

III. Operational Amplifiers

Amplifiers are two-port networks in which the output voltage or current is directly proportional to either input voltage or current. Four different kinds of amplifiers exist:

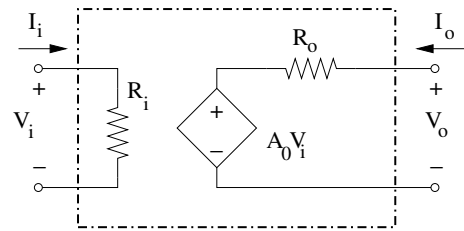
Voltage amplifier: $A_v = V_o/V_i = \text{constant}$

Current amplifier: $A_i = I_o/I_i = \text{constant}$

Transconductance amplifier: $G_m = I_o/V_i = \text{constant}$

Transresistance amplifier: $R_m = V_o/I_i = \text{constant}$

Our focus in this course is on voltage amplifiers (we also see a transconductance amplifier). Voltage amplifiers can be accurately modeled with three circuit elements as shown below. These circuit elements are related to transfer functions of two-port networks discussed before. R_i and R_o are, respectively, input and output resistances. Gain, A_0 , is the open-loop transfer function, $H_{vo}(j\omega)$. (For amplifiers the convention is to use A instead of $H(j\omega)$.)



Voltage Amplifier Model

A good voltage amplifier has a large input resistance, R_i , and a small output resistance, R_o . An ideal voltage amplifier has, $R_i \rightarrow \infty$ and $R_o \rightarrow 0$.

3.1 Feedback

Not only a good amplifier should have sufficient gain, its performance should be insensitive to environmental and manufacturing conditions, should have a large R_i , a small R_o , a sufficiently large bandwidth, *etc.* It is easy to make an amplifier with a very large gain. A typical transistor circuit can easily have a gain of 100 or more. A three-stage transistor amplifier can easily get gains of 10^6 . Other characteristics of a good amplifier are hard to achieve. For example, the β of a BJT changes with operating temperature making the gain of the three-stage amplifier vary widely. The circuit can be made to be insensitive to environmental and manufacturing conditions by the use of feedback.

Principle of feedback: The input to the circuit is modified by “feeding” a signal proportional to the output value “back” to the input. There are two types of feedback (remember the example of a car in the freeway discussed in the class):

1. Negative feedback: As the output is increased, the input signal is decreased and *vice versa*. Negative feedback stabilizes the output to the desired level. Linear system employs negative feedback.

2. Positive feedback: As the output is increased, the input signal is increased and *vice versa*. The output of an amplifier with a positive feedback is always at its limit (saturation voltages). (Positive feedback has its uses!)

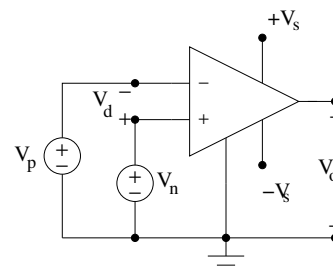
For feedback to work well, the amplifier gain should be large. In this limit, the response of the overall circuit is set by the feedback loop (and not by the gain of amplifier). As such, a voltage amplifier can be made into many other useful circuits such as active filter, integrator, *etc.* We will first explore the concept of feedback through operational amplifier circuits.

3.2 Operational Amplifiers

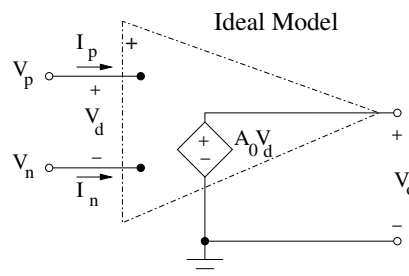
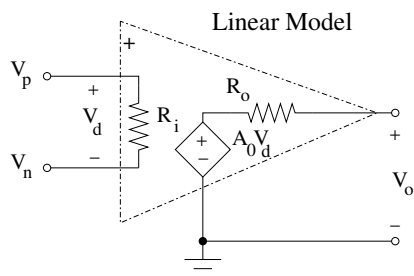
Operational amplifiers (OpAmps) are general purpose voltage amplifiers employed in a variety of circuits. OpAmps are “DC” amplifiers with a very large gain, A_0 (10^5 to 10^6), high input impedance ($> 10 - 100 \text{ M}\Omega$), and low output resistance ($< 100 \Omega$). They are constructed as a “difference” amplifier, *i.e.*, the output signal is proportional to the difference between the two input signals.

