

Exponential Accuracy Solutions of 2-D Electromagnetic Scattering From Multilayered Nonconcentric Elliptical Magnetodielectric Cylinders Under TE Illumination

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Abstract—In this article, we develop the fast computation algorithm for electromagnetic (EM) scattering from 2-D multilayered nonconcentric elliptical magnetodielectric cylinders illuminated by an EM wave with transverse electric (TE) polarization. The dielectric materials filling each homogeneous cylinder can be isotropic or biaxially anisotropic. Meanwhile, each elliptical cylinder is allowed to have its own axis ratio. The multilayer surface integral equations (SIEs) are first formed and then transformed into the spectral-domain expressions by expanding both the unknown boundary tangential field values and Green's functions with the Fourier series. The purpose of such a spectral integral method (SIM) is to lower the spatial sampling density (SD) along the multiple cylindrical boundaries and simultaneously maintain an exponentially decaying numerical error. Finally, the discretized matrix equations for SIM are assembled by letting the order of the Fourier series be equal to the number of total discretized points in each cylindrical boundary. Two numerical examples are presented to validate the computation accuracy and efficiency of the SIM when the material is either isotropic or biaxially anisotropic. It is found that the SIM has superiority over the traditional method of moments (MoM) for both the less computational time and exponentially decaying errors.

Index Terms—Anisotropic medium, multilayered magnetodielectric elliptical cylinders, spectral integral method (SIM), surface integral equations (SIEs).

I. INTRODUCTION

LECTROMAGNETIC (EM) scattering refers to the fundamental process that occurs when waves in a continuous medium interact with objects, which have obviously different dielectric properties [1]. This process will change the wave

propagation direction, its amplitude, phase, polarization, and other parameters [2]. Several numerical methods, such as finite-element method (FEM) [3], spectral-element method (SEM) [4], finite-difference time domain (FDTD) [5], and method of moments (MoM) [6], have been proposed to efficiently solve the EM scattering problems. They have also been successfully applied to geophysical exploration [7], microwave imaging [8], subsurface detection [9], periodic structure EM scattering evaluation [10], and so on.

Scattering by multilayered 2-D cylindrical structures is another typical application that has received intensive attention in recent years due to its wide applications in geophysical borehole detection, oil well logging, waveguide as well as artificial EM material design [11], [12], and so on. For example, in the transient EM nondestructive inspection via downhole casings, the transmitting and receiving coils are usually placed inside multicylindrically layered structures within which each layer has distinct permittivity, conductivity, and permeability [13]. In the nondestructive testing of oil and gas well casing, the downhole pulsed eddy-current system is also multicylindrically modeled to rapidly and accurately acquire the wide-frequency-range measurement data [14]. On the other hand, researchers have contributed a lot to the computation of Green's functions for multilayered cylindrical structures, which were applied to the layout of cylindrical microstrip antennas filled with certain dielectric materials [15], to the fast analysis of the radiation characteristics of parallel-plate cylindrical Luneberg lens antennas [16], and to the response simulation of multicomponent induction well logging [17]. Another popular application for the EM analysis of multilayered cylindrical structures in recent years is the metamaterial design to realize certain EM functions, e.g., cloaking. In [18], detailed formulas are derived to compute the EM scattering by cylindrical metasurfaces, which have arbitrary cross sections but are able to form 2-D porous cavities to achieve penetrable cloaking. In [19], a model-based approach is proposed to analyze the invisibility property of multilayered cylinders, which are filled by uniaxially anisotropic materials with spatial variations of constitutive parameters. Later studies show that even multilayered isotropic dielectric cylinders can also achieve the shielding effect as long as reasonable medium parameters and layer thickness are adopted [20]. Besides these

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74 contributions, pioneers have also conducted a lot of research
 75 on EM wave scattering from cylinders filled with composite
 76 materials, e.g., metal cylinders covered by plasma layers or
 77 other dielectrics [21], [22], [23].

78 In view of the aforementioned wide and important applications
 79 of EM scattering by multilayered cylindrical structures in
 80 different areas, researchers have proposed a series of analytical
 81 or numerical methods to precisely compute the scattered fields.
 82 The most straightforward analytical method is to approximate
 83 arbitrary EM fields inside a cylindrically multilayered medium
 84 to be the summation of a series of cylindrical vector wave
 85 functions in the spectral domain [24], [25], which has been
 86 applied to the computation of scattering from open-ended
 87 circular cavities with cylindrically periodic terminations [26].
 88 Unfortunately, the summation of cylindrical vector basis func-
 89 tions suffers from extreme arguments or high orders, which
 90 is especially severe when computing EM scattering from
 91 electrically large cylinders [27]. Most numerical methods can
 92 overcome this deficiency by avoiding directly computing the
 93 cylindrically layered Green's functions. For cylinders with
 94 homogeneous materials filled, the surface integral equation
 95 (SIE) can be directly formed and solved by MoM [28].
 96 However, when the cylinders are inhomogeneous, the whole
 97 domain instead of only the surface must be discretized. As a
 98 result, the FEM is preferred for solving the scattered EM
 99 fields from the cylinders [29]. Another efficient hybrid method
 100 to solve EM scattering by inhomogeneous cylinders with
 101 arbitrary cross sections is to combine the boundary integration
 102 and the FEM. The EM fields on the boundary are modeled by
 103 the integral equations and, thus, solved by MoM. By contrast,
 104 the interior region is discretized by the finite element, and the
 105 EM fields are solved by FEM [30]. Although these numerical
 106 methods have become rather mature in the past decades and
 107 have been applied to different EM scattering scenarios, the
 108 computational cost is restricted by a large number of sampling
 109 points per wavelength (PPW) in order to achieve a certain
 110 accuracy.

111 Bojarski [31] developed the fast solution of SIE with low
 112 sampling density (SD) for the smooth cylinder scattering of
 113 acoustic or EM waves via implementing boundary integration
 114 in the spectral domain. Also, this spectral integral method
 115 (SIM) was later applied to multilayered cylinder scattering
 116 by Schuster [32] and to sound-soft or sound-hard smooth
 117 cylinder scattering by Hu [33]. On the other hand, SIM has
 118 also been applied to the fast computation of EM scattering by
 119 periodic structures [34], by a 2-D cylinder embedded inside
 120 a layered medium [35], and by a smooth multilayered body
 121 with revolution [36]. In our previous work [37], we have
 122 applied SIM to the computation of EM scattering from
 123 multilayered magnetodielectric concentric circular cylinders.
 124 The 2-D cylinders are infinitely long in the \hat{z} -direction. The
 125 excitation plane wave is transverse magnetic (TM_z) polarized
 126 and, thus, has the E_z , H_x , and H_y components. Numerical
 127 simulations show that the computation error of SIM for
 128 the EM scattering by concentric circular cylinders decreases
 129 exponentially with the increase of the SD. Also, an SD with
 130 three PPW is able to maintain reliable computational accuracy.
 131 In this work, we further extend [37] and apply SIM to the

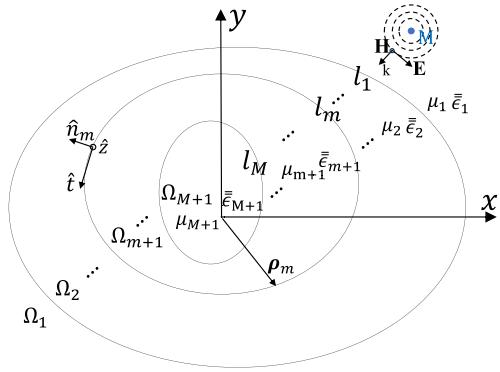


Fig. 1. Multilayered nonconcentric elliptical magnetodielectric cylinders illuminated by an EM wave that is excited by a magnetic dipole source M and has the TE_z polarization.

computation of EM scattering by multilayered nonconcentric elliptical magnetodielectric cylinders with transverse electric (TE) illumination. Compared with [37], this work has the following new contributions.

- 1) The multilayered elliptical cylinders are nonconcentric. Each cylinder is allowed to have its own axis ratio. The SIM sampling points in each cylinder circumference are freely chosen.
- 2) The dielectric medium in each elliptical cylinder is allowed to be biaxially anisotropic.
- 3) The excitation sources are magnetic dipoles. The incident wave is TE_z polarized with the H_z , E_x , and E_y components.

This article is organized as follows. In Section II, the SIE formulas for EM scattering by nonconcentric multilayered elliptical cylinders with TE_z illumination are first derived. Then, the SIM formulas are derived based on the SIE results. Meanwhile, 2-D Green's functions for isotropic and biaxially anisotropic media are evaluated. In order to expand Green's functions with Fourier series with lower orders, we must first carry out the singularity subtraction to smoothen them [33]. Finally, the spectral-domain operators are specifically defined, and the discretized algebraic equations are formulated. In Section III, two numerical examples are given to validate the correctness, efficiency, and cost of SIM for computing EM scattering from nonconcentric multilayered elliptical cylinders with isotropic and anisotropic materials, respectively. In Section IV, conclusions are drawn, and discussions are presented.

II. METHODS

In this section, the SIEs for multilayered nonconcentric elliptical cylinders illuminated by a TE_z wave are first derived. Then, by expanding both the magnetic fields and Green's functions with Fourier series in multiple elliptical boundaries, we obtain the multilayered spectral integral equations. Finally, the discretized matrix-form equations are formed.

A. 2-D SIEs for Multilayered Homogeneous Elliptical Cylinders With TE Illumination

As shown in Fig. 1, it is assumed that there are totally M nonconcentric elliptical cylindrical boundaries and $M + 1$

homogeneous materials. Their dielectric parameters from the outermost one to the innermost one are recorded as $(\mu_1, \bar{\epsilon}_1), \dots, (\mu_{M+1}, \bar{\epsilon}_{M+1})$ where

$$\bar{\epsilon}_m = \begin{bmatrix} \epsilon_{xm} & 0 \\ 0 & \epsilon_{ym} \end{bmatrix} + \frac{1}{j\omega\epsilon_0} \begin{bmatrix} \sigma_{xm} & 0 \\ 0 & \sigma_{ym} \end{bmatrix} \quad (1)$$

is the complex relative permittivity in the region Ω_m . The tensor is used to denote the 2-D biaxial anisotropy. Suppose an EM wave with TE_z polarization and $\mathbf{E}^{\text{inc}} = \hat{x}E_x^{\text{inc}} + \hat{y}E_y^{\text{inc}}$ and $\mathbf{H}^{\text{inc}} = \hat{z}H_z^{\text{inc}}$ impinges on the outermost boundary of the cylinders. According to the surface equivalence theorem, this EM scattering scenario can be described by the external and internal state equations

