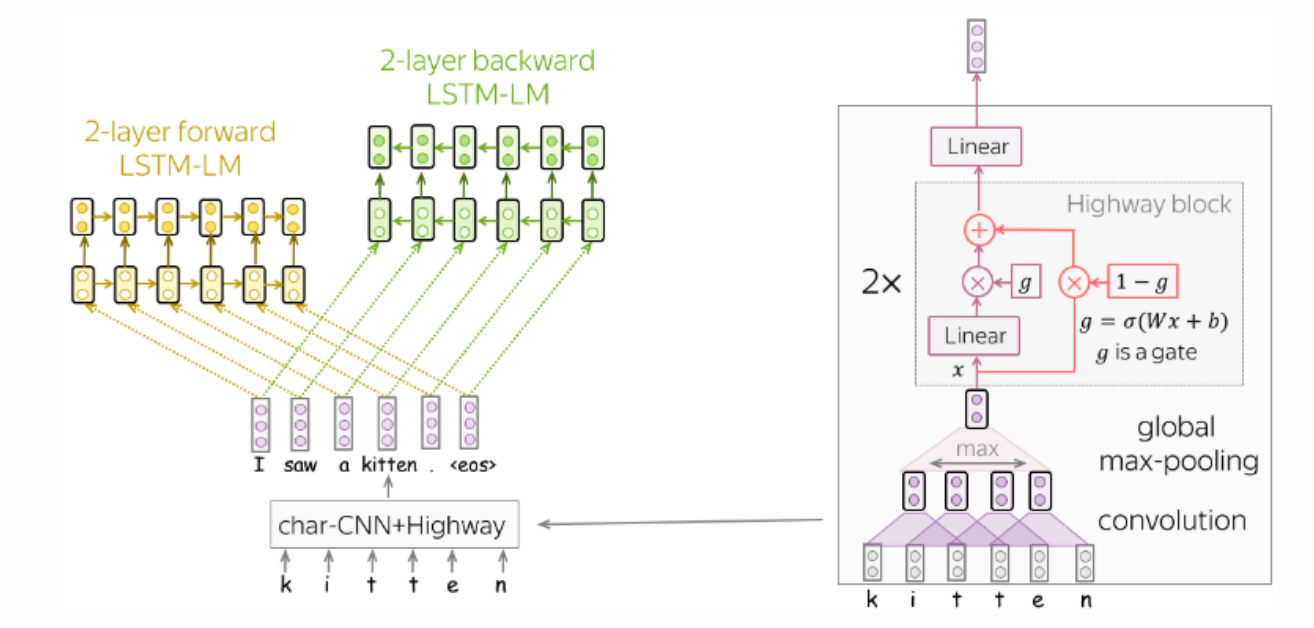


In models like *CoVe* and ELMo, we represent words **alongside the context they are used in**. This can be done by passing our words through one of these models, and which return *representations* (in context) to be passed to our actual model (which is slightly altered now).

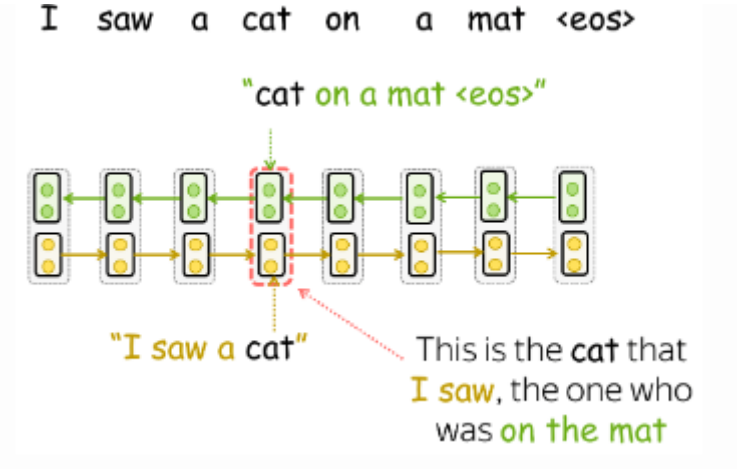


ELMo

***E**mbeddings from **L**anguage **Mo**dels *(ELMo) is a simple model, consisting of two-layer* LSTM *(not covered in course just trust the plan) models, forward and backward. Each character of a word is pooled by a CNN.*



To actually get the representations, we combine the individual LSTM representations and concatenate* the forward and backward vectors.



