```python
# -*- coding: utf-8 -*-
"""
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"""
import matplotlib
import matplotlib.pyplot as plt
import numpy as np
from numpy import linalg as LA
plt.style.use('classic') #Use a serif font.
from IPython.display import set_matplotlib_formats
set_matplotlib_formats('pdf', 'png')
plt.rcParams['savefig.dpi'] = 200
plt.rcParams['figure.autolayout'] = False
plt.rcParams['figure.figsize'] = 10, 6
plt.rcParams['axes.labelsize'] = 16
plt.rcParams['axes.titlesize'] = 20
plt.rcParams['font.size'] = 10
plt.rcParams['lines.linewidth'] = 2.0
plt.rcParams['lines.markersize'] = 6
plt.rcParams['legend.fontsize'] = 14
plt.rcParams['axes.facecolor'] = 'white'
plt.rcParams['savefig.facecolor']='white'
matplotlib.rcParams['xtick.direction'] = 'out'
matplotlib.rcParams['ytick.direction'] = 'out'
#This import is necessary for the isometric plot.
from mpl_toolkits.mplot3d import Axes3D
###############################################################################
# 1. Main Switchboard #########################################################
###############################################################################
#Choose an integer number of bodies to explore between 2 and 10.
N_ = 3
#Choose a stepping method. Choices are:
#   'euler' (Euler)
#   'ec'    (Euler-Cromer)
#   'rk2'   (2nd Order Runge-Kutta)
#   'vv'    (velocity Verlet)
#   'pv'    (position Verlet)
#   'vefrl' (velocity extended Forest-Ruth-like)
#   'pefrl' (position extended Forest-Ruth-like)
method_ = 'euler'

#Set time parameters for the simulation.
t0_ = 0.0 #start time in years
tf_ = 20.0 #final time in years
dt_ = 2.0 #time step in hours

###############################################################################
# 2. Initial Conditions #######################################################
###############################################################################
#Define the masses for a number of bodies (in kg).
m0 = 1.989e30 #mass of the sun
m1 = 3.285e23 #mass of Mercury
m2 = 4.867e24 #mass of Venus
m3 = 5.972e24 #mass of Earth
m4 = 6.417e23 #mass of Mars
m5 = 1.898e27 #mass of Jupiter
m6 = 5.683e26 #mass of Saturn
m7 = 8.681e25 #mass of Uranus
m8 = 1.024e26 #mass of Neptune
m9 = 1.309e22 #mass of Pluto
#Combine the masses into a global masses list.
m_ = [m0, m1, m2, m3, m4, m5, m6, m7, m8, m9]

#Create a list of colors that is at least as long as the mass list so each
#gravitationally interacting body has its own color.
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c0 = '#ff0000' #red
c1 = '#8c8c94' #gray
c2 = '#ffd56f' #pale yellow
c3 = '#005e7b' #sea blue
c4 = '#a06534' #reddish brown
c5 = '#404436' #rifle green
c6 = '#7a26e7' #magenta
c7 = '#5CCE2A' #pale green
c8 = '#000000' #black
c9 = '#4542f4' #purple
#Put all of the colors into a global colors list.
c_ = [c0, c1, c2, c3, c4, c5, c6, c7, c8, c9]

#What follows are two global initial conditions arrays. orbit() looks to this
#data, but only imports the appropriate amount of rows for the given N.

