Computational Methods: Project 0

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1 Given Equations and Formula

The given version of the quadratic formula was

$$x = -\frac{b}{2a} \pm \sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}}$$

The two polynomials were

$$3z^2 - 4.2z - 1.5 = 0 \tag{1}$$
$$-5.2z^2 + 176z + 1218 = 0 \tag{2}$$

2 Hand-calculated Evaluations

Polynomial (1)

$$a = 3, \ b = -4.2, \ c = -1.5$$

$$x = -\frac{-4.2}{2 \cdot 3} \pm \sqrt{\left(\frac{-4.2}{2 \cdot 3}\right)^2 - \frac{-1.5}{3}}$$

$$= \frac{4.2}{6} \pm \sqrt{\left(\frac{-4.2}{6}\right)^2 - \frac{-1.5}{3}}$$

$$= \frac{7}{10} \pm \sqrt{\left(\frac{7}{10}\right)^2 + \frac{1}{2}}$$

$$= \frac{7}{10} \pm \sqrt{\frac{49}{100} + \frac{1}{2}}$$

1

$$= \frac{7}{10} \pm \sqrt{\frac{99}{100}}$$

$$= \frac{7 + \sqrt{99}}{10}$$

$$x \approx 1.6949, \; -0.2949$$

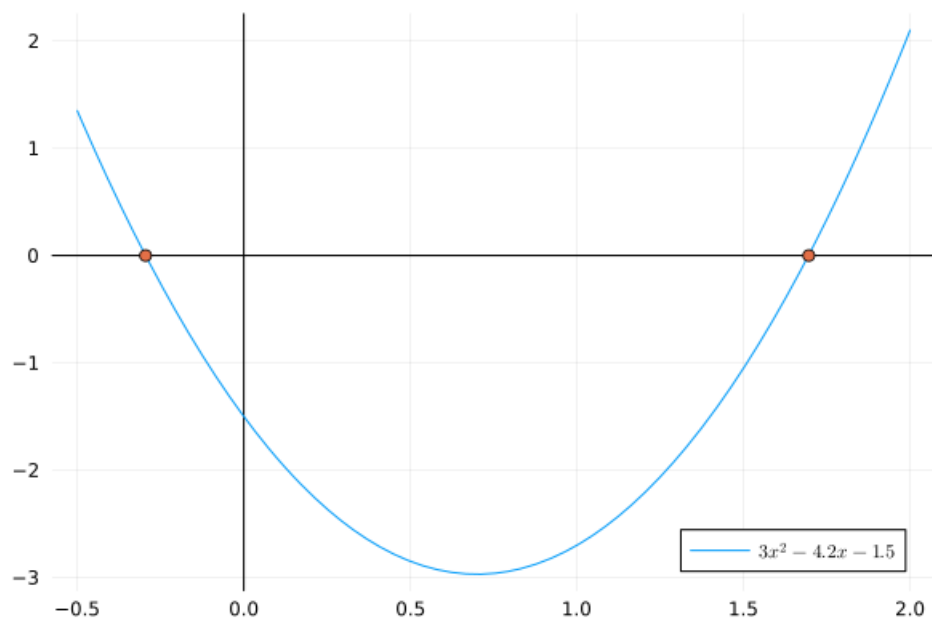## Polynomial (2)

$$a = -5.2, \; b = 176, \; c = 1218$$

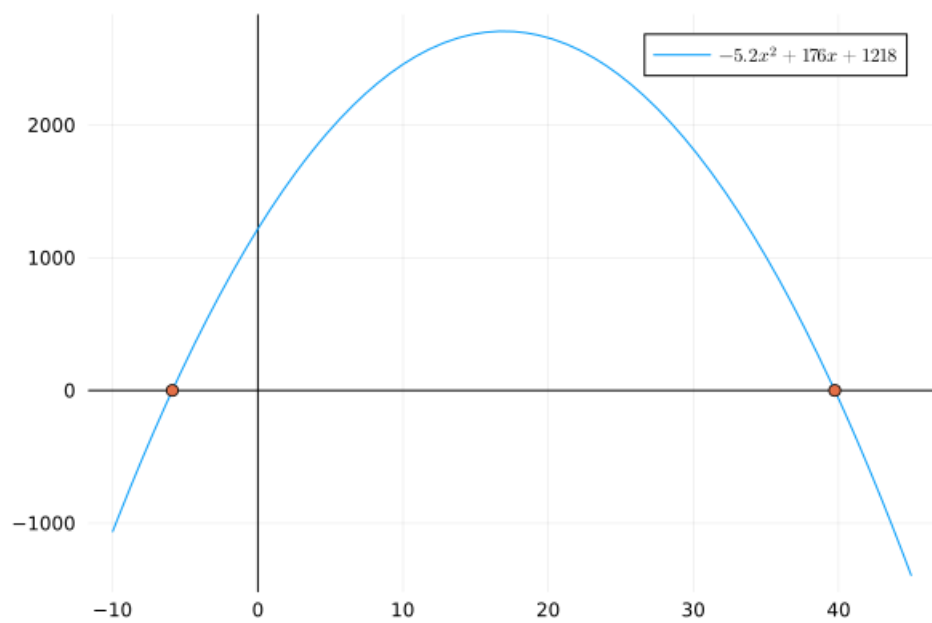$$x = -\frac{176}{-2 \cdot 5.2} \pm \sqrt{\left(\frac{176}{-2 \cdot 5.2}\right)^2 - \frac{1218}{-5.2}}$$

$$= \frac{176}{10.4} \pm \sqrt{\left(\frac{176}{-10.4}\right)^2 + \frac{1218}{5.2}}$$

$$= \frac{220}{13} \pm \sqrt{\left(-\frac{220}{13}\right)^2 + \frac{3045}{13}}$$

$$= \frac{220}{13} \pm \sqrt{\frac{220^2 + 3045 \cdot 13}{169}}$$

$$= \frac{220 \pm \sqrt{220^2 + 3045 \cdot 13}}{13}$$

$$= \frac{220 \pm \sqrt{87985}}{13}$$

$$x \approx -5.8940, \; 39.7402$$

3   Graphs

Polynomial (1)

