Informal Write-Up. The goal of this homework is to implement a new method for numerically solving ODEs, and use it to study either the damped, driven pendulum introduced in Chapter 3, or a gravitational system introduced in Chapter 4. You will have the freedom of deciding which system to explore, however, and you will also not need to complete a full formal write-up. Instead, you can submit a short write-up that includes figures to demonstrate that your implementation is operating correctly, and to show your result.

Problem 1. The Pendulum In this option, you will effectively complete Problems 3.18 and 3.20 in the text.

Procedure:

(a) Consulting the RK-2 handout and Appendix A, write a routine that uses RK-2 to simulate the motion of a damped, driven pendulum. You will submit this routine as part of your solution.

(b) To verify that your integrator is working correctly, reproduce the plots of θ and ω vs. time in Figure 3.6. Your formatting can be somewhat different; for instance, each trace can be shown in its own graph.

(c) While reproducing Figure 3.6, perform a convergence test and, separately, compute the difference between your traces of θ and ω vs. time with those I have posted on Canvas. Write a paragraph explaining what these tests tell you about the numerical stability of your solution.

(d) Use your routine, which should predict θ and ω values at each timestep, to reproduce Figure 3.8.

(e) Create a copy of, or a wrapper for, your pendulum routine so that you can obtain a single pair of (θ, ω) values once every driving period (T_D). Use this routine to recreate the Poincare section in Figure 3.9 (for $F_D = 1.2$).

(f) Use this routine to complete Problem 3.18 from the book, which has you construct Poincare sections for other drive strengths (F_D=1.4, 1.44 & 1.465, specifically). Make sure to remove transient behavior to achieve a clean Poincare section.

(g) Construct a final routine that wraps around the rest of your code to construct a full bifurcation diagram for a damped, driven pendulum with $1.35 < F_D < 1.5$ and (non-F_D) parameters as given at the end of the caption for Figure 3.6. Design your routine so that you can resolve the structure at least twice as well as shown in Figure 3.11 (and ideally better). Unless you are extremely clever – ok, just more clever than I – this will require plotting many values of θ for each value of F_D, even when they all lie directly on top of one another. Depending on the figure format you use, this can create a very large file: *be very careful that you don't fill up (or worse, overflow!) the storage space in your U drive / OneDrive / wherever else you are storing your files!*

When you are done, create a PDF file with your reproductions of figures 3.6, 3.8, 3.9, the figures required to complete problem 3.18 and the final bifurcation diagram. Submit those figures, with explanatory captions and the paragraph explaining the numerical precision + stability of your result, along with copies of your RK-2 routine and the code you used to reproduce Figure 3.9 in the text.

Problem 2. Planetary Motions In this option, you will use the 'Verlet method', a centered-difference ODE technique, to complete Problem 4.16 in the text.

Procedure:

(a) Consulting Appendix A.e, write a routine that uses the Verlet method to calculate the graviational interactions between three bodies, each of which is free to move. You will submit this routine as part of your solution.

(b) To verify that your integrator is working correctly, make a set of plots similar to those shown in Figures 4.12 and 4.13. Note that these plots assume a stationary Sun – to demonstrate that your system is similarly stable, you will need to set the initial conditions of the system to ensure that the net momentum is zero, as suggested in Problem 4.16.

(c) While producing the figures above, perform a convergence test and, separately, compute the difference between your orbits and those I have posted on Canvas for the Sun, Earth and Jupiter. Write a paragraph explaining what these tests tell you about the numerical precision & stability of your solution.

(d) Create a copy of, or a wrapper for, your orbital routine so that you can output a single pair of position-velocity coordinates (either orbital radius and linear velocity or angular position and velocity) for a single planet on a periodic basis. Use this routine to create 'gravitational Poincare sections' for Jupiter, Earth and the Sun, sampled on the natural orbital timescale for each body. Create diagrams that demonstrate the evolution of the system from initial conditions that produce perfectly circular orbits.

(e) Re-compute the 'Poincare sections' for each body in systems where Jupiter's mass has been increased by factors of 10, 100, and 1000 [remember each time to ensure that the system is initialized such that the net momentum is zero]. Write a paragraph summarizing the stability of each body, and how it is or is not affected by the change in Jupiter's mass.

When you are done, create a PDF file with your reproductions of figures 4.11, 4.12, and the 'gravitational Poincare' diagrams for the last two bullets above, Submit those figures, with explanatory captions and the paragraphs explaining the numerical precision + stability of your your calculation, as well as the physical stability of the system. Also include copies of your Verlet step routine and the code you used to create your Poincare diagrams.