#For r0_, the first row of data represents m0's (x,y,z) initial position, the
#second row represents m1's (x,y,z) initial position, etc.
r0_ = np.array([[  1.0,  3.0,  2.0 ], #0
                [  6.0, -5.0,  4.0 ], #1
                [  7.0,  8.0, -7.0 ], #2
                [  8.0,  6.0, -2.0 ], #3
                [  8.8,  9.8, -6.8 ], #4
                [  9.8,  3.8, -7.8 ], #5
                [ -3.8,  1.8,  4.8 ], #6
                [  7.8, -2.2,  1.8 ], #7
                [  6.8, -4.1,  3.8 ], #8
                [  5.8, -9.3,  5.8 ]])*1e11
#For v0_, the first row of data represents m0's (x,y,z) initial velocity, the
#second row represents m1's (x,y,z) initial velocity, etc.
v0_ = np.array([[  0.0,  0.0,  0.0 ], #0
                [  7.0,  0.5,  2.0 ], #1
                [ -4.0, -0.5, -3.0 ], #2
                [  7.0,  0.5,  2.0 ], #3
                [  4.8,  1.3,  4.8 ], #4
                [  1.8,  1.2, -5.8 ], #5
                [  2.8, 11.3,  1.4 ], #6
                [  3.8, 10.3,  2.4 ], #7
                [  4.8,  9.3, -1.4 ], #8
                [  5.8,  0.3, -2.4 ]])*1e3

###########################################################################
# 3. Main Function ########################################################
###########################################################################
"""
Purpose:
    orbit() calculates the orbital trajectories of N gravitationally
    interacting bodies given a set of mass and initial conditions data. Note
    both the mass list and initial conditions arrays must each contain data for
    at least N bodies, but may contain more. For example, orbit() could plot
    trajectories for the first 5 of 100 bodies in a large database.
Inputs:
    [0] N = the number of bodies to be considered in the calculation (integer)
    [1] t0 = the start time in years (number)
    [2] tf = the end time in years (number)
    [3] dt = the time step in hours (number)
    [4] m = list of masses of (at least) length N (list of numbers)
    [5] r0 = (at least) an Nx3 array of initial position data (2D numpy array)
    [6] v0 = (at least) an Nx3 array of initial velocity data (2D numpy array)
    [7] method = choice of stepping method (string)

Outputs:
    [0] r = position data (3D numpy array)
    [1] v = velocity data (3D numpy array)
    [2] p = momentum data (3D numpy array)
    [3] KE = kinetic energy data (3D numpy array)
    [4] T = total kinetic energy data (3D numpy array)
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    [5] t = time data (1D numpy array)
"""
def orbit(N, t0, tf, dt, m, r0, v0, method):
    G = 6.67e-11 #universal gravitational constant in units of m^3/kg*s^2
    t0 = t0*365.26*24*3600 # Convert t0 from years to seconds.
    tf = tf*365.26*24*3600 # Convert tf from years to seconds.
    dt = dt*3600.0 #Convert dt from hours to seconds.
    steps = int(abs(tf-t0)/dt) #Define the total number of time steps.
    #Multiply an array of integers [(0, 1, ... , steps-1, steps)] by dt to get
    #an array of ascending time values.
    t = dt*np.array(range(steps + 1))

    #If you print out either r or v below, you'll see several "layers" of Nx3
    #matrices. Which layer you are on represents which time step you are on.
    #Within each Nx3 matrix, the row denotes which body, while the columns 1-3
    #(indexed 0-2 in Python) represent x-, y-, z-positions respectively. In
    #essence, each number in r or v is associated with three indices:
    #step #, body #, and coordinate #.
    r = np.zeros([steps+1, N, 3])
    v = np.zeros([steps+1, N, 3])
    r[0] = r0[0:N] #Trim the initial conditions arrays to represent N bodies.
    v[0] = v0[0:N]

    """
    Purpose:
        accel() acts as a subroutine for orbit() by returning 3D acceleration
        vectors for a number of gravitationally interacting bodies given each of
        their positions.
