Faraday rotation experiment (Musterlösung)

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Best practice / standard solution for BK7n

The experiment is the evidence that electrons are responsible for optical properties of a medium. These, so called dispersion electrons, are few electrons per atom only. They are weakly bound valence electrons interacting easily with light waves. Using an oscillator model for the resonance the dielectric function and, hence, the complex refractive index can be obtained. Moreover, applying a magnetic field these dispersion electrons are subject of Lorentz force. The Zeeman splitting of the resonance in a dielectric medium in an axial magnetic field results in different refractive indices for left and right circular polarized light also in the transparent region far from the optical transition (resonance). Therefore, a linear polarized light wave feels the magnetic field induced double refraction if it is transmitted through a dielectric and nonmagnetic medium in the transparent region. This so called "magnetic field induced birefringence" leads to a rotation of the polarization plane of linear polarized light and is known as Faraday rotation.

In the experiment Faraday rotation angles are measured in dependence on the axial magnetic field strength at different wavelengths. To do this, various LEDs were used as light sources. The medium is a simple, about 7 cm long BK7n (and/or SF 10) glass cuboid arranged in a magnetic coil. A modulation technique have to be used to measure the in general small values of rotation angles and , hence, the Verdet constants at different wavelengths exactly.

From Verdet constants measured at different wavelengths the effective oscillator mass of the dispersion electrons and the number of dispersion electrons per atom are obtained.

Additionally, the experimental setup can be used to transfer information via light beam, like e.g. music, with light (LED or laser light) using polarization modulation due to magnetic field modulation.

Teaching and learning content / educational objectives:

- Dielectric function of a transparent, dielectric, nonmagnetic medium
- Dispersion theory, oscillator model
- Model resonance, Sellmeier equation
- Gradient of an axial magnetic field in a short coil
- Zeeman effect in an axial field
- Linear and circular polarized light, Malus law
- double refraction
- contrast, polarization degree, stress birefringence
- Modulation technique to measure small changes
- work with multimeter, oscilloscope and low-pass filter
- spectra of different light emitting diodes
- number of dispersion electrons per atom
- application of faraday rotation effect in optical isolators for high power laser

1. Determination of the wavelength position of the one-oscillator model resonance of BK7n-glass (Sellmeier equation)

Before starting measurements the dispersion $n = n(\lambda)$ of BK7n-glass given on the website http://refractiveindex.info/ has to be used to draw the function (Sellmeier equation)

$$\frac{1}{n^2-1} = \frac{am_e^*}{N\lambda_R^2} - \frac{am_e^*}{N}\left(\frac{1}{\lambda^2}\right) \qquad \text{with} \qquad a = \frac{4\pi^2 c^2 \varepsilon_0}{e^2} \quad .$$

The Sellmeier equation uses the idea to combine different glass absorption transitions occurring in the UV-region into one model resonance with the wavelength λ_R. As shown in Fig.1 this gives a good linear fit for wavelengths smaller than 720 nm. The deviations observed at longer wavelengths result on the influence of further glass absorption lines occurring in the infrared spectral region. With the obtained slope

$$\frac{am_e^*}{N} = 7.3393 \cdot 10^{-3}\ \mu\text{m}^2$$

and the zero crossing point

$$\frac{am_e^*}{N\lambda_R^2} = 0.79$$

the wavelength λ_R of the model resonance can be calculated easily to be

$$\lambda_R = \sqrt{\frac{7.3393 \cdot 10^{-3}\ \mu\text{m}^2}{0.79}} = 96.38\,\text{nm} \quad .$$

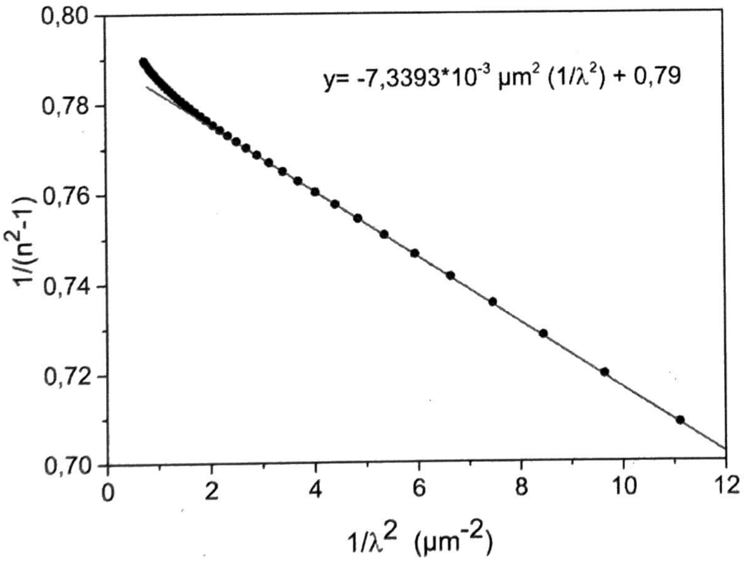

Fig 1. Application of BK7n-dispersion corresponding to Sellmeier equation

Later, if the effective oscillator mass m_e^* will be obtained from detailed measurements of Verdet constants the above obtained slope and the zero crossing point value are used to calculate the number N of dispersion electrons responsible for the optical properties of the glass.

