Peer prediction markets to elicit unverifiable information

Aurélien Baillon ¹ Cem Peker ² Sophie van der Zee ³

December 5, 2022

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Publications:

- Peker, C. (2022). Extracting the collective wisdom in probabilistic judgments. Theory and Decision. doi: 10.1007/s11238-022-09899-4
- Peker, C. (2022). Incentives for self-extremized expert judgments to alleviate the shared-information problem. *Decision*. doi.org: 10.1037/dec0000198

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Today's paper: Peer prediction markets to elicit unverifiable information (joint with A. Baillon and S. van der Zee)

More info: https://cempeker.github.io/

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Your answer is unverifiable!

Carefully considered and truthful answers

"Have you stood less than 6 feet apart from another person in a queue yesterday?"

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Peer Prediction (Miller et al., 2005):

"Yes" and "No" types have different beliefs on others' answers

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Peer Prediction (Miller et al., 2005):

- "Yes" and "No" types have different beliefs on others' answers
- Your prediction on others' answers is verifiable!

Peer-Prediction Market

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This paper: Peer-Prediction Market (PPM)

- One-shot market, buy/sell a single asset
- Trade ≡ A bet on others' answers

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This paper: Peer-Prediction Market (PPM)

- One-shot market, buy/sell a single asset
- Trade \equiv A bet on others' answers
- Trades reveal carefully considered and truthful answers
- Theory & evidence from 2 experimental studies

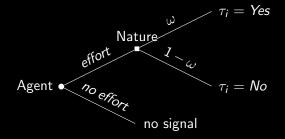
The Formal Framework

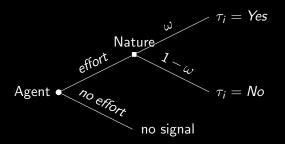
- $\bullet \ \ \textit{Center} \ \ \text{asks a binary question} \ \ \{\textit{Yes},\textit{No}\}$
- *N* risk-neutral *agents*

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- Each agent i can receive a costly signal $\tau_i \in \{Yes, No\}$. Signal cost $= c_i$

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- N risk-neutral agents
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- Signal $\tau_i \equiv$ Agent *i*'s honest answer

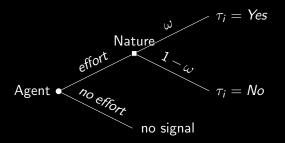
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"Have you stood less than 6 feet apart from another person in a queue yesterday?"

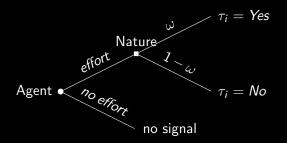
Signal $\tau_i \equiv$ honest answer. Why is τ_i costly?



"Have you stood less than 6 feet apart from another person in a queue yesterday?"

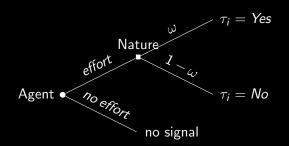
Signal $\tau_i \equiv$ honest answer. Why is τ_i costly?

Socially stigmatized behavior \rightarrow Unwilling to recall



Assumptions:

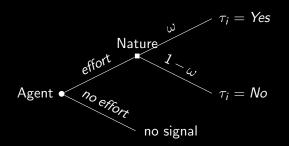
- Common prior expectation $E[\omega]$ on ω .
- $E[\omega]$ is public knowledge.
- Agents follow Bayesian updating.



 $\ensuremath{\mathtt{3}}$ groups of agents with posterior expectations:

$$E[\omega| ext{effort and } au_i = Yes]$$

 $E[\omega| ext{effort and } au_i = No]$
 $E[\omega| ext{no effort}] = ext{Prior} = E[\omega]$



Posterior expectations satisfy:

$$E[\omega|\tau_i = \textit{Yes}] > E[\omega] > E[\omega|\tau_i = \textit{No}]$$

"Yes"-types expect $\omega > E[\omega] = \text{prior.}$

"No"-types expect $\omega < E[\omega] = \text{prior.}$

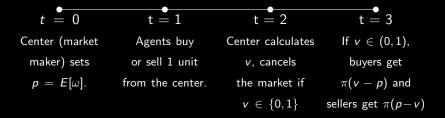
Peer prediction market

One-shot market

- Single asset
- Asset price $= p = E[\omega]$
- Asset value = v = proportion of agents who buy

