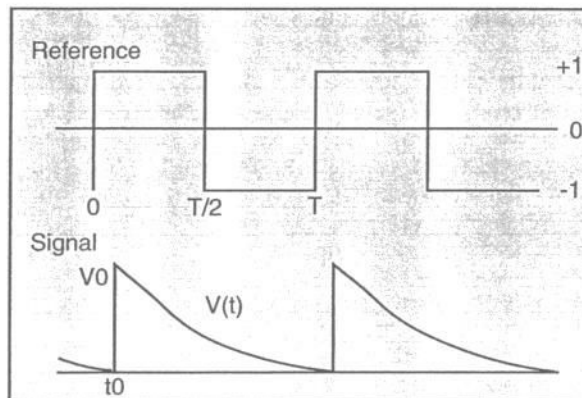# Lock-In Detection

Can a lock-in amplifier detect a single photon per reference cycle? In the previous photon counting experiment, the signal was about one photon per cycle at 50 cycles/second. This is a PMT current of 50Ae where A is the PMT gain. For $A=10^7$, the current is 80 pA, which is well within the capability of an SR510/530 lock-in amplifier.

A lock-in amplifier detects current by using a large resistor or current amplifier to convert the signal current into a voltage. When detecting single photon signals from a PMT, a well chosen termination resistor can provide signal to noise ratios governed entirely by the counting statistics of the photons.

Typically, a PMT output consists of a coaxial cable terminated by a 50Ω resistor. The output voltage appears across the resistor. Since the output cable is terminated in its characteristic impedance, the output voltage pulse will be $V_o \approx AeR/\Delta t$ where Δt is the pulse width of the PMT. For $A=10^7$ and $\Delta t=5$ ns, $V_o \approx 16$ mV. Now assume that 1 photon is detected per cycle at exactly the same time during each cycle. In the time domain, the signal is a periodic series of delta functions spaced by the reference period T where $T \gg \Delta t$. In the frequency domain, the signal spectrum is a series of delta functions spaced by 1/T and extending from dc out to 1/³t. In the case where $\Delta t=5$ ns, the spectrum extends to 200 MHz. A lock-in amplifier which is locked to f=1/T is not suited to detecting this signal, not because of the amplitude, but because of the frequency spectrum.

Now suppose the output of the PMT is terminated by a high resistance R≫50Ω. Because the cable is terminated in a high impedance, the cable can be modeled solely by its capacitance C. The charge from the PMT pulse is deposited on the capacitance in time t. The voltage on the cable will be $V_0=Ae/C$. The charge then bleeds away through R over many time constants $\tau =$ RC. Thus, a photon arriving at time t=0 results in an output voltage waveform $V(t)=V_o e^{-\tau/t}$. Note that the amplitude of the pulse does not vary with R. The large R serves to lengthen the pulse width and thereby change the frequency spectrum of the pulse. The frequency spectrum now has components from DC to $\approx 1/RC$. If C=100 pF and $R=10^7\Omega$, the pulse amplitude is 1.6 mV. The frequency spectrum extends from DC to ≈ 1 kHz and has a larger component at 50 Hz for the lock-in to detect.

The signal output of the lock-in can be estimated by considering a square wave multiplier and a periodic photon train at the reference frequency as shown below.

The DC output of the lock-in over one cycle is

$$S(t_0) = \frac{V_o}{T}\int_0^T e^{-t/\tau}dt \qquad \text{(Volts)}$$

Since real photons arrive at a random t0 between 0 and T/2, the response for a random photon is

$$S = \frac{2}{T}\int_0^{\frac{T}{2}} S(t_0)dt_0 \qquad \text{(Volts)}$$

$$S = \frac{\tau}{T}(1-e^{-\frac{T}{\tau}}) - \frac{4\tau^2}{T^2}(1-e^{-\frac{T}{2\tau}})^2 \qquad \text{(Volts)}$$

S is the response for an average of 1 photon arriving at a random time during each reference cycle. If $\tau \gg T$, S≈0 because the RC time constant of the PMT output attenuates signals at the reference frequency. If $\tau \ll$ 0, S≈0 because the signal extends to frequencies far greater than the reference frequency.

