OR PROGRAMMING EXERCISE

February 17, 2020

1 GETTING THE DATA SORTED

Before we get started, some info: We use python programming language. The problem is to apply a dynamic program to find the shortest path. We are given a text file, representing a given Direct Acyclic Graph. There are different ways of representing a graph, it could a matrix, or a list of lists. In our case we use a dictionary. A dictionary is tupled pair of data represented by a 'key' and its 'value'. For example, consider that node 1 have to out-going arcs to node 2 adn 3 with weith 4 and 5. Putting this in a dictionary gives: $\text{dict} = \{\text{`1':}[(\text{`2'}, 4), (\text{`3'}, 5)]\}$. Of course a complete dictionary has more enteries.

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[1]: def convert_text_to_dictionairy(filename):
         # PART ONE - IN THIS PART WE READ THE FILE LINE BY LINE AND SAVE IT TO
         # A LIST
         doc = \prod
         with open(filename) as file:
             for line in file:
                 t = line.strip().split('\t')
                 doc.append(t)
         # PART TWO - HERE WE FIND OUT WHAT MAIN-NODES WE HAVE,
         # I.E. THE FIRST ENTRY OF EVERY LINE
         main_nodes = []
         for line in doc[1:]:
             main_nodes.append(line[0])
         # PART THREE - YOU WILL NOTICE THAT IN THE PREVIOUS PART THE MAIN NODES
         # HAS ITEMS THAT OCCUR MULTIPLE TIMES HENCE WE NEED TO USE THE SET()
         # FUNCTION TO OPBTIAN THE UNIQUE ONES.
         set nodes = set(main nodes)
         set_nodes_list = list(set_nodes)
         # -> WE NEED TO SORT IT, BECAUSE THE SET() FUNCTION IS IMMUTABLE OBJECT
         # AND IS NOT SORTED, THATS WHY WE
         # -> CONVERT IT TO A LIST SO THAT WE CAN LATER ON ADD MORE NODES AND
              OFCOURSE SORT IT, AS THE ALGORITHM REQUIRES IT
         set_nodes_list.sort()
         # PART FOUR - WE CREATE AN EMPTY DICTIONARY WHERE WE CAN STORE
                       THE Acyclic Directed Graph.
         graph_dict = {x: [] for x in set_nodes_list}
         # PART FIVE - NOTE THAT WE HAVE SAVED THE DATA TO A LIST CALLED DOC.
                       WE USE A WHILE LOOP TO POP EVEY ITEM IN THE LIST,
                      BY POPPING EACH ITEM BY LOOKING BACK
                       WE CAN CREATE THE DICTIONAIRY
         while set_nodes_list:
             a = set_nodes_list.pop(0)
             for i in range(len(doc[1:])):
                 if a == doc[1:][i][0]:
                     #-> doc[1:][i][0] = begin node
                     \#-> doc[1:][i][1] = end node
                     \#-> doc[1:][i][2] = weight of that arc, between the two nodes
                     graph_dict[a].append( (doc[1:][i][1], doc[1:][i][2]) )
```

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# PART SIX - THIS PART IS IMPORTANT BECAUSE NOW THAT WE HAVE A DICTIONAIRY
# /GRAPH, BUT ONLY THE ONES THAT HAVE OUTGOING ARCS. IN ORDER FOR
# THE ALGORITHM TO WORK, WE HAVE TO ADD EMPTY OR DUMMT ARC, THAT HAVE ZERO
# WEIGHT/LENGTH AND WHERE THE BEGIN NODE AND END NODE ARE EQUAL.
# WE ARE GIVEN THAT THERE ARE N-NUMBER OF NODES IN THE TEXT FILE,
# SO WE RANGE OVER THE NUMBER OF NODES TO SEE WHICH ONES WE MISSM,
# AND PUT THEM IN A LIST
missing nodes = []
for node in range(1, int(doc[0][0])+1):
    if str(node) not in graph_dict.keys():
        missing_nodes.append(str(node))
# PART SEVEN - NOW THAT WE HAVE FOUND THE NODES THAT WE ARE MISSING,
# WE ADD THEM IN THE DICTIONAIRY USING THE TWO FOR LOOPS
for node in range(1, int(doc[0][0])+1):
    if str(node) in graph_dict.keys():
        pass
    else:
        graph_dict[str(node)] = (str(node), str(0))
for node in missing_nodes:
    graph_dict[node] = [graph_dict[node]]
new dict = {}
node_list = [str(x) for x in range(1, int(doc[0][0])+1)]
while node list:
    a = node_list.pop(0)
    for g in graph_dict.keys():
        if g == a:
            new_dict[a] = graph_dict[a]
# PART EIGHT - RETURN A DIRECTED ACYCLIC GRAPH
return new_dict
```

