Peer prediction markets to elicit unverifiable information

Aurélien Baillon [1] Cem Peker [2] Sophie van der Zee [3]

December 4, 2022

[1]Department of Quantitative Finance and Economics, Emlyon Business School

[2]School of Management, Polytechnic University of Milan

[3]Erasmus School of Economics, Erasmus University Rotterdam

Background

Education:

- PhD in Economics (30.03.2023, Tinbergen Institute, Netherlands)
- BSc in Industrial Engineering, Bogazici University

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Publications:

- Peker, C. (2022). Extracting the collective wisdom in probabilistic judgments. *Theory and Decision*. doi: 10.1007/s11238-022-09899-4
- Peker, C. (2022). Incentives for self-extremized expert judgments to alleviate the shared-information problem. *Decision*. doi.org: 10.1037/dec0000198

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Today's paper: Peer prediction markets to elicit unverifiable information (joint with A. Baillon and S. van der Zee)

More info: https://cempeker.github.io/

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Imagine a survey...

"Have you stood less than 6 feet apart from another person in a queue yesterday?"

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Your answer is unverifiable!

Carefully considered and truthful answers

"Have you stood less than 6 feet apart from another person in a queue yesterday?"

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- "Yes" and "No" types have different beliefs on others' answers

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- "Yes" and "No" types have different beliefs on others' answers
- **Your prediction on others' answers is verifiable!**

Peer-Prediction Market

Your prediction on others' answers is verifiable!

This paper: **Peer-Prediction Market (PPM)**

- One-shot market, buy/sell a single asset
- Trade $\equiv$ A bet on others' answers

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This paper: **Peer-Prediction Market (PPM)**

- One-shot market, buy/sell a single asset
- Trade $\equiv$ A bet on others' answers
- Trades reveal carefully considered and truthful answers
- Theory & evidence from 2 experimental studies

The Formal Framework

The framework

- *Center* asks a binary question $\{Yes, No\}$
- N risk-neutral *agents*

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- Each agent i can receive a costly signal $\tau_i \in \{Yes, No\}$.
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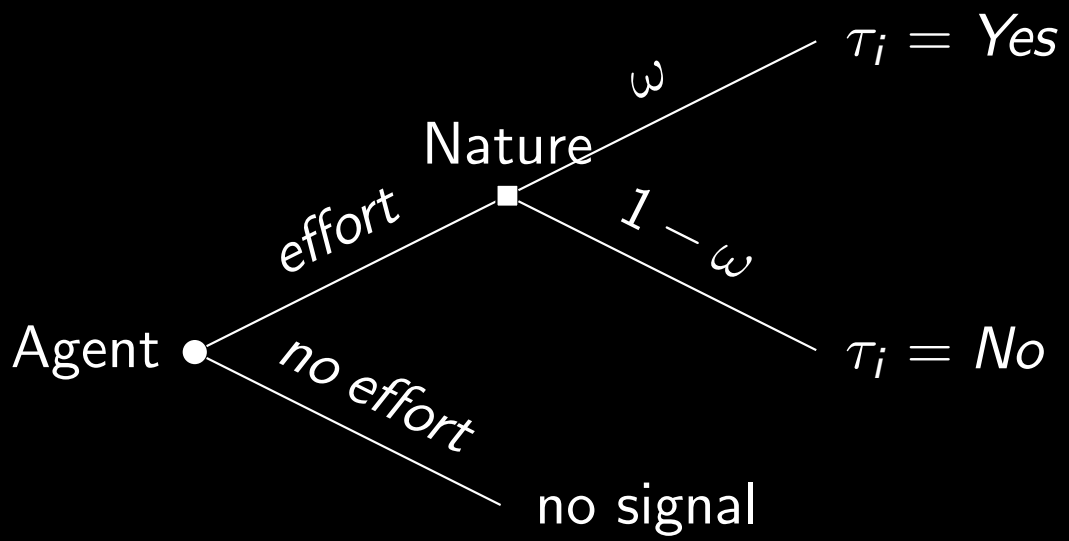

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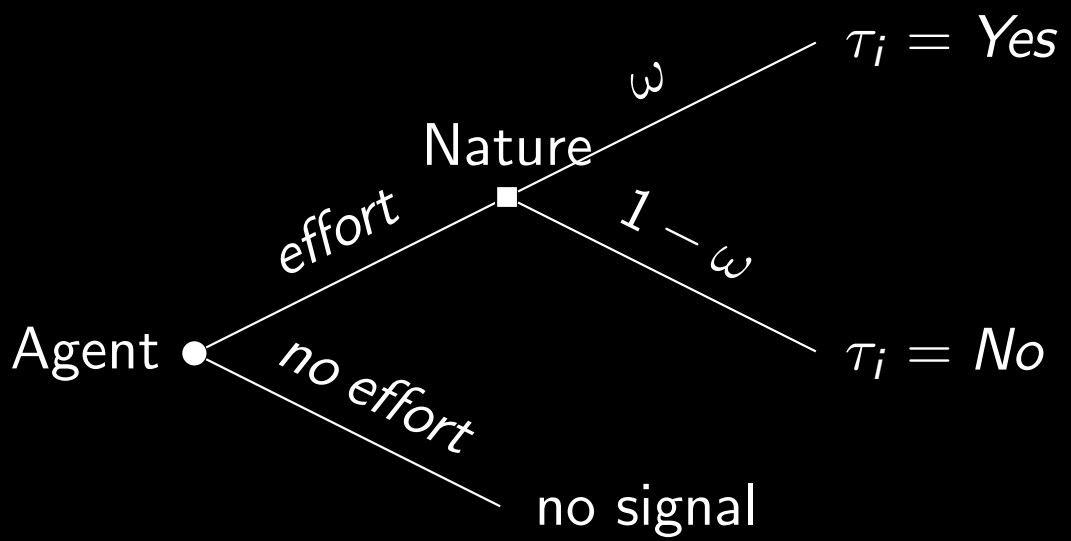

"Have you stood less than 6 feet apart from another person in a queue yesterday?"

Signal $\tau_i \equiv$ honest answer. Why is τ_i costly?

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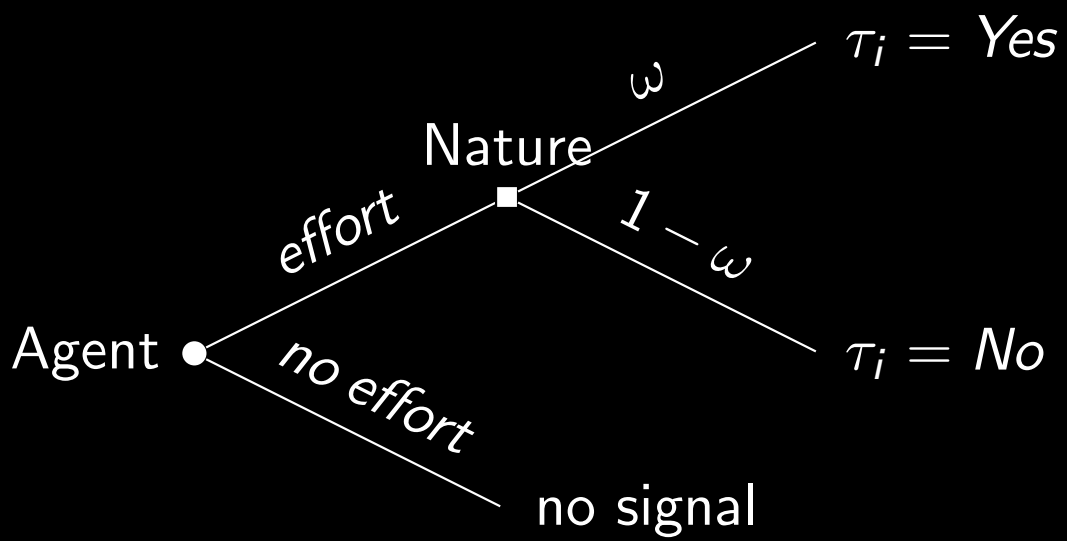

"Have you stood less than 6 feet apart from another person in a queue yesterday?"

