

# **Veamy**

An extensible object-oriented C++ library  
for the virtual element method

## **Veamy Primer**

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## 1 Features of Veamy

- Includes its own mesher based on the computation of the constrained Voronoi diagram. The meshes can be created in arbitrary two-dimensional domains, with or without holes, with procedurally generated points<sup>1</sup>.
- Meshes can also be read from OFF-style text files.
- Allows easy input of boundary conditions by constraining domain segments and nodes.
- The results of the computation can be either written into a file or used directly.
- PolyMesher meshes and boundary conditions can be read straightforwardly in Veamy to solve 2D linear elastostatic problems.

## 2 Source code

The source code is available to be downloaded from Veamy's web page:

<http://camlab.cl/research/software/veamy/>

Download the code before proceeding with the rest of this primer.

## 3 Up and running with Veamy

Veamy has been tested on Unix-based machines only. First of all, make sure that CMake is available in your machine. If it is not, install it before proceeding with the rest of this primer. To install CMake on Ubuntu machines, on a terminal type and execute:

```
sudo apt-get install cmake
```

Unpack the code to a folder of your choice. Fig. 1 shows the content of Veamy that was unpacked to “/home/Software/”

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<sup>1</sup> However, the constrained Voronoi mesher is part of a separate project and Veamy only makes use of this mesher. The full documentation of this mesher is available from its own repository: <https://github.com/capalvarez/Delynoi>

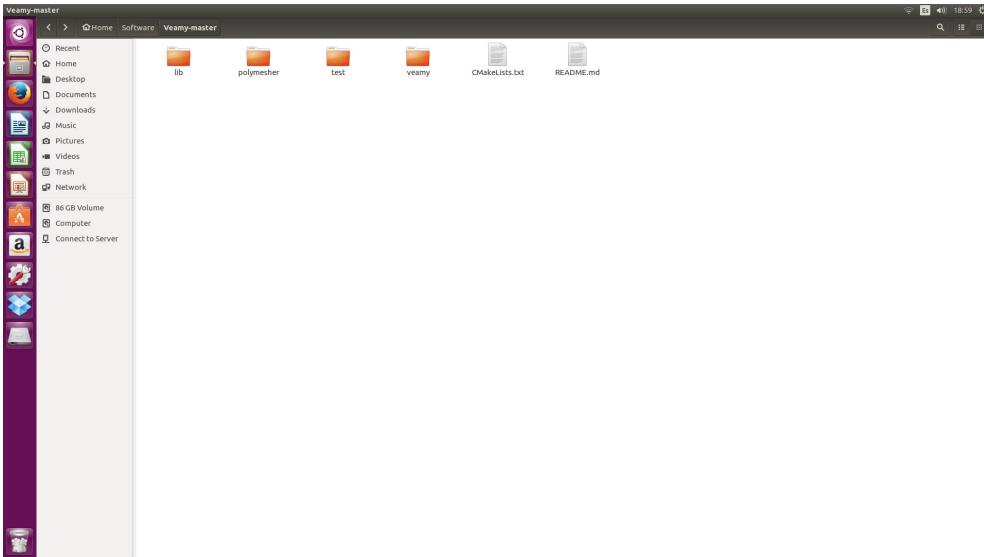


Fig. 1: Veamy source code.

Go inside “test” folder of Veamy’s root directory (see Fig. 2). This test folder is where the main C++ setup file implementing a problem of interest must be placed. In this example, a “cantilever beam subjected to a parabolic end load” will be solved in Veamy. This problem is part of the numerical examples provided in:

A. Ortiz-Bernardin, C. Alvarez, N. Hitschfeld-Kahler, A. Russo, R. Silva, E. Olate-Sanzana. Veamy: an extensible object-oriented C++ library for the virtual element method. [arXiv:1708.03438 \[cs.MS\]](https://arxiv.org/abs/1708.03438)

You may consult the details of the geometry and boundary conditions therein as in this primer we only refer to the final main C++ setup file to run the example.

The implementation of the cantilever beam subjected to a parabolic end load is provided in the main C++ setup file named “ParabolicMain.cpp” (see Fig. 2).

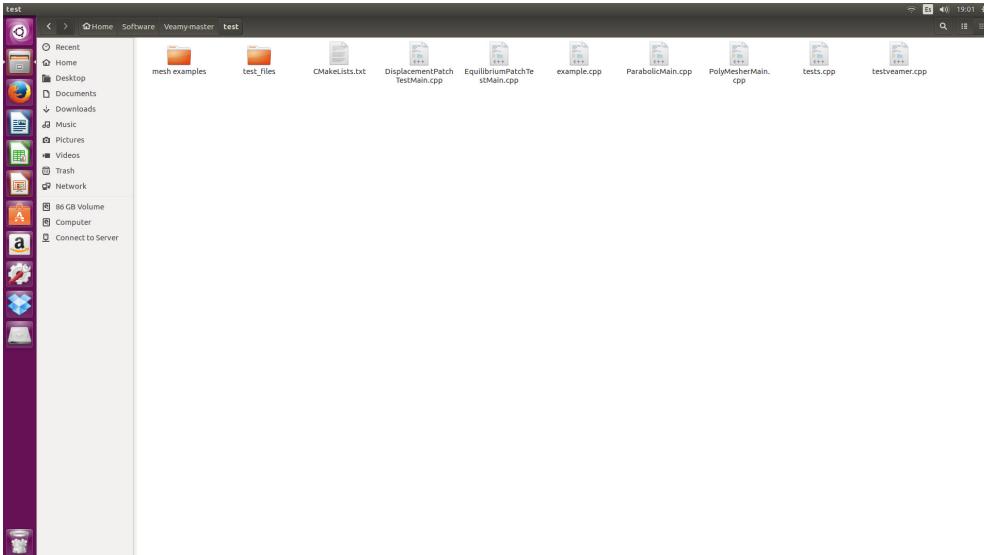


Fig. 2: Veamy’s test folder. The main C++ setup file implementing a problem of interest must be placed in this folder. Several main setup C++ files are shown. In this part of the primer, the C++ file “ParabolicMain.cpp” will be used.

Open “ParabolicMain.cpp” file. If you are interested, browse the code in this file to realize how a problem implementation is setup in Veamy. To run this problem is important to update the folder where the output files will be stored. In order to specify the output folder, check the instructions that are provided as comments in “ParabolicMain.cpp” (see Fig. 3). Modify accordingly, save and close the setup file.

```

double l = 8;
double I = std::pow(D,3)/12;
return I*y/((3*vBar*I)*((4*L - 3*x)*x + (2+vBar)*std::pow(y,2) - 3*std::pow(D,2)/2*(1+vBar)));
}

double uv(double x, double y){
    double P = -1980;
    double vBar = 1e7/(1 - std::pow(0.3,2));
    double D = 4;
    double l = 8;
    double I = std::pow(D,3)/12;
    return I/((vBar*I)*(3*vBar*std::pow(y,2)*(L-x) + (3*L-x)*std::pow(x,2)));
}

int main(){
    std::cout << "*** Starting Veamy ***" << std::endl;
    std::cout << "A cantilever beam subjected to a parabolic end load" << std::endl;
    std::cout << "... " << std::endl;

    // DEFINING PATH FOR THE OUTPUT FILES:
    // If the output files is not given, they are written to /Home directory by default.
    // Otherwise, include the path. For instance, for /Home/user/Documents/Veamy/output.txt , the path
    // can be defined as follows: std::string meshFileName = "/Home/user/Documents/Veamy/output.txt";
    // CAUTION: the path must exists either because it is already in your system or because it is created
    // by Veamy's configuration files. For instance, Veamy creates the folder "test" inside "build", so
    // can be defined as std::string meshFileName = "build/test/meshes/output.txt";
    // since "myn custom folder" won't be created by Veamy's configuration files.
    std::string meshFileName = 'Software/Veamy-master/build/test/parabolic_beam.mesh.txt';
    std::string displacementFileName = 'Software/Veamy-master/build/test/parabolic_beam_displacements.txt';

    std::cout << "Defining the domain ... ";
    std::vector<Point> rectangle4x8_rectangle4x8_points = {Point(0, -2), Point(8, -2), Point(8, 2), Point(0, 2)};
    Region rectangle4x8(rectangle4x8_rectangle4x8_points);
    std::cout << "done" << std::endl;

    std::cout << "% Generating polygonal mesh ... ";
    rectangle4x8.generatedSeedPoints(PointGenerator(functions::constantAlternating(), functions::constant()), 24, 12);
    std::vector<Point> seeds = rectangle4x8.getSeedPoints();
    std::vector<Constraint> meshConstraints = rectangle4x8.getConstraints();
    PolygonalMesh mesh = meshGenerator.getMesh(seeds, rectangle4x8);
    std::cout << "done" << std::endl;

    std::cout << "% Printing mesh to a file .... ";
    mesh.printInFile(meshFileName);
    std::cout << "done" << std::endl;

    std::cout << "% Defining Dirichlet and Neumann boundary conditions ... ";
    EssentialICConstraints essential;
    uXConstraint = new Function(ux);
    Function* uVConstraint = new Function(uv);

    PointSegment leftSide(Point(-2, 0));
    SegmentConstraint const1(leftSide, mesh.getPoints()), Constraint::Direction::Horizontal, uXConstraint);
    essential.addConstraint(const1, mesh.getPoints());
    SegmentConstraint const2 (leftSide, mesh.getPoints(), Constraint::Direction::Vertical, uVConstraint);
}

