3)

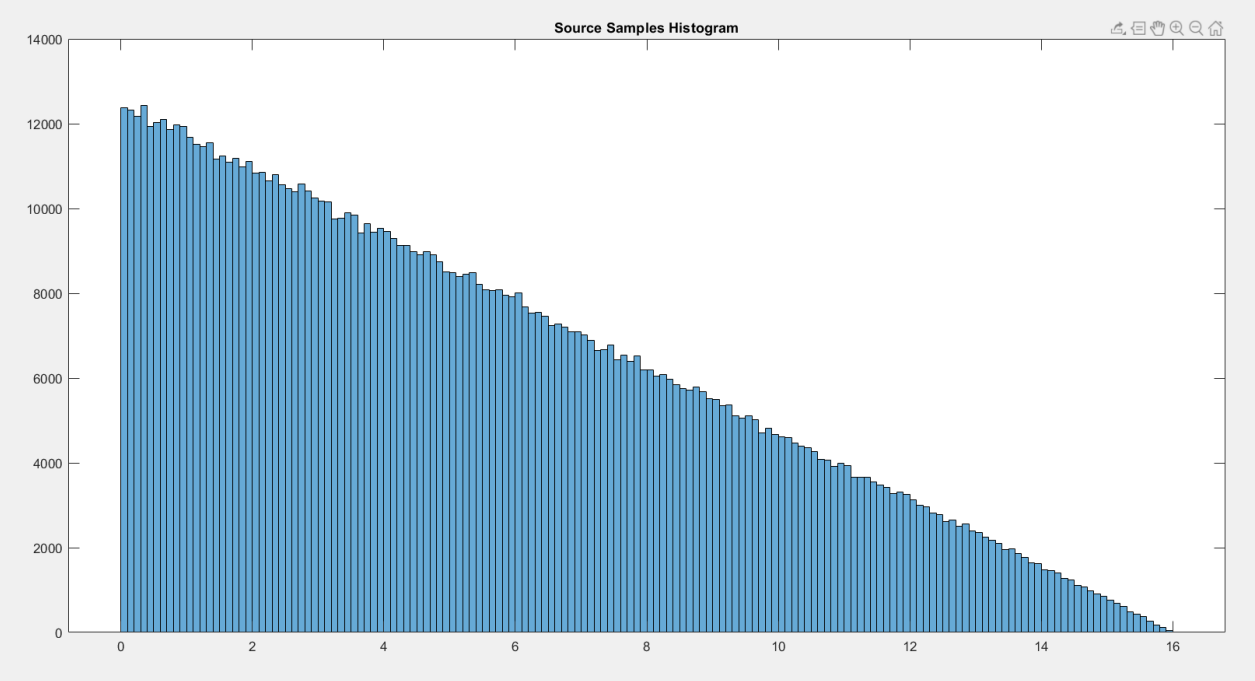

Figure 1: Source Sample Y's Histogram

In this part, 1 million source samples have constructed according to the inversion technique of the CDF and a uniform random variable is given as input [1].

$$F_Y(y) = \frac{8y - \frac{y^2}{4}}{64}$$

$$F^{-1}{}_Y(y) = x = \frac{8y - \frac{y^2}{4}}{64}, \quad 64x = 8y - \frac{y^2}{4}$$

$$8y - \frac{y^2}{4} - 64x = 0, \;\; y = -16\left(-1 + \sqrt{1-x}\right) \;\; where\; 0 \le x \le 1$$

The result is as expected because PDF of Y is a linear decreasing function which is higher near 0 and lower near 16.

4)

