# Peer prediction markets to elicit unverifiable information

Aurélien Baillon [1]    Cem Peker [2]    Sophie van der Zee [3]

December 26, 2022

[1] Department of Quantitative Finance and Economics, Emlyon Business School

[2] School of Management, Polytechnic University of Milan

[3] Erasmus School of Economics, Erasmus University Rotterdam

# Background

Education:

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**Publications:**

- Peker, C. (2022). Extracting the collective wisdom in probabilistic judgments. *Theory and Decision*. doi: 10.1007/s11238-022-09899-4
- Peker, C. (2022). Incentives for self-extremized expert judgments to alleviate the shared-information problem. *Decision*. doi.org: 10.1037/dec0000198

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**Today's paper: Peer prediction markets to elicit unverifiable information** (joint with A. Baillon and S. van der Zee)

More info: https://cempeker.github.io/

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Response is informative if you....

1. ...recall your experience accurately (cognitive effort)
2. ...report honestly (incentives to lie?)

# Imagine a Covid-19 survey…

"Have you stood less than 6 feet apart from another person in a queue yesterday?"

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Incentivize carefully considered and truthful answers?

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**Your answer is unverifiable!**

"Have you stood less than 6 feet apart from another person in a queue yesterday?"

**Incentivize carefully considered and truthful answers?**

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**Incentivize carefully considered and truthful answers?**

**Peer Prediction** (Miller et al., 2005):

- Your honest answer $\leftrightarrow$ Your prediction on others' answers

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- **Your prediction on others' answers is verifiable!**

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This paper: **Peer-Prediction Market (PPM)**

- One-shot market, buy/sell a single asset

- Trade $\equiv$ A bet on others' answers

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- Trades reveal carefully considered and truthful answers

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This paper: **Peer-Prediction Market (PPM)**

- One-shot market, buy/sell a single asset

- Trade $\equiv$ A bet on others' answers

- Trades reveal carefully considered and truthful answers

- Theory & evidence from 2 experimental studies

# The Formal Framework

- *Center* asks a binary question $\{Yes, No\}$
- *N* risk-neutral *agents*

## The framework

- *Center* asks a binary question $\{Yes, No\}$
- $N$ risk-neutral *agents*
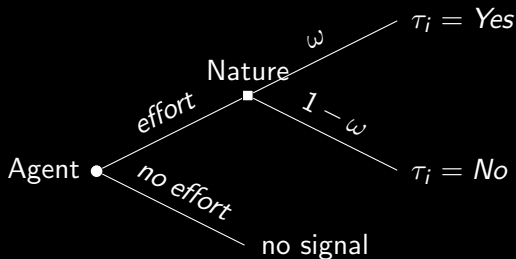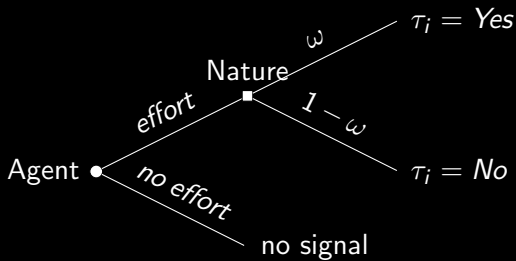- Each agent $i$ can receive a costly signal $\tau_i \in \{Yes, No\}$.
  Signal cost $= c_i$

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- *Center* asks a binary question $\{Yes, No\}$

- $N$ risk-neutral *agents*

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- Signal $\tau_i \equiv$ Agent $i$'s honest answer

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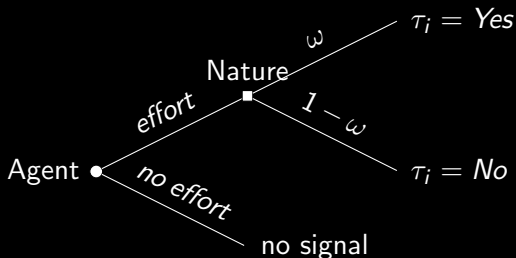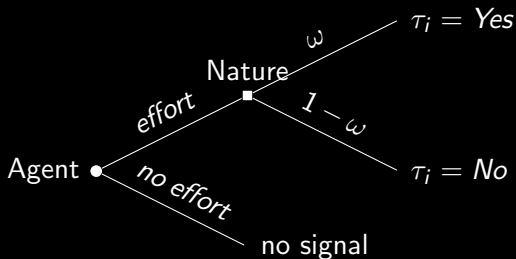
"Have you stood less than 6 feet apart from another person in a queue yesterday?"