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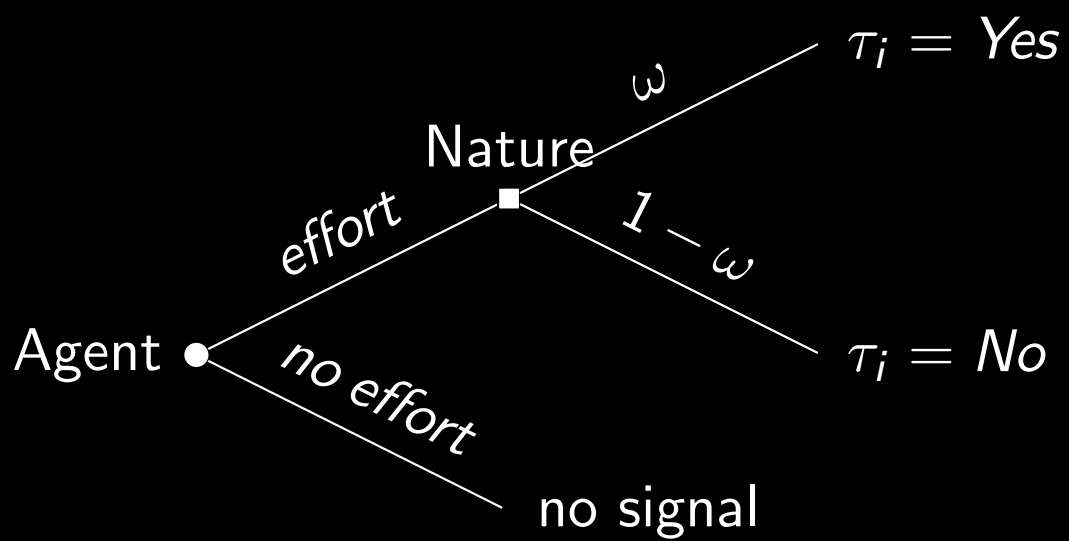
Socially stigmatized behavior $\rightarrow$ **Unwilling to recall**

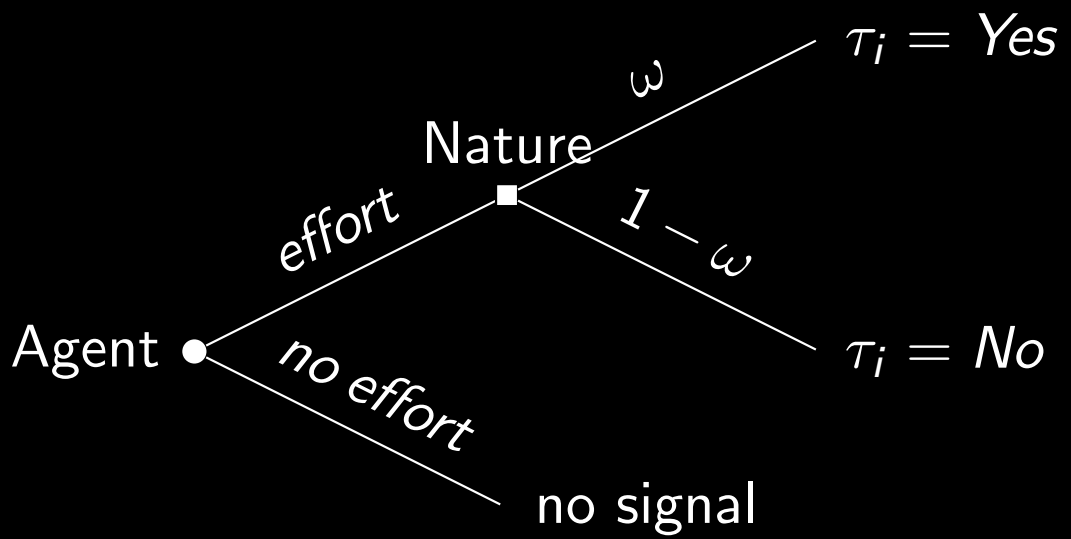
The framework



Assumptions:

- Common prior expectation $E[\omega]$ on ω.
- $E[\omega]$ is public knowledge.
- Agents follow Bayesian updating.

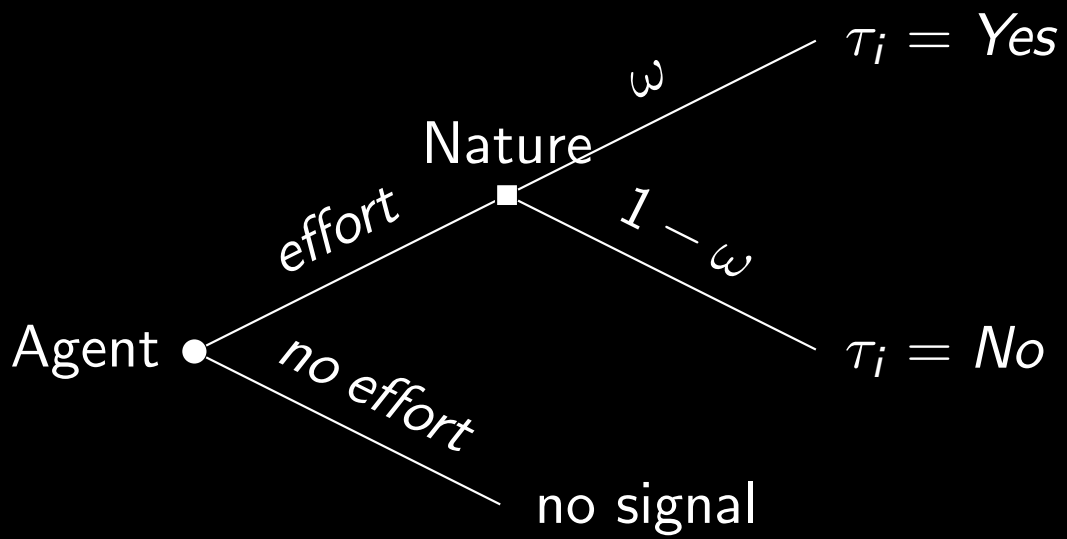
The framework



3 groups of agents with posterior expectations:

$E[\omega|\text{effort and } \tau_i = Yes]$
$E[\omega|\text{effort and } \tau_i = No]$
$E[\omega|\text{no effort}] = \text{Prior} = E[\omega]$

The framework



Posterior expectations satisfy:

$E[\omega|\tau_i = Yes] > E[\omega] > E[\omega|\tau_i = No]$

"Yes"-types expect $\omega > E[\omega] =$ prior.

"No"-types expect $\omega < E[\omega] =$ prior.

Peer prediction market

The mechanism

One-shot market

- Single asset
- Asset price $= p = E[\omega]$
- Asset value $= v =$ proportion of agents who buy

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One-shot market

- Single asset
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$t = 0$	t $= 1$	t $= 2$	t $= 3$
Center (market maker) sets $p = E[\omega]$.	Agents buy or sell 1 unit from the center.	Center calculates v, cancels the market if $v \in \{0, 1\}$	If $v \in (0, 1)$, buyers get $\pi(v - p)$ and sellers get $\pi(p - v)$

The mechanism

Numerical example:

- Currency is dollar, $\pi = 10$
- Price: $p = E[\omega] = 0.5$
- 40% of the participants buy, $v = 0.4$
- Buyer's payoff: $10\,(0.4 - 0.5) = -\$1$
- Seller's payoff: $10\,(0.5 - 0.4) = \$1$

The mechanism

Strategy = Effort or not + probability of buy in various situations

The mechanism

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Agent i's full strategy profile = $(e_i, R_i, R_i^{no}, R_i^{yes})$

- $e_i \in \{0, 1\}$ effort or no effort
- R_i probability of buy if $e_i = 0$,
- R_i^{no} probability of buy if $e_i = 1$ and $\tau_i = No$,
- R_i^{yes} probability of buy if $e_i = 1$ and $\tau_i = Yes$.