$3x^2 - 4.2x - 1.5$

Polynomial (2)

$-5.2x^2 + 176x + 1218$

4   Results

Polynomial (1)

1. Float32: (1.6949874, -0.29498744)

2. Float64: (1.69498743710662, -0.2949874371066199)

3. BigFloat: (1.694987437106620031812553861527703702449798583984375, -0.294987437106619898585790906508918851613998413085937 5)

Polynomial (2)

1. Float32: (39.740204, -5.8940506)

2. Float64: (39.74020430088564, -5.89405045473179)

3. BigFloat: (39.7402043008856367123371455818414688110351562 5, -5.894050454731790011919656535610556602478027343 75)

5   Error Computation

Since we are comparing the true value to Julia's approximate value in this assignment, and because we are measuring the severity of the error as scaled by the actual values, the appropriate error metric to use here would be relative true error.

$$\varepsilon_t = \frac{\text{True Value - Approximate Value}}{\text{True Value}}$$

However, since our true values are in the forms of radicals, there is no way to get a perfect measurement of relative true error. We need to approximate the true values as well. Thus, the actual approximation that we should use is relative approximate error.

$$\varepsilon_a = \frac{\text{Present - Previous}}{\text{Present}}$$

Polynomial (1)

1. Float32: $\varepsilon_a = 0.0$

2. Float64: $\varepsilon_a = 0.0$

3. BigFloat: $\varepsilon_a = 0.0$

Polynomial (2)

1. Float32: $\varepsilon_a = 0.0$

2. Float64: $\varepsilon_a = 0.0$

3. BigFloat: $\varepsilon_a = 0.0$

6 Code

Formula

```
function formula_float32(a, b, c)
    comp_1 = - Float32(b / (2 * a))
    comp_2 = Float32(sqrt( (b / (2 * a)) ^ 2 - c / a ))

    return (comp_1 + comp_2, comp_1 - comp_2)
end

function formula_float64(a, b, c)
    comp_1 = - Float64(b / (2 * a))
    comp_2 = Float64(sqrt( (b / (2 * a)) ^ 2 - c / a ))

    return (comp_1 + comp_2, comp_1 - comp_2)
end

function formula_bigfloat(a, b, c)
    comp_1 = - BigFloat(b / (2 * a))
    comp_2 = BigFloat(sqrt( (b / (2 * a)) ^ 2 - c / a ))

    return (comp_1 + comp_2, comp_1 - comp_2)
end
```

Polynomials

```
eq_1 = (3, -4.2, -1.5)
eq_2 = (-5.2, 176, 1218)


p_1 = x -> eq_1[1] * x ^ 2 + eq_1[2] * x + eq_1[3]
p_2 = x -> eq_2[1] * x ^ 2 + eq_2[2] * x + eq_2[3]
```

Plots

```
plot(p_1, -0.5, 2, label = L"3x^2 - 4.2x - 1.5", framestyle=:zerolines)
scatter!([1.69598, -0.29498], [0, 0], label="")
savefig("p1.png")


plot(p_2, -10, 45, label = L"-5.2x^2 + 176x + 1218", framestyle=:zerolines)
scatter!([39.7402, -5.8940], [0, 0], label="")
savefig("p2.png")
```

Calculation

```
println(formula_float32(eq_1[1], eq_1[2], eq_1[3]))
println(formula_float64(eq_1[1], eq_1[2], eq_1[3]))
println(formula_bigfloat(eq_1[1], eq_1[2], eq_1[3]))


println(formula_float32(eq_2[1], eq_2[2], eq_2[3]))
println(formula_float64(eq_2[1], eq_2[2], eq_2[3]))
println(formula_bigfloat(eq_2[1], eq_2[2], eq_2[3]))
```

Error

```
(Float32((7 + sqrt(99)) / 10) - formula_float32(eq_1[1], eq_1[2], eq_1[3])[1])
    / formula_float32(eq_1[1], eq_1[2], eq_1[3])[1]
(Float64((7 + sqrt(99)) / 10) - formula_float64(eq_1[1], eq_1[2], eq_1[3])[1])
    / formula_float64(eq_1[1], eq_1[2], eq_1[3])[1]
(BigFloat((7 + sqrt(99)) / 10) - formula_bigfloat(eq_1[1], eq_1[2], eq_1[3])[1])
    / formula_bigfloat(eq_1[1], eq_1[2], eq_1[3])[1]
```

```
(Float32((7 + sqrt(99)) / 10) - formula_float32(eq_2[1], eq_2[2], eq_2[3])[1])
    / formula_float32(eq_2[1], eq_2[2], eq_2[3])[1]
(Float64((7 + sqrt(99)) / 10) - formula_float64(eq_2[1], eq_2[2], eq_2[3])[1])
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(BigFloat((7 + sqrt(99)) / 10) - formula_bigfloat(eq_2[1], eq_2[2], eq_2[3])[1])
    / formula_bigfloat(eq_2[1], eq_2[2], eq_2[3])[2]
```