Signal $\tau_i \equiv$ honest answer. Why is $\tau_i$ costly?

Agent •
effort
Nature ■
ω
$\tau_i = Yes$
1 − ω
$\tau_i = No$
no effort
no signal

"Have you stood less than 6 feet apart from another person in a queue yesterday?"

Signal $\tau_i \equiv$ honest answer. Why is $\tau_i$ costly?
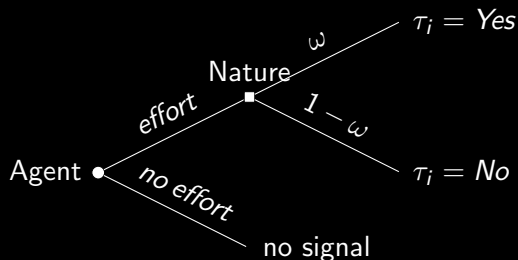
**Socially stigmatized behavior → Unwilling to recall**

Assumptions:

- Common prior expectation $E[\omega]$ on $\omega$.
- $E[\omega]$ is public knowledge.
- Agents follow Bayesian updating.
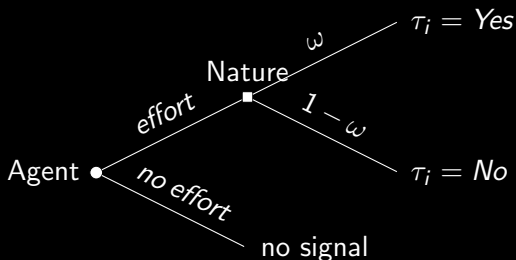
3 groups of agents with posterior expectations:

$E[\omega|\text{effort and } \tau_i = \text{Yes}]$
$E[\omega|\text{effort and } \tau_i = \text{No}]$
$E[\omega|\text{no effort}] = \text{Prior} = E[\omega]$

Posterior expectations satisfy:

$E[\omega|\tau_i = Yes] > E[\omega] > E[\omega|\tau_i = No]$

"Yes"-types expect $\omega > E[\omega] =$ prior.

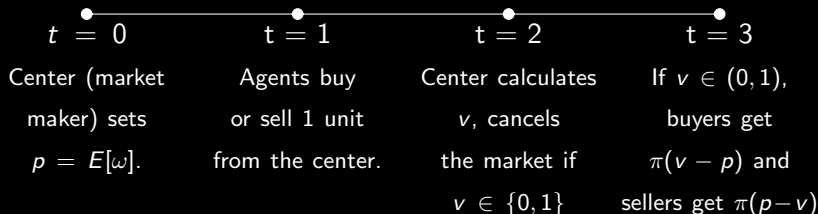"No"-types expect $\omega < E[\omega] =$ prior.

Peer prediction market

One-shot market

- Single asset
- Asset price $= p = E[\omega]$
- Asset value $= v =$ proportion of agents who buy

# The mechanism

One-shot market

- Single asset
- Asset price $= p = E[\omega]$
- Asset value $= v =$ proportion of agents who buy

| $t = 0$ | $t = 1$ | $t = 2$ | $t = 3$ |
|---|---|---|---|
| Center (market maker) sets $p = E[\omega]$. | Agents buy or sell 1 unit from the center. | Center calculates $v$, cancels the market if $v \in \{0, 1\}$ | If $v \in (0, 1)$, buyers get $\pi(v - p)$ and sellers get $\pi(p - v)$ |

Numerical example:

- Currency is dollar, $\pi = 10$

- Price: $p = E[\omega] = 0.5$

- 40% of the participants buy, $v = 0.4$

- Buyer's payoff: $10(0.4 - 0.5) = -\$1$

- Seller's payoff: $10(0.5 - 0.4) = \$1$

Strategy = Effort or not + probability of buy in various situations

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Agent $i$'s full strategy profile $= (e_i, R_i, R_i^{no}, R_i^{yes})$

- $e_i \in \{0, 1\}$ effort or no effort
- $R_i$ probability of buy if $e_i = 0$,
- $R_i^{no}$ probability of buy if $e_i = 1$ and $\tau_i = No$,
- $R_i^{yes}$ probability of buy if $e_i = 1$ and $\tau_i = Yes$.