2. Axial gradient of magnetic flux density in the coil and averaged *B*-field

Using an axial hall sensor the magnetic flux density B was measured along the symmetry axis of a coil with a length of L=7 cm. For a digital current of I = 0.8 A the curve $B(L)$ is shown in figure 2. The maximum value $B_{\max}$ was observed in the middle of the coil. As expected for a short coil the magnetic flux density B nearly halved at both sides. Due to the linear dependence of the Verdet-constant on magnetic field averaged values of magnetic flux density has to be used if the Faraday-rotation of a 7 cm long glass sample is investigated. From figure 2 the averaged magnetic flux density was calculated to be $\overline{B} = 6{,}08\ \mathrm{mT}$. Generally, it holds $\overline{B} = 0.8 B_{\max}$. The magnetic flux density does not change in lateral directions inside the 3 cm x 3 cm area of the coil.

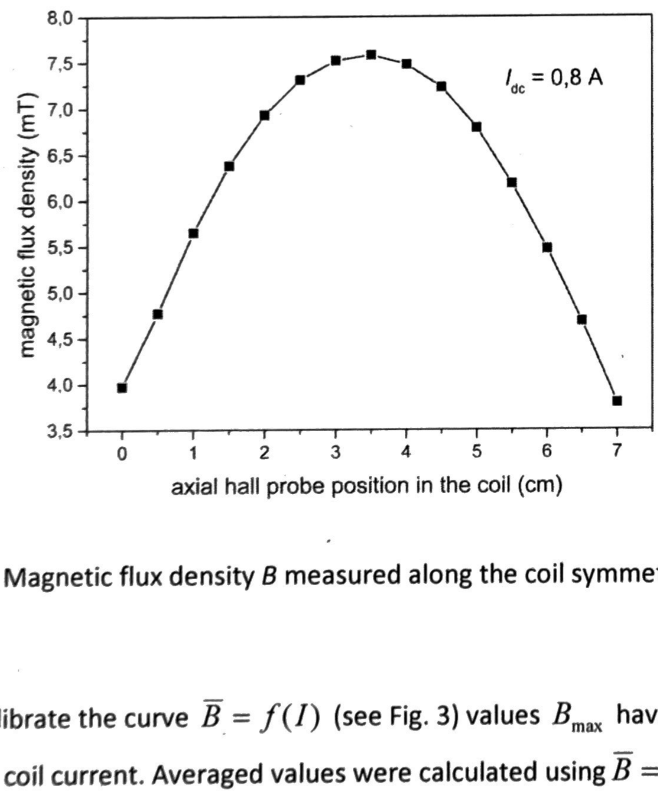

Fig.2 Magnetic flux density *B* measured along the coil symmetry axis at fixed dc-current

To calibrate the curve $\overline{B} = f(I)$ (see Fig. 3) values $B_{\max}$ have to be measured only in dependence on dc coil current. Averaged values were calculated using $\overline{B} = 0.8 B_{\max}$. The dc-current I_{dc} and a later used effective ac-current I_{eff} lead to the same result.

As shown in Fig.3 the slope of the curve $\overline{B} = f(I_{eff})$ is

$$\overline{B} = 7.57 \frac{\mathrm{mT}}{\mathrm{A}} \cdot I_{dc} = 7.57 \frac{\mathrm{mT}}{\mathrm{A}} \cdot I_{eff} \cdot$$