```

Fig. 3: Main C++ setup file for the cantilever beam subjected to a parabolic end load.

Now, the test folder contains a file named “CMakeLists.txt”. This file is important because it controls which main C++ setup file will be processed in Veamy. The file inside “test” folder is shown in Fig. 4.

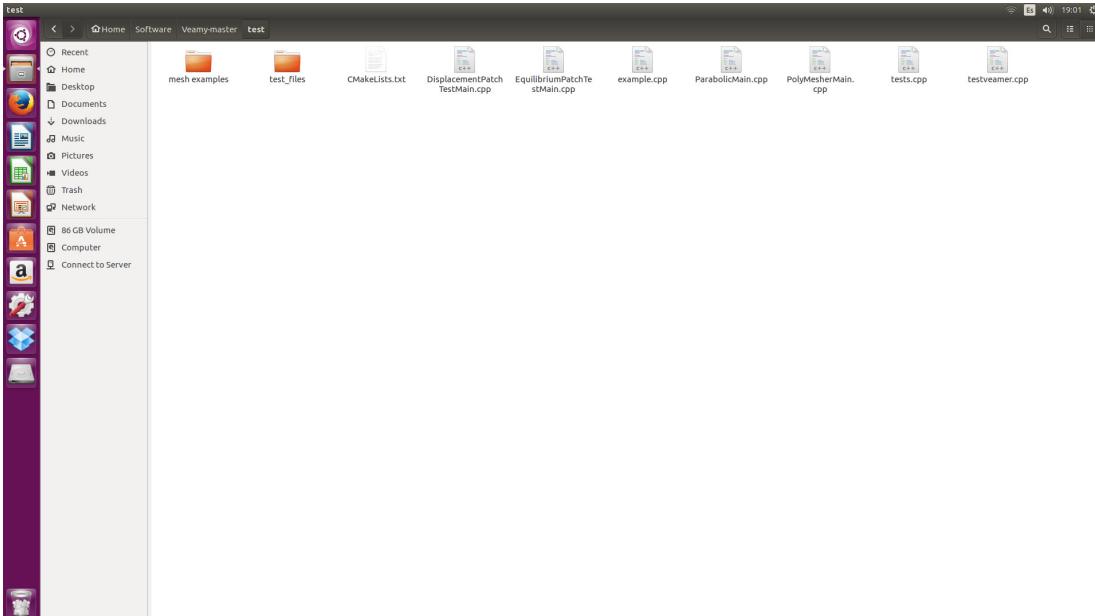
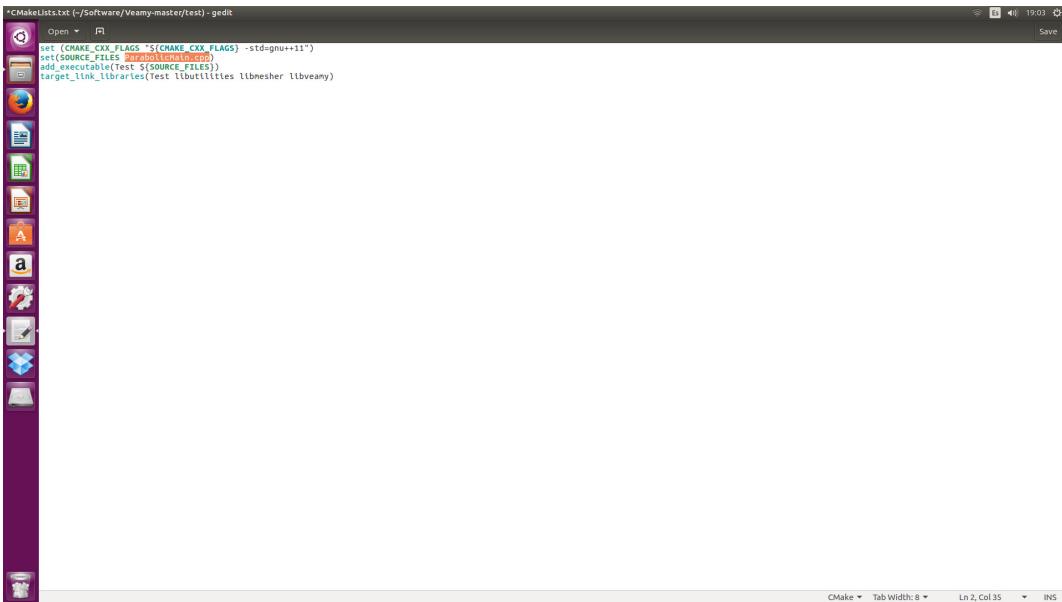


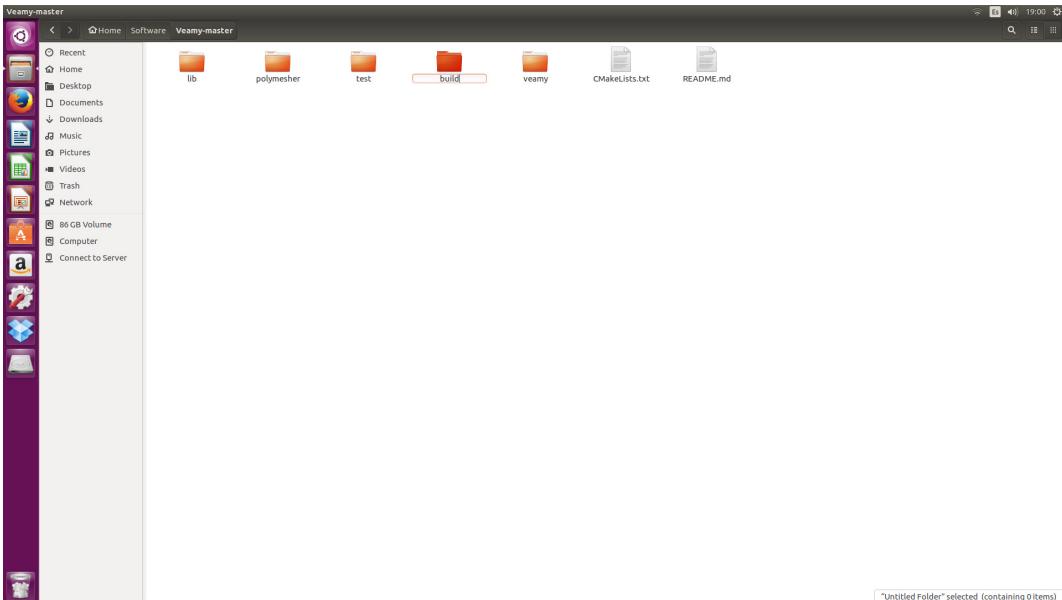
Fig. 4: CMakeLists.txt is located in test folder and controls which main C++ setup file is processed in Veamy.

Open “CMakeLists.txt” and on the highlighted zone, write the name of the main C++ setup problem file, in this case, “ParabolicMain.cpp,” as shown in Fig. 5. Save and close the file.



**Fig. 5:** Open “CMakeLists.txt” and on the highlighted zone, write the name of the main C++ setup problem file.

Go back to the Veamy’s root folder and there create a folder “build” (Fig. 6).