Strategy $=$ Effort or not $+$ probability of buy in various situations

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Center would like: $e_i = 1$, $R_i^{no} = 0$, and $R_i^{yes} = 1$.

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Center would like: $e_i = 1$, $R_i^{no} = 0$, and $R_i^{yes} = 1$.
$\implies$ **Truthful strategy**: Trades reflect honest answers.

**Assumption**. The following are **common knowledge**:

- The market mechanism
- Signal technology, beliefs, costs and the strategy space.
- Risk-neutrality and Bayesianism of agents.

Ensures that we have a *Bayesian game* (Osborne and Rubinstein, 1994, Definition 25.1).

For convenience, we let $N \to \infty$.

# Equilibrium analysis

**Truthful equilibrium**: For $N \to \infty$, truthful strategy is a Nash equilibrium if the rewards are scaled sufficiently high such that

$$\frac{c_i}{\pi} < E[\omega] \left( E[\omega | \tau_i = Yes] - E[\omega] \right) + (1 - E[\omega]) \left( E[\omega] - E[\omega | \tau_i = No] \right)$$

for all $i \in \{1, \dots, N\}$

**Truthful equilibrium**: For $N \to \infty$, truthful strategy is a Nash equilibrium if the rewards are scaled sufficiently high such that

$$\frac{c_i}{\pi} < E[\omega] \left( E[\omega | \tau_i = \text{Yes}] - E[\omega] \right) + (1 - E[\omega]) \left( E[\omega] - E[\omega | \tau_i = \text{No}] \right)$$

for all $i \in \{1, \ldots, N\}$

In the truthful equilibrium...

- All agents exert effort
- Yes-types buy, No-types sell
- Truthful answer $\equiv$ Equilibrium trade

**Truthful equilibrium**: Full effort, Yes-types buy, No-types sell

How?

**Truthful equilibrium**: Full effort, Yes-types buy, No-types sell

How? Suppose,

- Agent $i$ exerts effort ($e_i = 1$)
- All agents $j \neq i$ are truthful

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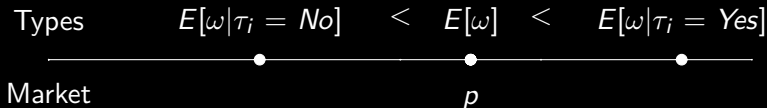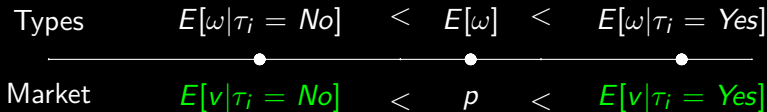- All agents $j \neq i$ are truthful

Types      $E[\omega|\tau_i = No]$   $<$   $E[\omega]$   $<$   $E[\omega|\tau_i = Yes]$

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Market $\qquad\qquad\qquad\qquad\qquad\qquad p$

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| Types | $E[\omega|\tau_i = No]$ | $<$ | $E[\omega]$ | $<$ | $E[\omega|\tau_i = Yes]$ |
|---|---|---|---|---|---|

| Market | $E[v|\tau_i = No]$ | $<$ | $p$ | $<$ | $E[v|\tau_i = Yes]$ |
|---|---|---|---|---|---|

Asset value ($v$) $\equiv$ Proportion of buyers $\rightarrow$ Proportion of Yes-type ($\omega$)

**Truthful equilibrium**: Full effort, Yes-types buy, No-types sell

How? Suppose,

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| Types | $E[\omega\|\tau_i = No]$ | $<$ | $E[\omega]$ | $<$ | $E[\omega\|\tau_i = Yes]$ |
|---|---|---|---|---|---|
| Market | $E[v\|\tau_i = No]$ | $<$ $\quad p \quad$ $<$ | | | $E[v\|\tau_i = Yes]$ |

Optimal: Buy if $\tau_i = Yes$, sell if $\tau_i = No$
Incentive to "learn" your type $\to e_i = 1$

**Multiple equilibria**

**No-effort equilibrium**: If $c_i > \pi$ for all $i \in \{1, \ldots, N\}$, then Nash equilibria are characterized by $e_i = 0$ and $R_i \in \{0, E[\omega], 1\}$. Expected payoffs are 0.