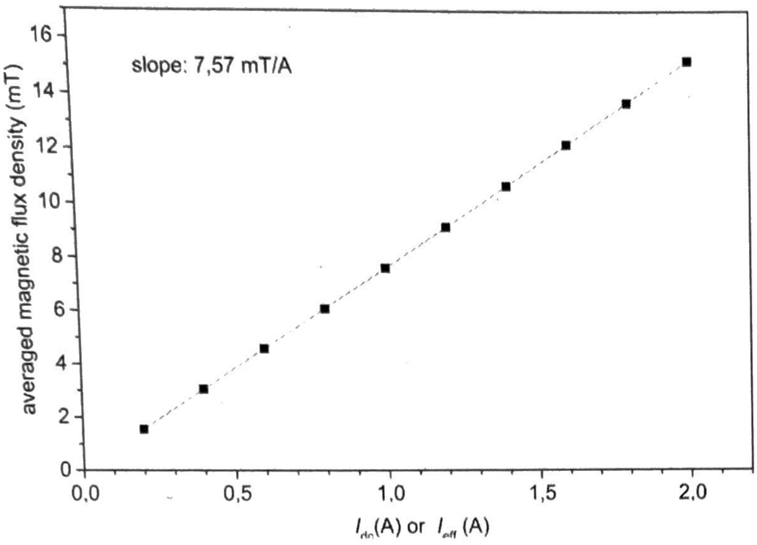

Fig.3 Averaged magnetic flux density $\overline{B}$ versus coil current

3. Verification of Malus law and measurements of contrast, polarization degree and of the rise of the characteristic line for modulation technique

To verify Malus law LED-light was linearly polarized (polarizer) and afterwards analyzed (analyzer) in dependence on the angle φ between polarizer and analyzer. These measurements were done without magnetic field $(B = 0)$. To do this, OPT 301 was used as detector (output voltage depends linearly on light intensity) and multimeter to measure the dc-voltage corresponding to the dc-light intensity I_{DC}. In figure 4 the output voltage (or light intensity in arbitrary units) is shown for red-LED light (628 nm) passing through the coil without magnetic field and without the BK7n-glass cuboid in dependence on analyzer angle. The measured curve was fitted very well with $y = 12\cos^2(x)$ in very good agreement with Malus law.

If light passes through the glass cuboid similar results were obtained but having smaller contrast values respectively smaller polarization degrees due to stress double refraction in the glass. In the table 1 contrast and polarization degree values are given obtained at different LED-wavelengths. Moreover, the rise values of corresponding Malus-curves are given measured at $\varphi = 45°$ important for the sensitivity of modulation measurements of Faraday rotation. All values are typical values, but they depend on adjustment. The obtained rise values depend on both intensities of different LEDs and the spectral sensitivity of the detector at used wavelength. Maximal signal voltage given by the detector is about 13.5 V (at operation voltages of plus and minus 15 V or 8 V only at operation voltages of plus and minus 9 V using two monobloc batteries for the OPT 301).

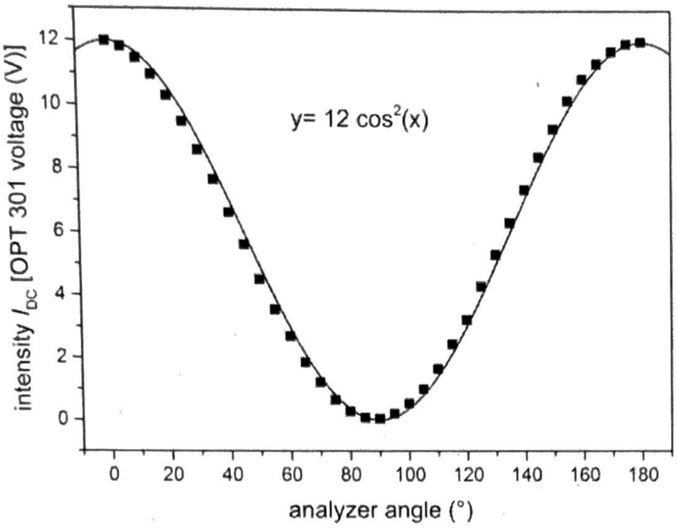

Fig. 4 Verification of Malus law: dc-light intensity I_{DC} of linearly polarized 628 nm LED light measured behind an analyzer in dependence on angle between polarizer and analyzer (without glass cuboid and without magnetic field) and fit curve.

Table 1 Contrast values, polarization degree values and rises of Malus curves at $\varphi = 45°$ measured at different LED-wavelengths if light passes through the BK7n-glass cuboid between polarizer and analyzer. For comparison values without glass cuboid are given at 628 nm.

	wavelength (nm)	contrast	Polarization degree	Rise at 45° (V/rad)
Without glass cuboid	628	$3.8 * 10^{-4}$	2600	12.1
With glass cuboid	628	0.956	44	10.01
	591	0.95	39.5	4.59
	508	0.941	32.9	8.17
	470	0.945	35.2	9.44
	400	0.981	106	9.45

4. Verification of the maximal modulation at analyzer position at 45°

Measurements were carried out at 628 nm linearly polarized LED-light transmitted through the BK7n-glass cuboid arranged in the coil. The axial *B*-field was sinusoidal modulated with modulation frequency of $\Omega = 60\,\mathrm{Hz}$ using $I_{\mathrm{eff}} = 1.2\,\mathrm{A}$ (compare Fig.2). Only the ac-light intensity (peak-to-peak signal I_{PP} was measured behind the analyzer in dependence on the angle φ between analyzer and polarizer. It was measured with OPT301-detector and oscilloscope at ac-modus. The curve shows the maximum of the modulation signal at $\varphi = 45°$. This value is expected theoretically, too, from Malus law $I = I_0 \sin^2 \varphi$ and its derivation $\frac{dI}{d\varphi} = 2I_0 \cos\varphi \sin\varphi$.