**Fig. 6:** In Veamy’s root folder create the folder “build”.

Go inside the “build” folder and on a terminal, type and execute:

```
cmake ..
```

to create the makefiles. Then, to compile the program, on a terminal type and execute:

```
make
```

Several files are created. Also, another folder called “test” is created inside “build”. The executable of the test problem is stored in this “test” folder and is called “Test”. Go inside “build/test/” folder (Fig. 7) and, on a terminal, type and execute:

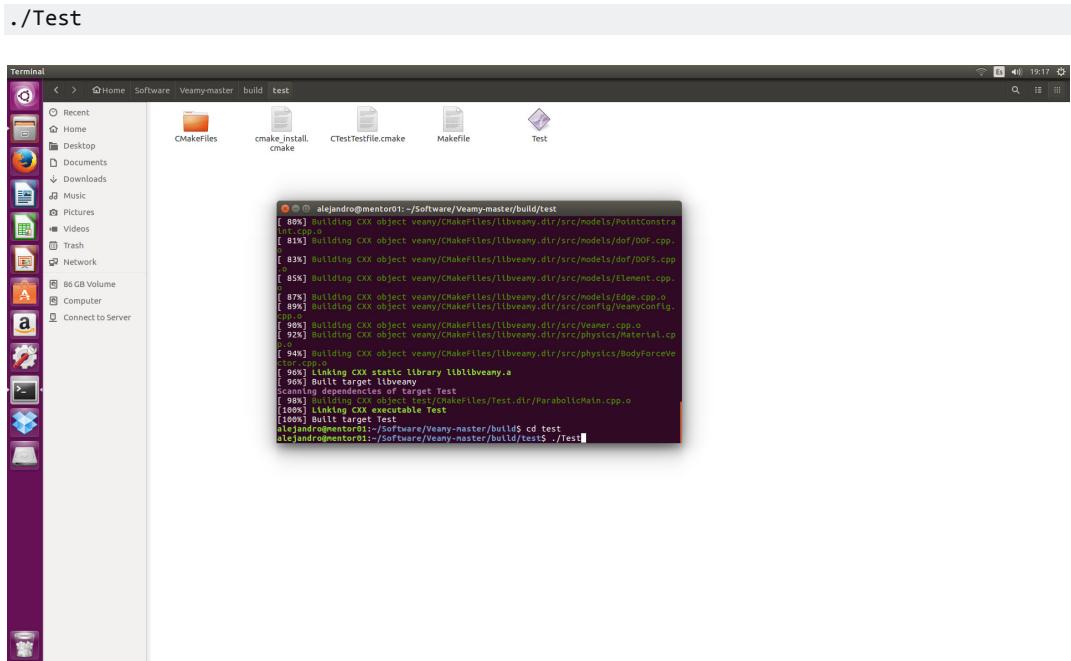


Fig. 7: Go inside “build/test/” folder and on a terminal type and execute ./Test

While running, Veamy prints out some messages on the screen indicating the progress of the simulation, as shown in Fig. 8.

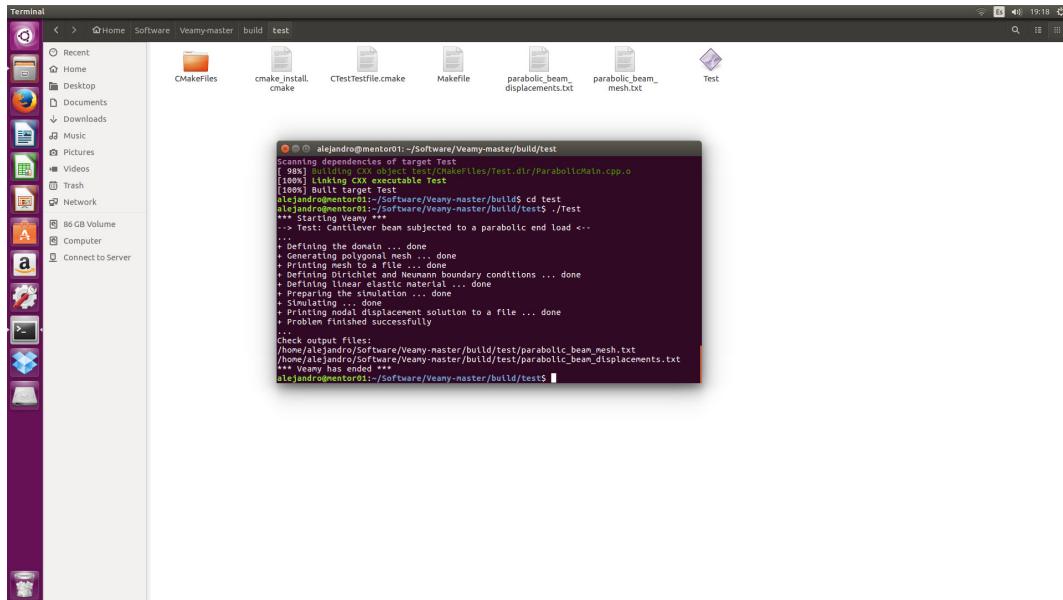


Fig. 8: Veamy prints out some messages while running the simulation.

The last lines of the printed out messages indicate the location of the output folders. The output files contain the mesh and the nodal displacement solution. The mesh can be visualized using the MATLAB function “plotPolyMesh.m” that is inside folder “Veamy\_root\_directory/lib/visualization/” or if you want to visualize both the mesh

and the displacement nodal solution, use the MATLAB function “`plotPolyMeshDisplacements.m`” that is also available in the “visualization” folder (see Fig. 9).

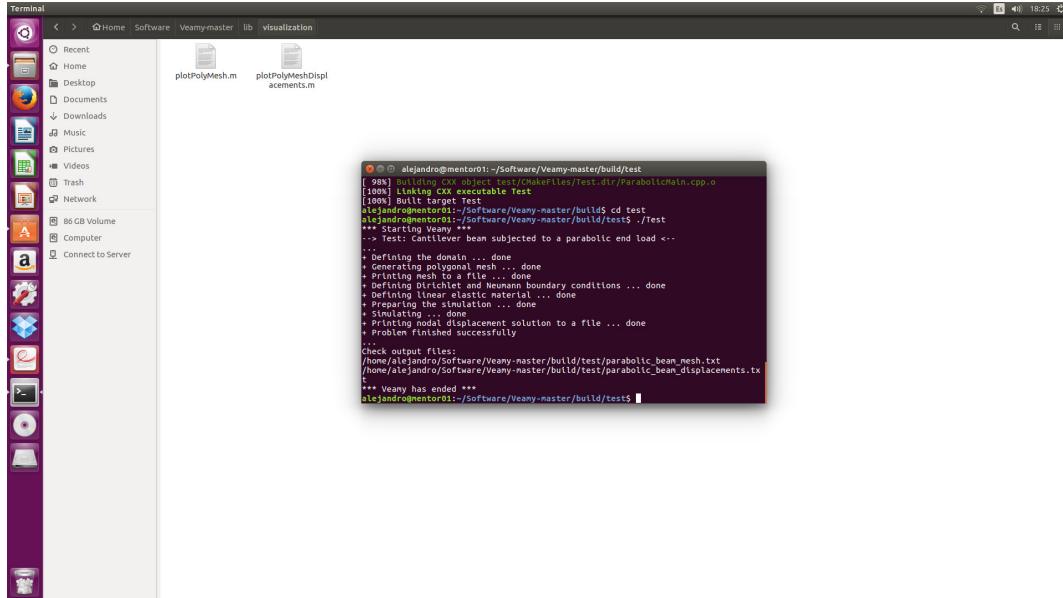


Fig. 9: Use “`plotPolyMesh.m`” to visualize the mesh or “`plotPolyMeshDisplacements.m`” to visualize both the mesh and the nodal displacement solution. Both MATLAB files are located inside folder “`Veamy_root_directory/lib/visualization/`”.