S maximizes for $\tau = T/6$ at which point S=0.065Vo. The factor 0.065 is due to the fact that the signal has many frequency components other than 1/T as well as a randomly shifting phase. Thus, the signal output of the lock-in is

$$\text{SIGNAL} = 0.065\, N_S Ae/C \qquad \text{(Volts)}$$

where N_s is the average number of signal photons per cycle and C is the cable capacitance. For T=20 ms (50 Hz), C=100 pF, R=30 MΩ, Ns=1, and A=10^7, the signal will be 1 mV.

The shunt resistor method is simple and easy to implement, however, phase information is lost. In many experiments, phase is not important. When phase measurements must be made, a current preamplifier is used instead. The current preamplifier eliminates the cable capacitance and the bandwidth of the amplifier is determined by the capacitance of the current gain resistor. Since this capacitance is much smaller, the time constant of the amplifier output pulse is much shorter than the case discussed above. Assuming that the reference period is much longer than this time constant, $T \gg \tau$, then the above formula applies and $S^2 V_o \tau / T$ and $V_o = Ae/C$ where C is the capacitance of the current conversion resistor. Since $\tau = RC$ where R is the current gain, then $S = AeR/T$ which is just the average current times the current gain resistor. The output signal will be

$$SIGNAL = N_s AeR/T \quad \text{(Volts)}$$

For the conditions stated above, the signal will be 2.4 mV. A disadvantage of this approach is that the output of the current preamplifier is a pulse of much greater amplitude and shorter duration than the simple shunt resistor. This requires the use of a higher dynamic reserve so that the high frequency components of the pulse do not overload the amplifier.

The output of the lock-in when there are background photons is

$$OUTPUT = (0.065Ae/C)(N_s + N_1 - N_2) + v_n \quad \text{(Volts)}$$

where N_1 is the number of background photons detected during the open cycle and N_2 is the number detected during the closed cycle. v_n is the noise voltage of the current conversion resistor.
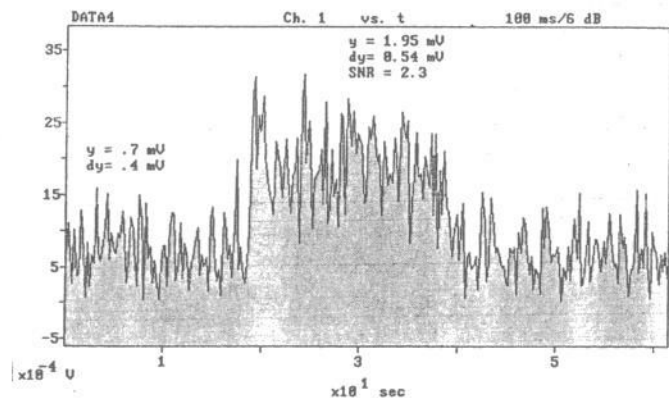
The signal to noise ratio is

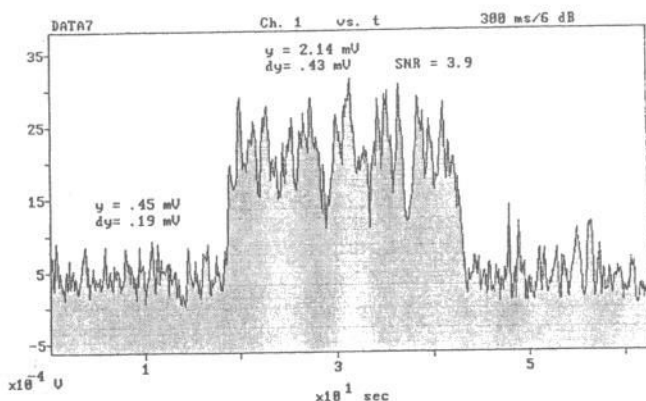$$SNR = \frac{N_s}{\left[N_s + N_b + \frac{v_n^2}{\Delta T (0.065Ae/C)^2} \right]^{\frac{1}{2}}}$$