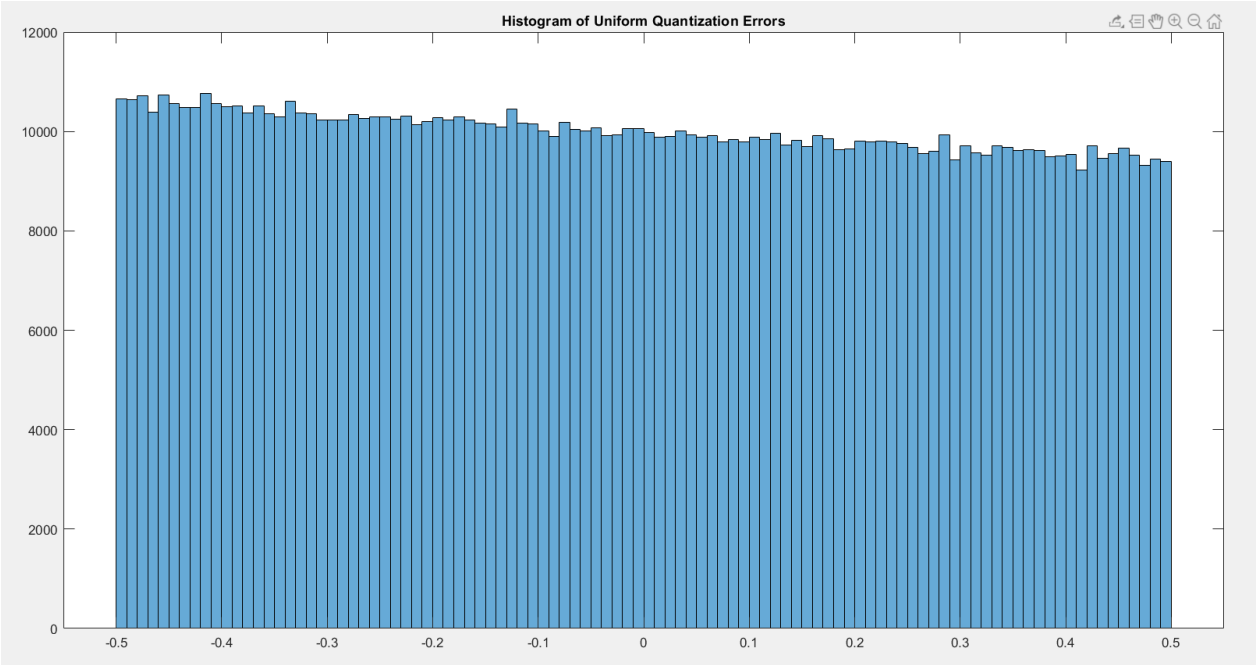

Figure 2: Uniform Quantization Errors' Histogram

Step size is chosen to be 1 since 16/16=1. 0 and 16 are decision boundaries since they are source samples boundaries. Quantization error is higher than expected but not a big amount considering the error. Expected error was 42.667 and implementation error is 42.691. This error is mainly caused by the approach to the source samples, since PDF of Y is not uniformly distributed, we tried to quantize it uniformly. The histogram that is obtained in *Figure 2* can be used for modelling PDF of quantization error. By normalizing the histogram, it will converge to the PDF of quantization error. SQNR of implementation is 27.086 dB.

The decision boundaries for uniform quantization are;

0.5000 1.5000 2.5000 3.5000 4.5000 5.5000 6.5000 7.5000 8.5000 9.5000 10.5000 11.5000 12.5000 13.5000 14.5000 15.5000

And the quantization levels are;

1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 14.0000

5)

In Lloyd-Max algorithm the quantization levels are;

$$\hat{y}_i = \frac{E[a_{i+1}] - E[a_i]}{F_Y(a_{i+1}) - F_Y(a_i)} \quad [2]$$

And the corresponding quantization boundaries are;

$$a_i = \frac{\hat{y}_i + \hat{y}_{i+1}}{2} \quad [2]$$

Initially quantization boundaries were uniform but after every iteration they changed and 1 million iteration is applied and we get;

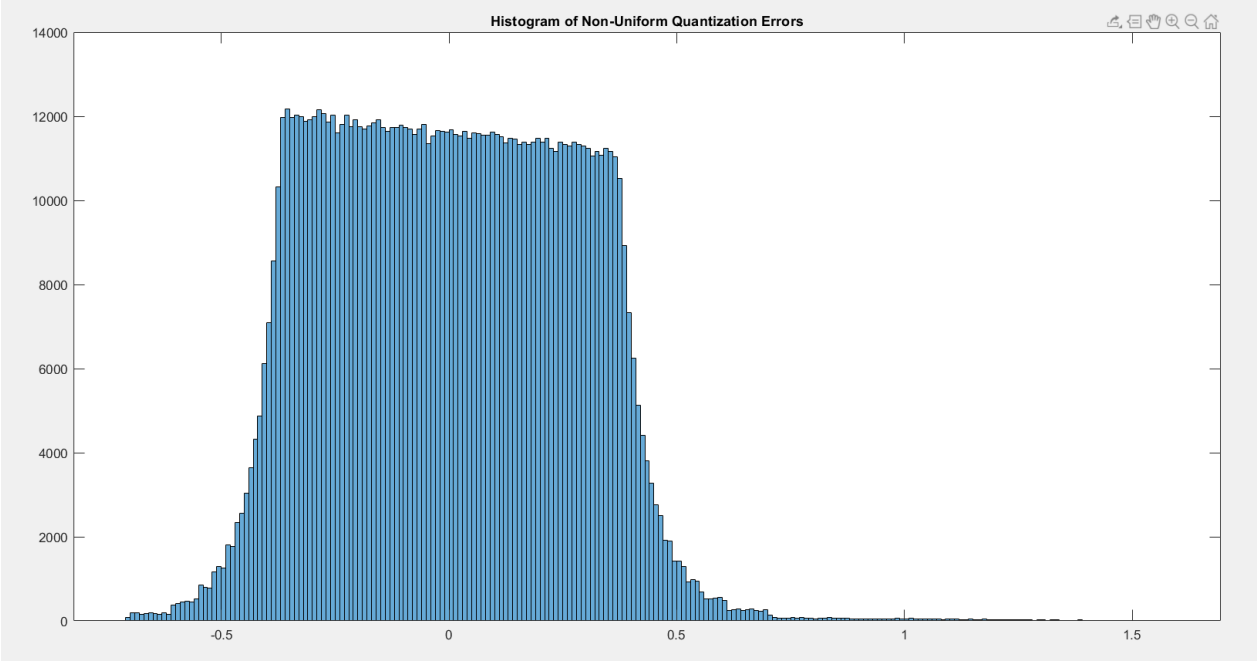

Figure 3: Non-Uniform Quantization Errors' Histogram

From the Figure 3, it can be seen that in higher probability areas there are more quantization levels which is a better approach for precise quantization. SQNR = 27.893 dB, it is higher than the uniform quantization SQNR value because Lloyd-Max algorithm minimizes the quantization error since quantization error is stable.

The decision boundaries for non-uniform quantization are;

0 0.7452 1.5027 2.2737 3.0595 3.8615
4.6816 5.5221 6.3858 7.2762 8.1982 9.1584
10.1662 11.2368 12.3971 13.7083 15.5000

The quantization levels are;

0.3696 1.1207 1.8847 2.6627 3.4562 4.2668
5.0965 5.9477 6.8238 7.7286 8.6678 9.6489 10.6835
11.7901 13.0041 14.4125