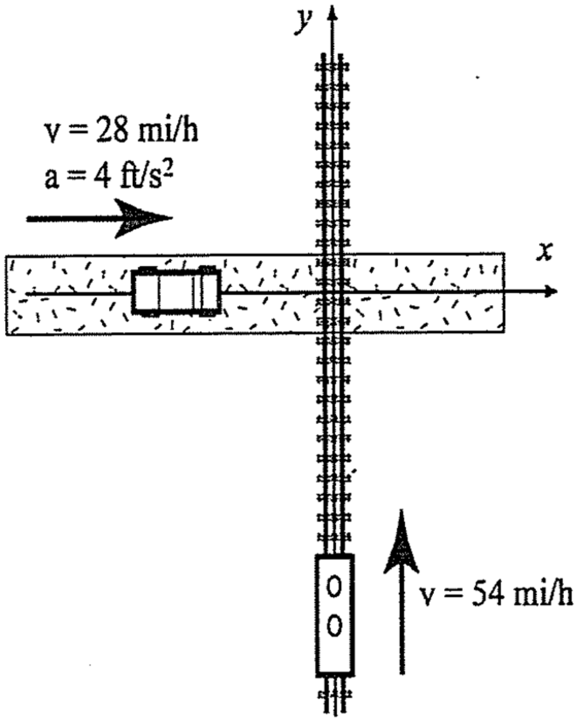
BIL 108E Intr. to Sci. & Eng.Computing

Res.Asst.Çiğdem Toparlı

EXERCISES -2

Example-1



A train and a car are approaching a road crossing. At $t = 0$ the train is 400 ft. south of the crossing traveling north at a constant speed of 54 mi/h. At the same time the car is 200 ft. west of the crossing traveling east at a speed of 28 mi/h and accelerating at 4 ft/s^2. Determine the positions of the train and the car, the distance between them, and the speed of the train relative to the car every second for the next 10 seconds.

To show the results, create an 11×6 matrix in which each row has the time in the first column and the train position, car position, distance between the train and the car, car speed, and the speed of the train relative to the car, in the next five columns, respectively.

Solution-1

The position of an object that moves along a straight line at a constant acceleration is given by $s = s_o + v_o t + \frac{1}{2}at^2$ where s_o and v_o are the position and velocity at $t = 0$, and a is the acceleration. Applying this equation to the train and the car gives:

$$y = -400 + v_{otrain}t \qquad \text{(train)}$$

$$x = -200 + v_{ocar}t + \frac{1}{2}a_{car}t^2 \qquad \text{(car)}$$

The distance between the car and the train is: $d = \sqrt{x^2 + y^2}$.
The velocity of the train is constant and in vector notation is: $\mathbf{v}_{train} = v_{otrain}\mathbf{j}$. The car is accelerating and its velocity at time t is given by: $\mathbf{v}_{car} = (v_{ocar} + a_{car}t)\mathbf{i}$. The velocity of the train relative to the car, $\mathbf{v}_{t/c}$ is given by: $\mathbf{v}_{t/c} = \mathbf{v}_{train} - \mathbf{v}_{car} = -(v_{ocar} + a_{car}t)\mathbf{i} + v_{otrain}\mathbf{j}$. The magnitude (speed) of this velocity is the length of the vector.

The problem is solved by first creating a vector `t` with 11 elements for the time from 0 to 10 s, and then calculating the positions of the train and the car, the distance between them, and the speed of the train relative to the car at each time element. The following are MATLAB commands that solve the problem.

Solution-1

```
>> v0train = 54*5280/3600; v0car = 28*5280/3600; acar = 4;
```

Create variables for the initial velocities (in ft/s) and the acceleration.

```
>> t = 0:10;
```

Create the vector `t`.

```
>> y = -400 + v0train*t;
>> x = -200 + v0car*t + 0.5*acar*t.^2;
```

Calculate the train and car positions.

```
>> d = sqrt(x.^2 + y.^2);
```

Calculate the distance between the train and car.

Solution-1

```
>> vcar = v0car + acar*t;
>> speed_trainRcar = sqrt(vcar.^2 + v0train^2);
>> table = [t' y' x' d' speed_trainRcar']
```

Calculate the car's velocity.