    Input:
        [0] r = 3D position vectors for all bodies at a certain time step
                (Nx3 numpy array)
    Output:
        [0] a = 3D acceleration vectors for each body (Nx3 numpy array)
    """
    def accel(r):
        a=np.zeros([N,3])
        #Each body's acceleration at each time step has to do with forces from
        #all other bodies. See: https://en.wikipedia.org/wiki/N-body_problem
        for i in range(N):
            #j is a list of indices of all bodies other than the ith body.
            j = list(range(i))+list(range(i+1,N))
            #Note each body's acceleration vector is a sum of N-1 terms, so for
            #each body, the terms are successively added to each other in a
            #running sum. Once all terms are added together, the ith body's
            #acceleration vector results.
            for k in range(N-1):
                a[i]=G*(m[j[k]]/LA.norm(r[i]-r[j[k]])**3*(r[j[k]]-r[i]))+a[i]
        return a

    #The simplest way to numerically integrate the accelerations into
    #velocities and then positions is with the Euler method. Note that this
    #method does not conserve energy.
    if method == 'euler':
        for i in range(steps):
            r[i+1] = r[i] + dt*v[i]
            v[i+1] = v[i] + dt*accel(r[i])

    #The Euler-Cromer method drives our next-simplest stepper.
    if method == 'ec':
        for i in range(steps):
            r[i+1] = r[i] + dt*v[i]
            v[i+1] = v[i] + dt*accel(r[i+1])

    #Getting slightly fancier, we employ the 2nd Order Runge-Kutta method.
    if method == 'rk2':
        for i in range(steps):
            v_iphalf = v[i] + accel(r[i])*(dt/2) # (i.e. v[i+0.5])
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            r_iphalf = r[i] + v[i]*(dt/2)
            v[i+1] = v[i] + accel(r_iphalf)*dt
            r[i+1] = r[i] + v_iphalf*dt

    #Here is a velocity Verlet implementation.
    #See: http://young.physics.ucsc.edu/115/leapfrog.pdf
    if method == 'vv':
        for i in range(steps):
            v_iphalf = v[i] + (dt/2)*accel(r[i])
            r[i+1] = r[i] + dt*v_iphalf
            v[i+1] = v_iphalf + (dt/2)*accel(r[i+1])

    #Next is a position Verlet implementation (found in the same pdf as 'vv').
    if method == 'pv':
        for i in range(steps):
            r_iphalf = r[i] + (dt/2)*v[i]
            v[i+1] = v[i] + dt*accel(r_iphalf)
            r[i+1] = r_iphalf + (dt/2)*v[i+1]

    #EFRL refers to an extended Forest-Ruth-like integration algorithm. Below
    #are three optimization parameters associated with EFRL routines.
    e = 0.1786178958448091e0
    l = -0.2123418310626054e0
    k = -0.6626458266981849e-1
    #First we do a velocity EFRL implementation (VEFRL).
    #See: https://arxiv.org/pdf/cond-mat/0110585.pdf
    if method == 'vefrl':
        for i in range(steps):
            v1 = v[i] + accel(r[i])*e*dt
            r1 = r[i] + v1*(1-2*l)*(dt/2)
            v2 = v1 + accel(r1)*k*dt
            r2 = r1 + v2*l*dt
            v3 = v2 + accel(r2)*(1-2*(k+e))*dt
            r3 = r2 + v3*l*dt
            v4 = v3 + accel(r3)*k*dt
            r[i+1] = r3 + v4*(1-2*l)*(dt/2)
            v[i+1] = v4 + accel(r[i+1])*e*dt

    #Next is a position EFRL (PEFRL) (found in the same pdf as 'vefrl').
    if method == 'pefrl':
        for i in range(steps):
            r1 = r[i] + v[i]*e*dt
            v1 = v[i] + accel(r1)*(1-2*l)*(dt/2)
            r2 = r1 + v1*k*dt
            v2 = v1 + accel(r2)*l*dt
            r3 = r2 + v2*(1-2*(k+e))*dt
            v3 = v2 + accel(r3)*l*dt
            r4 = r3 + v3*k*dt
            v[i+1] = v3 + accel(r4)*(1-2*l)*(dt/2)
            r[i+1] = r4 + v[i+1]*e*dt

    #Lastly, derive kinetic energy and momentum data from the velocity data.