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Center would like: $e_i = 1$, $R_i^{no} = 0$, and $R_i^{yes} = 1$.

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Center would like: $e_i = 1$, $R_i^{no} = 0$, and $R_i^{yes} = 1$.

$\Longrightarrow$ **Truthful strategy**: Trades reflect honest answers.

Bayesian game

Assumption. The following are **common knowledge**:

- The market mechanism
- Signal technology, beliefs, costs and the strategy space.
- Risk-neutrality and Bayesianism of agents.

Ensures that we have a *Bayesian game* (Osborne and Rubinstein, 1994, Definition 25.1).

For convenience, we let $N \to \infty$.

Equilibrium analysis

Equilibria

Truthful equilibrium: For $N \to \infty$, truthful strategy is a Nash equilibrium if the rewards are scaled sufficiently high such that

$$\frac{c_i}{\pi} < E[\omega]\left(E[\omega|\tau_i = Yes] - E[\omega]\right) + \left(1 - E[\omega]\right)\left(E[\omega] - E[\omega|\tau_i = No]\right)$$

for all $i \in \{1, \ldots, N\}$

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In the truthful equilibrium...

- All agents exert effort
- Yes-types buy, No-types sell
- Truthful answer $\equiv$ Equilibrium trade

Equilibria

Truthful equilibrium: Full effort, Yes-types buy, No-types sell

How?

Equilibria

Truthful equilibrium: Full effort, Yes-types buy, No-types sell

How? Suppose,

- Agent i exerts effort ($e_i = 1$)
- All agents $j \neq i$ are truthful

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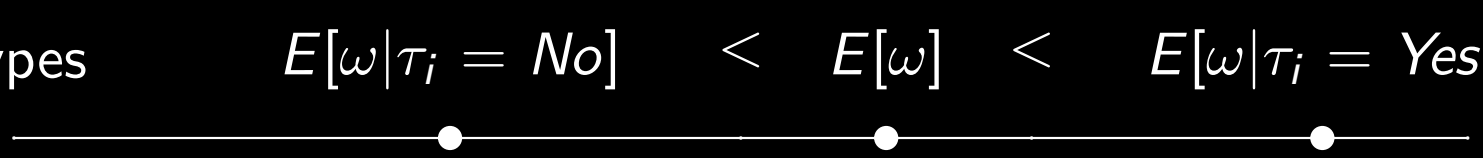

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Types	$E[\omega \mid \tau_i = No]$	$<$	$E[\omega]$	$<$	$E[\omega \mid \tau_i = Yes]$
Market	$E[v \mid \tau_i = No]$	$<$	p	$<$	$E[v \mid \tau_i = Yes]$

Asset value (v) $\equiv$ Proportion of buyers $\rightarrow$ Proportion of Yes-type (ω)

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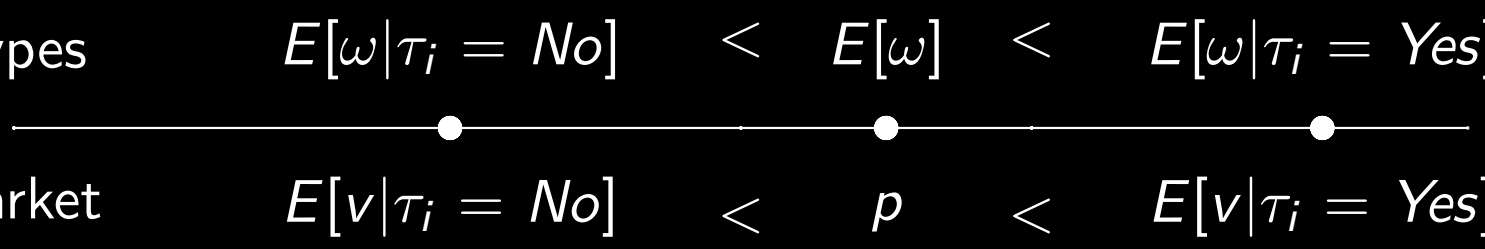

Optimal: Buy if $\tau_i = Yes$, sell if $\tau_i = No$

Incentive to "learn" your type $\rightarrow e_i = 1$

Equilibria

Multiple equilibria

No-effort equilibrium: If $c_i > \pi$ for all $i \in \{1, \ldots, N\}$, then Nash equilibria are characterized by $e_i = 0$ and $R_i \in \{0, E[\omega], 1\}$.

Expected payoffs are 0.

$\rightarrow$ No effort when costs are too high.

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Expected payoffs are 0.

$\rightarrow$ No effort when costs are too high.

Partial effort equilibrium: There are NE in which $K < N$ agents exert no effort and buy with probability $E[\omega]$ while the other agents are truthful.

$\rightarrow$ People with low cost exert effort, others do not.

Equilibria

Multiple equilibria

All-buy or all-sell: There exists Nash equilibria such that $e_i = 0$ and $R_i = 0$ or $R_i = 1$ for all i. Expected payoffs are 0.

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Truthful equilibrium: Strictly higher payoff than no-effort, all-buy and all-sell equilibria

Psychological costs

"Have you stood less than 6 feet apart from another person in a queue yesterday?"

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- Deception cost: The cost $d_i \geq 0$ of reporting "Yes" when $\tau_i = No$ or "No" when $\tau_i = Yes$.

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Truthful equilibrium if π is scaled appropriately

Experimental Evidence

Testing PPM

Two experimental studies.

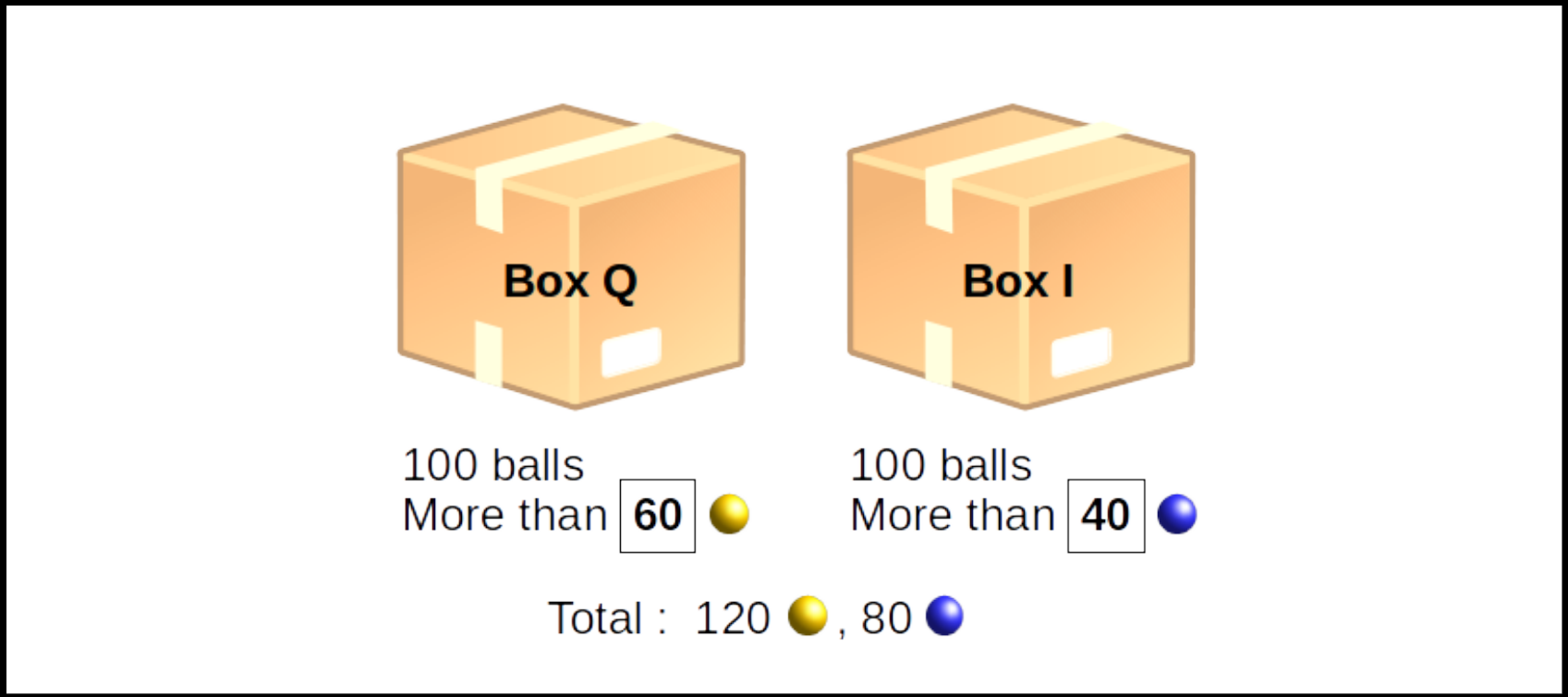
Study 1:

- Closely follows the theoretical model.
- Real effort task.

Study 2:

- Health survey, questions of 6-feet-apart type.
- Psychological costs & practical feasibility

Study 1 - A pair of boxes



One of the boxes has been selected (Q="more yellow" or I="less yellow").

Guess which one.

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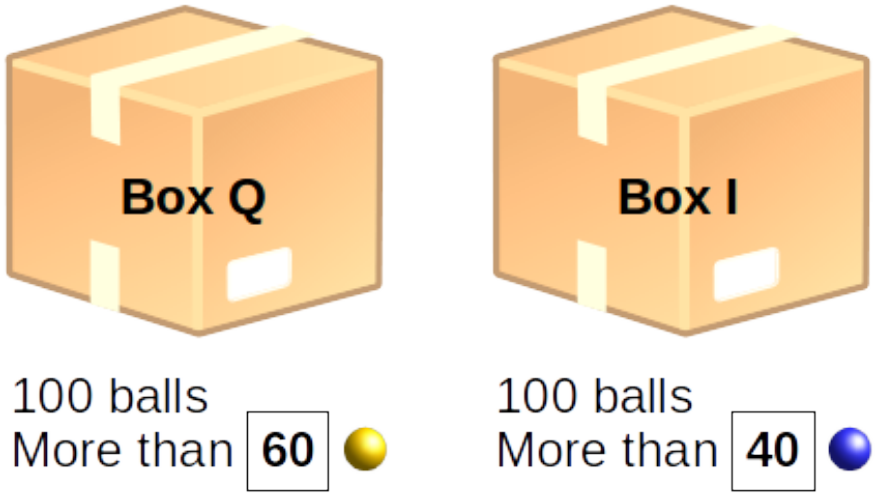


One of the boxes has been selected (Q="more yellow" or I="less yellow").

Guess which one. Want to see a ball from the selected box?

Study 1 - Real effort task

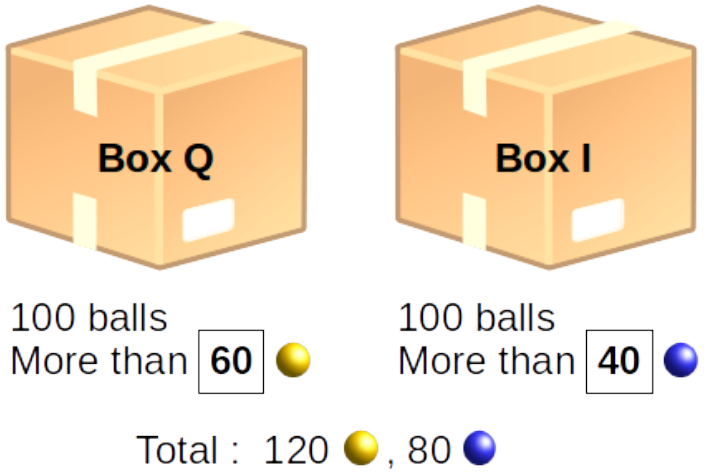
0	0	1	1	0	1
1	0	0	1	0	0
0	0	1	1	1	1
0	0	1	1	0	1

Count the number of 0s and you draw..

 OR 

Color of your draw $\equiv$ signal

Link with theory



0 0 1 1 0 1

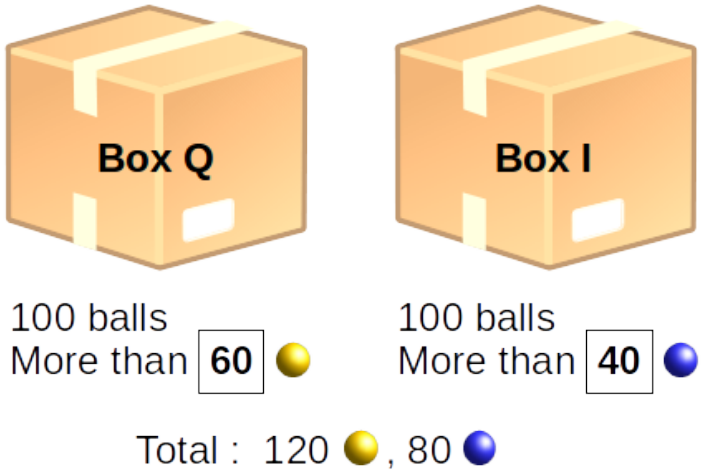
1 0 0 1 0 0

0 0 1 1 1 1

0 0 1 1 0 1

- Let's say a yellow draw is equivalent to ($\tau_i = Yes$).
- $E[\omega] = 0.6$ (common prior expectation on prop. yellow).

Link with theory



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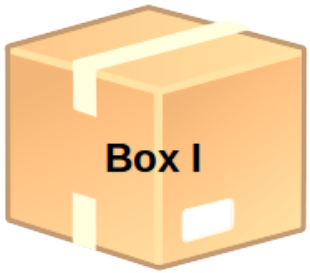
- c_i = cognitive effort of counting 1s in matrix.

Link with theory



100 balls
More than **60**

100 balls
More than **40**

Total : 120, 80

0	0	1	1	0	1
1	0	0	1	0	0
0	0	1	1	1	1
0	0	1	1	0	1

- Let's say a yellow draw is equivalent to ($\tau_i = Yes$).
- $E[\omega] = 0.6$ (common prior expectation on prop. yellow).
- c_i = cognitive effort of counting 1s in matrix.
- Picking Box Q $\equiv$ Buying $\equiv$ Reporting yellow ("Yes").

Study 1 - 3 treatments

- Flat fee: £3.25 completion fee.
- Accuracy incentives: £3.25 $\pm$ 0.20 per prediction task if the pick is correct or not.
- PPM: £3.25 + PPM incentives:
  Bonus: (% of people who pick the same box) − (prior).

