OR WEEK 4

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1 Exercise 9.8

Here we have to make three distinct Discrete Markov Chains, one without the fertilizer, one with and finally using a fertilizer for when conditions are poor.

Part a

State Variable: Let x(t) be the condition of the soil at time t.

State Space: $\mathcal{I} = \{x(t) \in \{\text{Good, Fair, Poor}\}\}$

Transition Steps:

$$\mathbf{P_{ij}} = \begin{array}{c} \begin{array}{ccc} \text{G} & \text{F} & \text{P} \end{array} \\ \begin{bmatrix} 0.2 & 0.5 & 0.3 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} \text{G} \\ \text{F} \\ \text{P} \end{array}$$

Note that $P_{PP} = 1$ is an absorbing state, meaning that in the long run the farmer only has poor quality of soil

Part b

State Variable: Let x(t) be the condition of the soil at time t using the fertilizer.

State Space: $\mathcal{I} = \{x(t) \in \{\text{Good, Fair, Poor}\}\}$

Transition Steps:

$$\mathbf{P_{ij}} = \begin{array}{c} \begin{array}{ccc} \text{G} & \text{F} & \text{P} \end{array} \\ \begin{bmatrix} 0.3 & 0.6 & 0.1 \\ 0.1 & 0.6 & 0.3 \\ 0.05 & 0.4 & 0.55 \end{bmatrix} \end{array} \begin{array}{c} \text{G} \\ \text{F} \\ \text{P} \end{array}$$

Steady States:

$$
\begin{aligned}
\pi_G &= 0.3\pi_G + 0.1\pi_F + 0.05\pi_P \\
\pi_F &= 0.6\pi_G + 0.6\pi_F + 0.4\pi_P \\
\pi_P &= 0.1\pi_G + 0.3\pi_F + 0.55\pi_P \\
1 &= \pi_G + \pi_F + \pi_P
\end{aligned}
$$

Doing Some algebra gives the following results: $\pi_G = \frac{2}{123}, \pi_F = \frac{682}{738}$ and $\pi_P = \frac{22}{369}$.

Part c

State Variable: Let x(t) be the condition of the ground at time t using the fertilizer conditionally on the poor states.

State Space: $\mathcal{I} = \{x(t) \in \{\text{Good, Fair, Poor}\}\}$

Transition Steps:

$$
\mathbf{P_{ij}} = \begin{array}{c c}
\begin{array}{ccc} \text{G} & \text{F} & \text{P} \end{array} & \\
\begin{bmatrix} 0.2 & 0.5 & 0.3 \\ 0 & 0.5 & 0.5 \\ 0.05 & 0.4 & 0.55 \end{bmatrix} & \begin{array}{c} \text{G} \\ \text{F} \\ \text{P} \end{array}
\end{array}
$$

Steady States:

$$
\begin{aligned}
\pi_G &= 0.2\pi_G + 0.05\pi_P \\
\pi_F &= 0.5\pi_G + 0.5\pi_F + 0.4\pi_P \\
\pi_P &= 0.1\pi_G + 0.3\pi_F + 0.55\pi_P \\
1 &= \pi_G + \pi_F + \pi_P
\end{aligned}
$$

Doing Some algebra gives the following results: $\pi_G = \frac{5}{154}, \pi_F = \frac{69}{154}$ and $\pi_P = \frac{80}{154}$.

2 10.16

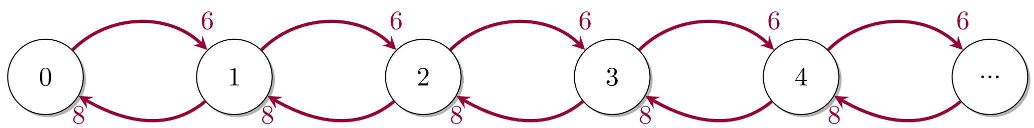

Figure 1: Visualization on how the system goes

We have a $/M/M/1$ Queuing system. Let us define the state variable $X(t)$, as the number of broken machines at time t. Given that we have unlimited number of machines, we have the following state space $\mathcal{I} = \{0, 1, 2, ..., \infty\}$. Ranging from 0 broken machines up to unlimited amount of broken machines. For every broken machine the company loses 10. What is the average loss due to broken machines.

We want to find L, that is what is the average number of broken machines in the system, and multiply it by 10. We know that $L = \sum_{n=0}^{\infty} n\pi_n$

We know that what comes in must come out. Thus $\pi_n = \frac{\lambda_{n-1} \dots \lambda_0}{\mu_{n-1} \dots \mu_0}\pi_0$. As μ and λ are the same in every time period, so $\pi_n = (\frac{\lambda}{\mu})^n \pi_0$. Moreover, we know that $\pi_0 = 1 - \rho = 1 - \frac{6}{1 \times 8} = \frac{1}{4}$.

$L = \sum_{n=0}^{\infty} n \times \pi_n = \sum_{n=0}^{\infty} n \times (\frac{\lambda}{\mu})^n \pi_0 = \pi_0 \sum_{n=0}^{\infty} n(\frac{\lambda}{\mu})^n = \frac{1}{4} \times \frac{\frac{3}{4}}{(1-\frac{3}{4})^2} = 3.$

Hence, on average there 3 machines broken per hour. So the loss per machine is total 30.