One-shot market

- Single asset
- Asset price $= p = E[\omega]$
- Asset value = v = proportion of agents who buy



Numerical example:

- Currency is dollar, $\pi=10$
- Price: $p = E[\omega] = 0.5$
- 40% of the participants buy, v = 0.4
- Buyer's payoff: 10(0.4 0.5) = -\$1
- Seller's payoff: 10(0.5 0.4) = \$1

 ${\sf Strategy} = {\sf Effort} \ {\sf or} \ {\sf not} \ + \ {\sf probability} \ {\sf of} \ {\sf buy} \ {\sf in} \ {\sf various} \ {\sf situations}$

 $\mathsf{Strategy} = \mathsf{Effort} \ \mathsf{or} \ \mathsf{not} + \mathsf{probability} \ \mathsf{of} \ \mathsf{buy} \ \mathsf{in} \ \mathsf{various} \ \mathsf{situations}$

Agent i's full strategy profile = $(e_i, R_i, R_i^{no}, R_i^{yes})$

- $e_i \in \{0,1\}$ effort or no effort
- R_i probability of buy if $e_i = 0$,
- R_i^{no} probability of buy if $e_i = 1$ and $\tau_i = No$,
- R_i^{yes} probability of buy if $e_i = 1$ and $\tau_i = Yes$.

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Center would like: $e_i = 1$, $R_i^{no} = 0$, and $R_i^{yes} = 1$.

Strategy = Effort or not + probability of buy in various situations

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Center would like: $e_i = 1$, $R_i^{no} = 0$, and $R_i^{yes} = 1$.

→ Truthful strategy: Trades reflect honest answers.

Bayesian game

Assumption. The following are **common knowledge**:

- The market mechanism
- Signal technology, beliefs, costs and the strategy space.
- Risk-neutrality and Bayesianism of agents.

Ensures that we have a *Bayesian game* (Osborne and Rubinstein, 1994, Definition 25.1).

For convenience, we let $N \to \infty$.

Equilibrium analysis

Truthful equilibrium: For $N \to \infty$, truthful strategy is a Nash equilibrium if the rewards are scaled sufficiently high such that

$$rac{c_i}{\pi} < E[\omega] \left(E[\omega | au_i = extsf{Yes}] - E[\omega]
ight) + \left(1 - E[\omega]
ight) \left(E[\omega] - E[\omega | au_i = extsf{No}]
ight)$$

for all $i \in \{1, \dots, N\}$

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ight)$$

for all
$$i \in \{1, \dots, N\}$$

In the truthful equilibrium...

- All agents exert effort
- Yes-types buy, No-types sell
- Truthful answer ≡ Equilibrium trade

Truthful equilibrium: Full effort, Yes-types buy, No-types sell

How?

Truthful equilibrium: Full effort, Yes-types buy, No-types sell

How? Suppose,

- Agent i exerts effort $(e_i = 1)$
- All agents $j \neq i$ are truthful

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How? Suppose,

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Types
$$E[\omega|\tau_i = No] \le E[\omega] \le E[\omega|\tau_i = Yes]$$

Truthful equilibrium: Full effort, Yes-types buy, No-types sell

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Types
$$E[\omega|\tau_i=No]$$
 $<$ $E[\omega]$ $<$ $E[\omega|\tau_i=Yes]$ Market

Truthful equilibrium: Full effort, Yes-types buy, No-types sell

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Types
$$E[\omega|\tau_i=No]$$
 $<$ $E[\omega]$ $<$ $E[\omega|\tau_i=Yes]$ Market $E[v|\tau_i=No]$ $<$ p $<$ $E[v|\tau_i=Yes]$

Asset value $(v) \equiv \mathsf{Proportion}$ of buyers $\to \mathsf{Proportion}$ of Yes-type (ω)

Truthful equilibrium: Full effort, Yes-types buy, No-types sell

How? Suppose,

- Agent i exerts effort $(e_i = 1)$
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Types
$$E[\omega|\tau_i=No]$$
 $<$ $E[\omega]$ $<$ $E[\omega|\tau_i=Yes]$ Market $E[v|\tau_i=No]$ $<$ p $<$ $E[v|\tau_i=Yes]$

Optimal: Buy if $\tau_i = Yes$, sell if $\tau_i = No$ Incentive to "learn" your type $\rightarrow e_i = 1$

Multiple equilibria

No-effort equilibrium: If $c_i > \pi$ for all $i \in \{1, ..., N\}$, then Nash equilibria are characterized by $e_i = 0$ and $R_i \in \{0, E[\omega], 1\}$. Expected payoffs are 0.