where N_s and N_b are the number of signal and background photons which occur during a lock-in output time constant ΔT and $v_n = 0.13\sqrt{R}$ nV/$\sqrt{Hz}$ is the Johnson noise density of the conversion resistor. If N_s or N_b is large, then the SNR is identical to the photon counting case described in the previous section where data is accumulated for M cycles and $\Delta T \approx M$ reference cycles. If v_n dominates, then the SNR is worse than pure counting statistics. However, the Johnson noise of large resistors is very small and does not limit many measurements. For example, a 30 MΩ resistor has a noise voltage of 2 µV (for $\Delta T=1$ s) while the signal due to 50 photons/sec is 1 mV. In fact, in this example, as long as 1 background photon is detected per second, the SNR will be dominated by counting statistics. In all cases, the SNR increases as $\sqrt{\Delta T}$ where ΔT is lock-in time constant. This is because more photons are detected and the statistical counting noise is reduced.

Experimental data is shown here. The signal source and PMT were the same as in the previous photon counting discussion. A 30 MΩ resistor was used to terminate the PMT output. An SR530 dual phase lock-in was used to measure signal magnitude. The resulting output was about 2 mV which agrees well with the calculations above for an average of 1 photon per cycle. When the PMT high voltage was off, there was no measurable output noise as expected. In all cases, the SNR is dominated by the background count rate which exceeded the signal rate by 2-3.
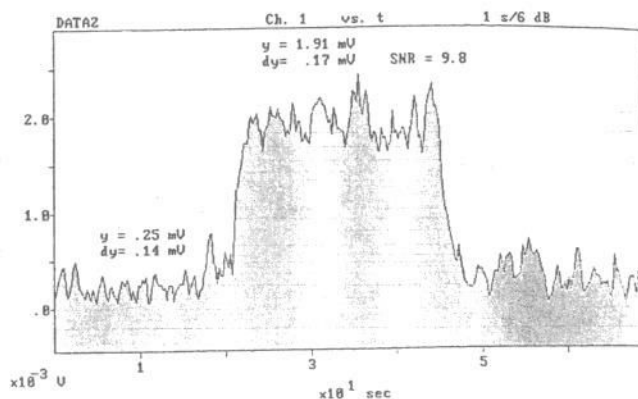
As seen from the data, a signal of 50 photons/sec at a reference of 50 Hz is easily detected with a lock-in amplifier to the same SNR as a photon counter.

Lock-In Amplifier Data: 100 ms Time Constant

Lock-In Amplifier Data: 300 ms Time Constant

Lock-In Amplifier Data: 1 s Time Constant

Conclusions

In general, large signals require the use of analog instruments such as boxcar averagers and lock-in amplifiers. While it is true that the theoretical achievable signal to noise ratio may not be as good as the counting statistics, the practical matter is that large signals take a lot less time to measure to a given SNR level.

When the signal is low, less than 1 photon per gate or cycle, the analog instruments, with the appropriate technique, can achieve photon counting signal to noise ratios. When the signal is much lower, photon counting is required.

In most experiments, the key to optimizing the measurement will lie in factors other than signal intensity. In all cases, the PMT quantum efficiency, gain, and noise are the most important factors. The initial gain from the PMT can never be replaced as well with an amplifier.

Low background or dark count rates are essential in low level measurements. External noise pickup in signal cables when lasers trigger or unstable background count rates (such as from an unstable glow discharge), can result in large fluctuations in signal amplitude far in excess of the counting statistics. These experimental factors can be the most important considerations when choosing an instrument.

For further reading;

Photomultiplier Handbook (publication PMT-62), RCA Corp., 1980.

The Art of Electronics, Horowitz and Hill, Cambridge University Press, Cambridge, 1982.