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DIGITAL CIRCUITS FINAL EXAM SOLUTIONS (Question 1)

QUESTION 1 (20 Points):

f(A,B,C,D)=A'B'CD+AB'CD+AC'D+AC'D'+A'B'CD'+ABCD+ACD'

$\quad$ =(A'+A)B'CD + AC'(D+D') + A'B'CD' + ABCD + ACD'$\quad$ (Inverse)

$\quad$ =(B'+AB)CD + AC' + A'B'CD' + ACD'$\quad$ (absorbtion)

$\quad$ = B'CD + A(CD + C') + (A'B'+A) CD' (absorbtion)

$\quad$ = B'CD + AD + AC' + B'CD' + ACD'

$\quad$ = B'CD + AD + B'CD' + A(C'+CD') (absorbtion)

$\quad$ = B'CD + AD + B'CD' + AC' + AD'

$\quad$ = B'C(D + D') + A(D+C'+D') (inverse)

$\quad$ = B'C + A

b)

f

CD AB	00	01	11	10
00	1	1	1	1
01	0	0	0	1
11	Φ	0	Φ	1
10	1	1	1	1

By considering 0 and Φ points we can obtain complement of f.

$$\bar{f} = B\bar{C} + BD$$

De Morgan:

$$f = \overline{B\bar{C} + BD} = \overline{B\bar{C}} \cdot \overline{BD}$$
$$= (\bar{B} + C) \cdot (\bar{B} + \bar{D})$$

Or by considering true (1) points:

$$f = \bar{B} + (C\bar{D})$$

Distributive Law:

$$f = (\bar{B} + C) \cdot (\bar{B} + \bar{D})$$

QUESTION 2 (40 Points):

a.

Z CD AB	00	01	11	10
00	1	0	1	1
01	1	1	1	0
11	Φ	0	0	0
10	1	0	1	1

Set of all prime implicants:

$$\bar{C}\bar{D}, \ \bar{B}C, \ \bar{B}\bar{D}, \ \bar{A}B\bar{C}, \ \bar{A}BD, \ \bar{A}CD$$
$$\text{A} \quad \text{B} \quad \text{C} \quad \text{D} \quad \text{E} \quad \text{F}$$

b. **Prime Implicant Chart:**

		0	2	3	4	5	7	8	10	11	Cost
√	A	X			X			X			6
√	B		X	X					X	(X)	5
	C	X	X					X	X		6
	D				X	X					8
√	E					X	X				7
	F			X			X				7

c.

Cheapest sufficient set of prime implicants:

A + B + E: Cost=18

Cheapest expression: $Z = \bar{C}\bar{D} + \bar{B}C + \bar{A}BD$

QUESTION 3

a) $Q_1^+ = A\bar{Q}_1 + \bar{B} Q_1$

$Q_0^+ = 1 \oplus Q_0 = \bar{Q}_0$

$Z = \bar{S}_1 S_0 + S_1 \bar{S}_0$

$\quad = S_1 \oplus S_0 = Q_1 \oplus Q_0$

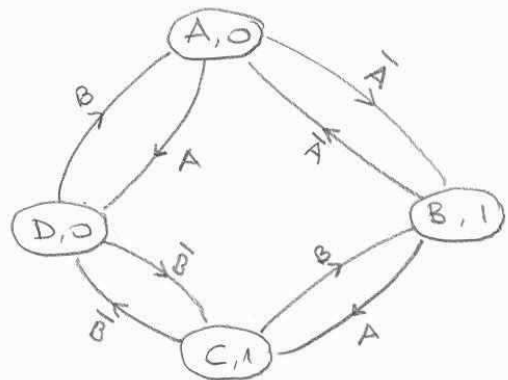Output is a function of

states $\longrightarrow$ Moore Model

$Q_1^+ Q_0^+$

$Q_1 Q_0$ \ AB	00	01	10	11	Z
00	01	01	11	11	0
01	00	00	10	10	1
10	11	01	11	01	1
11	10	00	10	00	0

A : 00
B : 01
C : 10
D : 11

b) $Q_1^+ = D_1 = A\bar{Q}_1 + \bar{B} Q_1$

$Q_0^+ = D_0 = \bar{Q}_0$

$Z = Q_1 \oplus Q_0$