 \rightarrow No effort when costs are too high.

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 \rightarrow No effort when costs are too high.

Partial effort equilibrium: There are NE in which K < N agents exert no effort and buy with probability $E[\omega]$ while the other agents are truthful.

 \rightarrow People with low cost exert effort, others do not.

Multiple equilibria

All-buy or all-sell: There exists Nash equilibria such that $e_i = 0$ and $R_i = 0$ or $R_i = 1$ for all i. Expected payoffs are 0.

Multiple equilibria

All-buy or all-sell: There exists Nash equilibria such that $e_i = 0$ and $R_i = 0$ or $R_i = 1$ for all i. Expected payoffs are 0.

Truthful equilibrium: Strictly higher payoff than no-effort, all-buy and all-sell equilibria

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Reporting "Yes" is shameful \rightarrow higher cost?

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- Asymmetric reporting cost: Cost $a_i \ge 0$ of reporting "Yes", no matter (presence of) signal.
- Deception cost: The cost $d_i \ge 0$ of reporting "Yes" when $\tau_i = No$ or "No" when $\tau_i = Yes$.

"Have you stood less than 6 feet apart from another person in a queue yesterday?"

Reporting "Yes" is shameful \rightarrow higher cost?

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Truthful equilibrium if π is scaled appropriately

Experimental Evidence

Testing PPM

Two experimental studies.

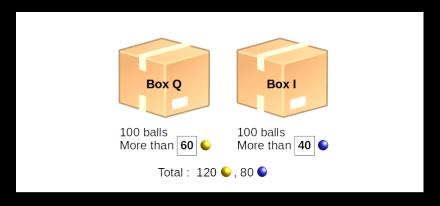
Study 1:

- Closely follows the theoretical model.
- Real effort task.

Study 2:

- Health survey, questions of 6-feet-apart type.
- Psychological costs & practical feasibility

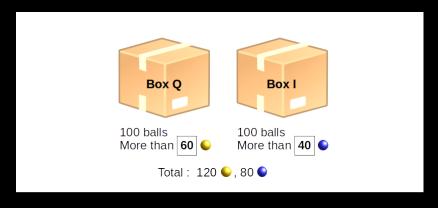
Study 1 - A pair of boxes



One of the boxes has been selected (Q= "more yellow" or I= "less yellow").

Guess which one.

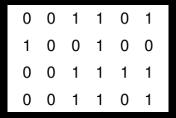
Study 1 - A pair of boxes



One of the boxes has been selected (Q= "more yellow" or I= "less yellow").

Guess which one. Want to see a ball from the selected box?

Study 1 - Real effort task



Count the number of 0s and you draw..

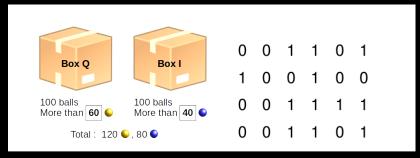


OR



Color of your draw \equiv signal

Link with theory



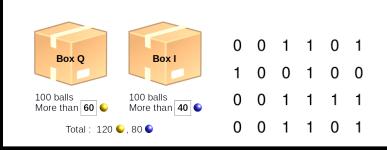
- Let's say a yellow draw is equivalent to $(\tau_i = Yes)$.
- $E[\omega] = 0.6$ (common prior expectation on prop. yellow).