2 ALGORITHM IMPLEMENTATION

Implementing a Reaching Algorithm (Dynamic Program)

The algorithm:

We prodive a simple describtion of the algorithm. Given a Direct Acyclic Graph G = (N, A), where V are the nodes and A are arcs. Let l(v) be the shortest path from s to v. Moreover, each arc has

cost or weight c_{uv}

- Step 1: sort all the nodes where $u < v, (u, v) \in A$ • Step 2: set l(1) = 0
- Step 3: for v = 2, ...n: $l(v) = min_{(u,v) \in A} \{l(u) + c_{uv}\}$

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[2]: def min_path_dynamic_program(graph):
         # NOTE THAT STEP 1 WAS DONE PREVIOUSLY IN PART FOUR.
         node_list = list(graph.keys())
         # THIS IS WHERE WE KEEP TRACK OF DISTANCES TRAVELLED TO EACH NODE,
         # AND THEIR PREDECESSORS.
         # STEP 2: INITIALLY WE SET THE ALL DISTANCES TRAVELLED TO ZERO.
         # RATHER THAN ONLY THE FIRST ONE
         min_path = 0
         distance = {str(x): 0 for x in range(1,len(node_list)+1)}
         pred = {str(x):'' for x in range(1,len(node_list)+1)}
         pred['1'] = '1'
         for v in range(1, len(node_list)):
             current_node = node_list[v]
             backtrack_node_dis = []
             min_value_id = ''
             # BACK TRACKING, WE ITERATIVE OVER THE NEXT NODE,
             # BUT LOOK BACKWARDS FOR POSSIBLE PATHS/CONNECTING ARCS
             for u in range(0, v):
                 back_track_node = node_list[u]
                 for s in graph[back_track_node]:
                     s id = s[0]
                     s_{value} = int(s[1])
                     if s_id == current_node:
                         backtrack_node_dis_append((back_track_node,(s_id, s_value)))
             min_distance = []
             for i in backtrack_node_dis:
                 min_distance.append(int(i[1][1]))
             min_value = min(min_distance)
```

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for i in backtrack_node_dis:
                 if i[1][1] == min_value:
                     min_value_id = i[0]
             distance[current_node] = distance[min_value_id] + int(min_value)
             pred[current_node] = min_value_id
         return distance, pred
    \section(RUNNING FOR ALL FILES)
[3]: list_of_files = ['P1-10.1.txt',
                      'P1-10.2.txt',
                      'P1-10.3.txt',
                      'P1-15.1.txt',
                      'P1-15.2.txt',
                      'P1-15.3.txt',
                      'P1-20.1.txt',
                      'P1-20.2.txt',
                      'P1-20.3.txt']
[4]: dictionary_of_graphs = {}
     for i,file in enumerate(list_of_files):
         dictionary_of_graphs['graph'+str(i)] = convert_text_to_dictionairy(file)
[]:
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[5]: list_of_results = []
     for g in dictionary_of_graphs.keys():
         pred, dis = min_path_dynamic_program(dictionary_of_graphs[g])
         list_of_results.append((g,pred, dis))
[6]: for result in list_of_results:
         print(result[0])
         print(f'dis: {result[1]}')
         print('\n')
         print(f'pred: {result[2]}')
```

```
graph0
dis: {'1': 0, '2': 4, '3': 10, '4': 18, '5': 12, '6': 14, '7': 18, '8': 10, '9':
17, '10': 19}
pred: {'1': '1', '2': '1', '3': '1', '4': '3', '5': '1', '6': '1', '7': '3',
'8': '2', '9': '1', '10': '9'}
graph1
dis: {'1': 0, '2': 13, '3': 17, '4': 20, '5': 21, '6': 28, '7': 5, '8': 2, '9':
15, '10': 7}
pred: {'1': '1', '2': '1', '3': '1', '4': '2', '5': '3', '6': '5', '7': '1'.
'8': '1', '9': '2', '10': '8'}
graph2
dis: {'1': 0, '2': 19, '3': 6, '4': 10, '5': 27, '6': 5, '7': 29, '8': 9, '9':
15, '10': 30}
pred: {'1': '1', '2': '1', '3': '1', '4': '3', '5': '2', '6': '1', '7': '5',
'8': '6', '9': '8', '10': '5'}
graph3
dis: {'1': 0, '2': 10, '3': 2, '4': 6, '5': 15, '6': 21, '7': 8, '8': 23, '9':
23, '10': 8, '11': 10, '12': 3, '13': 13, '14': 25, '15': 10}
pred: {'1': '1', '2': '1', '3': '1', '4': '3', '5': '3', '6': '5', '7': '3',
'8': '6', '9': '6', '10': '4', '11': '4', '12': '3', '13': '10', '14': '9',
'15': '7'}
graph4
dis: {'1': 0, '2': 3, '3': 3, '4': 6, '5': 10, '6': 6, '7': 13, '8': 7, '9': 7,
'10': 9, '11': 17, '12': 3, '13': 4, '14': 20, '15': 4}
pred: {'1': '1', '2': '1', '3': '1', '4': '3', '5': '2', '6': '3', '7': '6',
'8': '4', '9': '2', '10': '8', '11': '7', '12': '1', '13': '12', '14': '11',
'15': '2'}
```