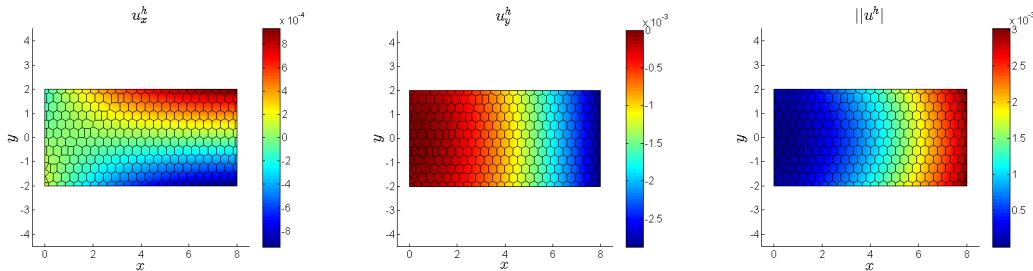


Fig. 10: Mesh and nodal displacements for the beam problem are plotted using the “`plotPolyMeshDisplacements.m`” MATLAB function.

## 4 Using a PolyMesher mesh and boundary conditions in Veamy

Now, we show how to use a mesh and boundary conditions obtained from PolyMesher. This primer assumes that the user knows how to use PolyMesher. This problem is part of the numerical examples provided in:

A. Ortiz-Bernardin, C. Alvarez, N. Hitschfeld-Kahler, A. Russo, R. Silva, E. Olate-Sanzana. Veamy: an extensible object-oriented C++ library for the virtual element method. arXiv:1708.03438 [cs.MS]

You may consult the details of the geometry and boundary conditions therein as in this primer we only refer to the final main C++ setup file to run the example.

The procedure is straightforward. In PolyMesher add a call to the MATLAB function “`PolyMesher2Veamy.m`”. This function is located in “`Veamy_root_directory/polymesher/`”, as shown in Fig. 11. The call to this function is done on the last line of the

“PolyMesher.m” function, as shown in Fig. 12. After defining a model and boundary conditions, and performing the meshing copy the file “polymesher2veamy.txt” to a folder of your choice to be used in Veamy. In the source code of Veamy, the example file containing the PolyMesher mesh and boundary conditions is located inside the folder “Veamy\_root\_directory/test/test\_files/”.

The implementation of the PolyMesher to Veamy example is provided in the main C++ setup file named “PolyMesherMain.cpp” (see Fig. 13). This setup file as usual is inside “Veamy\_root\_directory/test/” folder. Go to this folder and open “PolyMesherMain.cpp” (see Fig. 13). Explore this file to see details about its implementation. The function that reads the PolyMesher mesh and boundary conditions is “initProblemFromFile”. You will have to provide the path to the folder where the PolyMesher mesh and boundary conditions are located. Update the output folders (check the instructions that are provided as comments). Modify the paths accordingly, save and close the setup file.

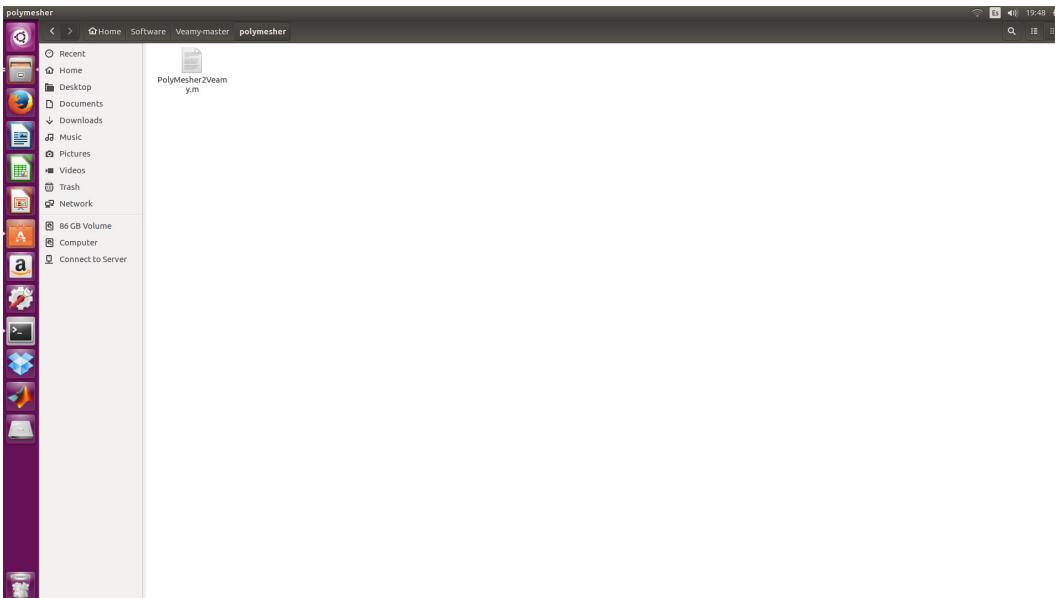


Fig. 11: The MATLAB function “PolyMesher2Veamy.m” is located in folder “Veamy\_root\_directory/polymesher/”.

```
PolyMesher.m
%
%     elements written in Matlab", Struct Multidisc Optim, 2012,
%     DOI 10.1007/s00158-011-0706-z
%
% Ref2: A Pereira, C Talischi, GH Paulino, IFM Menezes, MS Carvalho,
% "Implementation of fluid flow topology optimization in PolyTop",
% Struct Multidisc Optim, 2013, DOI XX.XXXX/XXXXX-XXX-XXX-X
%-----
function [Node,Element,Supp,Load,P] = PolyMesher(Domain,NElem,MaxIter,P)
if ~exist('P','var'), P=PolyMshr_RndPtSet(NElem,Domain); end
NElem = size(P,1);
Tol=5e-6; It=0; Err=1; c=1.5;
BdBox = Domain('BdBox'); PFix = Domain('PFix');
Area = (BdBox(2)-BdBox(1))*(BdBox(4)-BdBox(3));
Pc = P; figure;
while(It<=MaxIter && Err>Tol)
    Alpha = c*sqrt(Area/NElem);
    P = Pc; %Lloyd's update
    R_P = PolyMshr_Rflct(P,NElem,Domain,Alpha); %Generate the reflections
    [P,R_P] = PolyMshr_FixedPoints(P,R_P,PFix); % Fixed Points
    [Node,Element] = voronoin([P;R_P]); %Construct Voronoi diagram
    [Pc,A] = PolyMshr_CntrdPly(Element,Node,NElem);
    Area = sum(abs(A));
    Err = sqrt(sum((A.^2).*sum((Pc-P).*(Pc-P),2)))*NElem/Area^1.5;
    fprintf('It: %3d Err: %1.3e\n',It,Err); It=It+1;
    if NElem<=2000, PolyMshr_PlotMsh(Node,Element,NElem); end;
end
[Node,Element] = PolyMshr_ExtrNds(NElem,Node,Element); %Extract node list
[Node,Element] = PolyMshr_ClpsEdgs(Node,Element,0.1); %Remove small edges
[Node,Element] = PolyMshr_RsqNsDs(Node,Element); %Reorder Nodes
BC=Domain('BC',{Node,Element}); Supp=BC{1}; Load=BC{2}; %Recover BC arrays
PolyMshr_PlotMsh(Node,Element,NElem,Supp,Load); %plot mesh and BCs
PolyMesher2Veamy(Node,Element,NElem,Supp,Load); %plot mesh to a Veamy mesh format
%----- GENERATE RANDOM POINTSET
```