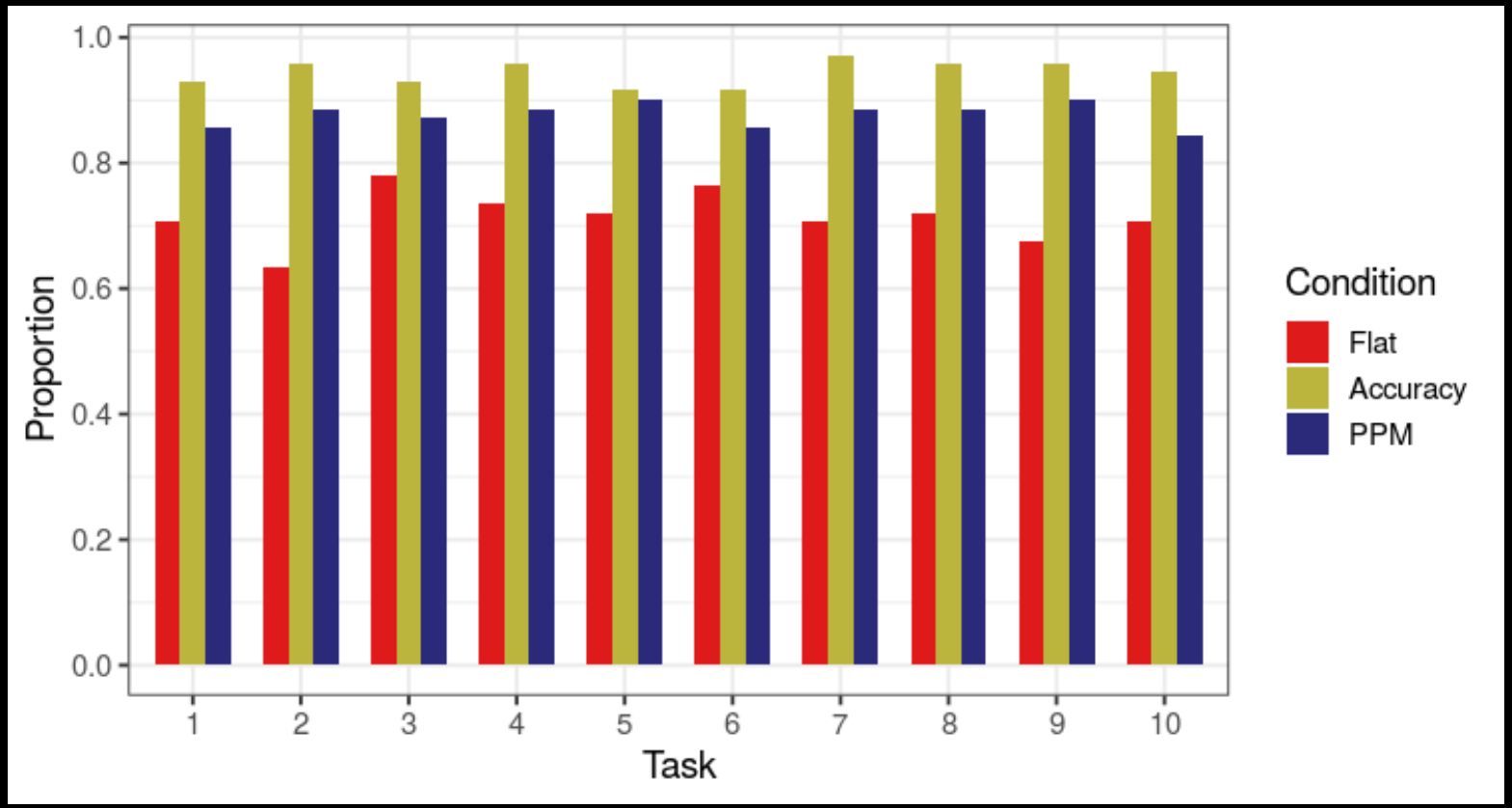
"Accuracy" is a benchmark for verifiable tasks.

Participants

- Online experiment, May 2020
- 210 U.S. citizens, students, recruited on Prolific
- 10 tasks (10 pairs of boxes, 10 matrices).

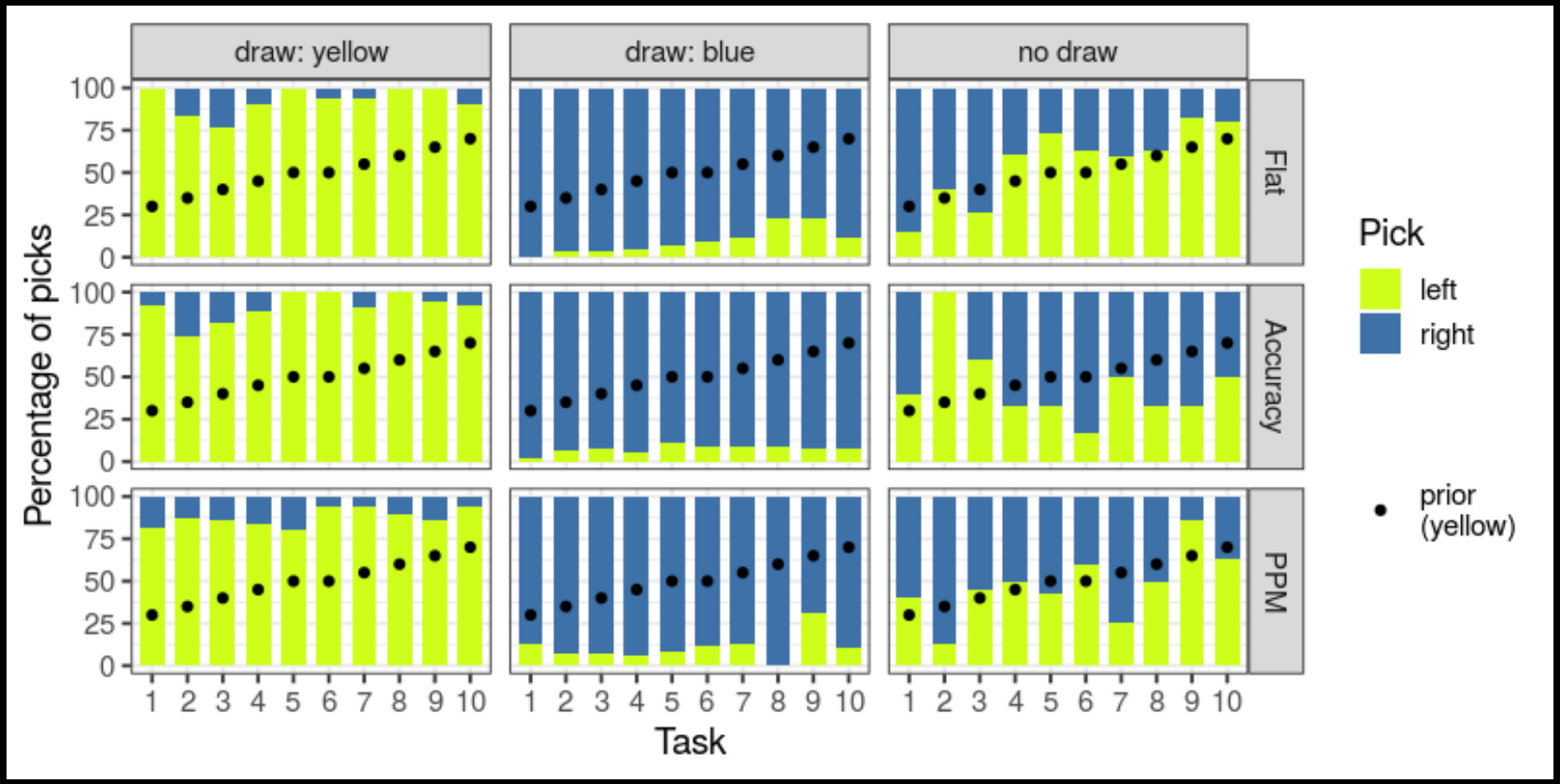
Study 1 - Effort

Proportion of subjects who complete the effort task



Accuracy > *PPM* > *Flat* in effort elicitation

Study 1 - Picks



Picks are as predicted by the truthful equilibrium.