    T = np.zeros((steps+1,N,1)) #array for total kinetic energy data
    KE = np.zeros((steps+1,N,3)) #array for 3D kinetic energy data
    p = np.zeros((steps+1,N,3)) #array for 3D momentum data
    v2 = v**2 #Square all velocities for use in the energy calculation.
    for i in range(steps):
        for j in range(N):
            KE[i,j,:] = (m[j]/2)*v2[i,j,:]
            T[i,j,0] = sum(KE[i,j,:])
            p[i,j,:] = m[j]*v[i,j,:]

    return r, v, p, KE, T, t

################################################################################
# 5. Data Generation ###########################################################
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r, v, p , KE, T, t = orbit(N_, t0_, tf_, dt_, m_, r0_, v0_, method_)

#############################################################################
# 6. Figures ################################################################
#############################################################################
#Select a body index from 0, 1, ..., N-1. Phase space and energy plots will be
#generated for the selected body.
mf = 1
mf_lab = '$m_'+str(mf)+'$' #Create a LaTeX-wrapped label for mf.

#Generate an ascending list of integers from 2 to N and then change the
#elements into strings for use in figure titles.
words = list(range(2,N_+1))
words = [str(x) for x in words]
#Wrap each string in LaTeX so it has a serif font on the plot.
for i in range(len(words)):
    words[i] = '$\mathrm{'+words[i]+'\ }$'
Nstr = words[N_-2] #Note the index shift because words[0]='2'.

labs = [None]*N_ #Create a list to store labels for the masses (m0, m1, etc.).
#Wrap each label with LaTeX math mode so it prints with a serif font.
for i in range(N_):
    labs[i] = r'$m_'+str(i)+'$'

a_ = 0.7 #Set a global transparency value so we can see where orbits overlap.
#----------------------------------------------------------------------------
fig1 = plt.figure(1, facecolor='white') #3D plot of orbital trajectories
ax1 = fig1.add_subplot(1,1,1, projection='3d')
plt.title(r'%s'%Nstr+r'$\mathrm{Orbiting\ Bodies}$', y=1.05)
ax1.set_xlabel(r'$\mathrm{x-Position}\ \mathrm{(m)}$', labelpad=10)
ax1.set_ylabel(r'$\mathrm{y-Position}\ \mathrm{(m)}$', labelpad=10)
ax1.set_zlabel(r'$\mathrm{z-Position}\ \mathrm{(m)}$', labelpad=10)
for i in range(N_): #For all times, plot mi's (x,y,z) data.
    ax1.plot(r[:, i, 0], r[:, i, 1], r[:, i, 2], color=c_[i], label=labs[i],
          alpha=a_)
ax1.axis('equal')
plt.legend(loc='upper left')
plt.savefig('fig1', bbox_inches='tight')
#----------------------------------------------------------------------------
fig2 = plt.figure(2, facecolor='white') #fig1 as viewed from the +x-axis
ax2 = fig2.add_subplot(111)
plt.title(r'%s'%Nstr+r'$\mathrm{Orbiting\ Bodies\ }$'+'\n'
          +r'$\mathrm{as\ Viewed \ From\ the\ Positive\ x-Axis}$', y=1.05)
ax2.set_xlabel(r'$\mathrm{y-Position}\ \mathrm{(m)}$')
ax2.set_ylabel(r'$\mathrm{z-Position}\ \mathrm{(m)}$')
for i in range(N_): #For all times, plot mi's (y,z) data.