Fig. 12: Call to “PolyMesher2Veamy.m” in “PolyMesher.m” is done on its last line.

```
PolyMesherMain.cpp (-/Software/Veamy-master/test) - gedit
Open ▾
Save
#include <veamy/veamy.h>
#include <utilities/utilities.h>

int main()
{
    // Start Veamy
    std::cout << "*** Starting Veamy ***" << std::endl;
    std::cout << " *** Test Using PolyMesher mesh and boundary conditions ***" << std::endl;
    std::cout << " ***" << std::endl;

    // DEFINING PATH FOR THE OUTPUT FILES:
    // If the path for the output files is not given, they are written to /home directory by default.
    // Otherwise, include the path. For instance, for /home/user/Documents/Veamy/output.txt, the path
    // CAUTION: the path must exists either because it is already in your system or because it is created
    // by Veamy's configuration files. For instance, Veamy creates the Folder "/test" inside "/build", so
    // since "/vcustom/folder" won't be created by Veamy's configuration files.
    std::string meshFileName = Software/Veamy-master/build/test/polyMesher_test_mesh.txt;
    std::string dispFileName = Software/Veamy-master/build/test/polyMesher_test_displacements.txt;

    // File the configuration PolyMesher mesh and boundary conditions. Use Matlab Function
    std::string polyMesherVeamyR = generate this file
    std::string polyMesherVeamyR = "Software/Veamy-master/test/test_files/polyMesher2veamy.txt";

    std::cout << " * Defining linear elastic material ... ";
    Material m(1e-9, 0.3);
    std::cout << "done" << std::endl;

    std::cout << " * Preparing the simulation from a PolyMesher mesh and boundary conditions ... ";
    PolyMesherMesh mesh = v.readProblemFromfile(polyMesherMeshFileName, m);
    std::cout << "done" << std::endl;

    std::cout << " * Printing mesh to a file ... ";
    mesh.printMesh(meshFileName);
    std::cout << "done" << std::endl;

    std::cout << " * Simulating ... ";
    Eigen::Vector3d x = v.simulate(mesh);
    std::cout << "done" << std::endl;

    std::cout << " * Printing nodal displacement solution to a file ... ";
    v.printDisplacements(x);
    std::cout << "done" << std::endl;
    std::cout << " * Problem finished successfully" << std::endl;
    std::cout << " * Check number files!" << std::endl;
    std::string path1 = utilities::getPath();
    std::string path2 = utilities::getPath();
    path1 += meshFileName;
    path2 += dispFileName;
    std::cout << path1 << std::endl;
    std::cout << path2 << std::endl;
    std::cout << "*** Veamy has ended ***" << std::endl;
}
```

Fig. 13: Main C++ setup file for the PolyMesher mesh and boundary condition example.

From now on, the procedure to run the PolyMesher problem in Veamy is identical to the one performed for the beam problem.

Go inside the “Veamy\_root\_directory/build/” folder and on a terminal, type and execute to update the makefiles:

```
cmake ..
```

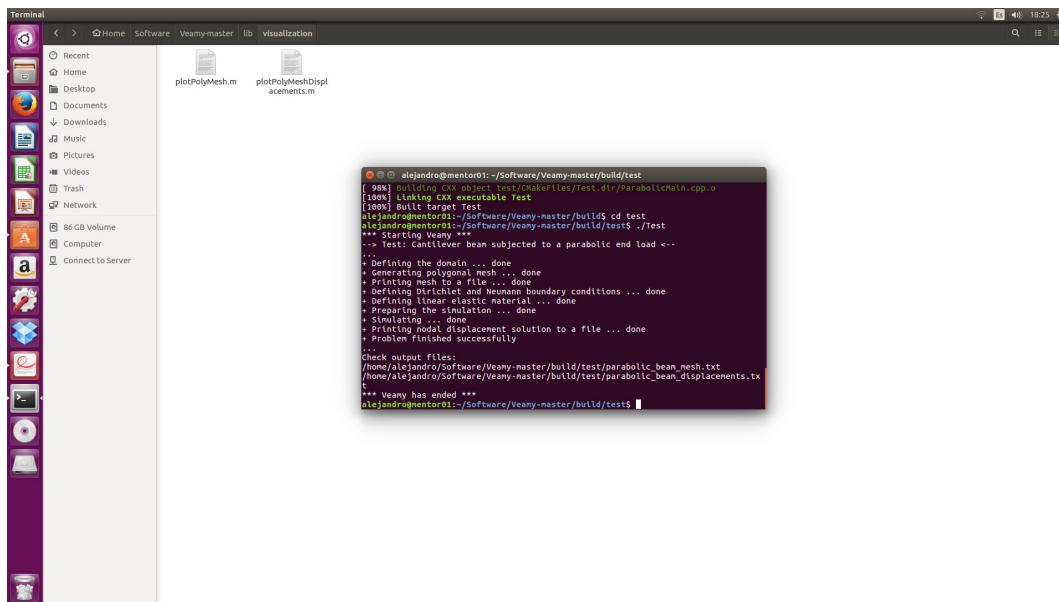
Then, to compile the program, on a terminal type and execute:

```
make
```

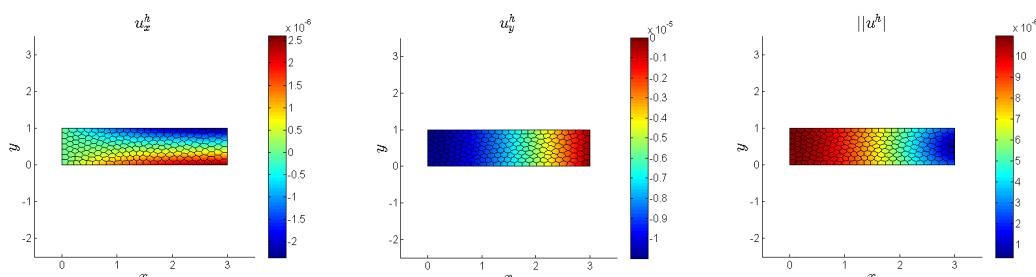
If this procedure has been done several times before, many of the libraries are likely to be already compiled, so the compilation procedure is quite short in comparison with the first time compilation. The executable of the test problem is stored in the “build/test/” folder and is called “Test”. Go inside “build/test/” folder and, on a terminal, type and execute:

```
./Test
```

The output screen for the PolyMesher problem is shown in Fig. 14. The last lines of the printed out messages indicate the location of the output folders. The output files contain the mesh and the nodal displacement solution. The mesh can be visualized using the MATLAB function “plotPolyMesh.m” that is inside folder “Veamy\_root\_directory/lib/visualization/” or if you want to visualize both the mesh and the displacement nodal solution, use the MATLAB function “plotPolyMeshDisplacements.m” that is also available in the “visualization” folder. The mesh and the nodal displacements for the PolyMesher example are shown in Fig. 15.



**Fig. 14:** Output screen for the PolyMesher example. Use “plotPolyMesh.m” to visualize the mesh or “plotPolyMeshDisplacements.m” to visualize both the mesh and the nodal displacement solution. Both MATLAB files are located inside folder “Veamy\_root\_directory/lib/visualization/”.



**Fig. 15:** Nodal displacements for the PolyMesher example are plotted using the “plotPolyMeshDisplacements.m” MATLAB function.

## 5 Using a generic mesh file

Reading a generic mesh file is very similar to the process of using a PolyMesher mesh. The only difference is that boundary conditions are not provided with the mesh file. That is, the mesh is read from a file, but the boundary conditions must be provided in Veamy similarly as done in the cantilever beam problem of Section 3. An example of this is provided in the main C++ setup file “Veamy\_root\_directory/test/EquilibriumPatchTestMain.cpp”. In this main C++ setup file, the external test mesh, which is the file “Veamy\_root\_directory/test/test\_files/equilibriumTest\_mesh.txt”, is read by the function “createFromFile”:

```
std::string externalMeshFileName =
    "Software/Veamy-master/test/test_files/equilibriumTest_mesh.txt";
PolygonalMesh mesh;
mesh.createFromFile(externalMeshFileName);
```

As you can confirm by exploring the external mesh file “equilibriumTest\_mesh.txt”, it contains the nodal coordinates of the mesh and the element connectivity.

## 6 Additional examples

These additional examples require the user to have read the previous sections of this primer.

### 6.1 Perforated Cook’s membrane

The implementation of the perforated Cook’s membrane is provided in the main C++ setup file named “CookTestMain.cpp”. This setup file as usual is inside “Veamy\_root\_directory/test/” folder. Go to this folder and open “CookTestMain.cpp” (see Fig. 16). Explore this file to understand its implementation. Be sure you update the path to the output files. The important lines of code are highlighted. They provide the information for the four points that define the geometry and three circular holes on it.