Marginal effects, logistic regression

Dep. var.: P(effort task completed)				
	(whole sample)		*(filtered sample)*	
	(1)	(2)	(3)	(4)
PPM	0.10**	0.09**	0.10**	0.08**
	(0.03)	(0.03)	(0.03)	(0.03)
Accuracy	0.18***	0.18***	0.18***	0.18***
	(0.03)	(0.03)	(0.03)	(0.03)
Age		−0.00		−0.00
		(0.00)		(0.00)
Female?		0.04		0.04
		(0.03)		(0.03)
US resident?		−0.02		−0.02
		(0.06)		(0.06)
Num. obs.	2100	2070	2060	2030

$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$; $^{+}p < 0.1$

PPM can elicit effort when incentives for accuracy are not feasible (unverifiable questions).

Study 2

Study 1: Simple task, carefully controlled setup.

Study 2: Online field experiment

- Health & safety guidelines during the Covid-19 pandemic
- Did people follow them?

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- Did people follow them? **(Difficult to measure)**

Study 2

Study 1: Simple task, carefully controlled setup.

Study 2: Online field experiment

- Health & safety guidelines during the Covid-19 pandemic
- Did people follow them? **(Difficult to measure)**
- Would they self-report their unsafe behavior? **(Unverifiable)**
- A health survey with PPM incentives

Study 2 - Covid Survey

Question 2 of 8 (show instructions)

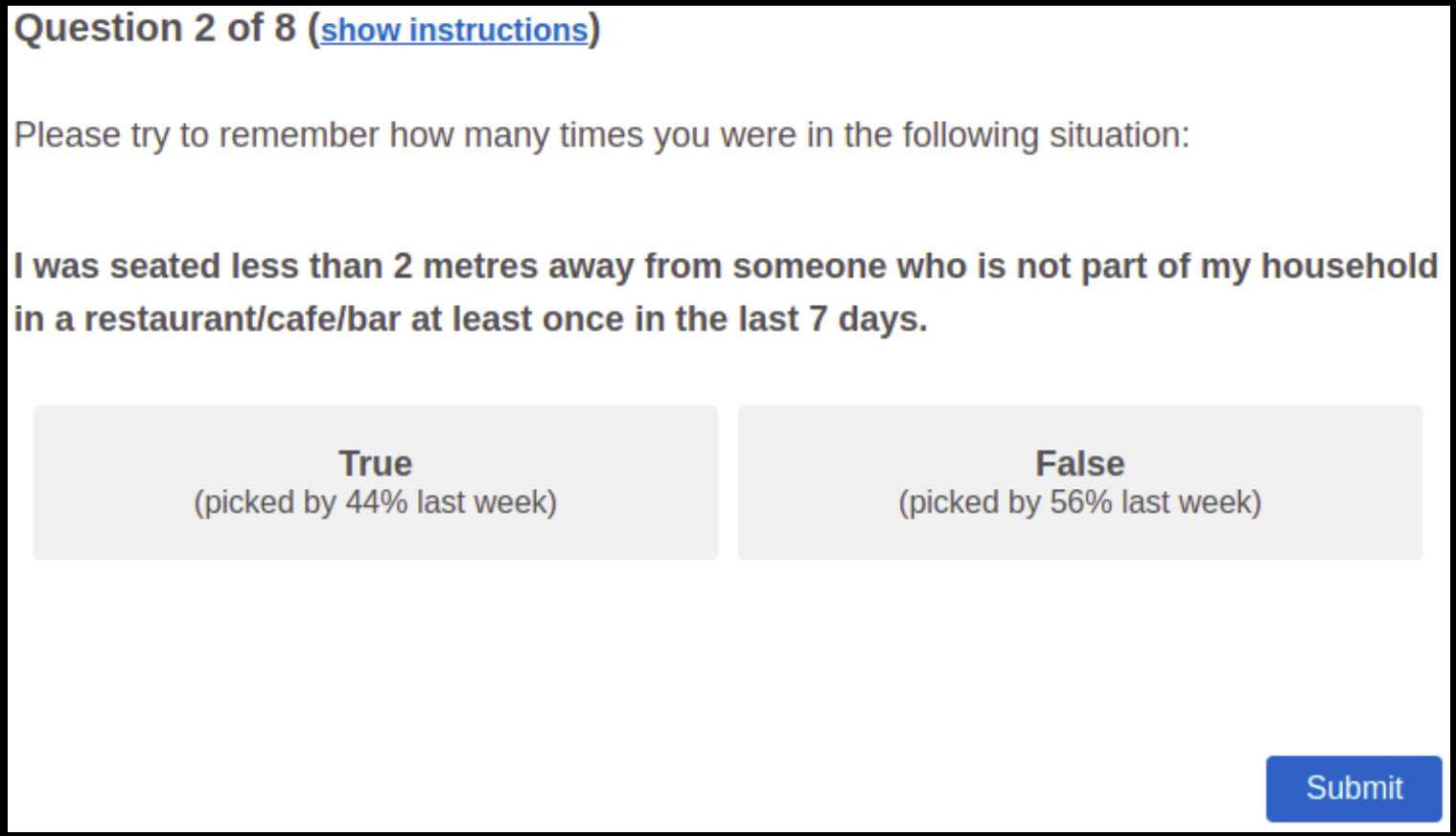
Please try to remember how many times you were in the following situation:

I was seated less than 2 metres away from someone who is not part of my household in a restaurant/cafe/bar at least once in the last 7 days.

True	False
(picked by 44% last week)	(picked by 56% last week)

Submit

Study 2 - Covid Survey



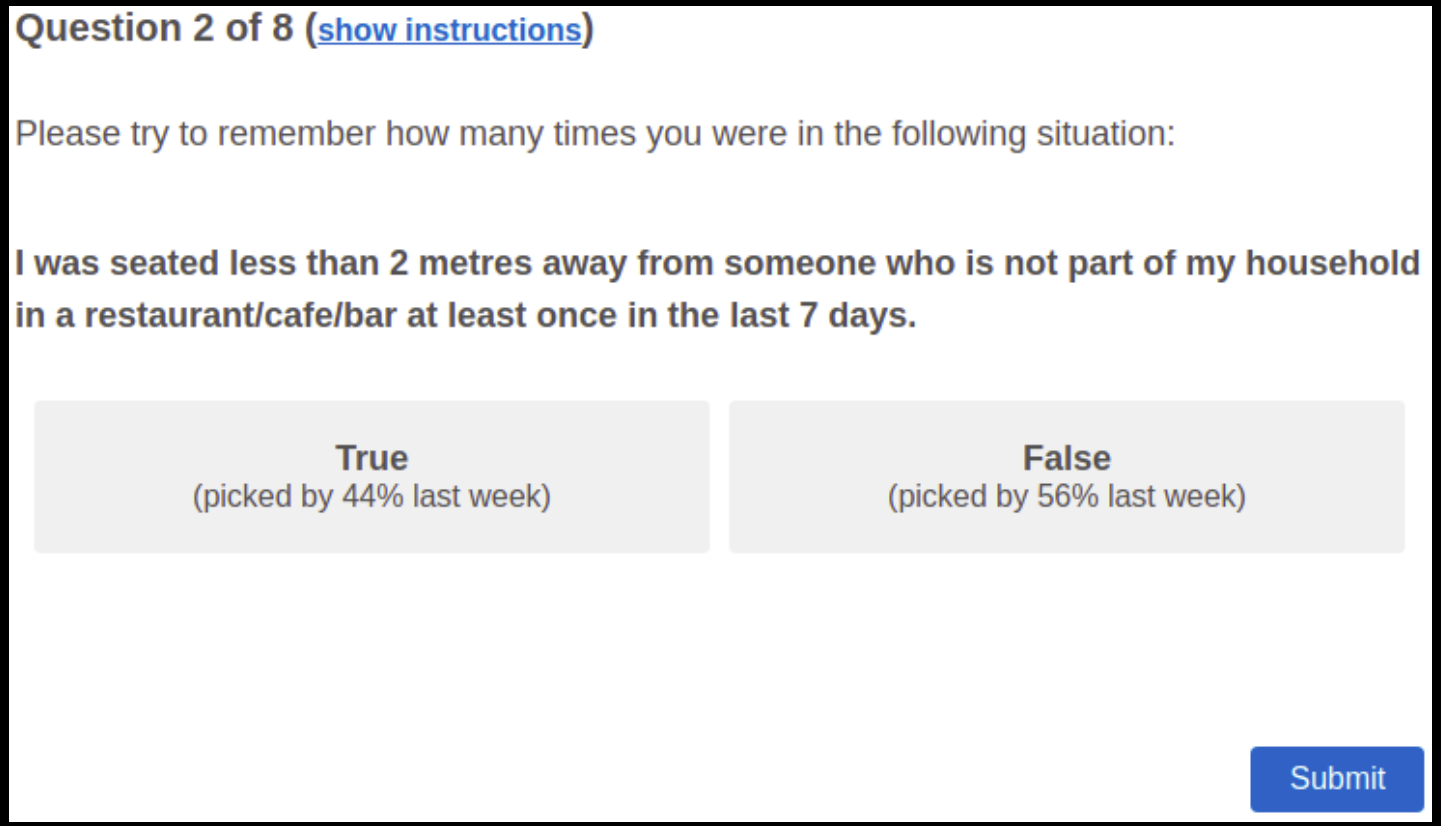
"True" could be underreported.