Calculate the speed of the train relative to the car.

Create a table (see note below).

```
table =
        0 -400.0000 -200.0000 447.2136  41.0667  89.2139
   1.0000 -320.8000 -156.9333 357.1284  45.0667  91.1243
   2.0000 -241.6000 -109.8667 265.4077  49.0667  93.1675
   3.0000 -162.4000  -58.8000 172.7171  53.0667  95.3347
   4.0000  -83.2000   -3.7333  83.2837  57.0667  97.6178
   5.0000   -4.0000   55.3333  55.4777  61.0667 100.0089
   6.0000   75.2000  118.4000 140.2626  65.0667 102.5003
   7.0000  154.4000  185.4667 241.3239  69.0667 105.0849
   8.0000  233.6000  256.5333 346.9558  73.0667 107.7561
   9.0000  312.8000  331.6000 455.8535  77.0667 110.5075
  10.0000  392.0000  410.6667 567.7245  81.0667 113.3333
>>
```

Time (s) | Train position (ft) | Car position (ft) | Car-train distance (ft) | Car speed (ft/s) | Train speed relative to the car; (ft/s)

Example-2

Given are a 5×6 matrix A, a 3×6 matrix B, and a 9 element long vector v.

$$A = \begin{bmatrix} 2 & 5 & 8 & 11 & 14 & 17 \\ 3 & 6 & 9 & 12 & 15 & 18 \\ 4 & 7 & 10 & 13 & 16 & 19 \\ 5 & 8 & 11 & 14 & 17 & 20 \\ 6 & 9 & 12 & 15 & 18 & 21 \end{bmatrix} \qquad B = \begin{bmatrix} 5 & 10 & 15 & 20 & 25 & 30 \\ 30 & 35 & 40 & 45 & 50 & 55 \\ 55 & 60 & 65 & 70 & 75 & 80 \end{bmatrix}$$

$$v = \begin{bmatrix} 99 & 98 & 97 & 96 & 95 & 94 & 93 & 92 & 91 \end{bmatrix}$$

Create the three arrays in the Command Window, and then, by writing one command, replace the last four columns of the 1st and 3rd rows of A with the first four columns of the first two rows of B, the last four columns of the 4th row of A with the elements 5 through 8 of v, and the last four columns of the 5th row of A with columns 3 through 5 of the third row of B.

Solution-2

```
>> A = [2:3:17; 3:3:18; 4:3:19; 5:3:20; 6:3:21]
A =
    2    5    8   11   14   17
    3    6    9   12   15   18
    4    7   10   13   16   19
    5    8   11   14   17   20
    6    9   12   15   18   21
>> B = [5:5:30; 30:5:55; 55:5:80]
B =
    5   10   15   20   25   30
   30   35   40   45   50   55
   55   60   65   70   75   80
>> v = [99:-1:91]
v =
   99   98   97   96   95   94   93   92   91
>> A([1 3 4 5],3:6) = [B([1 2],1:4); v(5:8); B(3,2:5)]
```

4 × 4 matrix made of columns 3 through 6 of rows 1, 3, 4, and 5.

4 × 4 matrix. The first two rows are columns 1 through 4 of rows 1 and 2 of matrix B. The third row are elements 5 through 8 of vector v. The fourth row are columns 2 through 5 of row 3 of matrix B.

```
A =
    2    5    5   10   15   20
    3    6    9   12   15   18
    4    7   30   35   40   45
    5    8   95   94   93   92
    6    9   60   65   70   75
```

Example-3

Create a 5×7 matrix in which the first row are the numbers 1 2 3 4 5 6 7, the second row are the numbers 8 9 10 11 12 13 14, the third row are the numbers 15 through 21, and so on. From the this matrix create a new 3×4 matrix that is made from rows 2 through 4, and columns 3 through 6 of the first matrix.