This problem is part of the numerical examples provided in:

A. Ortiz-Bernardin, C. Alvarez, N. Hitschfeld-Kahler, A. Russo, R. Silva, E. Olate-Sanzana. Veamy: an extensible object-oriented C++ library for the virtual element method. arXiv:1708.03438 [cs.MS]

You may consult the details of the geometry and boundary conditions therein as in this primer we only refer to the final main C++ setup file to run the example.

```

#include <vector>
#include <mesher/models/basic/Point.h>
#include <mesher/models/Region.h>
#include <mesher/models/hole/CircularHole.h>
#include <mesher/models/mesh_generator/functions.h>
#include <mesher/tools/triangleMeshGenerator.h>
#include <veamy/Veamy.h>
#include <veamy/models/constraints/values/Constant.h>
#include <utilities/utilities.h>

int main(){
    std::cout << "**** Starting Veamy ****" << std::endl;
    std::cout << "--> Test: Cook's membrane <--" << std::endl;
    std::cout << "... " << std::endl;

    // DEFINING PATH FOR THE OUTPUT FILES:
    // If the path for the output files is not given, they are written to /Home directory by default.
    // Otherwise, include the path. For instance, for /Home/user/Documents/Veamy/output.txt , the path
    // must be "Documents/Veamy/output.txt"
    // CAUTION: the path must exists either because it is already in your system or because it is created
    // by Veamy's configuration files. For instance, Veamy creates the folder "/test" inside "/build", so
    // you can save the output files to "/build/test/" folder, but not to "/build/test/nycustom_folder",
    // since "/nycustom_folder" won't be in the Veamy's configuration files.
    std::string meshFileName = "Software/Veamy-master/build/test/cook_membrane_mesh.txt";
    std::string dispFileName = "Software/Veamy-master/build/test/cook_membrane_displacements.txt";
    std::string geoFileName = "Software/Veamy-master/build/test/cook_membrane_geometry.txt";

    std::cout << "+ Defining the domain ... ";
    std::vector<Point> TBeam_points = {Point(0,0), Point(48,44), Point(48,64), Point(0,44)};
    Region TBeam(TBeam_points);

    Hole hole1 = CircularHole(Point(0,30), 5);
    Hole hole2 = CircularHole(Point(24,40), 4);
    Hole hole3 = CircularHole(Point(48,50), 3);
    TBeam.addHole(hole1);
    TBeam.addHole(hole2);
    TBeam.addHole(hole3);
    std::cout << "done" << std::endl;

    std::cout << "+ Printing geometry to a file ... ";
    TBeam.printInFile(geoFileName);
    std::cout << "done" << std::endl;

    std::cout << "+ Generating polygonal mesh ... ";
    TBeam.generatePoints(PointGenerator(functions::constantAlternating(), functions::constant()), 16, 16);
    std::vector<Point> seeds = TBeam.getSeedPoints();
    TriangleMeshGenerator::create(seeds, TBeam);
}

```

Fig. 16: Main C++ setup file for the perforated Cook's membrane example.

In order to run the test, follow the same steps described in the previous examples. Once you have compiled the problem, go inside “build/test/” folder and, on a terminal, type and execute:

```
./Test
```

The output files are visualized, as in the previous examples, using the MATLAB function “plotPolyMeshDisplacements.m”. The plots are shown in Fig. 17.

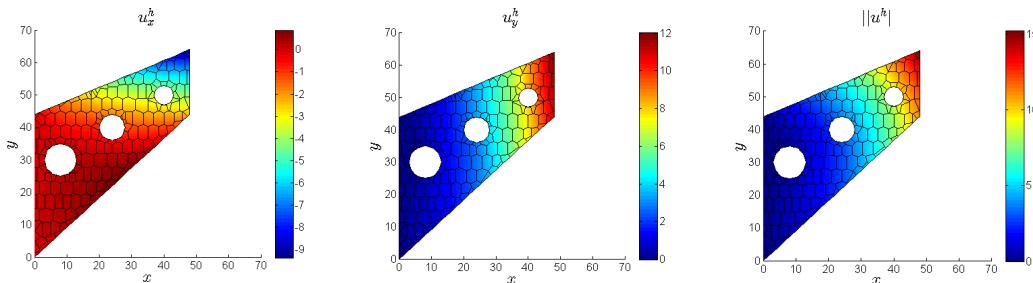


Fig. 17: Nodal displacements for the perforated Cook's membrane problem are plotted using the “plotPolyMeshDisplacements.m” MATLAB function.

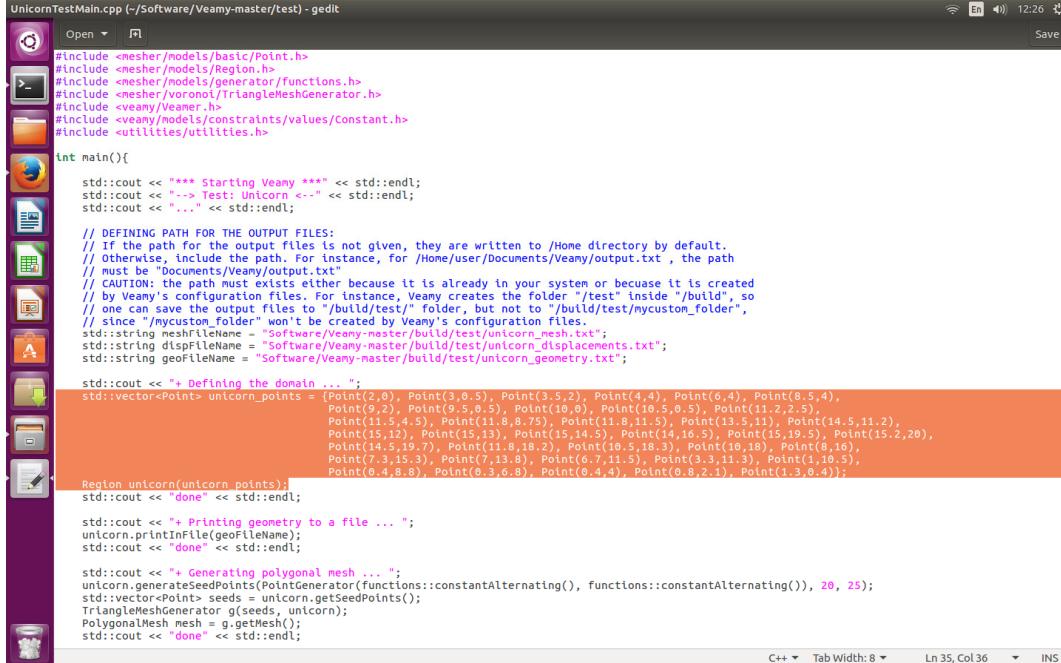
## 6.2 A toy example

In this example, a Unicorn loaded on its back and fixed at its feet is solved using Veamy. This problem is part of the numerical examples provided in:

A. Ortiz-Bernardin, C. Alvarez, N. Hitschfeld-Kahler, A. Russo, R. Silva, E. Olate-Sanzana. Veamy: an extensible object-oriented C++ library for the virtual element method. arXiv:1708.03438 [cs.MS]

You may consult the details of the geometry and boundary conditions therein as in this primer we only refer to the final main C++ setup file to run the example.

The implementation of the Unicorn problem is provided in the main C++ setup file named “UnicornTestMain.cpp”. This setup file as usual is inside “Veamy\_root\_directory/test/” folder. Go to this folder and open “UnicornTestMain.cpp” (see Fig. 18). Be sure you update the path to the output files. The important lines of code are highlighted. They provide the information for the points that define the boundary of the Unicorn.



```

UnicornTestMain.cpp (~Software/Veamy-master/test) - gedit
Open Save
#include <mesher/models/basic/Point.h>
#include <mesher/models/Region.h>
#include <mesher/models/generator/functions.h>
#include <mesher/voronoi/TriangleMeshGenerator.h>
#include <veamy/Veamer.h>
#include <veamy/models/constraints/values/Constant.h>
#include <utilities/utility.h>

int main()
{
    std::cout << "*** Starting Veamy ***" << std::endl;
    std::cout << "--- Test: Unicorn ---" << std::endl;
    std::cout << "..."

    // DEFINING PATH FOR THE OUTPUT FILES:
    // If the path for the output files is not given, they are written to /Home directory by default.
    // Otherwise, indicate the path. For instance, for /home/user/Documents/Veamy/output.txt , the path
    // will be "/Documents/Veamy/output.txt"
    // CAUTION: the path must exist either because it is already in your system or because it is created
    // by Veamy's configuration files. For instance, Veamy creates the folder "/test" inside "/build", so
    // one can save the output files to "/build/test/" folder, but not to "/build/test/mycustom_folder",
    // since "/mycustom_folder" won't be created by Veamy's configuration files.
    std::string meshFileName = "Software/Veamy-master/build/test/unicorn_mesh.txt";
    std::string dispFileName = "Software/Veamy-master/build/test/unicorn_displacements.txt";
    std::string geoFileName = "Software/Veamy-master/build/test/unicorn_geometry.txt";

    std::cout << "+ Defining the domain ... ";
    std::vector<Point> unicorn_points = {Point(2,0), Point(3,0.5), Point(3.5,2), Point(4,4), Point(6,4), Point(8.5,4),
                                         Point(9,2), Point(9.5,0.5), Point(10,0), Point(10.5,0.5), Point(11.2,2.5),
                                         Point(11.5,4.5), Point(11.8,8.75), Point(11.8,11.5), Point(13.5,11), Point(14.5,11.2),
                                         Point(15,12), Point(15,13), Point(15,14.5), Point(14,16.5), Point(15,19.5), Point(15.2,20),
                                         Point(14.5,19.7), Point(14.8,18.2), Point(10.5,18.3), Point(10,18), Point(8,16),
                                         Point(7.3,15.3), Point(7,13.8), Point(6.7,11.5), Point(3.3,11.3), Point(1,10.5),
                                         Point(0.4,8.8), Point(0.3,6.8), Point(0.4,4), Point(0.8,2.1), Point(1.3,0.4)};
    Region unicorn(unicorn_points);
    std::cout << "done" << std::endl;

    std::cout << "+ Printing geometry to a file ... ";
    unicorn.printInFile(geoFileName);
    std::cout << "done" << std::endl;

    std::cout << "+ Generating polygonal mesh ... ";
    unicorn.generateSeedPoints(PointGenerator(functions::constantAlternating(), functions::constantAlternating()), 20, 25);
    std::vector<Point> seeds = unicorn.getSeedPoints();
    TriangleMeshGenerator g(seeds, unicorn);
    PolygonalMesh mesh = g.getMesh();
    std::cout << "done" << std::endl;
}