$\rightarrow$ No effort when costs are too high.

**Multiple equilibria**

**No-effort equilibrium**: If $c_i > \pi$ for all $i \in \{1, \ldots, N\}$, then Nash equilibria are characterized by $e_i = 0$ and $R_i \in \{0, E[\omega], 1\}$. Expected payoffs are 0.

    $\rightarrow$ No effort when costs are too high.

**Partial effort equilibrium**: There are NE in which $K < N$ agents exert no effort and buy with probability $E[\omega]$ while the other agents are truthful.

    $\rightarrow$ People with low cost exert effort, others do not.

**Multiple equilibria**

**All-buy or all-sell**: There exists Nash equilibria such that $e_i = 0$ and $R_i = 0$ or $R_i = 1$ for all $i$. Expected payoffs are 0.

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**All-buy or all-sell**: There exists Nash equilibria such that $e_i = 0$ and $R_i = 0$ or $R_i = 1$ for all $i$. Expected payoffs are 0.

**Truthful equilibrium: Strictly higher payoff than no-effort, all-buy and all-sell equilibria**

"Have you stood less than 6 feet apart from another person in a queue yesterday?"

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Reporting "Yes" is shameful $\rightarrow$ higher cost?

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- Asymmetric reporting cost: Cost $a_i \geq 0$ of reporting "Yes", no matter (presence of) signal.
- Deception cost: The cost $d_i \geq 0$ of reporting "Yes" when $\tau_i = No$ or "No" when $\tau_i = Yes$.

"Have you stood less than 6 feet apart from another person in a queue yesterday?"

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**Truthful equilibrium if $\pi$ is scaled appropriately**

# Experimental Evidence

Two experimental studies.

Study 1:

- Closely follows the theoretical model.
- Real effort task.

Study 2:

- Health survey, questions of 6-feet-apart type.
- Psychological costs & practical feasibility

One of the boxes has been selected (Q= "more yellow" or I= "less yellow").

Guess which one.

One of the boxes has been selected (Q="more yellow" or I="less yellow").

Guess which one. Want to see a ball from the selected box?

Count the number of 0s and you draw..

OR

Color of your draw $\equiv$ signal

- Let's say a yellow draw is equivalent to ($\tau_i = Yes$).
- $E[\omega] = 0.6$ (common prior expectation on prop. yellow).

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- $E[\omega] = 0.6$ (common prior expectation on prop. yellow).
- $c_i$ = cognitive effort of counting 0s in matrix.
- Picking Box Q $\equiv$ Buying $\equiv$ Reporting yellow ("Yes").

## Study 1 - 3 treatments

- **Flat** fee: £3.25 completion fee.

- **Accuracy** incentives: £3.25 $\pm 0.20$ per prediction task if the pick is correct or not.

- **PPM**: £3.25 + PPM incentives:
  Bonus: (% of people who pick the same box) − (prior).

"Accuracy" is a benchmark for verifiable tasks.

- Online experiment, May 2020

- 210 U.S. citizens, students, recruited on Prolific

- 10 tasks (10 pairs of boxes, 10 matrices).