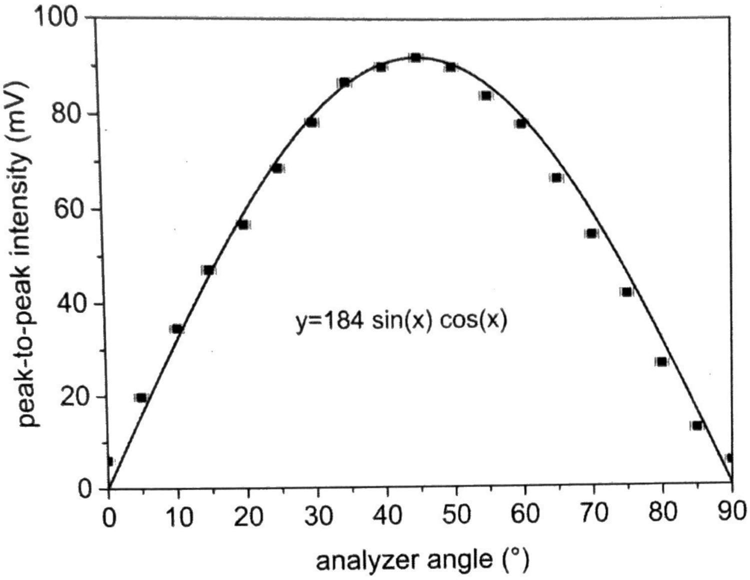

Fig.5 ac-peak-to-peak intensity I_{PP} in dependence on the analyzer angle φ measured with OPT 301-photodetector in the case if linearly polarized 628 nm-LED-light passes the glass cuboid in an axial magnetic field which was sinusoidal modulated with 60 Hz at $I_{eff} = 1.2\mathrm{A}$. The fit curve corresponds to the derivation of Malus law.

5. Faraday rotation angles in dependence on the magnetic field amplitude measured at different wavelengths

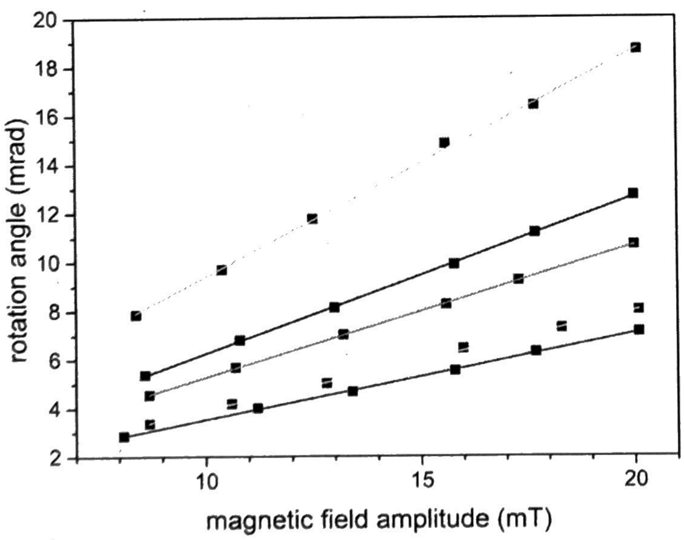

Fig.6 Faraday rotation angles of linear polarized light transmitted through the BK7n glass cuboid in dependence on the amplitude of the axial magnetic field measured at different wavelengths (628, 591, 508, 470, 400 nm from bottom to top)

For the measurement of Faraday rotation angles with a modulated magnetic axial field (frequency between 60 and 90 Hz) the angle between polarizer and analyzer have to be 45° exactly. To adjust this, at first both the dc-light intensities $I_{\mathrm{DC}}(\varphi = 0°)$ and $I_{\mathrm{DC}}(\varphi = 90°)$ had to be measured with multimeter. The analyzer position of $\varphi = 45°$ can be adjusted exactly if the multimeter value shows the mean value $(I_{\mathrm{DC}}(\varphi = 0°) + I_{\mathrm{DC}}(\varphi = 90°))/2$.

