MLSP 2022 HW1

DFT

Deadline: 2022/03/21 23:55

Requirements:

1. The code should be written in python or matlab.
2. Zip code (.py/.m) and reports (PDF) and name the zipped file as "HW1_YOURSTUDENTID.zip" and submit it to the E3 system.
3. We will deduct a late penalty of 20% per additional late day.
4. Note that you cannot use any well-developed library. e.g. numpy.fft, scipy.fft, fft2 in matlab

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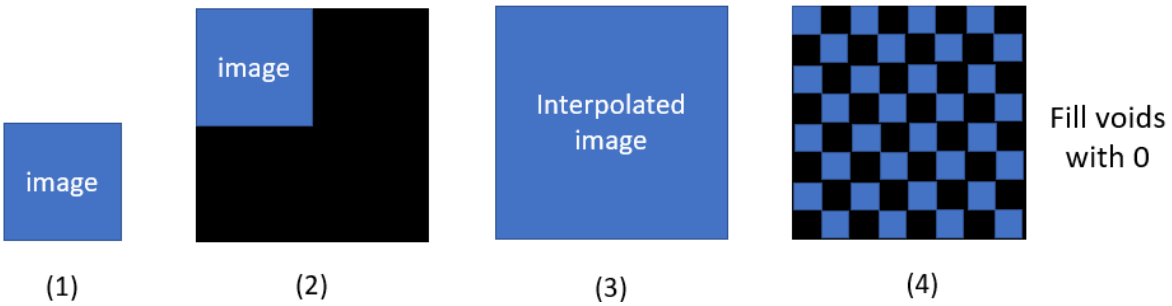
In this assignment, we look into (a) the frequency domain representation of a gray image, specifically via Discrete Fourier Transform (DFT) due to the discrete 2d input data and the simplicity of DFT. (b) We further explore how resizing an image affects the frequency domain representation. The following 4 problems should be answered with codes (4 separate code files) and explanation (1 unified PDF file).

1) Given a gray image (channel number=1) of arbitrary size below 256*256 pixels (does not have to be square), implement DFT on your own (calling DFT packages is not allowed) on the image. Show the frequency response and discuss how you implement the DFT on the report.
2) Enlarge the size of the same image by a factor of 2 while keeping the content exactly the same. The extended area should be filled with zeros. That is, after doing this, the output image will be of size (2H, 2W) and is a black image while the upper-left part (H, W) is the original image. Do the same DFT on this image. Show the frequency response and discuss the difference from 1.
3) Enlarge the original image by a factor of 2 via interpolation (you can call packages for interpolation). Do the same DFT on this image. Show the frequency response and discuss the difference from 2.
4) Enlarge the original image by a factor of 2 by filling the voids with zeros instead of interpolation. Do the same DFT on this image. Show the frequency response and discuss the difference from 2, 3.



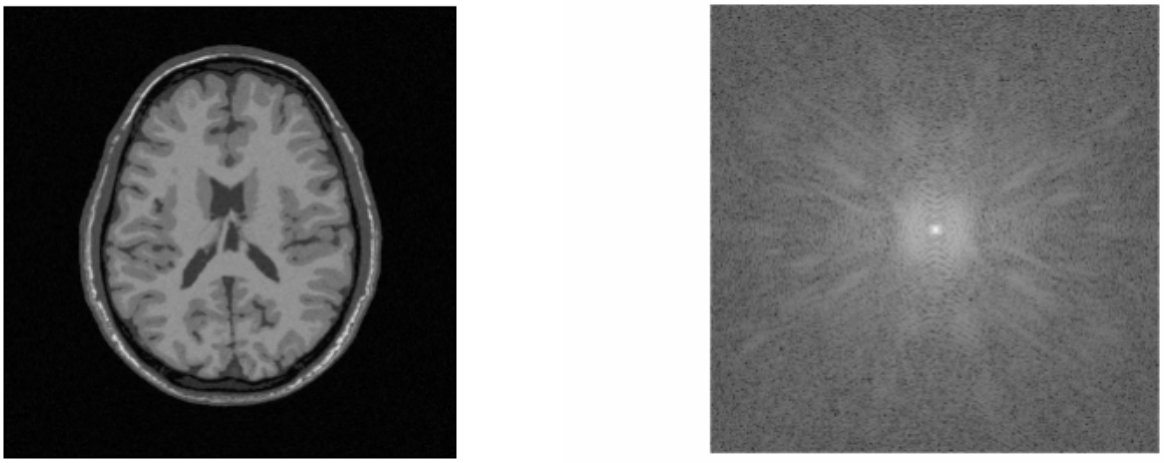
Requirement of report:

The report should contains two part:

- Part I: Explain each step of your implementation in detail
- Part II: Result of given images and your own images (at least one) for question (1) to (4). (contain original input image and upsample image and DFT result.)

  example:

  Input image ($f[m,n]$) DFT result ($F[k,l]$)



Hint: The DFT is the sampled Fourier Transform and therefore does not contain all frequencies forming an image, but only a set of samples which is large enough to fully describe the spatial domain image. The number of frequencies corresponds to the number of pixels in the spatial domain image, i.e. the image in the spatial and Fourier domain are of the same size.

For an image of size M×N, the two-dimensional DFT is given by:

$$F[k,l]=\frac{1}{MN}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}f[m,n]e^{-j2\pi\left(\frac{k}{M}m+\frac{l}{N}n\right)}$$

where $f[m, n]$ is the image in the spatial domain and the exponential term is the basis function corresponding to each point $F[k, l]$ in the Fourier space. The equation can be interpreted as: the value of each point $F[k, l]$ is obtained by multiplying the spatial image with the corresponding base function and summing the result.