Proportion of subjects who complete the effort task
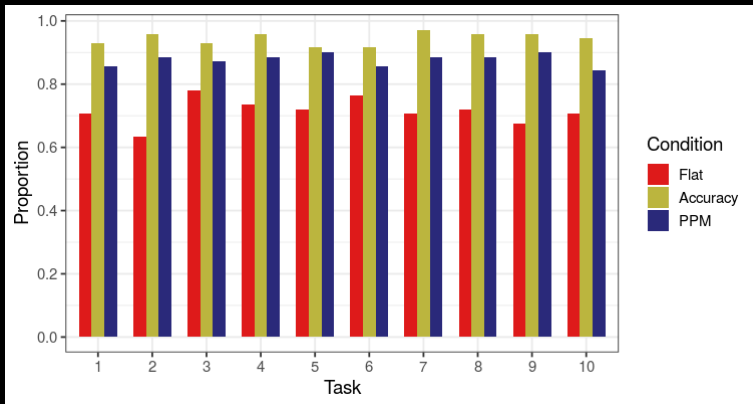


*Accuracy > PPM > Flat* in effort elicitation
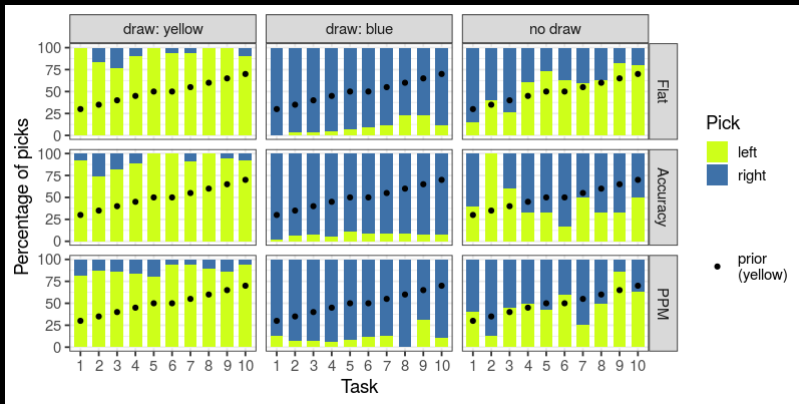
Picks are as predicted by the truthful equilibrium.

# Marginal effects, logistic regression

| Dep. var.: P(effort task completed) | | | | |
|---|---|---|---|---|
| | (whole sample) | | (filtered sample) | |
| | (1) | (2) | (3) | (4) |
| PPM | 0.10** | 0.09** | 0.10** | 0.08** |
| | (0.03) | (0.03) | (0.03) | (0.03) |
| Accuracy | 0.18*** | 0.18*** | 0.18*** | 0.18*** |
| | (0.03) | (0.03) | (0.03) | (0.03) |
| Age | | −0.00 | | −0.00 |
| | | (0.00) | | (0.00) |
| Female? | | 0.04 | | 0.04 |
| | | (0.03) | | (0.03) |
| US resident? | | −0.02 | | −0.02 |
| | | (0.06) | | (0.06) |
| Num. obs. | 2100 | 2070 | 2060 | 2030 |

***$p < 0.001$; **$p < 0.01$; *$p < 0.05$; +$p < 0.1$

PPM can elicit effort when incentives for accuracy are not feasible (unverifiable questions).

Study 1: Simple task, carefully controlled setup.

Study 2: Online field experiment

- Health & safety guidelines during the Covid-19 pandemic
- Did people follow them? **(Difficult to measure)**

Study 1: Simple task, carefully controlled setup.

Study 2: Online field experiment

- Health & safety guidelines during the Covid-19 pandemic
- Did people follow them? **(Difficult to measure)**

- Would they self-report their unsafe behavior? **(Unverifiable)**
- A health survey with PPM incentives

# Study 2 - Covid Survey

Please try to remember how many times you were in the following situation:

**I was seated less than 2 metres away from someone who is not part of my household in a restaurant/cafe/bar at least once in the last 7 days.**

| **True** (picked by 44% last week) | **False** (picked by 56% last week) |
|---|---|

Submit

# Study 2 – Covid Survey

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Submit

**"True" could be underreported.**

**Would a PPM elicit a higher % of "True" responses?**

# Study 2 - True/False statements

| 1. | I have been in an elevator with another person in it at least once in the last 7 days |
|----|---|
| 2. | I may have stood less than 2 metres away from the person in front in a queue at least once in the last 7 days |
| 3. | I was seated less than 2 metres away from someone who is not part of my household in a restaurant/cafe/bar at least once in the last 7 days |
| 4. | I have been in a social gathering with more than 6 people who are not part of my household at least once in the last 7 days |
| 5. | I have been in a busy shop/market with no restrictions on number of customers at least once in the last 7 days |
| 6. | I participated in an indoor activity with more than 6 people who are not part of my household at least once in the last 7 days |
| 7. | I have been in a shop/market where one or more of the staff did not wear a mask at least once in the last 7 days |
| 8. | I had an interaction with someone experiencing high body temperature, persistent cough or loss of taste/smell at least once in the last 7 days |