Using modulation method the rotation angle of polarization plane can be simply obtained by the formula $\theta = (1/K)(I_{\mathrm{PP}}/4I_{\mathrm{DC}})$, where *K* is the measured contrast, I_{PP} the peak-to-peak value of the modulation induced ac-signal measured with oscilloscope, and I_{DC} the steady-state signal occurring at polarizer-analyzer angle $\varphi = 45°$ measured with multimeter. The amplitude of the magnetic flux density $\overline{B}_{\mathrm{ampl}}$ was determined from the measured effective coil current I_{eff} and using the calibration curve given in Fig.2 to get $\overline{B}$ as well as by formula $\overline{B}_{\mathrm{ampl}} = \sqrt{2}\overline{B}$. Typical values of I_{PP} are in the region between 100 and 400 mV. Therefore uncertainties of measured rotation angles depend on uncertainties of measuring instruments only and are smaller than 5 %.

The spectral sensitivity of the OPT 301 (Si) detector and different brightness's of various LEDs must not take into account because both signals I_{PP} and I_{DC} are here influenced in the same way.

6. Verdet-constants at different wavelengths and calculation of the effective oscillator mass

The curves of Fig. 6 were fitted linearly. From the slopes of the curves in Fig. 6 the Verdet constants at different wavelengths can be calculated using $V = \theta / BL$ by taking into account the glass cuboid length of *L*= 6.9 cm. The effective oscillator mass m_{e}^{*} of dispersion electrons was calculated from the formula

$$V = -\frac{1}{2}\frac{e}{m_{\mathrm{e}}^{*}c}\lambda\frac{\mathrm{d}n}{\mathrm{d}\lambda}$$

using Verdet constants measured experimentally and the values $\mathrm{d}n/\mathrm{d}\lambda$ given on the website http://refractiveindex.info/ for BK7n glass.

Table 2 Slopes obtained from Fig. 6, Verdet constants, and quotients $m_{\mathrm{e}}^{*}/m_{\mathrm{e}}$ at different wavelengths

wavelength (nm)	half width (nm)	LED color	Slope (rad/T)	zero crossing point (mrad)	*V* (mrad/T*m)	$m_{\mathrm{e}}^{*}/m_{\mathrm{e}}$
628	17	red	0.354 (+/- 0.005)	0.017 (+/- 0.07)	5.13	1.25
591	14	yellow	0.4076 (+/- 0.005)	-0.16 (+/- 0.07)	5.91	1.21
508	40	green	0.5396 (+/-0.003)	-0.125 (+/-0.044)	7.82	1.21
470	34	blue	0.6395 (+/-0.006)	-0.148 (+/-0.085)	9.27	1.19
400	12	UV	0.926 (+/-0.015)	0.12 (+/-0.2)	13.4	1.15

The standard deviations of the slopes are very small and in the region between 0.5% and 1.6%. To determine values of Verdet constants and of the mass ratios, main sources of errors may be due to different half widths of the LEDs and the fact that $\mathrm{d}n/\mathrm{d}\lambda$ increases strongly with shorter wavelengths.

7. Number of dispersion electrons

Starting with results $m_e^* \approx 1.2 m_e$ and $\lambda_R = 96.4\,\mathrm{nm}$ (see point 1) and using the zero crossing point of Sellmeier equation fit

$$\frac{a m_e^*}{N \lambda_R^2} = 0.79 \qquad \text{with} \qquad a = \frac{4\pi^2 c^2 \varepsilon_0}{e^2}$$

one obtains $N_{dispersionelectrons} = 1.65 \cdot 10^{23}\ \mathrm{electrons} \cdot \mathrm{cm}^{-3}$ as density of dispersion electrons in BK7n glass.

The density of the BK7n glass cuboid was simply measured to be $\rho = 2.51 \frac{\mathrm{g}}{\mathrm{cm}^3}$. The number of SiO_2 moleculs was estimated using the mol mass of SiO_2 (60 g) and the Avogadro constant to be $N_{\mathrm{SiO2\,moleculs}} = 0.25 \cdot 10^{23}\ \mathrm{SiO_2\ moleculs} \cdot \mathrm{cm}^{-3}$. Because a molecule SiO_2 has 30 electrons, the whole number of electrons is $N_{\mathrm{electrons}} = 7.38 \cdot 10^{23}\ \mathrm{electrons} \cdot \mathrm{cm}^{-3}$.

Therefore, only round about one fourth (1.65/7.38) of electrons are responsible for the optical properties of glass. These are round about one half of the more weakly bound valence electrons interacting with the light wave.