```

Fig. 18: Main C++ setup file for the Unicorn example.

In order to run the test, follow the same steps described in the previous examples. Once you have compiled the problem, go inside “build/test/” folder and, on a terminal, type and execute:

```
./Test
```

The output files are visualized, as in the previous examples, using the MATLAB function “plotPolyMeshDisplacements.m”. The plots are shown in Fig. 19.

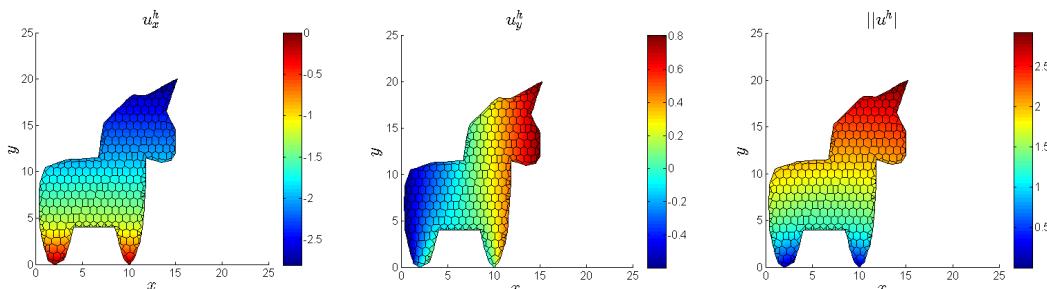


Fig. 19: Nodal displacements for the Unicorn problem are plotted using the “plotPolyMeshDisplacements.m” MATLAB function.

## 7 Geometry definition and mesh generation

Geometry definition and polygonal mesh generation in Veamy are handled using Delynoi, an object oriented C++ library for the generation of polygonal meshes that is based on the constrained Voronoi diagram. Delynoi depends on two external open source libraries, whose code is included in the repository:

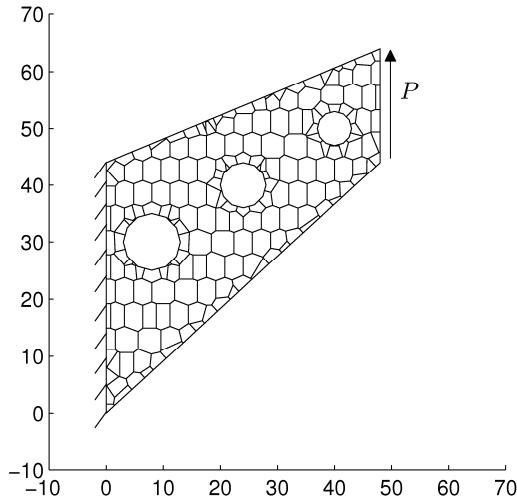
- Triangle - A Two-Dimensional Quality Mesh Generator and Delaunay Triangulator.
- Clipper - an open source freeware library for clipping and offsetting lines and polygons.

The source code, several examples and the Delynoi primer are accessible from the GitHub repository:

<https://github.com/capalvarez/Delynoi>

Nevertheless, few examples are presented in what follows.

### Example: Perforated Cook's membrane



#### Define the corner points of the Cook's membrane

```
std::vector<Point> TBeam_points = {Point(0,0), Point(48,44), Point(48,64), Point(0,44)};
```

#### Define the region formed by the points

```
Region TBeam(TBeam_points);
```

#### Define holes

```
Hole hole1 = CircularHole(Point(8,30), 5); Hole hole2 = CircularHole(Point(24,40), 4);
Hole hole3 = CircularHole(Point(40,50), 3);
```

#### Add holes to the region

```
TBeam.addHole(hole1); TBeam.addHole(hole2); TBeam.addHole(hole3);
```

#### Generate seeds points in the region

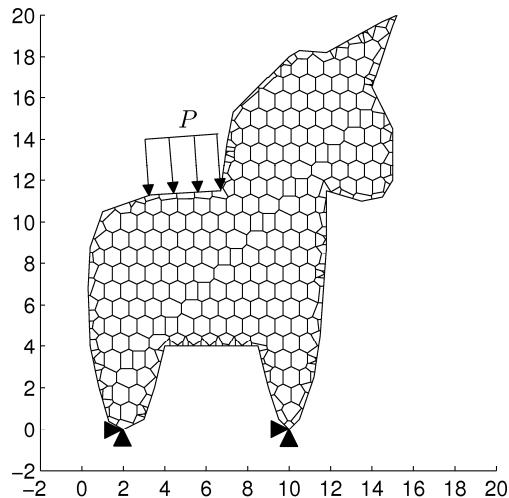
```
TBeam.generateSeedPoints(PointGenerator(functions::constantAlternating(),
                                         functions::constant()), 16, 16);
std::vector<Point> seeds = TBeam.getSeedPoints();
```

#### Use Triangle to generate a Delaunay tessellation

```
TriangleMeshGenerator g(seeds, TBeam);
```

#### Compute the polygonal mesh using the constrained Voronoi diagram

```
PolygonalMesh mesh = g.getMesh();
```

**Example: Unicorn****Define the points of the Unicorn boundary**

```
std::vector<Point> unicorn_points = {Point(2,0), Point(3,0.5), Point(3.5,2), Point(4,4),
Point(6,4), Point(8.5,4), Point(9,2), Point(9.5,0.5), Point(10,0), Point(10.5,0.5),
Point(11.2,2.5), Point(11.5,4.5), Point(11.8,8.75), Point(11.8,11.5), Point(13.5,11),
Point(14.5,11.2), Point(15,12), Point(15,13), Point(15,14.5), Point(14,16.5), Point(15,19.5),
Point(15.2,20), Point(14.5,19.7), Point(11.8,18.2), Point(10.5,18.3), Point(10,18),
Point(8,16), Point(7.3,15.3), Point(7,13.8), Point(6.7,11.5), Point(3.3,11.3), Point(1,10.5),
Point(0.4,8.8), Point(0.3,6.8), Point(0.4,4), Point(0.8,2.1), Point(1.3,0.4)};
```

**Define the region formed by the points**

```
Region unicorn(unicorn_points);
```

**Generate seeds points in the region**

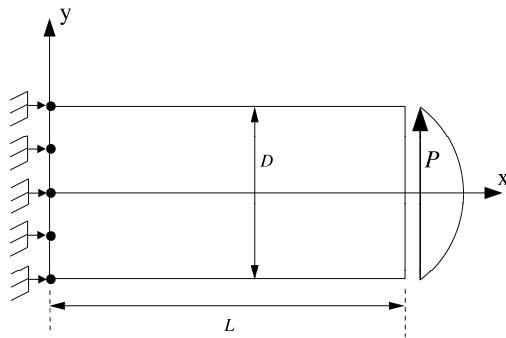
```
unicorn.generateSeedPoints(PointGenerator(functions::constantAlternating(),
functions::constantAlternating()), 20, 25);
std::vector<Point> seeds = unicorn.getSeedPoints();
```