$$\begin{aligned} \mathbf{H}_m^{\text{inc}} &= \mathbf{H}_m(\rho_m) \\ &- \oint_{l_m} \bar{\mathbf{G}}_{\mathbf{HJ}}(\mu_m, \bar{\epsilon}_m, \rho_m, \rho'_m) \cdot \mathbf{J}_{m,o}(\rho'_m) dt' \\ &- \oint_{l_m} \bar{\mathbf{G}}_{\mathbf{HM}}(\mu_m, \bar{\epsilon}_m, \rho_m, \rho'_m) \cdot \mathbf{M}_{m,o}(\rho'_m) dt' \\ &- \oint_{l_{m-1}} \bar{\mathbf{G}}_{\mathbf{HJ}}(\mu_m, \bar{\epsilon}_m, \rho_m, \rho'_{m-1}) \cdot \mathbf{J}_{m-1,i}(\rho'_{m-1}) dt' \\ &- \oint_{l_{m-1}} \bar{\mathbf{G}}_{\mathbf{HM}}(\mu_m, \bar{\epsilon}_m, \rho_m, \rho'_{m-1}) \cdot \mathbf{M}_{m-1,i}(\rho'_{m-1}) dt' \end{aligned} \quad (2a)$$

$$\begin{aligned} \mathbf{0} &= \mathbf{H}_m(\rho_m) \\ &- \oint_{l_m} \bar{\mathbf{G}}_{\mathbf{HJ}}(\mu_{m+1}, \bar{\epsilon}_{m+1}, \rho_m, \rho'_m) \cdot \mathbf{J}_{m,i}(\rho'_m) dt' \\ &- \oint_{l_m} \bar{\mathbf{G}}_{\mathbf{HM}}(\mu_{m+1}, \bar{\epsilon}_{m+1}, \rho_m, \rho'_m) \cdot \mathbf{M}_{m,i}(\rho'_m) dt' \\ &- \oint_{l_{m+1}} \bar{\mathbf{G}}_{\mathbf{HJ}}(\mu_{m+1}, \bar{\epsilon}_{m+1}, \rho_m, \rho'_{m+1}) \cdot \mathbf{J}_{m+1,o}(\rho'_{m+1}) dt' \\ &- \oint_{l_{m+1}} \bar{\mathbf{G}}_{\mathbf{HM}}(\mu_{m+1}, \bar{\epsilon}_{m+1}, \rho_m, \rho'_{m+1}) \cdot \mathbf{M}_{m+1,o}(\rho'_{m+1}) dt' \end{aligned} \quad (2b)$$

where $m = 1, \dots, M$. $\mathbf{H}_m^{\text{inc}}$ and \mathbf{H}_m are, respectively, the incident and total magnetic fields in the m th boundary. $\bar{\mathbf{G}}_{\mathbf{HJ}}$ and $\bar{\mathbf{G}}_{\mathbf{HM}}$ are 2-D Green's functions whose expressions are given in Appendixes A and B for both isotropic and biaxially anisotropic materials. The ρ_m and ρ'_m , respectively, denote the field point position and equivalent source point position in the m th boundary. $\mathbf{J}_{m,o}$ and $\mathbf{J}_{m,i}$, respectively, denote the equivalent electric current in the outside and inside sides of the m th boundary. In a similar way, $\mathbf{M}_{m,o}$ and $\mathbf{M}_{m,i}$, respectively, denote the equivalent magnetic current in the outside and inside sides of the m th boundary. Finally, it should be emphasized that only $\mathbf{H}_1^{\text{inc}} = \mathbf{H}^{\text{inc}}$ and $\mathbf{H}_2^{\text{inc}}, \dots, \mathbf{H}_M^{\text{inc}}$ are zero in our problem. Meanwhile, the integrals \oint_{l_0} and $\oint_{l_{M+1}}$ are automatically set as zero, since the zeroth and $(M+1)$ th boundaries do not exist. Because the tangential components of the electric or magnetic fields in each boundary are continuous, we have

$$\mathbf{J}_{m,o} = -\mathbf{J}_{m,i} = \hat{n}'_m \times \mathbf{H}_m \quad (3a)$$

$$\mathbf{M}_{m,o} = -\mathbf{M}_{m,i} = -\hat{n}'_m \times \mathbf{E}_m \quad (3b)$$

where $\hat{n}'_m = \hat{x}n'_{xm} + \hat{y}n'_{ym}$ is the outward unit normal vector in the m th boundary. Equation (3), therefore, can be rewritten as follows:

$$J_{xm,o} = -J_{xm,i} = n'_{ym} H_{zm} \quad (4a)$$

$$J_{ym,o} = -J_{ym,i} = -n'_{xm} H_{zm} \quad (4b)$$

$$M_{zm,o} = -M_{zm,i} = -E_{tm} \quad (4c)$$

where H_{zm} is the \hat{z} component of the total magnetic field in the m th boundary, while E_{tm} is the tangential component of the total electric field. In order to facilitate further derivation, we first define the new variable \bar{E}_t and the new symbolic operators \mathcal{K}_{pq}^h and \mathcal{L}_{pq}^h

$$\bar{E}_t = \frac{1}{\eta_0} E_t \quad (5a)$$

$$\mathcal{K}_{pq}^{h,s}\{H_{zq}\} = \oint_{l_q} G_{HJ}^{zx}(\mu_s, \bar{\epsilon}_s, \rho_p, \rho'_q) n'_{yq} \{H_{zq}(\rho'_q)\} dt' \quad (226)$$

$$- \oint_{l_q} G_{HJ}^{zy}(\mu_s, \bar{\epsilon}_s, \rho_p, \rho'_q) n'_{xq} \{H_{zq}(\rho'_q)\} dt' \quad (227)$$

$$\mathcal{L}_{pq}^{h,s}\{\bar{E}_{tq}\} = -\eta_0 \oint_{l_q} G_{HM}^{zz}(\mu_s, \bar{\epsilon}_s, \rho_p, \rho'_q) \{\bar{E}_{tq}(\rho'_q)\} dt' \quad (229)$$

where η_0 denotes the intrinsic impedance of the free space, the superscript h means the magnetic field is evaluated, $s = 1, \dots, M+1$ is the material index, the subscript $p = 1, \dots, M$ is the index of the boundary in which the field point locates, $q = 1, \dots, M$ is the index of the boundary in which the equivalent source point locates, and \oint is the principal-value integral with the singularity extracted when $p = q$ and ρ_p and ρ'_q overlap in the q th boundary. Note the scaling of the electric field in (5a) is to guarantee that the solved electric field values and magnetic field values are commensurate.

Then, by substituting (4) and $\bar{\mathbf{G}}_{\mathbf{HJ}}$ as well as $\bar{\mathbf{G}}_{\mathbf{HM}}$ given in Appendixes A and B into (2), considering the singularity extraction when the source point and field point overlap in the m th boundary [38], and making use of the definitions given in (5), we obtain the compact form of the state equation (6) for the m th boundary, which is shown at the bottom of the next page. Finally, we let m change from 1 to M , assemble all M state equations similar to (6) together, and obtain the multilayered boundary integral equations with the unknowns $H_{z,1,\dots,M}$ and $\bar{E}_{t,m=1,\dots,M}$, which can be easily discretized and solved by MoM [28].

The scattered magnetic fields at the receiver array can be obtained by applying the operators \mathcal{K}^h and \mathcal{L}^h in (5) to the total field vectors on the boundaries if we replace ρ_p with the receiver position ρ_r and also replace \oint with \oint , since the receivers do not locate in the layer boundaries. However, in order to compute the scattered electric fields at the receiver array, we need to define the new symbolic operators \mathcal{K}_{rq}^{ex} , \mathcal{K}_{rq}^{ey} , \mathcal{L}_{rq}^{ex} , and \mathcal{L}_{rq}^{ey}

$$\mathcal{K}_{rq}^{ex,s}\{H_{zq}\} = \oint_{l_q} G_{EJ}^{xx}(\mu_s, \bar{\epsilon}_s, \rho_r, \rho'_q) n'_{yq} \{H_{zq}(\rho'_q)\} dt' \quad (260)$$

$$-\oint_{l_q} G_{EJ}^{xy}(\mu_s, \bar{\epsilon}_s, \rho_r, \rho'_q) n'_{xq} \{H_{zq}(\rho'_q)\} dt' \quad (7a)$$

$$\begin{aligned} \mathcal{K}_{rq}^{ey,s} \{H_{zq}\} &= \oint_{l_q} G_{EJ}^{yx}(\mu_s, \bar{\epsilon}_s, \rho_r, \rho'_q) n'_{yq} \{H_{zq}(\rho'_q)\} dt' \\ &\quad - \oint_{l_q} G_{EJ}^{yy}(\mu_s, \bar{\epsilon}_s, \rho_r, \rho'_q) n'_{xq} \{H_{zq}(\rho'_q)\} dt' \end{aligned} \quad (7b)$$

$$\mathcal{L}_{rq}^{ex,s} \{\bar{E}_{tq}\} = -\eta_0 \oint_{l_q} G_{EM}^{xz}(\mu_s, \bar{\epsilon}_s, \rho_r, \rho'_q) \{\bar{E}_{tq}(\rho'_q)\} dt' \quad (7c)$$

$$\mathcal{L}_{rq}^{ey,s} \{\bar{E}_{tq}\} = -\eta_0 \oint_{l_q} G_{EM}^{yz}(\mu_s, \bar{\epsilon}_s, \rho_r, \rho'_q) \{\bar{E}_{tq}(\rho'_q)\} dt' \quad (7d)$$

where s is the index of the material in which the receiver locates, the superscript ex means the \hat{x} component of the electric field, while ey means the \hat{y} component of the electric field, and the subscript r denotes the receiver. In addition, the scattered fields at a certain receiver are only contributed by the equivalent current in two adjacent boundaries.

The operators defined in (5) and (7) are all in the spatial domain. The MoM to solve the multilayered SIEs is also implemented in the spatial domain. In this work, we adopt the pulse basis and point matching method. Because it is rather trivial, MoM will not be discussed here. Usually, at least a ten-PPW SD for the discretization and implementation of MoM is required in the elliptical boundaries to maintain reliable numerical accuracy [28]. Unfortunately, this high SD will lead to an unaffordable computational cost when the elliptical scatterer has a large electrical size and numerous layer boundaries. One of the feasible methods to lower the computational cost is to implement surface integrals in the spectral domain. In other words, we expand the operators defined in (5) with Fourier series and directly solve for the Fourier coefficients of H_z and \bar{E}_t . Consequently, the necessary SD for the discretization along the elliptical boundaries is expected to be significantly lowered according to the Nyquist sampling theorem. This will be discussed in detail in the following.