Would a PPM elicit a higher % of "True" responses?

Study 2 - True/False statements

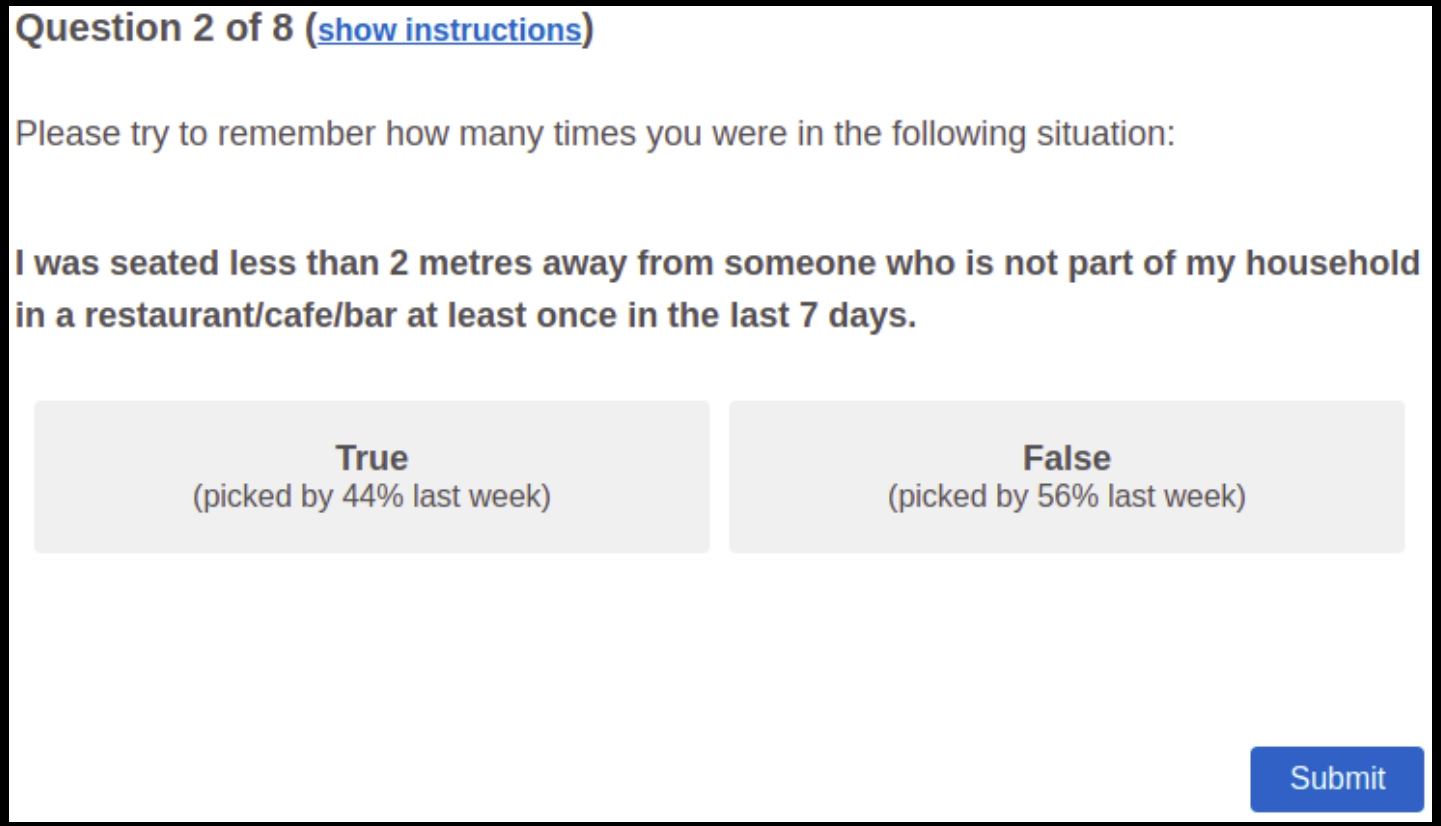
1.	I have been in an elevator with another person in it at least once in the last 7 days
2.	I may have stood less than 2 metres away from the person in front in a queue at least once in the last 7 days
3.	I was seated less than 2 metres away from someone who is not part of my household in a restaurant/cafe/bar at least once in the last 7 days
4.	I have been in a social gathering with more than 6 people who are not part of my household at least once in the last 7 days
5.	I have been in a busy shop/market with no restrictions on number of customers at least once in the last 7 days
6.	I participated in an indoor activity with more than 6 people who are not part of my household at least once in the last 7 days
7.	I have been in a shop/market where one or more of the staff did not wear a mask at least once in the last 7 days
8.	I had an interaction with someone experiencing high body temperature, persistent cough or loss of taste/smell at least once in the last 7 days