Solution-3

```
>> A=[1 2 3 4 5 6 7;
8 9 10 11 12 13 14;
15 16 17 18 19 20 21;
22 23 24 25 26 27 28;
29 30 31 32 33 34 35]

A =

     1     2     3     4     5     6     7
     8     9    10    11    12    13    14
    15    16    17    18    19    20    21
    22    23    24    25    26    27    28
    29    30    31    32    33    34    35

>> B=A(2:4,3:6)

B =

    10    11    12    13
    17    18    19    20
    24    25    26    27
```

Example-4

Use matrix operations to solve the following system of linear equations.

$$\begin{aligned} 4x - 2y + 6z &= 8 \\ 2x + 8y + 2z &= 4 \\ 6x + 10y + 3z &= 0 \end{aligned}$$

Solution-4

Using the rules of linear algebra demonstrated earlier, the above system of equations can be written in the matrix form $AX = B$ or in the form $XC = D$:

$$\begin{bmatrix} 4 & -2 & 6 \\ 2 & 8 & 2 \\ 6 & 10 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \\ 0 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} 4 & 2 & 6 \\ -2 & 8 & 10 \\ 6 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 4 & 0 \end{bmatrix}$$

```
>> A = [4 -2 6; 2 8 2; 6 10 3];
>> B = [8; 4; 0];
>> X = A\B
X =
  -1.8049
   0.2927
   2.6341
```

Solving the form $AX = B$.

```
>> Xb = inv(A)*B
Xb =
  -1.8049
   0.2927
   2.6341
```

Solving by using the inverse of A $X = A^{-1}B$.

```
>> C = [4 2 6; -2 8 10; 6 2 3];
>> D = [8 4 0];
```

Solving the form $XC = D$.

```
>> Xc = D/C
Xc =
  -1.8049    0.2927    2.6341
```

Solving by using right division $X = D/C$.

```
>> Xd = D*inv(C)
Xd =
  -1.8049    0.2927    2.6341
```

Solving by using the inverse of C, $X = D \cdot C^{-1}$.

Example-5

Two vectors are given:

$\mathbf{u} = 4\mathbf{i} + 9\mathbf{j} - 5\mathbf{k}$ and $\mathbf{v} = -3\mathbf{i} + 6\mathbf{j} - 7\mathbf{k}$

Use MATLAB to calculate the dot product $\mathbf{u} \cdot \mathbf{v}$ of the vectors in two ways:

a) Define **u** as a row vector and **v** as a column vector, and then use matrix multiplication.

b) Use the `dot` function.

Solution-5

```
>> u=[4 9 -5]

u =

    4    9   -5

>> v=[-3 6 -7]

v =

   -3    6   -7

>> v=[-3;6;-7]

v =

   -3
    6
   -7

>> A=u*v

A =

   77

>> B=v*u

B =

  -12  -27   15
   24   54  -30
  -28  -63   35

>> dot(u,v)

ans =

   77
```

Example-6

When several resistors are connected in an electrical circuit in series, the voltage across each of them is given by the voltage divider rule:

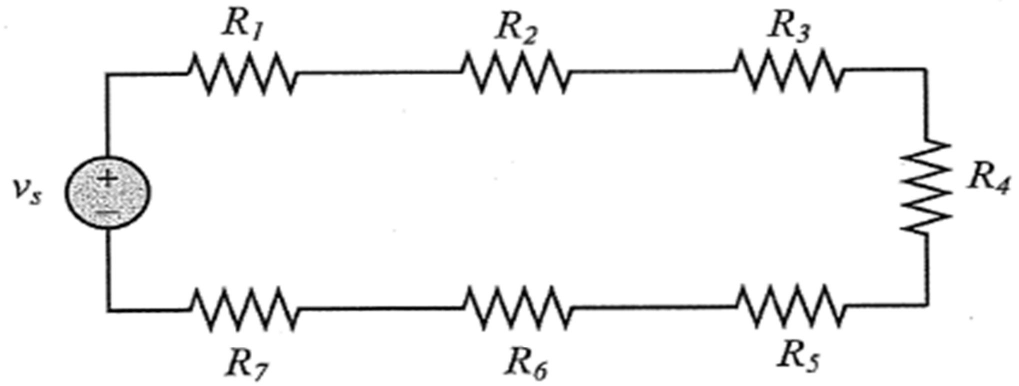
$$v_n = \frac{R_n}{R_{eq}} v_s$$

where v_n and R_n are the voltage across resistor n and its resistance, respectively, $R_{eq} = \Sigma R_n$ is the equivalent resistance, and v_s is the source voltage. The power dissipated in each resistor is given by:

$$P_n = \frac{R_n}{R_{eq}^2} v_s^2$$

Example-6

The figure below shows, for example, a circuit with seven resistors connected in series.