B. SIM for EM Scattering by 2-D Multilayered Elliptical Cylinders With TE Illumination

Since the EM fields along the smooth cylindrical boundaries are periodic functions of the azimuthal angle θ with the cycle of 2π , they can be approximated by the truncated Fourier

series as follows:

$$H_{zm}(\theta) = \sum_{n=-\frac{N_m}{2}}^{\frac{N_m}{2}-1} \tilde{H}_{zm}^n e^{-jn\theta} \quad (8a)$$

$$\bar{E}_{tm}(\theta) = \sum_{n=-\frac{N_m}{2}}^{\frac{N_m}{2}-1} \tilde{\bar{E}}_{tm}^n e^{-jn\theta} \quad (8b)$$

where θ is the azimuthal angle for the m th elliptical boundary, $(N_m/2)$ is the order of the Fourier series, and \tilde{H}_{zm}^n and $\tilde{\bar{E}}_{tm}^n$ are the corresponding n th Fourier coefficients. We then redefine the integral along the m th elliptical boundary $\oint_{l_m} \{\} dt' = \int_0^{2\pi} \{\} (d\rho'_m/d\theta') d\theta'$, apply the operators $\mathcal{K}_{mm}^{h,m}$ and $\mathcal{L}_{mm}^{h,m}$, respectively, to the boundary fields H_{zm} and \bar{E}_{tm} in (8), exchange the order of summation and integration, and, finally, write the spectral-domain operation as follows:

$$\begin{aligned} \mathcal{K}_{mm}^{h,m} \{H_{zm}\} &= \sum_{n=-\frac{N_m}{2}}^{\frac{N_m}{2}-1} \tilde{H}_{zm}^n \int_0^{2\pi} e^{-jn\theta'} G_{HJ}^{zx}(\theta_m, \theta'_m) n'_{ym} \left| \frac{d\rho'_m}{d\theta'} \right| d\theta' \\ &\quad - \sum_{n=-\frac{N_m}{2}}^{\frac{N_m}{2}-1} \tilde{H}_{zm}^n \int_0^{2\pi} e^{-jn\theta'} G_{HJ}^{zy}(\theta_m, \theta'_m) n'_{xm} \left| \frac{d\rho'_m}{d\theta'} \right| d\theta' \end{aligned} \quad (9a)$$

$$\begin{aligned} \mathcal{L}_{mm}^{h,m} \{\bar{E}_{tm}\} &= - \sum_{n=-\frac{N_m}{2}}^{\frac{N_m}{2}-1} \tilde{\bar{E}}_{tm}^n \eta_0 \int_0^{2\pi} e^{-jn\theta'} G_{HM}^{zz}(\theta_m, \theta'_m) \left| \frac{d\rho'_m}{d\theta'} \right| d\theta' \end{aligned} \quad (9b)$$

where θ_m and θ'_m are, respectively, the field point and the equivalent source point in the m th boundary but are denoted by the azimuthal angle, and $n'_{x,m}$ and $n'_{y,m}$ also vary with θ' . The integrals in (9) with respect to the azimuth angle are actually to compute the n th Fourier coefficients of Green's functions multiplied with some variables. It is well known that Green's functions usually show strong singularity when θ_m and θ'_m approach, which will lead to the generation of high-order Fourier coefficients. In order to suppress the high-order components, we take the same smoothening procedure given in [33], since Green's functions in this work are also combinations of the first-order and second-order Hankel functions. Therefore, we can rewrite three Green's functions in (9) as the summation of smooth terms and logarithmic terms. They are, respectively, expressed as follows:

$$\begin{aligned} G_{HJ}^{zx}(\theta_m, \theta'_m) &= \bar{G}_{HJ}^{zx}(\theta_m, \theta'_m) \\ &\quad + C_{HJ}^{zx}(\theta_m, \theta'_m) \cdot \frac{2}{\pi j} \ln \left| 2 \sin \left(\frac{\theta_m - \theta'_m}{2} \right) \right| \end{aligned} \quad (331)$$

$$\begin{bmatrix} -\mathcal{K}_{m,m-1}^{h,m} & -\mathcal{L}_{m,m-1}^{h,m} & \frac{1}{2} - \mathcal{K}_{mm}^{h,m} & -\mathcal{L}_{mm}^{h,m} \\ 0 & 0 & \frac{1}{2} - \mathcal{K}_{mm}^{h,m+1} & -\mathcal{L}_{mm}^{h,m+1} \end{bmatrix} \begin{bmatrix} H_{z,m-1} \\ \bar{E}_{t,m-1} \\ H_{zm} \\ \bar{E}_{tm} \\ -\mathcal{K}_{m,m+1}^{h,m+1} \\ -\mathcal{L}_{m,m+1}^{h,m+1} \end{bmatrix} = \begin{bmatrix} H_{zm}^{\text{inc}} \\ 0 \\ H_{z,m+1} \\ \bar{E}_{t,m+1} \end{bmatrix} \quad (6)$$

$$\cdot J_1(KR) \quad (10a)$$

$$G_{HJ}^{zy}(\theta_m, \theta'_m) = \overline{G}_{HJ}^{zy}(\theta_m, \theta'_m) + C_{HJ}^{zy}(\theta_m, \theta'_m) \cdot \frac{2}{\pi j} \ln \left| 2 \sin \left(\frac{\theta_m - \theta'_m}{2} \right) \right| \quad (10b)$$

$$\cdot J_1(KR) \quad (10b)$$

$$G_{HM}^{zz}(\theta_m, \theta'_m) = \overline{G}_{HM}^{zz}(\theta_m, \theta'_m) + C_{HM}^{zz} \cdot \frac{2}{\pi j} \ln \left| 2 \sin \left(\frac{\theta_m - \theta'_m}{2} \right) \right| \cdot J_0(KR) \quad (10c)$$

$$(10c)$$

where $KR = k_m |\rho_m(\theta_m) - \rho'_m(\theta'_m)|$ for the isotropic medium but is equal to $k_0(\mu_m)^{1/2}([\epsilon_{ym}(x(\theta_m) - x'(\theta'_m))^2 + \epsilon_{xm}(y(\theta_m) - y'(\theta'_m))^2])^{1/2}$ for the biaxially anisotropic medium. The three coefficients in front of the logarithmic terms are expressed as follows:

$$C_{HJ}^{zx} = -\frac{jk_m}{4} \sin[\phi(\theta_m, \theta'_m)] \quad (11a)$$

$$C_{HJ}^{zy} = \frac{jk_m}{4} \cos[\phi(\theta_m, \theta'_m)] \quad (11b)$$

$$C_{HM}^{zz} = -\frac{\omega \epsilon_0 \epsilon_m}{4} \quad (11c)$$

for the isotropic medium but are expressed as follows:

$$C_{HJ}^{zx} = -\frac{jk_{xm}}{4} \left(\frac{\epsilon_{ym}}{\epsilon_{xm}} \right)^{\frac{1}{2}} \sin[\phi_x(\theta_m, \theta'_m)] \quad (12a)$$

$$C_{HJ}^{zy} = \frac{jk_{ym}}{4} \left(\frac{\epsilon_{xm}}{\epsilon_{ym}} \right)^{\frac{1}{2}} \cos[\phi_y(\theta_m, \theta'_m)] \quad (12b)$$

$$C_{HM}^{zz} = -\frac{\omega \epsilon_0 \sqrt{\epsilon_{xm} \epsilon_{ym}}}{4} \quad (12c)$$

for the biaxially anisotropic medium. By substituting (10) into (9), we come to the final forms of the operators

$$\begin{aligned} & \tilde{\mathcal{K}}_{mm}^{h,m} \{ H_{zm} \} \\ &= \sum_{n=-\frac{N_m}{2}}^{\frac{N_m}{2}-1} \tilde{H}_{zm}^n \int_0^{2\pi} e^{-jn\theta'} \left(\overline{G}_{HJ}^{zx}(\theta_m, \theta'_m) n'_{ym} \left| \frac{d\rho'_m}{d\theta'} \right| \right. \\ & \quad \left. + \frac{2}{\pi j} \ln \left| 2 \sin \left(\frac{\theta_m - \theta'_m}{2} \right) \right| \right. \\ & \quad \times J_1(KR) C_{HJ}^{zx} n'_{ym} \left| \frac{d\rho'_m}{d\theta'} \right| \right) d\theta' \\ & - \sum_{n=-\frac{N_m}{2}}^{\frac{N_m}{2}-1} \tilde{H}_{zm}^n \int_0^{2\pi} e^{-jn\theta'} \left(\overline{G}_{HJ}^{zy}(\theta_m, \theta'_m) n'_{xm} \left| \frac{d\rho'_m}{d\theta'} \right| \right. \\ & \quad \left. + \frac{2}{\pi j} \ln \left| 2 \sin \left(\frac{\theta_m - \theta'_m}{2} \right) \right| \right. \\ & \quad \times J_1(KR) C_{HJ}^{zy} n'_{xm} \left| \frac{d\rho'_m}{d\theta'} \right| \right) d\theta' \end{aligned} \quad (13a)$$

$$\begin{aligned} & \tilde{\mathcal{L}}_{mm}^{h,m} \{ \bar{E}_{tm} \} \\ &= - \sum_{n=-\frac{N_m}{2}}^{\frac{N_m}{2}-1} \tilde{E}_{tm}^n \eta_0 \int_0^{2\pi} e^{-jn\theta'} \left(\overline{G}_{HM}^{zz}(\theta_m, \theta'_m) \left| \frac{d\rho'_m}{d\theta'} \right| \right. \\ & \quad \left. + \frac{2}{\pi j} C_{HM}^{zz} \ln \left| 2 \sin \left(\frac{\theta_m - \theta'_m}{2} \right) \right| \right. \\ & \quad \times J_0(KR) \left| \frac{d\rho'_m}{d\theta'} \right| \right) d\theta'. \end{aligned} \quad (13b)$$

Two points must be emphasized here. First, the aforementioned smoothening procedure is valid for the external state equation of the m th boundary. For the internal state equation described by the operators $\tilde{\mathcal{K}}_{mm}^{h,m+1}$ and $\tilde{\mathcal{L}}_{mm}^{h,m+1}$, the EM wave propagates in the $(m+1)$ th medium. Equations (10)–(12) still can be used to construct them as long as we replace k_m with k_{m+1} , k_{xm} with $k_{x,m+1}$, k_{ym} with $k_{y,m+1}$, ϵ_{xm} with $\epsilon_{x,m+1}$, and ϵ_{ym} with $\epsilon_{y,m+1}$. Second, when we construct the spectral-domain operators $\tilde{\mathcal{K}}_{m,m-1}^{h,m}$, $\tilde{\mathcal{K}}_{m,m+1}^{h,m}$, $\tilde{\mathcal{L}}_{m,m-1}^{h,m}$, and $\tilde{\mathcal{L}}_{m,m+1}^{h,m}$ to evaluate the magnetic fields at the m th boundary contributed by the equivalent current source at two adjacent boundaries, the aforementioned smoothening procedure becomes unnecessary. In this situation, the singularity of Green's functions is significantly weakened, since the source points and field points locate in different elliptical boundaries. Consequently, we can directly construct these four spectral-domain operators using similar definitions to (9) but replace the medium dielectric parameters in the m th layer with those in the $(m-1)$ th or $(m+1)$ th layer when computing Green's functions, ρ_m with ρ_{m-1} or ρ_{m+1} , \hat{n}'_m with \hat{n}'_{m-1} or \hat{n}'_{m+1} , \tilde{H}_{zm} with $\tilde{H}_{z,m-1}$ or $\tilde{H}_{z,m+1}$, and \tilde{E}_{tm} with $\tilde{E}_{t,m-1}$ or $\tilde{E}_{t,m+1}$.