Study 2 - Link with theory



If you report True, bonus > 0 if % True this week > 0.44

Study 2 - Link with theory

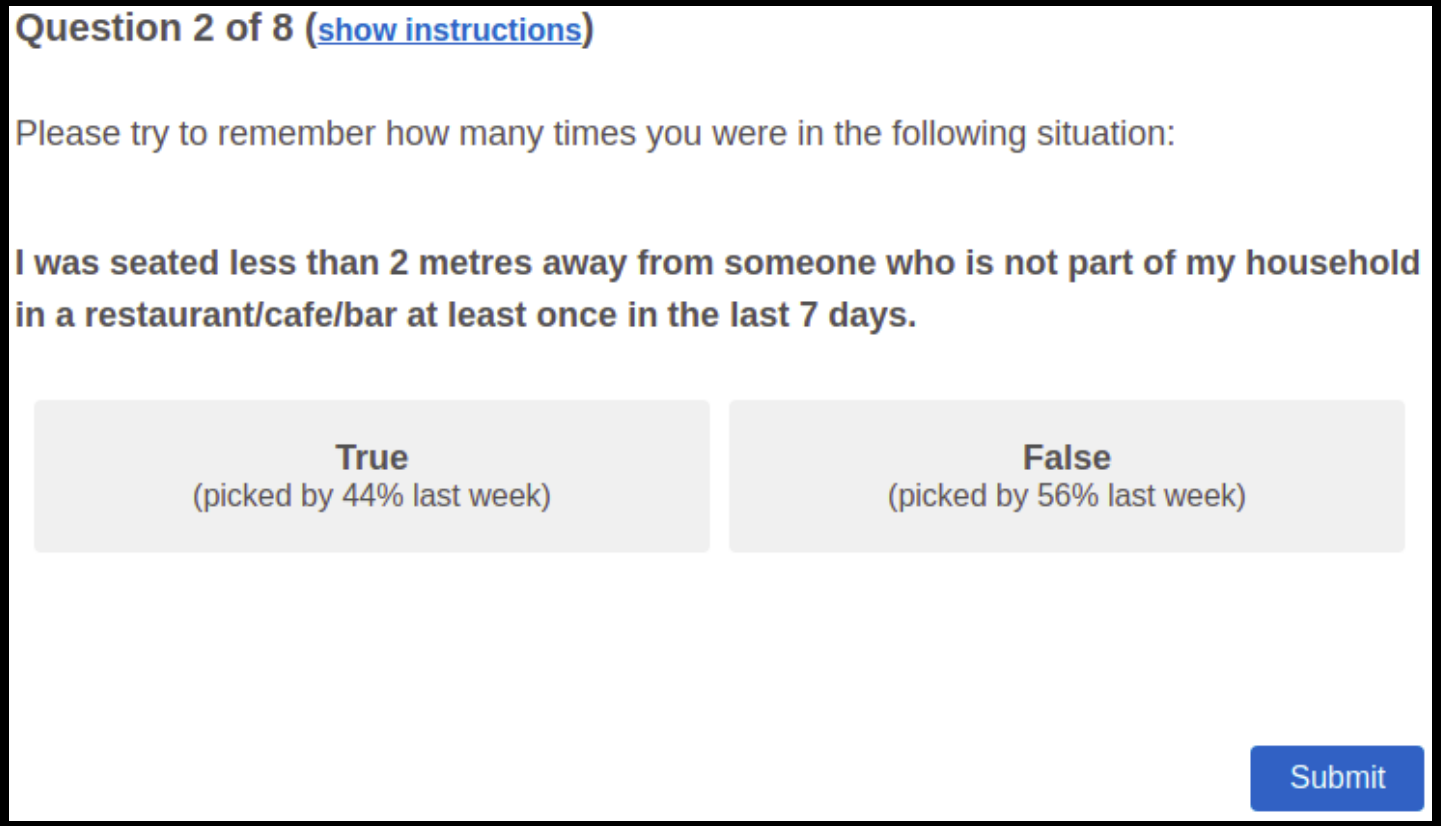


If you report True, bonus > 0 if % True this week > 0.44

Analogous to PPM:

- $v =$ % True this week
- $p = E[\omega] = 0.44$ (common prior on True)

Study 2 - Link with theory



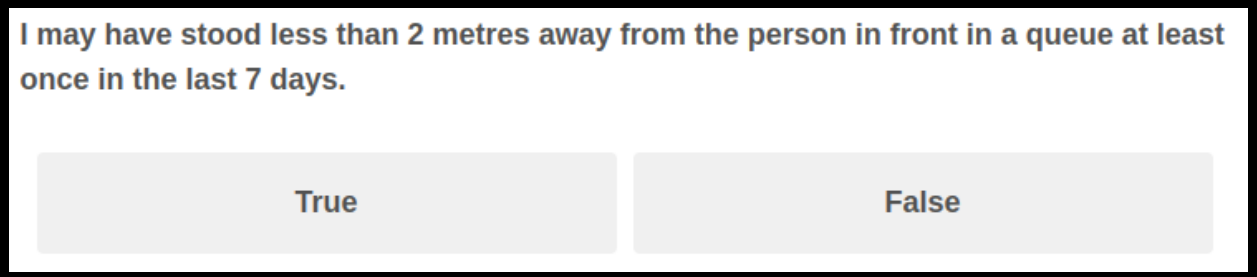
- Mental cost of remembering.
- Shame of answering "True" or lying.

Study 2 - Experimental conditions

Weekly survey in the UK, 3 weeks, November 2020

3 Experimental conditions

Control-1 (flat fee)

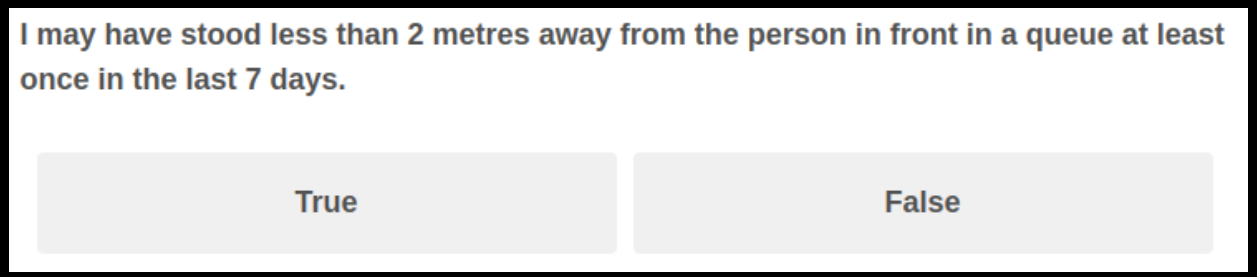


Study 2 - Experimental conditions

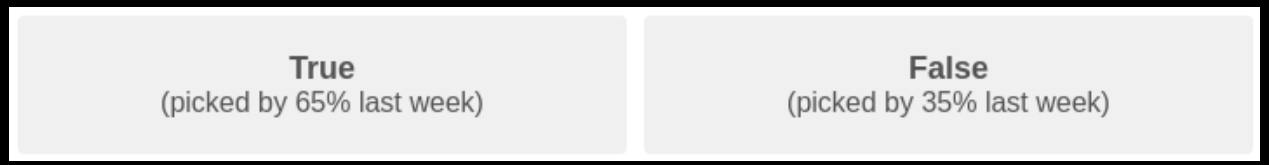
Weekly survey in the UK, 3 weeks, November 2020

3 Experimental conditions

Control-1 (flat fee)



Treatment (PPM incentives), Control-2 (flat fee)*



* tests the effect of just showing the last week's %s.

Study 2 - Marginal effects, Pr(Response = "True")

	(week 2)		*(week 3)*	
	(1)	(2)	(3)	(4)
(Intercept)				
Control-2	0.05	0.04	−0.01	−0.00
	(0.04)	(0.04)	(0.04)	(0.04)
PPM	0.11***	0.10**	0.08*	0.08*
	(0.03)	(0.03)	(0.04)	(0.04)
Age		−0.00		−0.00
		(0.00)		(0.00)
Female?		0.02		−0.02
		(0.03)		(0.03)
UK citizen?		−0.00		0.03
		(0.03)		(0.04)
Num. obs.	1259	1259	1279	1279

$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$; $^{+}p < 0.1$

Higher rate of self-reported unsafe behavior in the PPM treatment.

Contribution

Conclusion

Mechanism design literature: Explored ways to reveal private signals (Crémer and McLean, 1988).

Peer-prediction markets: Novel solution for eliciting effort & truthful revelation.

Conclusion

Mechanism design literature: Explored ways to reveal private signals (Crémer and McLean, 1988).

Peer-prediction markets: Novel solution for eliciting effort & truthful revelation.

Peer prediction method (Miller et al., 2005): similar framework, but

- The complete prior must be known
- Scoring is not transparent

Bayesian truth-serum (Prelec, 2004) and follow-ups:

- Detail-free (implementer needs less), but more demanding from respondents (answer + prediction)

Usually, costly effort to acquire signal not modelled.

Conclusion

Peer prediction markets: Transparent, easy to implement.

Conclusion

Peer prediction markets: Transparent, easy to implement.

Strong assumptions, but same as or weaker than in the literature.

Limitations: Binary questions only, multiple equilibria.

Thank you!

https://cempeker.github.io/

References

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Miller, N., Resnick, P., and Zeckhauser, R. (2005). Eliciting informative feedback: The peer-prediction method. *Management Science*, 51(9):1359–1373.

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