$$V_o = A_0 V_d = A_0 (V_p - V_n)$$

+ and – terminals of the OpAmp are called, respectively, non-inverting and inverting terminals. V_s and $-V_s$ are power supply attachments. An OpAmp chip should be powered for it to work, *i.e.*, power supply attachments are necessary. These connections, however, are not usually shown in the circuit diagram.



3.2.1 OpAmp Models



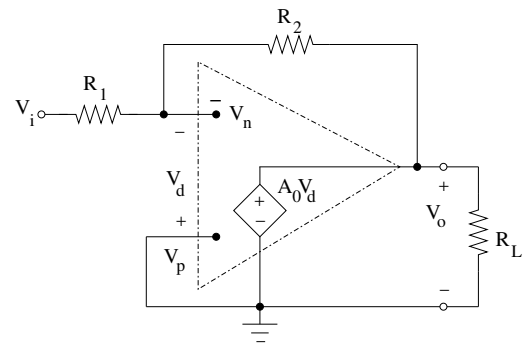
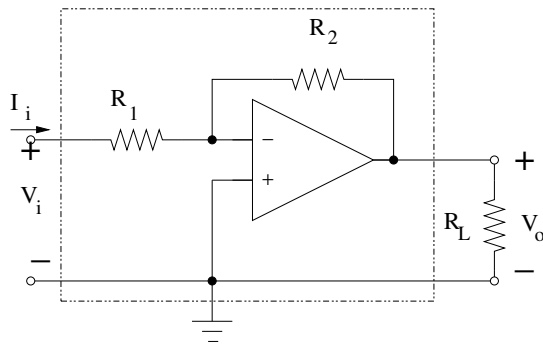
Because R_i is very large and R_o is very small, ideal model of the OpAmp assumes $R_i \rightarrow \infty$ and $R_o \rightarrow 0$. Ideal model is usually a very good model for OpAmp circuits. Very large input resistance also means that the input current into an OpAmp is very small:

First Golden Rule of OpAmps: $I_p \approx I_n \approx 0$ (Also called “Virtual Open Principle”)

As discussed before, concepts of very large and very small employed above require a frame of reference. A “rule of thumb” for ignoring R_i and R_o and employing the ideal model for the OpAmp (and using the first golden rule) is to ensure that all impedances connected to the OpAmp circuit are much smaller than R_i and much larger than R_o (for a “typical” OpAmp, all impedances should be between $1 \text{ k}\Omega$ and $1 \text{ M}\Omega$)

Another important feature of OpAmp is that the OpAmp will be in saturation without negative feedback because its gain is very high. For example, take an OpAmp with a gain of 10^5 and $V_{sat} = 15$ V. Then, for OpAmp to be in linear region, $V_i \leq 15 \times 10^{-5} = 150 \mu\text{V}$ (a very small value). As such, OpAmps are rarely used by themselves. They are always part of a circuit which employ either negative feedback (*e.g.*, linear amplifiers, active filters) or positive feedback (*e.g.*, comparators). Examples below shows several OpAmp circuits with negative feedback.

3.2.2 Inverting Amplifier



The first step in solving OpAmp circuits is to replace the OpAmp with its circuit model (ideal model is usually very good).

$$V_p = 0, \quad V_o = A_0 V_d = A_0 (V_p - V_n) = -A_0 V_n$$

Using node-voltage method and noting $I_n \approx 0$:

$$\frac{V_n - V_i}{R_1} + \frac{V_n - V_o}{R_2} = 0$$

Substituting for $V_n = -V_o/A_0$ and multiplying the equation by R_2 , we have:

$$-\frac{R_2}{A_0 R_1} V_o - \frac{R_2}{R_1} V_i - \frac{V_o}{A_0} - V_o = 0 \quad \rightarrow \quad V_o \left[1 + \frac{1}{A_0} + \frac{R_2}{A_0 R_1} \right] = -\frac{R_2}{R_1} V_i$$

$$\frac{V_o}{V_i} = -\frac{R_2/R_1}{1 + \frac{1}{A_0} \left(1 + \frac{R_2}{R_1} \right)}$$