Once the spectral-domain operators $\tilde{\mathcal{K}}$ and $\tilde{\mathcal{L}}$ are constructed, we can also derive the spectral-domain state equation similar to (6) for the m th boundary. Finally, we let m change from 1 to M , assemble all M spectral-domain state equations together, and obtain the multilayered spectral integral equations with the unknowns \tilde{H}_{zm} and \tilde{E}_{tm} with $m = 1, \dots, M$. In order to evaluate the scattered EM fields recorded by a certain receiver, we assume that it locates at a fictitious circular boundary, and its position is denoted by θ_r . The spectral-domain operators $\tilde{\mathcal{K}}_{rq}^{h,s}$ and $\tilde{\mathcal{L}}_{rq}^{h,s}$ can be constructed similar to (9) if we replace θ_m with θ_r , θ'_m with θ'_q , ρ'_m with ρ'_q , \hat{n}'_m with \hat{n}'_q , \tilde{H}_{zm} with \tilde{H}_{zq} , and \tilde{E}_{tm} with \tilde{E}_{tq} , and compute Green's functions using the dielectric parameters of the s th medium. The spectral-domain operators $\tilde{\mathcal{K}}_{rq}^{ex,s}$, $\tilde{\mathcal{K}}_{rq}^{ey,s}$, $\tilde{\mathcal{L}}_{rq}^{ex,s}$, and $\tilde{\mathcal{L}}_{rq}^{ey,s}$ can be constructed following the similar procedure but referring to (7). Finally, it should be emphasized that the smoothening procedure in (10)–(12) is unnecessary for the operators used to compute the scattered fields.

C. Discretization of the 2-D Spectral Integral Equations

In the spectral-domain operators $\tilde{\mathcal{K}}$ and $\tilde{\mathcal{L}}$, both the field point denoted by θ and the source point denoted by θ' vary continuously along the elliptical circumference. For the m th boundary, we can discretize it

$$\theta_{mi} = 2\pi \left(i + \frac{1}{2} \right) / N_m, \quad i = 0, 1, \dots, N_m - 1 \quad (14a)$$

$$\theta'_{mi'} = 2\pi i' / N_m, \quad i' = 0, 1, \dots, N_m - 1 \quad (14b)$$

where the source point and the field point have misaligned half grid in the circumference to avoid evaluating the Bessel

functions when $\theta = \theta'$. In addition, the discretization point number must remain the same as the Fourier series order used in (8), since only in this way can the system matrix be square. Then, the integral $\int_0^{2\pi} \{\} d\theta'$ to compute the Fourier coefficients in (13) can be approximated by algebraic summation. Thus, for a certain field point θ_{mi} , we define the coefficients

$$\begin{aligned} 421 \quad 2\pi f_{mi,m}^n &= \int_0^{2\pi} e^{-jn\theta'} \bar{G}_{HJ}^{zx}(\theta_{mi}, \theta'_m) n'_{ym} \left| \frac{d\rho'_m}{d\theta'} \right| d\theta' \\ 422 \quad &\approx \frac{2\pi}{N_m} \sum_{i'=0}^{N_m-1} e^{-jn\theta'_{mi'}} \bar{G}_{HJ}^{zx}(\theta_{mi}, \theta'_{mi'}) n'_{ym} \left| \frac{d\rho'_m}{d\theta'}(\theta'_{mi'}) \right| \end{aligned} \quad (15a)$$

$$\begin{aligned} 424 \quad 2\pi g_{mi,m}^n &= \int_0^{2\pi} e^{-jn\theta'} \bar{G}_{HJ}^{zy}(\theta_{mi}, \theta'_m) n'_{xm} \left| \frac{d\rho'_m}{d\theta'} \right| d\theta' \\ 425 \quad &\approx \frac{2\pi}{N_m} \sum_{i'=0}^{N_m-1} e^{-jn\theta'_{mi'}} \bar{G}_{HJ}^{zy}(\theta_{mi}, \theta'_{mi'}) n'_{xm} \left| \frac{d\rho'_m}{d\theta'}(\theta'_{mi'}) \right| \end{aligned} \quad (15b)$$

$$\begin{aligned} 427 \quad 2\pi h_{mi,m}^n &= \int_0^{2\pi} e^{-jn\theta'} \bar{G}_{HM}^{zz}(\theta_{mi}, \theta'_m) \left| \frac{d\rho'_m}{d\theta'} \right| d\theta' \\ 428 \quad &\approx \frac{2\pi}{N_m} \sum_{i'=0}^{N_m-1} e^{-jn\theta'_{mi'}} \bar{G}_{HM}^{zz}(\theta_{mi}, \theta'_{mi'}) \left| \frac{d\rho'_m}{d\theta'}(\theta'_{mi'}) \right| \end{aligned} \quad (15c)$$

where the subscript (mi, m) means the equivalent source locating at the m th boundary contributes to the i th discretized field point at the m th boundary. Following the similar procedure, we can also define Fourier coefficients $f_{mi,m-1}^n, g_{mi,m-1}^n, h_{mi,m-1}^n, f_{mi,m+1}^n, g_{mi,m+1}^n$, and $h_{mi,m+1}^n$. Note, in these cases, that the smooth \bar{G} in (15) must be replaced with original the Green's function G , since the smoothening procedure in (10)–(12) is unnecessary when the source points and field points locate in different elliptical boundaries. The other three remaining integral terms in (13) are actually to compute the Fourier coefficients of the multiplication of the logarithmic term and the Bessel function term. They can be evaluated by the convolution of their respective Fourier coefficients. Therefore, we first define the Fourier coefficients of Bessel function terms

$$\begin{aligned} 445 \quad 2\pi \hat{u}_{mi,m}^n &= \int_0^{2\pi} e^{-jn\theta'} J_1(KR) C_{HJ}^{zx} n'_{ym} \left| \frac{d\rho'_m}{d\theta'} \right| d\theta' \\ 446 \quad &\approx \frac{2\pi}{N_m} \sum_{i'=0}^{N_m-1} e^{-jn\theta'_{mi'}} J_1(KR) C_{HJ}^{zx} n'_{ym} \left| \frac{d\rho'_m}{d\theta'}(\theta'_{mi'}) \right| \end{aligned} \quad (16a)$$

$$\begin{aligned} 448 \quad 2\pi \hat{v}_{mi,m}^n &= \int_0^{2\pi} e^{-jn\theta'} J_1(KR) C_{HJ}^{zy} n'_{xm} \left| \frac{d\rho'_m}{d\theta'} \right| d\theta' \\ 449 \quad &\approx \frac{2\pi}{N_m} \sum_{i'=0}^{N_m-1} e^{-jn\theta'_{mi'}} J_1(KR) C_{HJ}^{zy} n'_{xm} \left| \frac{d\rho'_m}{d\theta'}(\theta'_{mi'}) \right| \end{aligned} \quad (16b)$$

$$451 \quad 2\pi \hat{w}_{mi,m}^n = \int_0^{2\pi} e^{-jn\theta'} C_{HM}^{zz} J_0(KR) \left| \frac{d\rho'_m}{d\theta'} \right| d\theta'$$

$$\approx \frac{2\pi}{N_m} \sum_{i'=0}^{N_m-1} e^{-jn\theta'_{mi'}} C_{HM}^{zz} J_0(KR) \left| \frac{d\rho'_m}{d\theta'}(\theta'_{mi'}) \right|. \quad (16c)$$

Then, we expand the logarithmic term in (13) with Fourier coefficients in the m th elliptical boundary

$$\ln \left| 2 \sin \left(\frac{\theta_m - \theta'_m}{2} \right) \right| = \sum_{n=-\infty}^{\infty} a_n e^{jn(\theta_m - \theta'_m)} \quad (16d)$$

$$= \sum_{n=-\infty}^{\infty} a_n e^{jn\theta_m} \cdot e^{-jn\theta'_m} \quad (16e)$$

where $a_0 = 0$, $a_n = -(1/2|n|)$ for $n \neq 0$, and $a_n e^{jn\theta_m}$ is the n th coefficient. Finally, the three integral terms in (13) for the multiplication of the logarithmic term and the Bessel function term can be, respectively, expressed as the following convolution sums of the aforementioned two groups of Fourier coefficients:

$$\begin{aligned} 463 \quad \frac{2}{\pi j} \int_0^{2\pi} e^{-jn\theta'} \ln \left| 2 \sin \left(\frac{\theta_m - \theta'_m}{2} \right) \right| J_1 \cdot C_{HJ}^{zx} n'_{ym} \left| \frac{d\rho'_m}{d\theta'} \right| d\theta' \\ 464 \quad = -4j \sum_{k=-N_m/2}^{N_m/2-1} \hat{u}_{mi,m}^k a_{n-k} e^{j(n-k)\theta_m} \\ 465 \quad \equiv -4j u_{mi,m}^n \end{aligned} \quad (18a)$$

$$\begin{aligned} 466 \quad \frac{2}{\pi j} \int_0^{2\pi} e^{-jn\theta'} \ln \left| 2 \sin \left(\frac{\theta_m - \theta'_m}{2} \right) \right| J_1 \cdot C_{HJ}^{zy} n'_{xm} \left| \frac{d\rho'_m}{d\theta'} \right| d\theta' \\ 467 \quad = -4j \sum_{k=-N_m/2}^{N_m/2-1} \hat{v}_{mi,m}^k a_{n-k} e^{j(n-k)\theta_m} \\ 468 \quad \equiv -4j v_{mi,m}^n \end{aligned} \quad (18b)$$

$$\begin{aligned} 469 \quad \frac{2\eta_0}{\pi j} \int_0^{2\pi} e^{-jn\theta'} C_{HM}^{zz} \ln \left| 2 \sin \left(\frac{\theta_m - \theta'_m}{2} \right) \right| J_0(KR) \left| \frac{d\rho'_m}{d\theta'} \right| d\theta' \\ 470 \quad = -4j \eta_0 \sum_{k=-N_m/2}^{N_m/2-1} \hat{w}_{mi,m}^k a_{n-k} e^{j(n-k)\theta_m} \\ 471 \quad \equiv -4j \eta_0 w_{mi,m}^n \end{aligned} \quad (18c)$$

where $i = 0, \dots, N_m - 1$ is the field point index in the m th elliptical boundary and $n = -N_m/2, \dots, N_m/2 - 1$ is the order of the Fourier series. The convolution sums can be evaluated by the fast Fourier transform. Details can be found in Sections II–IV of [33] and will not be repeated here. We now update $\tilde{\mathcal{K}}_{mm}^{h,m}$ and $\tilde{\mathcal{L}}_{mm}^{h,m}$ in (13) using the definitions given in (15) and (18), construct $\tilde{\mathcal{K}}_{m,m-1}^{h,m}$, $\tilde{\mathcal{K}}_{m,m+1}^{h,m}$, $\tilde{\mathcal{L}}_{m,m-1}^{h,m}$, and $\tilde{\mathcal{L}}_{m,m+1}^{h,m}$ using $f_{mi,m-1}^n, g_{mi,m-1}^n, h_{mi,m-1}^n, f_{mi,m+1}^n, g_{mi,m+1}^n$, and $h_{mi,m+1}^n$, and, respectively, come to the external and internal discretized spectral-domain integral equations for the m th elliptical boundary