Write a program in a script file that calculates the voltage across each resistor, and the power dissipated in each resistor, in a circuit that has resistors connected in series. When the script file is executed it requests the user to first enter the source voltage and then to enter the resistance of the resistors in a vector. The program displays a table with the resistances listed in the first column, the voltage across the resistor in the second, and the power dissipated in the resistor in the third column. Following the table, the program displays the current in the circuit, and the total power.

Run the file and enter the following data for v_s and the R's.

$v_s = 24\text{V}$, $R_1 = 20\Omega$, $R_2 = 14\Omega$, $R_3 = 12\Omega$, $R_4 = 18\Omega$, $R_5 = 8\Omega$, $R_6 = 15\Omega$, $R_7 = 10\Omega$.

Solution-6

A script file that solves the problem is shown below.

```
% The program calculates the voltage across each resistor
% in a circuit that has resistors connected in series.
vs = input('Please enter the source voltage ');
Rn = input('Enter the values of the resistors as elements in a row vector\n');
Req = sum(Rn);                    Calculate the equivalent resistance.
vn = Rn*vs/Req;                   Apply the voltage divider rule.
Pn = Rn*vs^2/Req^2;               Calculate the power in each resistor.
i = vs/Req;                       Calculate the current in the circuit.
Ptotal = vs*i;                    Calculate the total power in the circuit.
Table = [Rn', vn', Pn'];          Create a variable table with the
                                  vectors Rn, vn, and Pn as columns.
disp(' ')
disp(' Resistance  Voltage   Power')     Display headings for the columns.
disp('   (Ohms)    (Volts)  (Watts)')
disp(' ')                         Display an empty line.
disp(Table)                       Display the variable Table.
disp(' ')
fprintf('The current in the circuit is %f Amps.',i)
fprintf('\nThe total power dissipated in the circuit is %f Watts.',Ptotal)
```

Solution-6

The Command Window where the script file was executed is:

```
>> VoltageDivider                                              Name of the script file.
Please enter the source voltage 24        <--   Source voltage entered by the user.
Enter the value of the resistors as elements in a row vector
[20 14 12 18 8 15 10]                      <--   Resistor values entered as a vector.

 Resistance  Voltage   Power
   (Ohms)    (Volts)  (Watts)
  20.0000     4.9485   1.2244

  14.0000     3.4639   0.8571
  12.0000     2.9691   0.7346
  18.0000     4.4536   1.1019
   8.0000     1.9794   0.4897
  15.0000     3.7113   0.9183
  10.0000     2.4742   0.6122

The current in the circuit is 0.247423 Amps.
The total power dissipated in the circuit is 5.938144 Watts.
>>
```

Example-7

Suppose a and b are defined as follows: a = [2 -1 5 0]; b = [3 2 -1 4]; Evaluate by hand the vector c in the following statements. Check your answers with MATLAB.

a. c = a - b; b. c = b + a - 3; c. c = 2 * a + a .^ b;

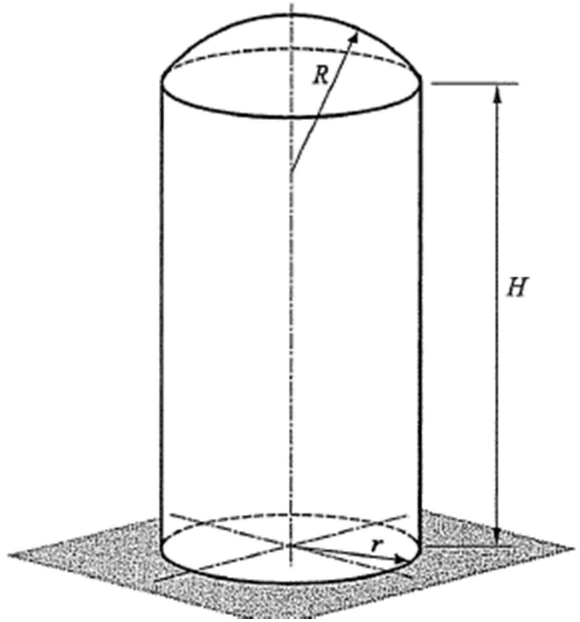
d. c = b ./ a; e. c = b .\ a; f. c = a .^ b;

g. c = 2.^ b+a; h. c = 2*b/3.*a; i. c = b*2.*a;

Example-8

A cylindrical silo with radius r has a spherical cap roof with radius R. The height of the cylindrical portion is H. Write a program in a script file that determines the height H for given values of r, R, and the volume V. In addition, the program also calculates the surface area of the silo.