    ax2.plot(r[:, i, 1], r[:, i, 2], color=c_[i], label=labs[i], alpha=a_)
ax2.axis('equal')
ax2.legend(loc='lower right')
plt.savefig('fig2', bbox_inches='tight')
#----------------------------------------------------------------------------
fig3 = plt.figure(3, facecolor='white') #fig1 as viewed from the +y-axis
ax3 = fig3.add_subplot(111)
plt.title(r'%s'%Nstr+r'$\mathrm{Orbiting\ Bodies\ }$'+'\n'
          +r'$\mathrm{as\ Viewed \ From\ the\ Positive\ y-Axis}$', y=1.05)
ax3.set_xlabel(r'$\mathrm{x-Position}\ \mathrm{(m)}$')
ax3.set_ylabel(r'$\mathrm{z-Position}\ \mathrm{(m)}$')
for i in range(N_): #For all times, plot mi's (x,z) data.
    ax3.plot(r[:, i, 0], r[:, i, 2], color=c_[i], label=labs[i], alpha=a_)
ax3.axis('equal')
ax3.legend(loc='lower right')
plt.savefig('fig3', bbox_inches='tight')
#----------------------------------------------------------------------------
fig4 = plt.figure(4, facecolor='white') #fig1 as viewed from the +z-axis
ax4 = fig4.add_subplot(111)
plt.title(r'%s'%Nstr+r'$\mathrm{Orbiting\ Bodies\ }$'+'\n'
          +r'$\mathrm{as\ Viewed \ From\ the\ Positive\ z-Axis}$', y=1.05)
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ax4.set_xlabel(r'$\mathrm{x-Position}\ \mathrm{(m)}$')
ax4.set_ylabel(r'$\mathrm{y-Position}\ \mathrm{(m)}$')
for i in range(N_): #For all times, plot mi's (x,y) data.
    ax4.plot(r[:, i, 0], r[:, i, 1], color=c_[i], label=labs[i], alpha=a_)
ax4.axis('equal')
ax4.legend(loc='lower right')
plt.savefig('fig4', bbox_inches='tight')

#------------------------------------------------------------------------------
#x-Component Momentum Phase Space for mf
fig5 = plt.figure(5, facecolor='white')
ax5 = fig5.add_subplot(111)
plt.title(r'%s'%Nstr+r'$\mathrm{Orbiting\ Bodies\ }$'+'\n'
          +r'$\mathrm{x-Component\ Momentum\ Phase\ Space\ for\ }$'
          +r'%s'%mf_lab, y=1.05)
ax5.set_xlabel(r'$\mathrm{x-Position}\ \mathrm{(m)}$')
ax5.set_ylabel(r'$\mathrm{x-Momentum}\ \mathrm{(kg\cdot\frac{m}{s})}$')
ax5.plot(r[:, mf, 0], p[:, mf, 0], color=c_[mf], label=labs[mf], alpha=a_)
plt.savefig('fig5', bbox_inches='tight')
#------------------------------------------------------------------------------
#y-Component Momentum Phase Space for mf
fig6 = plt.figure(6, facecolor='white')
ax6 = fig6.add_subplot(111)
plt.title(r'%s'%Nstr+r'$\mathrm{Orbiting\ Bodies\ }$'+'\n'
          +r'$\mathrm{y-Component\ Momentum\ Phase\ Space\ for\ }$'
          +r'%s'%mf_lab, y=1.05)
ax6.set_xlabel(r'$\mathrm{y-Position}\ \mathrm{(m)}$')
ax6.set_ylabel(r'$\mathrm{y-Momentum}\ \mathrm{(kg\cdot\frac{m}{s})}$')
ax6.plot(r[:, mf, 1], p[:, mf, 1], color=c_[mf], label=labs[mf], alpha=a_)
plt.savefig('fig6', bbox_inches='tight')
#------------------------------------------------------------------------------
#z-Component Momentum Phase Space for mf
fig7 = plt.figure(7, facecolor='white')
ax7 = fig7.add_subplot(111)
