

# UCK363E Automatic Control-II Term Project

**Article:** *LQR* control for speed and torque of internal combustion engines

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### 1. Introduction

A test bed is needed in the testing process of conventional internal combustion engines. By means of these test beds, it should be possible to measure how much torque the engine produces with which gas input in which rev range. In order to put a real load on the engine, the test setup uses a braking mechanism that knows how much torque it applies. This braking mechanism is directly connected to the shaft of the engine.

In this work, a control system for engine speed and torque, based on a LQR, has been developed. It was designed with first order transfer functions with delay, and it was successfully implemented and validated in Renault Spark Ignition engine. Although controllers were created for 2 different types (diesel and gasoline) engines in the article, only the gasoline engine was studied in this project assignment. While it would be desirable for the people who wrote the article, it would be pointless as it would make the project work repetitive.

## 2. Methodology

### a. Obtaining Transfer Function

The system's inputs are the magnetic brake current and the percentage of throttle opening, while its outputs are the engine speed and torque. In order to obtain the dynamic model of the engine, various tests were applied to the engine and the speed-torque data were collected. Once these experimental data were obtained, the parameters of each curve were determined using the IDENT toolbox in Matlab R. Each subsystem model was created by averaging eight curves spread out along the operational range (speed and torque).

The first-order model with delay obtained for the spark ignition engine was:

$$Y_R(s) = \begin{bmatrix} \frac{350e^{-s}}{s+1} & \frac{-27e^{-s}}{s+1} \\ \frac{0.4e^{-s}}{s+1} & \frac{0.8}{s+1} \end{bmatrix} \cdot U(s)$$

Equation 1

# **b.** Obtaining State Space

In this case, a simple and practical method of state space transformation developed by Pota



(1996) has been implemented. The transformation process started introducing state-space entries into the matrix of the transfer functions of equation (1) with a change of variable, and applying the inverse Laplace transform, result:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$C_{Renault} = \begin{bmatrix} 17.54 & -3.18 \\ 1.16 & 2.32 \end{bmatrix}$$

Equation 2 Equation 3

With this procedure, the states lost some of their physical meaning by making the transformation to time. In both engines most of the transfer functions coincided in the delay, it was chosen an average value:  $\tau$  Renault = 0.15 s

## c. LQR controller

The LQR technique calculates a matrix K which must be allowed for feedback control

$$\dot{x} = A \cdot x + B \cdot u$$
$$u = -K \cdot x$$

Equation 4

furthermore, this technique minimizes a cost function called "J" which is decreases the error in the system by applying a correction proportional to the size of the error.

$$J = \frac{1}{2} \int_0^\infty \left( x^T Q x + u^T R u \right) dt$$

Equation 5

On the article, the gain of the engine speed was decreased because it was larger, as the same as the torque response time because it had a faster actuator. The engine speed and torque did not varied significantly with different values of Q and R, but the actuators were sensitive to this changes and tended to oscillate, even when the system seemed stable, then those matrices must be carefully chosen.



$$Q_R = \begin{bmatrix} 0.01 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 \\ 0 & 0 & 0.05 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

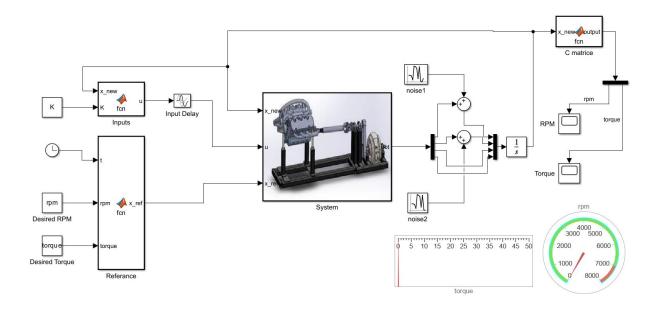
**Equation 6** 

In the article The Q matrice is given in as Eq. 6 and the R matrice were identity.

### d. Simulink

I set up the structure of the system in the given state space form in simulink. Then, by adding the C matrix to the output of the system, I was able to observe the physical rpm and torque values.

In order to add realism to the system; I added delay to the input part and distortion to the output of the system. I also added two rpm and torque gauges that you can observe when you run the system in real time (simulation pacing).



### 3. Results

To test my controller, I tried to simulate the same of "figure 5" presented in the article.



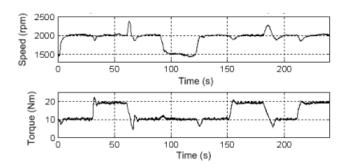
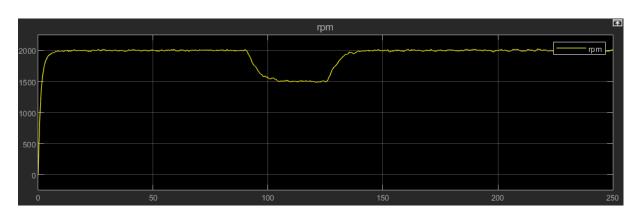
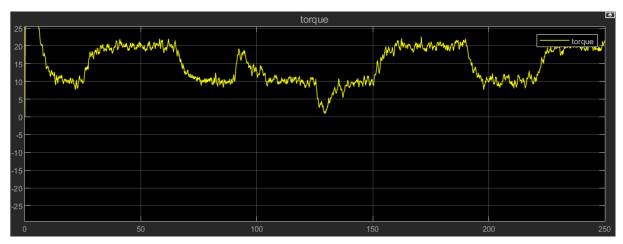


Fig. 5. Spark ignition engine on the linear region. Several fixed size speed and toque steps were performed between 1.500-2.000 rpm and 10-20 Nm with an expected response on each case

### And I saw my controller succeed:





Since states are not physically variable the system noise can not implemented smoothly. Maybe it is quite much on torque.

# 4. Discussion



With this application, I understood better how to control a system with state space. Understanding that we do it by both changing the polars of the system and deciding how much we want to apply it in which direction to which parameter helped me to understand LQR better. Working in Simulink allowed me to instantly follow any signal I wanted and to find faults more easily. At the same time, I have designed a system that can respond to every

reference request in a partially user-friendly interface.