Use the program to calculate the height and surface area of a silo with $r = 30$ ft., $R = 45$ ft., and a volume of 120,000 ft^3. Assign values for r, R, and V in the Command Window.



Solution-8

```
theta = asin(r/R);                                Calculating θ.
h = R*(1 - cos(theta));                           Calculating h.
```

```
Vcap = pi*h^2*(3*R - h)/3;                        Calculating the volume of the cap.
H = (V - Vcap)/(pi*r^2);                          Calculating H.
S = 2*pi*(r*H + R*h);                             Calculating the surface area S.
fprintf('The height H is: %f ft.',H)
fprintf('\nThe surface area of the silo is: %f square ft.',S)
```

The Command Window where the script file, named silo, was executed is:

```
>> r = 30; R = 45; V = 200000;                    Assigning values to r, R, and V.
>> silo                                           Running the script file named silo.
The height H is: 64.727400 ft.
The surface area of the silo is: 15440.777753 square ft.
```

Example-9

Write Matlab code that will evaluate and plot the following functions:

(a) $y = 5\cos(3\pi x)$ for 101 equally spaced points on the interval $0 \leq x \leq 1$.

(b) $y = \dfrac{1}{1+x^2}$ for 101 equally spaced points on the interval $-5 \leq x \leq 5$.

Solution-9

```
>> x = linspace(0,1,101);
>> y = 5 * cos(3 * pi * x);
>> plot(x, y)
>> plot(x, y, '--rs')
```

```
>> x = linspace(-5, 5, 101);
>> y = 1 ./ (1 + x .* x);
>> plot(x, y)
```

Example-10

Water freezes at 32° and boils at 212° on the Fahrenheit scale. If C and F are Celsius and Fahrenheit temperatures, the formula $F = 9C/5 + 32$, converts from Celsius to Fahrenheit. Use the MATLAB command line to convert a temperature of $37^\circ C$ (normal human temperature) to Fahrenheit (98.6°).

Solution-10

```
>> F = 9 * C / 5 + 32

F =

   98.6000

>> C = 0;
>> F = 9 * C / 5 + 32

F =

    32

>> C = 100;
>> F = 9 * C / 5 + 32

F =

   212
```

Example-11

Chebyshev polynomials are used in a variety of engineering applications. The jth Chebyshev polynomial

$T_j(x)$ is defined by

$$T_j(x) = \cos(j \arccos(x)),\ -1 \leq x \leq 1 .$$

Plot, in the same figure, the Chebyshev polynomials for j = 1, 3, 5, 7.

```
>> x = -1.0:0.1:1.0;
>> T1 = cos(1 * acos(x));
>> T3 = cos(3 * acos(x))

T3 =

  Columns 1 through 9

   -1.0000   -0.2160    0.3520    0.7280    0.9360    1.0000    0.9440    0.7920    0.5680

  Columns 10 through 18

    0.2960   -0.0000   -0.2960   -0.5680   -0.7920   -0.9440   -1.0000   -0.9360   -0.7280

  Columns 19 through 21

   -0.3520    0.2160    1.0000

>> T5 = cos(5 * acos(x));
>> T7 = cos(7 * acos(x));
>> plot(x, T1, x, T3, x, T5, x, T7)
>> plot(x, T1)
>> plot(x, T3)
>> plot(x, T5)
>> plot(x, T7)
>> plot(x, T1, x, T3, x, T5, x, T7, '--rs')
>> plot(x, T1, x, T3, x, T5, x, T7, '--rs', 'LineWidth', 2)
>> figure; plot(x, T1, x, T3, x, T5)
```