$$\begin{aligned} 483 \quad &- 2\pi \sum_{n=-\frac{N_m-1}{2}}^{\frac{N_m-1}{2}-1} \tilde{H}_{z,m-1}^n (f_{mi,m-1}^n - g_{mi,m-1}^n) \\ 484 \quad &+ 2\pi \eta_0 \sum_{n=-\frac{N_m-1}{2}}^{\frac{N_m-1}{2}-1} \tilde{E}_{t,m-1}^n h_{mi,m-1}^n + \frac{1}{2} \sum_{n=-\frac{N_m}{2}}^{\frac{N_m}{2}-1} \tilde{H}_{zm}^n e^{-jn\theta_m} \end{aligned}$$

$$\begin{aligned}
& -2\pi \sum_{n=-\frac{N_m}{2}}^{\frac{N_m}{2}-1} \tilde{H}_{zm}^n (f_{mi,m}^n - g_{mi,m}^n) \\
& + 4j \sum_{n=-\frac{N_m}{2}}^{\frac{N_m}{2}-1} \tilde{H}_{zm}^n (u_{mi,m}^n - v_{mi,m}^n) \\
& + \eta_0 \sum_{n=-\frac{N_m}{2}}^{\frac{N_m}{2}-1} \tilde{E}_{tm}^n (2\pi h_{mi,m}^n - 4j w_{mi,m}^n) \\
& = H_{zm}^{\text{inc}}(\theta_{mi}) \tag{19a}
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2} \sum_{n=-\frac{N_m}{2}}^{\frac{N_m}{2}-1} \tilde{H}_{zm}^n e^{-jn\theta_{mi}} - 2\pi \sum_{n=-\frac{N_m}{2}}^{\frac{N_m}{2}-1} \tilde{H}_{zm}^n (f_{mi,m}^n - g_{mi,m}^n) \\
& + 4j \sum_{n=-\frac{N_m}{2}}^{\frac{N_m}{2}-1} \tilde{H}_{zm}^n (u_{mi,m}^n - v_{mi,m}^n) \\
& + \eta_0 \sum_{n=-\frac{N_m}{2}}^{\frac{N_m}{2}-1} \tilde{E}_{tm}^n (2\pi h_{mi,m}^n - 4j w_{mi,m}^n) \\
& - 2\pi \sum_{n=-\frac{N_{m+1}}{2}}^{\frac{N_{m+1}}{2}-1} \tilde{H}_{z,m+1}^n (f_{mi,m+1}^n - g_{mi,m+1}^n) \\
& + 2\pi \eta_0 \sum_{n=-\frac{N_{m+1}}{2}}^{\frac{N_{m+1}}{2}-1} \tilde{E}_{t,m+1}^n h_{mi,m+1}^n \\
& = 0 \tag{19b}
\end{aligned}$$

where $i = 0, \dots, N_m - 1$. Note that when we compute Green's functions to construct the internal state equation, the dielectric parameters of the $(m + 1)$ th material must be used.

D. Algebraic System Matrix Assembly

If we let the field point index i in (19) change from 0 to $N_m - 1$, (19) can be written as the compact matrix form

$$[\mathbf{Z}_{m-1} \quad \mathbf{Z}_m \quad \mathbf{Z}_{m+1}] \begin{bmatrix} \mathbf{I}_{m-1} \\ \mathbf{I}_m \\ \mathbf{I}_{m+1} \end{bmatrix} = \mathbf{V}_m \tag{20}$$

where $\mathbf{Z}_m \in \mathbb{C}^{2N_m \times 2N_m}$, $\mathbf{Z}_{m-1} \in \mathbb{C}^{2N_m \times 2N_{m-1}}$, $\mathbf{Z}_{m+1} \in \mathbb{C}^{2N_m \times 2N_{m+1}}$, $\mathbf{I}_m \in \mathbb{C}^{2N_m \times 1}$, $\mathbf{I}_{m-1} \in \mathbb{C}^{2N_{m-1} \times 1}$, $\mathbf{I}_{m+1} \in \mathbb{C}^{2N_{m+1} \times 1}$, and $\mathbf{V}_m \in \mathbb{C}^{2N_m \times 1}$. The first N_m elements of \mathbf{I}_m are \tilde{H}_{zm}^n with n increasing from $-(N_m/2)$ to $(N_m/2) - 1$, and its last N_m elements are \tilde{E}_{tm}^n with n increasing from $-(N_m/2)$ to $(N_m/2) - 1$. \mathbf{I}_{m-1} and \mathbf{I}_{m+1} can be assembled in a similar way if we replace m with $m - 1$ or $m + 1$ in \mathbf{I}_m . In order to show the assembly of \mathbf{Z}_m , we first decompose it into

$$\mathbf{Z}_m = \begin{bmatrix} \mathbf{Z}_m^{11} & \mathbf{Z}_m^{12} \\ \mathbf{Z}_m^{21} & \mathbf{Z}_m^{22} \end{bmatrix} \tag{21}$$

where the dimensions of \mathbf{Z}_m^{11} , \mathbf{Z}_m^{12} , \mathbf{Z}_m^{21} , and \mathbf{Z}_m^{22} are all $N_m \times N_m$. The element in the i th row and n th column of \mathbf{Z}_m^{11} and \mathbf{Z}_m^{12} is, respectively,

$$\mathbf{Z}_m^{11}(i, n) = \frac{1}{2} \exp \left[-j \left(n - 1 - \frac{N_m}{2} \right) \theta_{m,i-1} \right] \delta_{n,i}$$

$$\begin{aligned}
& -2\pi \left(f_{m(i-1),m}^{n-1-\frac{N_m}{2}} - g_{m(i-1),m}^{n-1-\frac{N_m}{2}} \right) \\
& + 4j \left(u_{m(i-1),m}^{n-1-\frac{N_m}{2}} - v_{m(i-1),m}^{n-1-\frac{N_m}{2}} \right) \tag{22a}
\end{aligned}$$

$$\mathbf{Z}_m^{12}(i, n) = \eta_0 \left(2\pi h_{m(i-1),m}^{n-1-\frac{N_m}{2}} - 4j w_{m(i-1),m}^{n-1-\frac{N_m}{2}} \right) \tag{22b}$$

where $\delta_{n,i}$ is the Kronecker symbol. \mathbf{Z}_m^{21} has the same expression as that of \mathbf{Z}_m^{11} , and \mathbf{Z}_m^{22} has the same expression as that of \mathbf{Z}_m^{12} . However, the dielectric parameters of the $(m + 1)$ th material must be used when we evaluate Green's functions. \mathbf{Z}_{m-1} and \mathbf{Z}_{m+1} can be assembled in a similar way by referring to (19). However, \mathbf{Z}_{m-1}^{21} , \mathbf{Z}_{m-1}^{22} , \mathbf{Z}_{m+1}^{11} , and \mathbf{Z}_{m+1}^{12} are zeros. The first N_m elements of \mathbf{V}_m are $H_{zm}^{\text{inc}}(\theta_{mi})$ with i increasing from 0 to $N_m - 1$. The last N_m elements are zero. Finally, we let m change from 1 to M and, respectively, cascade \mathbf{Z}_m , \mathbf{I}_m , and \mathbf{V}_m , and obtain the whole matrix equation

$$\mathbf{ZI} = \mathbf{V} \tag{23}$$

where \mathbf{Z} is a complex matrix with the dimensions of $2\sum_{m=1}^M N_m \times 2\sum_{m=1}^M N_m$. \mathbf{I} and \mathbf{V} are complex vectors with the dimensions of $2\sum_{m=1}^M N_m \times 1$. \mathbf{Z} is a block-by-block tridiagonal matrix, since the total magnetic fields in the m th boundary are only contributed by equivalent current in the $(m - 1)$ th, m th, and $(m + 1)$ th boundaries. However, each block is a dense matrix. Equation (23) can be directly solved by an iterative method. Once \tilde{H}_z and \tilde{E}_t in all elliptical boundaries are obtained, they can be substituted into (8) to compute the corresponding spatial-domain field values. On the other hand, the scattered fields at the receiver array can be evaluated by first applying the discretized spectral-domain operators $\tilde{\mathcal{K}}_{rq}^{h,s}$, $\tilde{\mathcal{L}}_{rq}^{h,s}$, $\tilde{\mathcal{K}}_{rq}^{ex,s}$, $\tilde{\mathcal{K}}_{rq}^{ey,s}$, $\tilde{\mathcal{L}}_{rq}^{ex,s}$, and $\tilde{\mathcal{L}}_{rq}^{ey,s}$ to \tilde{H}_z and \tilde{E}_t and then performing the inverse Fourier transform, such as (8).

III. NUMERICAL RESULTS

In this section, we present two numerical examples to verify the expected exponential accuracy of the proposed SIM for EM scattering from multilayered nonconcentric elliptical cylinders by comparing its computation results with those from MoM as well as FEM, which is implemented through the commercial software COMSOL. Note that the FEM results are used as criteria in our work, since FEM can achieve reliable computation accuracy as long as the discretized mesh is small enough. It is assumed that the elliptical cylinders are immersed in a vacuum. The magnetic dipoles are used to excite TE_z EM waves. The operating frequency is 300 MHz. All the simulations and numerical computations are performed on a workstation with a 48-core Intel Xeon 6248R 3.0G CPU and 1024-GB RAM. COMSOL automatically uses parallel computation. In the SIM, the Fourier transforms are realized by paralleled fast algorithms.

A. Three-Layer Isotropic Nonconcentric Elliptical Cylinders

As shown in Fig. 2, three isotropic elliptical cylinders are illuminated by TE_z-polarized EM waves, which are simultaneously excited by three unit magnetic dipole transmitters. The geometry parameters of each elliptical boundary and the

TABLE I

GEOMETRY PARAMETERS OF EACH ELLIPSE AND THE ISOTROPIC DIELECTRIC PARAMETERS OF THE MEDIUM ON ITS INNER SIDE

Parameters	Ellipse	#1	#2	#3
center coordinate		(-1.0, 0.0)	(-0.5, -1.5)	(0.2, -2.0)
horizontal half axis length		6.0	4.5	3.0
vertical half axis length		8.0	4.5	2.0
ϵ		1.9	3.0	5.2
σ		1.0	2.0	3.0
μ		1.5	2.0	0.9

Remark: the units of position and length are meters; the unit of σ is mS/m.

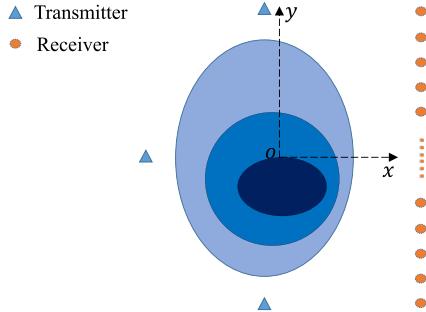


Fig. 2. Three nonconcentric elliptical magnetodielectric isotropic cylinders illuminated by TE_z -polarized EM waves, which are excited by three magnetic dipole transmitters.

isotropic dielectric parameters of the medium on its inner side are listed in Table I. Note that the ellipses are numbered from the outermost one to the innermost one. Three transmitters are located at $(-9.0, 0.0)$, $(-1.0, 10.0)$, and $(-1.0, -10.0)$ m, respectively. Totally, 49 receivers are evenly placed at a straight line in the \hat{y} -direction. The lowermost one is located at $(9.0, -10.0)$ m, and the increment between the two receivers is 0.41666 m.