BERT

***B**idirectional **E**ncoder **R**epresentations from **T**ransformers *is the encoder for Transformers. It gets trained on sentenes with special token separators. It also gets Positional encoding, so an input is* token, positional*, and* segment embeddings*. These segment embeddings can be used to teach the model about* sentence relations*.

> [Input]: ***[CLS] the man went to [MASK] store [SEP] he bought a gallon [MASK] milk [SEP]***
> **isNext**

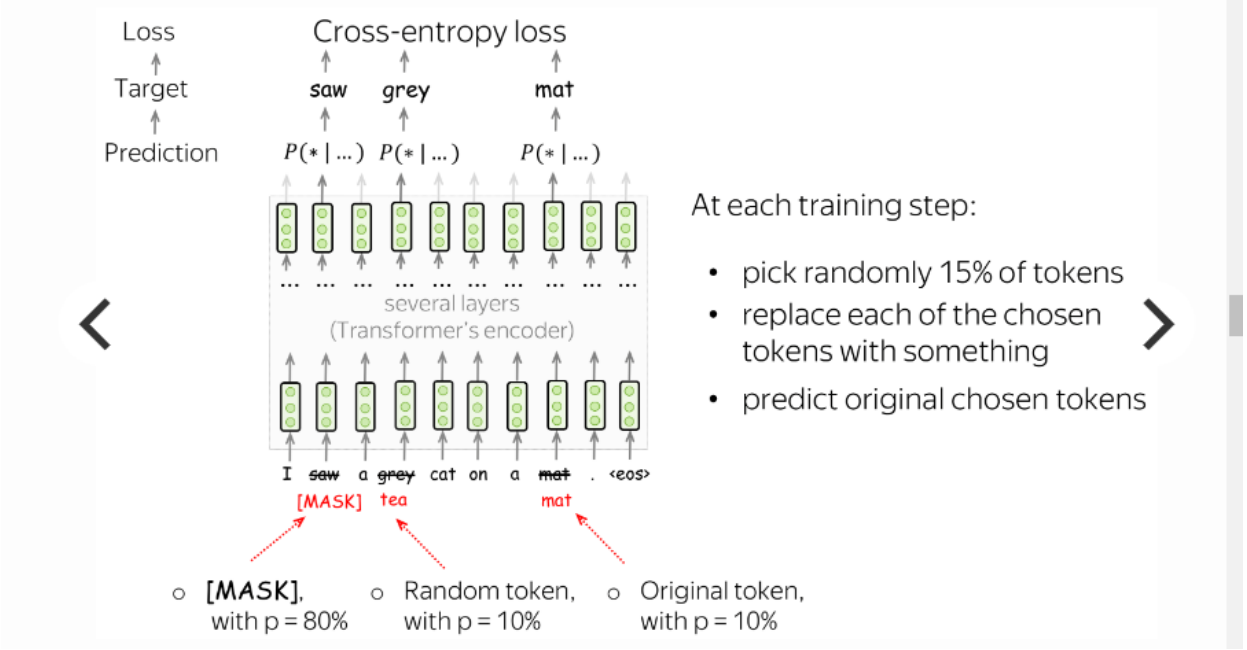
> [Input]: ***[CLS] the man went to [MASK] store [SEP] penguin [MASK] are flight ##less birds [SEP]***
> **notNext**

Masked Language Modelling

BERT also uses *masked language modelling*.

At each training step, BERT:

- randomly picks 15% of the tokens
- replaces each of these chosen tokens with something
- and then predicts the original chosen tokens



This lets it compare if it predicted a different word to the one *falsely* supplied, which lets it work out *missing word problems*.

> The woman walked across the street, checking for traffic over _ shoulder. [coreference]

> Overall, the value I got from the two hours watching it was the sum total of the popcorn and the drink. The movie was _. [sentiment]

> John went into the kitchen to make some tea. Standing next to John, Jake pondered his destiny. Jake left the _. [reasoning]

THE END.