print('\n')

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graph5
dis: {'1': 0, '2': 7, '3': 15, '4': 15, '5': 8, '6': 19, '7': 17, '8': 22, '9':
10, '10': 16, '11': 25, '12': 18, '13': 17, '14': 5, '15': 18}
pred: {'1': '1', '2': '1', '3': '2', '4': '1', '5': '2', '6': '3', '7': '4',
'8': '7', '9': '5', '10': '9', '11': '6', '12': '3', '13': '10', '14': '1',
'15': '10'}
graph6
dis: {'1': 0, '2': 14, '3': 18, '4': 27, '5': 30, '6': 40, '7': 20, '8': 41,
'9': 32, '10': 28, '11': 30, '12': 27, '13': 34, '14': 44, '15': 44, '16': 21,
'17': 25, '18': 22, '19': 32, '20': 36}
pred: {'1': '1', '2': '1', '3': '2', '4': '2', '5': '4', '6': '5', '7': '3',
'8': '4', '9': '5', '10': '7', '11': '4', '12': '7', '13': '10', '14': '8',
'15': '8', '16': '7', '17': '7', '18': '16', '19': '10', '20': '13'}
graph7
dis: {'1': 0, '2': 6, '3': 19, '4': 9, '5': 4, '6': 7, '7': 6, '8': 9, '9': 11,
'10': 11, '11': 8, '12': 11, '13': 10, '14': 11, '15': 11, '16': 10, '17': 12,
'18': 20, '19': 8, '20': 6}
pred: {'1': '1', '2': '1', '3': '2', '4': '2', '5': '1', '6': '1', '7': '5',
'8': '5', '9': '4', '10': '7', '11': '7', '12': '6', '13': '8', '14': '11',
'15': '11', '16': '5', '17': '12', '18': '3', '19': '7', '20': '5'}
graph8
dis: {'1': 0, '2': 14, '3': 16, '4': 26, '5': 32, '6': 33, '7': 2, '8': 29, '9':
5, '10': 18, '11': 3, '12': 16, '13': 18, '14': 21, '15': 3, '16': 5, '17': 20,
'18': 36, '19': 22, '20': 22}
pred: {'1': '1', '2': '1', '3': '2', '4': '3', '5': '4', '6': '5', '7': '1',
'8': '4', '9': '7', '10': '3', '11': '7', '12': '2', '13': '3', '14': '10',
'15': '7', '16': '11', '17': '13', '18': '6', '19': '13', '20': '17'}
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