**Use Triangle to generate a Delaunay tessellation**

```
TriangleMeshGenerator g(seeds, unicorn);
```

**Compute the polygonal mesh using the constrained Voronoi diagram**

```
PolygonalMesh mesh = g.getMesh();
```

**Example: Cantilever beam subjected to a parabolic end load****Define the corner points of the beam**

```
std::vector<Point> rectangle4x8_points={Point(0, -2), Point(8, -2), Point(8, 2), Point(0, 2)};
```

**Define the region formed by the points**

```
Region rectangle4x8(rectangle4x8_points);
```

**Generate seeds points in the region**

```
rectangle4x8.generateSeedPoints(PointGenerator(functions::constantAlternating(),
    functions::constant()), 24, 12);
std::vector<Point> seeds = rectangle4x8.getSeedPoints();
```

**Use Triangle to generate a Delaunay tessellation**

```
TriangleMeshGenerator meshGenerator = TriangleMeshGenerator(seeds, rectangle4x8);
```

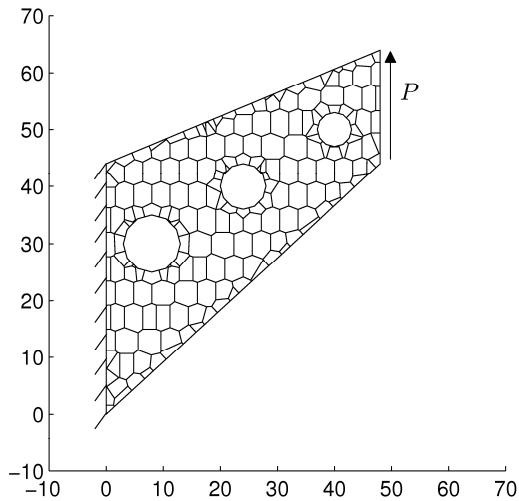
**Compute the polygonal mesh using the constrained Voronoi diagram**

```
PolygonalMesh mesh = meshGenerator.getMesh();
```

## 8 Essential and natural boundary conditions

Boundary conditions are assigned by constraining domain segments and nodes. Some examples follow.

**Example: Perforated Cook's membrane**



**Essential boundary conditions on the left edge:**

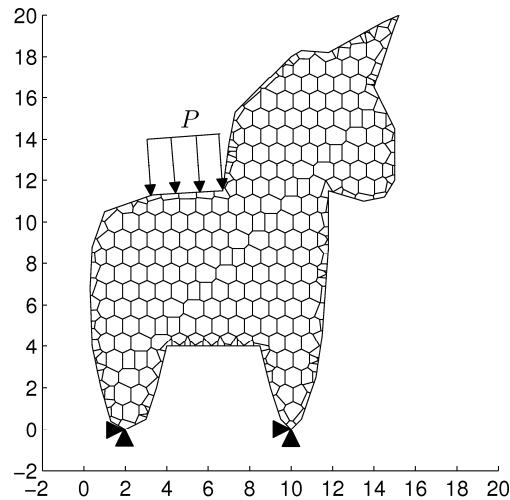
```
EssentialConstraints essential;
PointSegment leftSide(Point(0,0), Point(0,44));
SegmentConstraint left(leftSide, mesh.getPoints(), Constraint::Direction::Total,
new Constant(0));
essential.addConstraint(left, mesh.getPoints());
```

**Natural boundary condition on the right edge:**

```
NaturalConstraints natural;
PointSegment rightSide(Point(48,44), Point(48,64));
SegmentConstraint right(rightSide, mesh.getPoints(), Constraint::Direction::Vertical,
new Constant(6.25));
natural.addConstraint(right, mesh.getPoints());
```

**Add boundary conditions to the model:**

```
ConstraintsContainer container;
container.addConstraints(essential, mesh);
container.addConstraints(natural, mesh);
```

**Example: Unicorn****Essential boundary conditions at Unicorn's feet:**

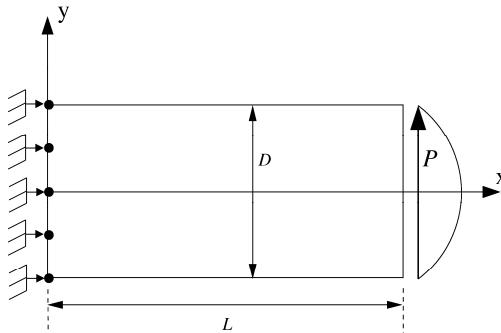
```
EssentialConstraints essential;
Point leftFoot(2,0);
PointConstraint left(leftFoot, Constraint::Direction::Total, new Constant(0));
Point rightFoot(10,0);
PointConstraint right(rightFoot, Constraint::Direction::Total, new Constant(0));
essential.addConstraint(left);
essential.addConstraint(right);
```

**Natural boundary condition on Unicorn's back:**

```
NaturalConstraints natural;
PointSegment backSegment(Point(6.7,11.5), Point(3.3,11.3));
SegmentConstraint back (backSegment, mesh.getPoints(), Constraint::Direction::Total,
new Constant(-200));
natural.addConstraint(back, mesh.getPoints());
```

**Add boundary conditions to the model:**

```
ConstraintsContainer container;
container.addConstraints(essential, mesh);
container.addConstraints(natural, mesh);
```

**Example: Cantilever beam subjected to a parabolic end load****User defined functions:**

```
double tangencial(double x, double y){
    double P = -1000; double D = 4;
    double I = std::pow(D,3)/12; double value = std::pow(D,2)/4-std::pow(y,2);
    return P/(2*I)*value;
}
double uX(double x, double y){
    double P = -1000; double Ebar = 1e7/(1 - std::pow(0.3,2));
    double vBar = 0.3/(1 - 0.3); double D = 4;
    double L = 8; double I = std::pow(D,3)/12;
    return -P*y/(6*Ebar*I)*((6*L - 3*x)*x + (2+vBar)*std::pow(y,2) -
        3*std::pow(D,2)/2*(1+vBar));
}
double uY(double x, double y){
    double P = -1000; double Ebar = 1e7/(1 - std::pow(0.3,2));
    double vBar = 0.3/(1 - 0.3); double D = 4;
    double L = 8; double I = std::pow(D,3)/12;
    return P/(6*Ebar*I)*(3*vBar*std::pow(y,2)*(L-x) + (3*L-x)*std::pow(x,2));
}
```

**Essential boundary conditions on the left edge:**

```
EssentialConstraints essential;
Function* uXConstraint = new Function(uX);
Function* uYConstraint = new Function(uY);
PointSegment leftSide(Point(0,-2), Point(0,2));
SegmentConstraint const1 (leftSide, mesh.getPoints(), Constraint::Direction::Horizontal,
uXConstraint);
essential.addConstraint(const1, mesh.getPoints());
SegmentConstraint const2 (leftSide, mesh.getPoints(), Constraint::Direction::Vertical,
uYConstraint);
essential.addConstraint(const2, mesh.getPoints());
```

**Natural boundary condition on the right edge:**

```
NaturalConstraints natural;
Function* tangencialLoad = new Function(tangencial);
PointSegment rightSide(Point(8,-2), Point(8,2));
SegmentConstraint const3 (rightSide, mesh.getPoints(), Constraint::Direction::Vertical,
tangencialLoad);
natural.addConstraint(const3, mesh.getPoints());
```

**Add boundary conditions to the model:**

```
ConstraintsContainer container;
container.addConstraints(essential, mesh);
container.addConstraints(natural, mesh);
```

## 9 Material definition

Material is specified either as a plane strain or plane stress material using the following lines of code:

```
Material* material = new MaterialPlaneStrain(240, 0.3);
ProblemConditions conditions(container, material);
```

for plane strain condition, and

```
Material* material = new MaterialPlaneStress(240, 0.3);
ProblemConditions conditions(container, material);
```

for plane stress condition.

The arguments in both “MaterialPlaneStrain” and “MaterialPlaneStress” above are the Young’s modulus (240 in the example) and Poisson’s ratio (0.3 in the example) of the material.

## 10 Veamy’s website

Check Veamy’s website for newer versions:

<http://camlab.cl/research/software/veamy/>

--- THE END ---