First, let us verify the computation accuracy of the tangential fields H_z and E_t along three elliptical boundaries. Here, we pick up 61 representative sampling points in three boundaries, and the corresponding EM field values are displayed in Fig. 3. The FEM results by COMSOL are used as a reference. As can be seen, SIM can achieve reliable accuracy even when the spatial sampling densities along the elliptical boundaries are lowered to 2.5 PPW. Unfortunately, the traditional MoM must increase the SD to 19.8 PPW in order to reach good matches with the FEM results. Precise calculations show that the relative errors of H_z for SIM with SD = 2.5 PPW, MoM with SD = 19.8 PPW, and MoM with SD = 2.5 PPW compared with FEM results are 0.19%, 0.34%, and 5.3%, respectively. Similarly, the relative errors of E_t for the aforementioned three situations are 0.06%, 0.37%, and 5.7%, respectively. These error data indicate that SIM only needs less than 3-PPW SD to achieve reliable computation accuracy. However, MoM needs to increase the SD to near 20 PPW in order to reach the same accuracy.

Then, let us verify the computation accuracy of scattered fields at the receiver array. We also compare the EM field values for SIM with SD = 2.5 PPW, MoM with SD = 19.8 PPW, and MoM with SD = 2.5 PPW. Fig. 4 shows the corresponding results. Numerical calculations show that

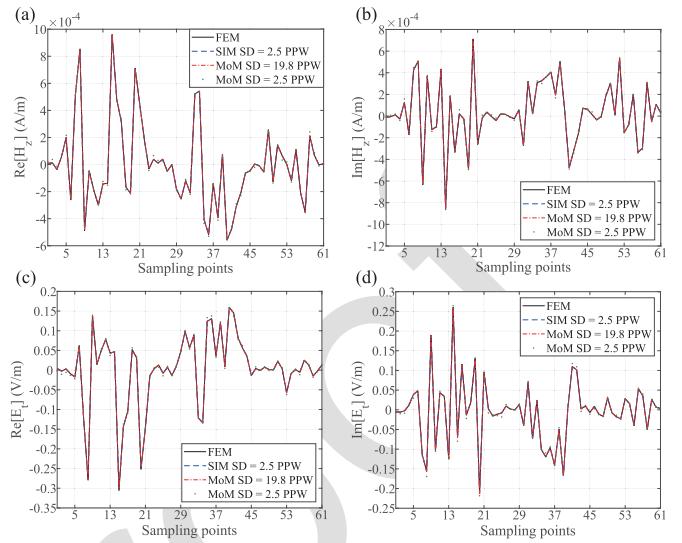


Fig. 3. Comparisons of the tangential EM fields along three elliptical boundaries computed by FEM, MoM, and SIM when the spatial SD takes different values. (a) Real part of H_z . (b) Imaginary part of H_z . (c) Real part of E_t . (d) Imaginary part of E_t .

the relative errors of E_x^{sct} for the three situations are 0.09%, 0.18%, and 3.0%, respectively, the relative errors of E_y^{sct} are 0.11%, 0.22%, and 3.5%, respectively, and the relative errors of H_z^{sct} are 0.10%, 0.22%, and 3.3%, respectively. It can be seen that the scattered fields at the receiver array computed by SIM can still achieve the 0.1% error when the SD is lowered to 2.5 PPW. However, the computation error of MoM is in the order of one hundredth for the same SD. In addition, COMSOL takes 178.3 s and consumes 59.8-GB memory to complete the computation. By contrast, SIM only takes 2.0 s and consumes 2.4-GB memory to compute the scattered fields when the SD = 2.5 PPW. When SD = 19.8 PPW, MoM takes 52.1 s and consumes 4.1-GB memory to reach the same computation accuracy. Clearly, SIM outperforms both FEM and MoM for computation efficiency.

Finally, let us check the exponential accuracy property of SIM when the spatial SD in the elliptical boundaries gradually increases. Fig. 5(a)-(c) shows the variations of relative errors of the scattered fields at the receiver array when SD increases from 1.27 to 24.8 PPW. For SIM, when SD is less than 5.0, the error decreases exponentially. However, when SD is larger than 5.0, the error almost remains unchanged with the order of 10^{-13} . This means that the 5.0 PPW SD for the spectral-domain integral method is saturated when the multilayered elliptical cylinders are isotropic. Another possible reason for this nearly constant error is the numerical calculation errors of both SIM and FEM. They are numerical solvers instead of analytical ones. As a result, the relative error between them cannot decrease to infinitesimal. Compared with SIM, the errors of MoM decay rather slow as SD increases. Even when we increase its value to near 25 PPW, the relative errors of the scattered fields are in the order of 0.1%. Clearly, MoM can never reach the same accuracy as SIM, since the sampling points in the elliptical boundaries are independent if the integral equation is solved in the spatial domain. Fig. 5(d)

TABLE II
GEOMETRY PARAMETERS OF EACH ELLIPSE AND THE ANISOTROPIC DIELECTRIC PARAMETERS OF THE MEDIUM ON ITS INNER SIDE

Parameters	Ellipse		#1	#2	#3	#4	#5	#6	#7	#8	#9	#10
	center coordinate		(-1.1, -1.1)	(-1.0, -0.9)	(-1.1, -1.0)	(-0.9, -0.4)	(-1.3, -0.5)	(-1.0, 0.3)	(-1.2, -1.2)	(-1.4, -1.5)	(-1.8, -1.2)	(-3.0, -1.0)
horizontal half axis length		17.0	14.0	12.0	9.0	8.0	7.0	6.0	4.0	3.0	1.0	
vertical half axis length		14.0	12.5	12.0	10.0	9.0	7.0	4.0	3.0	2.0	1.0	
ε_x		1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	
ε_y		1.4	1.8	2.2	2.6	3.0	3.4	3.8	4.2	4.6	5.0	
σ_x		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	
σ_y		0.2	0.1	0.3	0.5	0.4	0.5	0.2	0.3	0.4	0.5	
μ		1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	

Remark: the units of position and length are meters; the unit of σ is mS/m.

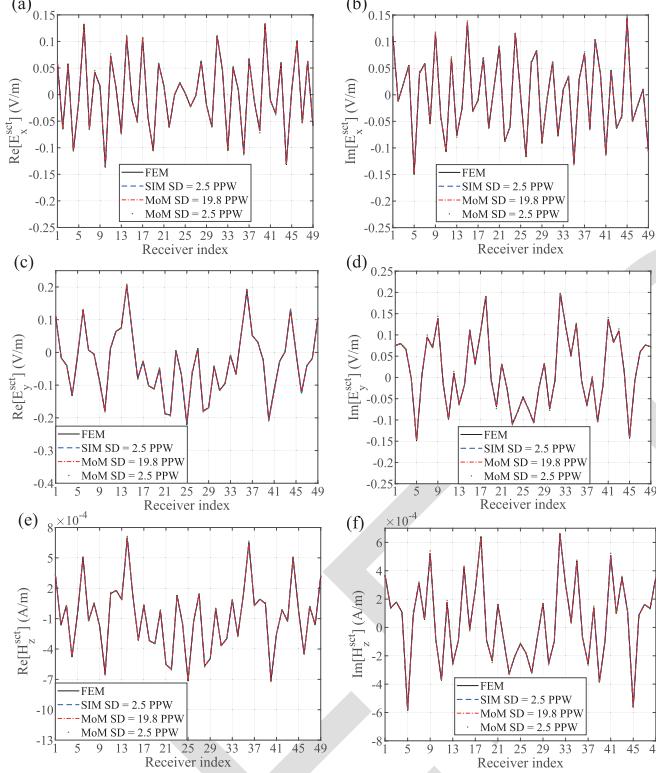


Fig. 4. Comparisons of the scattered EM fields at the receiver array computed by FEM, MoM, and SIM when the spatial sampling densities along the elliptical boundaries take different values. (a) Real part of E_x^{scat} . (b) Imaginary part of E_x^{scat} . (c) Real part of E_y^{scat} . (d) Imaginary part of E_y^{scat} . (e) Real part of H_z^{scat} . (f) Imaginary part of H_z^{scat} .

shows the CPU time and the number of unknowns in the discretized integral equations versus the mean error of the scattered fields of three components when SD increases from 1.27 to 24.8 PPW. It is obvious that the error of SIM is much lower than that of MoM for the same CPU time or the number of unknowns. In other words, MoM will consume much more computation resources than SIM if we try to use both of them to reach the same accuracy.

B. Ten-Layer Electrically Large Biaxially Anisotropic Nonconcentric Elliptical Cylinders

In this case, we increase the layer number and also use the biaxially anisotropic medium for the cylinders, which raises the complexity of the spatial medium distribution. Meanwhile, the size of the scatterer is also increased. The purpose of these changes is to test the adaptability and verify

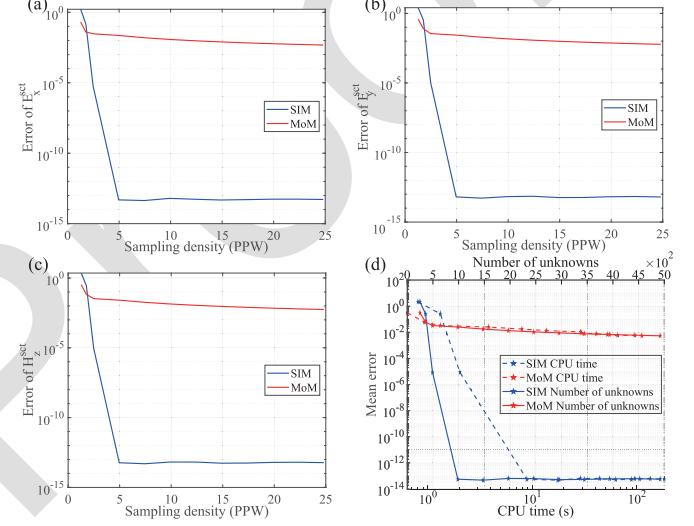


Fig. 5. Comparisons of the computation error variations of scattered fields with SD, CPU time, and the number of unknowns in the integral equations for SIM and MoM. (a) Error of E_x^{scat} changes with SD. (b) Error of E_y^{scat} changes with SD. (c) Error of H_z^{scat} changes with SD. (d) Mean error versus CPU time and the number of unknowns.

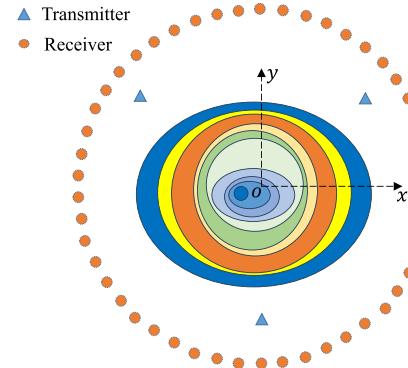


Fig. 6. Ten nonconcentric elliptical magnetodielectric anisotropic cylinders illuminated by TE_z -polarized EM waves, which are excited by three magnetic dipoles transmitters.

the superiority of the proposed SIM. As shown in Fig. 6, ten anisotropic nonconcentric elliptical cylinders are illuminated by the TE_z -polarized EM waves, which are excited by three unit magnetic dipoles located at $(15.8, 11.0)$, $(-18.6, 11.0)$, and $(-0.6, -18.8)$ m, respectively. The specific parameters of ten elliptical cylinders are listed in Table II. Note that the outermost ellipse has a perimeter of nearly 100.0 m, which is about 100 times the wavelength in the background medium.