Link with theory



- Let's say a yellow draw is equivalent to $(\tau_i = Yes)$.
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Link with theory



- Let's say a yellow draw is equivalent to $(\tau_i = Yes)$.
- $E[\omega] = 0.6$ (common prior expectation on prop. yellow).
- $c_i = \text{cognitive effort of counting 0s in matrix.}$
- Picking Box $Q \equiv Buying \equiv Reporting yellow ("Yes").$

Study 1 - 3 treatments

- Flat fee: £3.25 completion fee.
- Accuracy incentives: £3.25 \pm 0.20 per prediction task if the pick is correct or not.
- PPM: £3.25 + PPM incentives:
 Bonus: (% of people who pick the same box) (prior).

"Accuracy" is a benchmark for verifiable tasks.

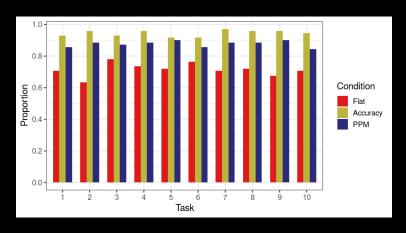
Participants

- Online experiment, May 2020
- 210 U.S. citizens, students, recruited on Prolific

• 10 tasks (10 pairs of boxes, 10 matrices).

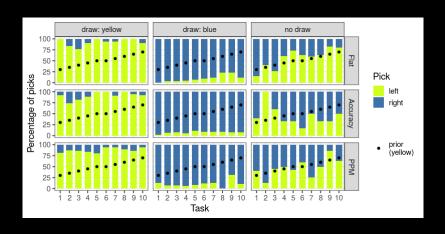
Study 1 - Effort

Proportion of subjects who complete the effort task



Accuracy > PPM > Flat in effort elicitation

Study 1 - Picks



Picks are as predicted by the truthful equilibrium.

Marginal effects, logistic regression

	D/ · CC · · ·		1.1.1)	
Dep. var.: P(effort task completed)				
	(whole sample)		(filtered sample)	
	(1)	(2)	(3)	(4)
PPM	0.10**	0.09**	0.10**	0.08**
	(0.03)	(0.03)	(0.03)	(0.03)
Accuracy	0.18***	0.18***	0.18***	0.18***
	(0.03)	(0.03)	(0.03)	(0.03)
Age		-0.00		-0.00
		(0.00)		(0.00)
Female?		0.04		0.04
		(0.03)		(0.03)
US resident?		-0.02		-0.02
		(0.06)		(0.06)
Num. obs.	2100	2070	2060	2030
*** $p < 0.001$; ** $p < 0.01$; * $p < 0.05$; $p < 0.05$; * $p < 0.05$				

PPM can elicit effort when incentives for accuracy are not feasible (unverifiable questions).

Study 2

Study 1: Simple task, carefully controlled setup.

Study 2: Online field experiment

- Health & safety guidelines during the Covid-19 pandemic
- Did people follow them?

Study 2

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Study 2: Online field experiment

- Health & safety guidelines during the Covid-19 pandemic
- Did people follow them? (Difficult to measure)

Study 2

Study 1: Simple task, carefully controlled setup.

Study 2: Online field experiment

- Health & safety guidelines during the Covid-19 pandemic
- Did people follow them? (Difficult to measure)
- Would they self-report their unsafe behavior? (Unverifiable)
- A health survey with PPM incentives

Study 2 - Covid Survey

Question 2 of 8 (show instructions)

Please try to remember how many times you were in the following situation:

I was seated less than 2 metres away from someone who is not part of my household in a restaurant/cafe/bar at least once in the last 7 days.

True False (picked by 44% last week) (picked by 56% last week)

Submit

Study 2 - Covid Survey

Question 2 of 8 (show instructions) Please try to remember how many times you were in the following situation: I was seated less than 2 metres away from someone who is not part of my household in a restaurant/cafe/bar at least once in the last 7 days. True False (picked by 44% last week) (picked by 56% last week) Submit

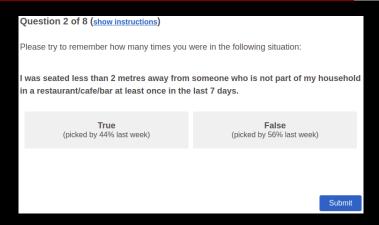
"True" could be underreported.

Would a PPM elicit a higher % of "True" responses?