plt.title(r'%s'%Nstr+r'$\mathrm{Orbiting\ Bodies\ }$'+'\n'
          +r'$\mathrm{z-Component\ Momentum\ Phase\ Space\ for\ }$'
          +r'%s'%mf_lab, y=1.05)
ax7.set_xlabel(r'$\mathrm{z-Position}\ \mathrm{(m)}$')
ax7.set_ylabel(r'$\mathrm{z-Momentum}\ \mathrm{(kg\cdot\frac{m}{s})}$')
ax7.plot(r[:, mf, 2], p[:, mf, 2], color=c_[mf], label=labs[mf], alpha=a_)
plt.savefig('fig7', bbox_inches='tight')

#------------------------------------------------------------------------------
#x-Component Kinetic Energy Phase Space for mf
fig8 = plt.figure(8, facecolor='white')
ax8 = fig8.add_subplot(111)
plt.title(r'%s'%Nstr+r'$\mathrm{Orbiting\ Bodies}$'+'\n'
          +r'$\mathrm{x-Component\ Kinetic\ Energy\ Phase\ Space\ for\ }$'
          +r'%s'%mf_lab, y=1.05)
ax8.set_xlabel(r'$\mathrm{x-Position}\ \mathrm{(m)}$')
ax8.set_ylabel(r'$\mathrm{x-Kinetic\ Energy}\ \mathrm{(J)}$')
ax8.plot(r[:, mf, 0], KE[:, mf, 0], color=c_[mf], label=labs[mf], alpha=a_)
plt.savefig('fig8', bbox_inches='tight')
#------------------------------------------------------------------------------
#y-Component Kinetic Energy Phase Space for mf
fig9 = plt.figure(9, facecolor='white')
ax9 = fig9.add_subplot(111)
plt.title(r'%s'%Nstr+r'$\mathrm{Orbiting\ Bodies}$'+'\n'
          +r'$\mathrm{y-Component\ Kinetic\ Energy\ Phase\ Space\ for\ }$'
          +r'%s'%mf_lab, y=1.05)
ax9.set_xlabel(r'$\mathrm{y-Position}\ \mathrm{(m)}$')
ax9.set_ylabel(r'$\mathrm{y-Kinetic\ Energy}\ \mathrm{(J)}$')
ax9.plot(r[:, mf, 1], KE[:, mf, 1], color=c_[mf], label=labs[mf], alpha=a_)
plt.savefig('fig9', bbox_inches='tight')
#------------------------------------------------------------------------------
#z-Component Kinetic Energy Phase Space for mf
fig10 = plt.figure(10, facecolor='white')
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```
ax10 = fig10.add_subplot(111)
plt.title(r'%s'%Nstr+r'$\mathrm{Orbiting\ Bodies}$'+'\n'
          +r'$\mathrm{z-Component\ Kinetic\ Energy\ Phase\ Space\ for\ }$'
          +r'%s'%mf_lab, y=1.05)
ax10.set_xlabel(r'$\mathrm{z-Position}\ \mathrm{(m)}$')
ax10.set_ylabel(r'$\mathrm{z-Kinetic\ Energy}\ \mathrm{(J)}$')
ax10.plot(r[:, mf, 2], KE[:, mf, 2], color=c_[mf], label=labs[mf], alpha=a_)
plt.savefig('fig10', bbox_inches='tight')

#-----------------------------------------------------------------------------
#Total Kinetic Energy over Time for All Bodies
fig11 = plt.figure(11, facecolor='white')
ax11 = fig11.add_subplot(111)
plt.title(r'%s'%Nstr+r'$\mathrm{Orbiting\ Bodies\ }$'+'\n'
          +r'$\mathrm{Total\ Kinetic\ Energy\ vs.\ Time}$', y=1.05)
ax11.set_xlabel(r'$\mathrm{Time}\ \mathrm{(s)}$')
ax11.set_ylabel(r'$\mathrm{Total\ Kinetic\ Energy}\ \mathrm{(J)}$')
for i in range(N_):
    ax11.plot(t, T[:, i, :], color=c_[i], label=labs[i], alpha=a_)
ax11.legend(loc='lower right')
plt.savefig('fig11', bbox_inches='tight')
```