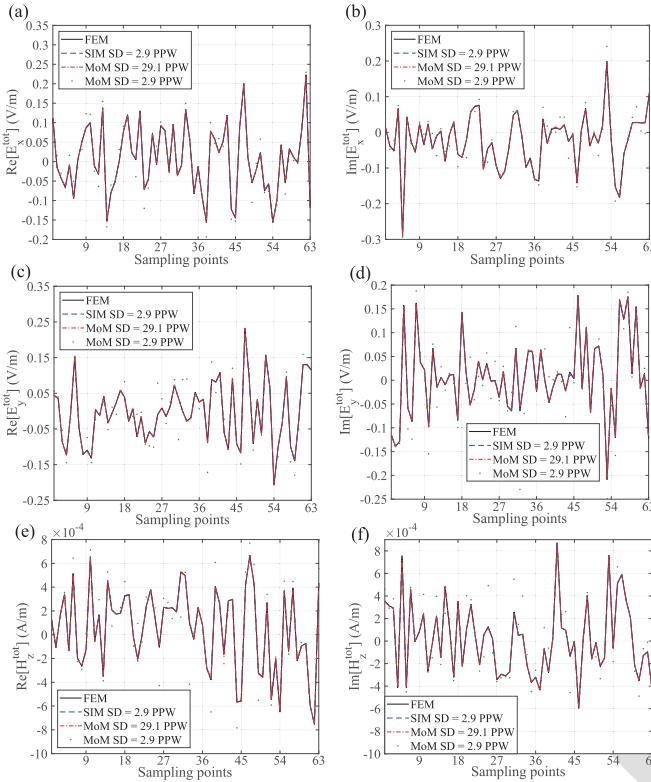


Fig. 7. Comparisons of the total EM fields at 63 typical sampling points inside the computational domain computed by FEM, MoM, and SIM when the spatial sampling densities along the elliptical boundaries take different values. (a) Real part of E_x^{tot} . (b) Imaginary part of E_x^{tot} . (c) Real part of E_y^{tot} . (d) Imaginary part of E_y^{tot} . (e) Real part of H_z^{tot} . (f) Imaginary part of H_z^{tot} .

Totally, 49 receivers are evenly placed on a circle whose center is located at $(1.1, -1.4)$ m, and the radius is 24.0 m.

Now, let us verify the computation accuracy of the total fields E_x^{tot} , E_y^{tot} , and H_z^{tot} at 7×9 sampling points inside the computational domain. The sampling point located at the bottom left corner has the coordinate $(-21.2, -16.0)$ m. The increment between two sampling points in either the \hat{x} -direction or the \hat{y} -direction is 5.0 m. Fig. 7 shows the comparisons of the total EM fields at 63 sampling points computed by FEM, SIM, and MoM. It is obvious that SIM still can achieve good matches with FEM when the SD is lower than 3.0 PPW. However, MoM must increase the SD to ten times larger in order to reach reliable matches. Precise calculations show that the relative errors of E_x^{tot} for SIM with SD = 2.9 PPW, MoM with SD = 29.1 PPW, and MoM with SD = 2.9 PPW compared with FEM results are 0.09%, 1.5%, and 28%, respectively, the errors of E_y^{tot} for these three situations are 0.07%, 1.8%, and 45%, respectively, and the errors of H_z^{tot} for these three situations are 0.10%, 1.8%, and 40%, respectively. By comparing these error data with those for the isotropic case discussed in Section III-A, we can see that SIM still can achieve similar computation accuracy even for electrically large anisotropic elliptical scatterers when the SD is low. Unfortunately, the computation of MoM is not reliable if the SD is lowered to near 3.0 PPW. Even when it is increased to near 30.0 PPW, MoM can only achieve around one hundredth computation error. These discrepancies are further validated by the computation of the scattered fields

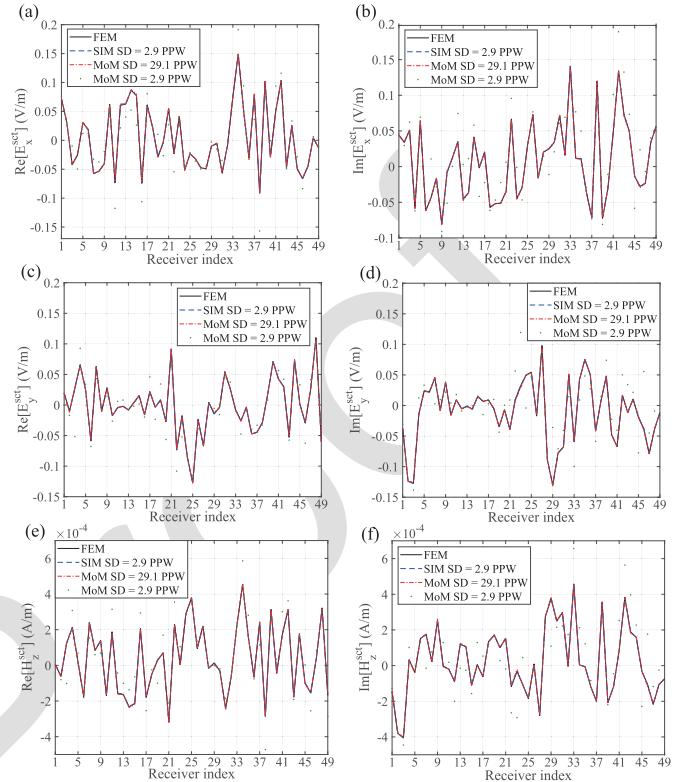


Fig. 8. Comparisons of the scattered EM fields at the receiver array computed by FEM, MoM, and SIM when the spatial sampling densities along the elliptical boundaries take different values. (a) Real part of E_x^{scat} . (b) Imaginary part of E_x^{scat} . (c) Real part of E_y^{scat} . (d) Imaginary part of E_y^{scat} . (e) Real part of H_z^{scat} . (f) Imaginary part of H_z^{scat} .

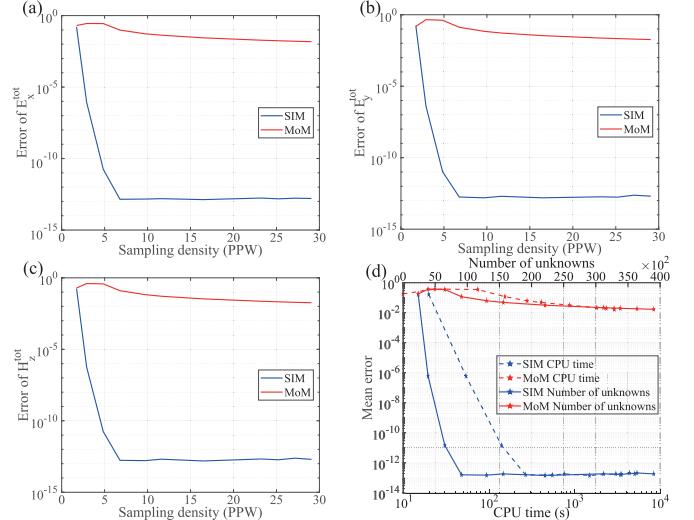


Fig. 9. Comparisons of the computation error variations of total fields at 63 typical sampling points with SD, CPU time, and the number of unknowns in the integral equations for SIM and MoM. (a) Error of E_x^{tot} changes with SD. (b) Error of E_y^{tot} changes with SD. (c) Error of H_z^{tot} changes with SD. (d) Mean error versus CPU time and the number of unknowns.

at the receiver array, which are shown in Fig. 8. The relative errors of the scattered fields are in the order of one thousandth, several tens percent, and one hundredth for SIM with SD = 2.9 PPW, MoM with SD = 29.1 PPW, and MoM with SD = 2.9 PPW, respectively. In addition, for this electrically large anisotropic case, COMSOL takes 788 s and consumes 245-GB

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memory to complete the computation. MoM takes 2918 s and consumes 186-GB memory for the SD of 29.1 PPW. However, SIM only takes 53 s and consumes 4.5-GB memory to achieve reliable computation accuracy.

Fig. 9(a)–(c) shows the variations of relative errors of the total EM fields at the 63 sampling points for different SD values. We can see that SIM still has exponential accuracy for electrically large anisotropic elliptical cylindrical scatterers. However, compared with the error variations shown in Fig. 5(a)–(c) for isotropic scatterers, the errors of both SIM and MoM decay slower for anisotropic scatterers. Such a difference may be caused by the complexity of the spatial medium distribution. Fig. 9(d) shows the CPU time and the number of unknowns versus the mean error of the total fields. The variations of errors have similar trends as those shown in Fig. 5(d). However, SIM outperforms MoM more prominently in both time and accuracy for this anisotropic case compared with the previous isotropic case.

IV. CONCLUSION AND DISCUSSION

In this work, we developed the SIM for EM scattering from multilayered nonconcentric elliptical magnetodielectric cylinders illuminated by TE_z -polarized waves. The dielectric materials filling each cylinder can either be isotropic or biaxially anisotropic. Both the unknown tangential EM fields along the multiple elliptical boundaries and the 2-D Green's functions are expanded by the Fourier series. A subtraction method is adopted to smoothen the strong singularity of Green's functions caused by the proximity of the source point and the field point in the elliptical boundary. This guarantees Green's functions can be correctly expanded by the Fourier series with low orders. Consequently, the whole discretized integral equation is solved in the spectral domain with a limited number of Fourier coefficients.

Two numerical examples are used to show the superiority of the proposed SIM to the traditional MoM. It is found that SIM can achieve reliable computation accuracy, i.e., the error is in the order of one thousandth, as long as the spatial SD is higher than 3.0 PPW. By contrast, the traditional MoM needs at least 20.0 PPW to reach the same computation accuracy for isotropic scatterers. When the multilayered elliptical cylinders become anisotropic and their electrical sizes are huge, MoM can only lower the computation error to the order of one hundredth even if we increase the SD to 30.0 PPW. SIM is able to maintain computation accuracy with low computational cost for both simple and complex elliptical magnetodielectric cylinders.

Finally, we want to emphasize three points here. First, both the formula derivation and numerical examples are given for smooth elliptical boundaries in this work. However, the proposed SIM can be directly applied to any homogeneous scatterers having smooth boundaries as long as the EM fields and Green's functions in the boundaries can be expanded by Fourier series with low orders. Second, we use the magnetic dipoles as transmitters in the numerical examples for the convenience of verification via FEM. SIM is also applicable to the scenario with plane wave excitation. Third, the proposed

SIM can also be directly applied to a smooth metallization boundary as long as we discard \mathbf{M} in (2).