Study 2 - True/False statements

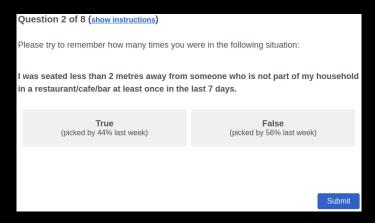
1.	I have been in an elevator with another person in it at least once
	in the last 7 days
2.	I may have stood less than 2 metres away from the person in front
	in a queue at least once in the last 7 days
3.	I was seated less than 2 metres away from someone who is not part
	of my household in a restaurant/cafe/bar at least once in the last
	7 days
4.	I have been in a social gathering with more than 6 people who are
	not part of my household at least once in the last 7 days
5.	I have been in a busy shop/market with no restrictions on number
	of customers at least once in the last 7 days
6.	I participated in an indoor activity with more than 6 people who
	are not part of my household at least once in the last 7 days
7.	I have been in a shop/market where one or more of the staff did
	not wear a mask at least once in the last 7 days
8.	I had an interaction with someone experiencing high body temper-
	ature, persistent cough or loss of taste/smell at least once in the
	last 7 days

Study 2 - Link with theory



If you report True, bonus > 0 if % True this week > 0.44

Study 2 - Link with theory

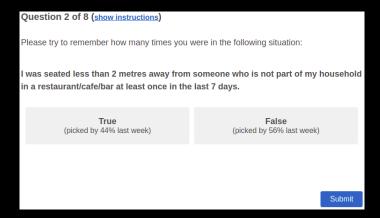


If you report True, bonus > 0 if % True this week > 0.44

Analogous to PPM:

- v = % True this week
- $p = E[\omega] = 0.44$ (common prior on True)

Study 2 - Link with theory



- Mental cost of remembering.
- Shame of answering "True" or lying.

Study 2 - Experimental conditions

Weekly survey in the UK, 3 weeks, November 2020

3 Experimental conditions

Control-1 (flat fee)

I may have stood less than 2 metres away from the person in front in a queue at least ${\sf I}$				
once in the last 7 days.				
True	False			

Study 2 - Experimental conditions

Weekly survey in the UK, 3 weeks, November 2020

3 Experimental conditions

Control-1 (flat fee)

I may have stood less than 2 metres away from the person in front in a queue at least once in the last 7 days.

True False

Treatment (PPM incentives), Control-2 (flat fee)*



* tests the effect of just showing the last week's %s.

Study 2 - Marginal effects, Pr(Response = "True")

	(week 2)		(week 3)		
	(1)	(2)	(3)	(4)	
(Intercept)					
Control-2	0.05	0.04	-0.01	-0.00	
	(0.04)	(0.04)	(0.04)	(0.04)	
PPM	0.11***	0.10**	0.08*	0.08*	
	(0.03)	(0.03)	(0.04)	(0.04)	
Age		-0.00		-0.00	
		(0.00)		(0.00)	
Female?		0.02		-0.02	
		(0.03)		(0.03)	
UK citizen?		-0.00		0.03	
		(0.03)		(0.04)	
Num. obs.	1259	1259	1279	1279	
*** $p < 0.001$; ** $p < 0.01$; * $p < 0.05$; $p < 0.1$					

Higher rate of self-reported unsafe behavior in the PPM treatment.



Mechanism design literature: Explored ways to reveal private signals (Crémer and McLean, 1988).

Peer-prediction markets: Novel solution for eliciting effort & truthful revelation.

Mechanism design literature: Explored ways to reveal private signals (Crémer and McLean, 1988).

Peer-prediction markets: Novel solution for eliciting effort & truthful revelation.

Peer prediction method (Miller et al., 2005): similar framework, but

- The complete prior must be known
- Scoring is not transparent

Bayesian truth-serum (Prelec, 2004) and follow-ups:

 Detail-free (implementer needs less), but more demanding from respondents (answer + prediction)

Usually, costly effort to acquire signal not modelled.

Peer prediction markets: Transparent, easy to implement.

Peer prediction markets: Transparent, easy to implement.

Strong assumptions, but same as or weaker than in the literature.

Limitations: Binary questions only, multiple equilibria.

Thank you!

https://cempeker.github.io/

References

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