# Study 2 - Link with theory

Please try to remember how many times you were in the following situation:

**I was seated less than 2 metres away from someone who is not part of my household in a restaurant/cafe/bar at least once in the last 7 days.**

| True | False |
|------|-------|
| (picked by 44% last week) | (picked by 56% last week) |

Submit

If you report True, bonus = % True this week - 44

Please try to remember how many times you were in the following situation:

**I was seated less than 2 metres away from someone who is not part of my household in a restaurant/cafe/bar at least once in the last 7 days.**

| True (picked by 44% last week) | False (picked by 56% last week) |
|---|---|

Submit

If you report True, bonus = % True this week - 44

Analogous to PPM:

- $v$ = Proportion of "True" this week
- $p = E[\omega] = 0.44$ (common prior on True)

# Study 2 – Link with theory

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| True | False |
|------|-------|
| (picked by 44% last week) | (picked by 56% last week) |

Submit

- Mental cost of remembering.
- Shame of answering "True" or lying.

Weekly survey in the UK, 3 weeks, November 2020

3 Experimental conditions

Control-1 (flat fee)

**I may have stood less than 2 metres away from the person in front in a queue at least once in the last 7 days.**

| True | False |
|------|-------|

Weekly survey in the UK, 3 weeks, November 2020

3 Experimental conditions

Control-1 (flat fee)

I may have stood less than 2 metres away from the person in front in a queue at least once in the last 7 days.

| True | False |
|------|-------|

Treatment (PPM incentives), Control-2 (flat fee)*

| True (picked by 65% last week) | False (picked by 35% last week) |
|--------------------------------|---------------------------------|

* tests the effect of just showing the last week's %s.

## Study 2 - Marginal effects, Pr(Response = "True")

|  | (week 2) | | (week 3) | |
|---|---|---|---|---|
|  | (1) | (2) | (3) | (4) |
| (Intercept) |  |  |  |  |
|  |  |  |  |  |
| Control-2 | 0.05 | 0.04 | −0.01 | −0.00 |
|  | (0.04) | (0.04) | (0.04) | (0.04) |
| PPM | 0.11*** | 0.10** | 0.08* | 0.08* |
|  | (0.03) | (0.03) | (0.04) | (0.04) |
| Age |  | −0.00 |  | −0.00 |
|  |  | (0.00) |  | (0.00) |
| Female? |  | 0.02 |  | −0.02 |
|  |  | (0.03) |  | (0.03) |
| UK citizen? |  | −0.00 |  | 0.03 |
|  |  | (0.03) |  | (0.04) |
| Num. obs. | 1259 | 1259 | 1279 | 1279 |

$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$; $^{+}p < 0.1$

**Higher rate of self-reported unsafe behavior in the PPM treatment.**

# Contribution

Mechanism design literature: Explored ways to reveal private signals (Crémer and McLean, 1988).

Sender-Receiver games, Bayesian Elicitation (Whitmeyer, 2019)

Mechanism design literature: Explored ways to reveal private signals (Crémer and McLean, 1988).

Sender-Receiver games, Bayesian Elicitation (Whitmeyer, 2019)

Peer prediction method (Miller et al., 2005): similar framework, but

- The complete prior must be known
- Scoring is not transparent

Bayesian truth-serum (Prelec, 2004) and follow-ups:

- Detail-free (implementer needs less), but more demanding from respondents (answer + prediction)

**Usually, costly effort to acquire signal not modelled.**

Peer prediction markets: Transparent, easy to implement.

Peer prediction markets: Transparent, easy to implement.

Strong assumptions, but same as or weaker than in the literature.

Limitations: Binary questions only, multiple equilibria.

# Thank you!

https://cempeker.github.io/

# References

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