APPENDIX A

It is assumed both the EM fields and the background medium are invariant along the \hat{z} -direction in the 2-D problem. As a result, we have $\nabla = \hat{x}(\partial/\partial x) + \hat{y}(\partial/\partial y)$. In the unbounded homogeneous isotropic background medium with the complex relative permittivity ϵ_b and the relative permeability μ_b , the 2-D source point locates at $\rho' = \hat{x}x' + \hat{y}y'$, while the 2-D field point locates at $\rho = \hat{x}x + \hat{y}y$. For the TE_z mode, we have $\mathbf{E} = \hat{x}E_x + \hat{y}E_y$, $\mathbf{H} = \hat{z}H_z$, $\mathbf{J} = \hat{x}J_x + \hat{y}J_y$, and $\mathbf{M} = \hat{z}M_z$. The 2-D scalar Green's function is written as follows:

$$g(\rho, \rho') = -\frac{j}{4} H_0^{(2)}(k_b \rho) \quad (\text{A1})$$

where $H_0^{(2)}$ is the zeroth-order Hankel function of the second kind, $k_b = k_0(\epsilon_b \mu_b)^{1/2}$ is the wavenumber of the background medium, and $\rho = |\rho - \rho'|$ is the distance between the field point and the source point. We then define the auxiliary magnetic vector potential $\mathbf{A} = \mu_b \mu_0 [g \hat{x} \hat{x} + g \hat{y} \hat{y}] \cdot \mathbf{J}$ and the auxiliary electric vector potential $\mathbf{F} = \epsilon_b \epsilon_0 [g \hat{z} \hat{z}] \cdot \mathbf{M}$. The EM fields are related to them by [39]

$$\mathbf{E} = -j\omega \mathbf{A} - j \frac{1}{\omega \mu_0 \epsilon_0 \mu_b \epsilon_b} \nabla (\nabla \cdot \mathbf{A}) - \frac{1}{\epsilon_0 \epsilon_b} \nabla \times \mathbf{F} \quad (\text{A2a})$$

$$\mathbf{H} = \frac{1}{\mu_0 \mu_b} \nabla \times \mathbf{A} - j\omega \mathbf{F} - j \frac{1}{\omega \mu_0 \epsilon_0 \mu_b \epsilon_b} \nabla (\nabla \cdot \mathbf{F}). \quad (\text{A2b})$$

By substituting g in (A1) in to (A2), we obtain the final 2-D dyadic Green's function components for an isotropic medium

$$G_{EJ}^{xx} = -\frac{\omega \mu_b \mu_0}{4} H_0^{(2)}(k_b \rho) + \frac{k_b}{4\omega \epsilon_0 \epsilon_b \rho} H_1^{(2)}(k_b \rho) - \frac{k_b^2}{4\omega \epsilon_0 \epsilon_b} H_2^{(2)}(k_b \rho) \cos^2 \phi \quad (\text{A3a})$$

$$G_{EJ}^{yx} = -\frac{k_b^2}{8\omega \epsilon_0 \epsilon_b} H_2^{(2)}(k_b \rho) \sin 2\phi \quad (\text{A3b})$$

$$G_{EJ}^{xy} = G_{EJ}^{yx} \quad (\text{A3c})$$

$$G_{EJ}^{yy} = -\frac{\omega \mu_b \mu_0}{4} H_0^{(2)}(k_b \rho) + \frac{k_b}{4\omega \epsilon_0 \epsilon_b \rho} H_1^{(2)}(k_b \rho) - \frac{k_b^2}{4\omega \epsilon_0 \epsilon_b} H_2^{(2)}(k_b \rho) \sin^2 \phi \quad (\text{A3d})$$

$$G_{HJ}^{zx} = G_{EM}^{xz} = -\frac{jk_b}{4} H_1^{(2)}(k_b \rho) \sin \phi \quad (\text{A3e})$$

$$G_{HJ}^{zy} = G_{EM}^{yz} = \frac{jk_b}{4} H_1^{(2)}(k_b \rho) \cos \phi \quad (\text{A3f})$$

$$G_{HM}^{zz} = -\frac{\omega \epsilon_0 \epsilon_b}{4} H_0^{(2)}(k_b \rho) \quad (\text{A3g})$$

where ϕ is the angle between the vector $\rho - \rho'$ and the \hat{x} -axis with $\cos \phi = ((x - x')/\rho)$ and $\sin \phi = ((y - y')/\rho)$. In addition, the 2-D dyadic Green's functions for the TM_z mode can be directly evaluated based on (A3) according to the duality theorem.

APPENDIX B

For the TE_z mode, it is assumed that the unbounded homogeneous biaxially anisotropic background medium has the complex relative permittivity $\bar{\epsilon}_b = \hat{x}\hat{x}\epsilon_{xb} + \hat{y}\hat{y}\epsilon_{yb}$ and the relative permeability μ_b . By taking the similar method given in [40, Sec. V], we construct the auxiliary magnetic vector potential \mathbf{A} and the auxiliary electric vector potential \mathbf{F} as follows:

$$\mathbf{A} = \hat{x} \left[-\frac{j\mu_b\mu_0}{4} \sqrt{\frac{\epsilon_{yb}}{\epsilon_{xb}}} H_0^{(2)}(k_{xb}\rho_x) \right] J_x \\ + \hat{y} \left[-\frac{j\mu_b\mu_0}{4} \sqrt{\frac{\epsilon_{xb}}{\epsilon_{yb}}} H_0^{(2)}(k_{yb}\rho_y) \right] J_y \quad (\text{B1a})$$

$$\mathbf{F} = \hat{z} \left[-\frac{j\epsilon_0\sqrt{\epsilon_{xb}\epsilon_{yb}}}{4} H_0^{(2)}(k_{xb}\rho_x) \right] M_z \quad (\text{B1b})$$

where $k_{xb} = k_0(\mu_b\epsilon_{xb})^{1/2}$, $\rho_x = ((\epsilon_{yb}/\epsilon_{xb})(x-x')^2 + (y-y')^2)^{1/2}$, $k_{yb} = k_0(\mu_b\epsilon_{yb})^{1/2}$, $\rho_y = ((x-x')^2 + (\epsilon_{xb}/\epsilon_{yb})(y-y')^2)^{1/2}$, and $k_{xb}\rho_x = k_{yb}\rho_y = k_0(\mu_b)^{1/2}[(\epsilon_{yb}(x-x')^2 + \epsilon_{xb}(y-y')^2)]^{1/2}$. Different from (A2), in a 2-D biaxially anisotropic medium, the EM fields are related to the vector potentials by

$$\mathbf{E} = -j\omega\mathbf{A} - j\frac{1}{\omega\mu_0\epsilon_0\mu_b\epsilon_{xb}\epsilon_{yb}} \nabla(\nabla \cdot \bar{\epsilon}_b \mathbf{A}) - \frac{\bar{\epsilon}_b^{-1}}{\epsilon_0} \nabla \times \mathbf{F} \quad (\text{B2a})$$

$$\mathbf{H} = \frac{1}{\mu_0\mu_b} \nabla \times \mathbf{A} - j\omega\mathbf{F} - j\frac{\bar{\epsilon}_b^{-1}}{\omega\mu_0\epsilon_0\mu_b} \nabla(\nabla \cdot \mathbf{F}) \quad (\text{B2b})$$

where $\epsilon_{xb}\epsilon_{yb}$ in the denominator is the gauge for an anisotropic medium suggested by Chew [41]. By substituting (B1) into (B2), we obtain the final 2-D dyadic Green's function components for a biaxially anisotropic medium

$$G_{EJ}^{xx} = -\frac{\omega\mu_b\mu_0}{4} \left(\frac{\epsilon_{yb}}{\epsilon_{xb}} \right)^{\frac{1}{2}} H_0^{(2)}(k_{xb}\rho_x) \\ + \frac{k_{xb}}{4\omega\epsilon_0\epsilon_{yb}\rho_x} \left(\frac{\epsilon_{yb}}{\epsilon_{xb}} \right)^{\frac{3}{2}} H_1^{(2)}(k_{xb}\rho_x) \\ - \frac{k_{xb}^2}{4\omega\epsilon_0\epsilon_{yb}} \left(\frac{\epsilon_{yb}}{\epsilon_{xb}} \right)^{\frac{5}{2}} H_2^{(2)}(k_{xb}\rho_x) \cos^2 \phi_x \quad (\text{B3a})$$

$$G_{EJ}^{yx} = -\frac{k_{xb}^2}{8\omega\epsilon_0\epsilon_{yb}} \left(\frac{\epsilon_{yb}}{\epsilon_{xb}} \right)^{\frac{3}{2}} H_2^{(2)}(k_{xb}\rho_x) \sin 2\phi_x \quad (\text{B3b})$$

$$G_{EJ}^{xy} = -\frac{k_{yb}^2}{8\omega\epsilon_0\epsilon_{xb}} \left(\frac{\epsilon_{xb}}{\epsilon_{yb}} \right)^{\frac{3}{2}} H_2^{(2)}(k_{yb}\rho_y) \sin 2\phi_y \quad (\text{B3c})$$

$$G_{EJ}^{yy} = -\frac{\omega\mu_b\mu_0}{4} \left(\frac{\epsilon_{xb}}{\epsilon_{yb}} \right)^{\frac{1}{2}} H_0^{(2)}(k_{yb}\rho_y) \\ + \frac{k_{yb}}{4\omega\epsilon_0\epsilon_{xb}\rho_y} \left(\frac{\epsilon_{xb}}{\epsilon_{yb}} \right)^{\frac{3}{2}} H_1^{(2)}(k_{yb}\rho_y) \\ - \frac{k_{yb}^2}{4\omega\epsilon_0\epsilon_{xb}} \left(\frac{\epsilon_{xb}}{\epsilon_{yb}} \right)^{\frac{5}{2}} H_2^{(2)}(k_{yb}\rho_y) \sin^2 \phi_y \quad (\text{B3d})$$

$$G_{HJ}^{zx} = G_{EM}^{xz} = -\frac{jk_{xb}}{4} \left(\frac{\epsilon_{yb}}{\epsilon_{xb}} \right)^{\frac{1}{2}} H_1^{(2)}(k_{xb}\rho_x) \sin \phi_x \quad (\text{B3e})$$

$$G_{HJ}^{zy} = G_{EM}^{yz} = \frac{jk_{yb}}{4} \left(\frac{\epsilon_{xb}}{\epsilon_{yb}} \right)^{\frac{1}{2}} H_1^{(2)}(k_{yb}\rho_y) \cos \phi_y \quad (\text{B3f})$$

$$G_{HM}^{zz} = -\frac{\omega\epsilon_0\sqrt{\epsilon_{xb}\epsilon_{yb}}}{4} H_0^{(2)}(k_{xb}\rho_x) \quad (\text{B3g})$$

where $\cos \phi_x = ((x-x')/\rho_x)$, $\cos \phi_y = ((y-y')/\rho_y)$, $\sin \phi_x = ((y-y')/\rho_x)$, and $\sin \phi_y = ((y-y')/\rho_y)$. The TM_z mode 2-D dyadic Green's functions for a biaxially anisotropic medium can also be evaluated based on (B3) according to the duality theorem.

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