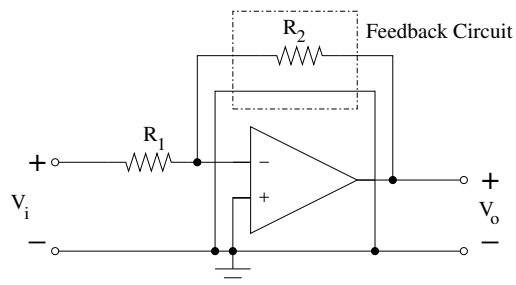
If R_2 and R_1 are chosen such that their ratio is not very large, $1 + R_2/R_1 \ll A_0$, then the voltage transfer function of the OpAmp is

$$\frac{V_o}{V_i} \approx -\frac{R_2}{R_1}$$

This circuit is called an inverting amplifier because the voltage transfer function is “negative.” (A “negative” sinusoidal function looks inverted.) The negative sign actually means that there is 180° phase shift between input and output signals.

Note that the voltage transfer function of the circuit is “independent” of the OpAmp gain, A_0 , and is set by the values of the resistors R_1 and R_2 . While A_0 is quite sensitive to environmental and manufacturing conditions (can vary by a factor of 10 to 100), the resistor values are quite insensitive and, thus, the gain of the system is quite stable.

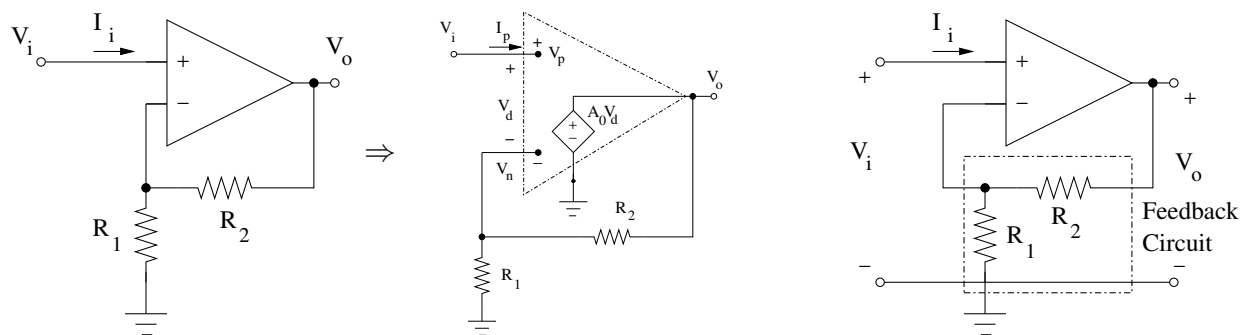
This stability is achieved by a negative feedback (see figure). The output voltage is sampled via R_2 and is applied in parallel to the input signal (via R_1) to the input terminals of the OpAmp (called “parallel-parallel” feedback). If V_o increases, this resistor forces V_n to increase, reducing $V_d = V_p - V_n$ and $V_o = A_0 V_d$, and stabilizes the OpAmp output.



This is a negative feedback as V_o is connected to the inverting terminal of OpAmp. Also, it is obvious that R_1 is needed, otherwise feedback would not work as $V_n = V_i$ is a fixed value and the input to OpAmp chip would not change when V_o changes.

Feedback can be applied in a different manner. The output voltage is sampled and is applied in series with V_i to the input terminals of the OpAmp (called “parallel-series” feedback, *i.e.*, parallel to the output, in series with in the input). We explore this feedback configuration below.

3.2.3 Non-inverting Amplifier



The left circuit above is a non-inverting amplifier. It employs a “parallel-series” feedback (see figure to the right above) as the output is sampled through a combination of R_2 and R_1 resistors and is applied in series to V_i to the input terminals of the OpAmp. Furthermore, as the output V_o is connected to the inverting terminal, it is a negative feedback.

Replacing the OpAmp with its ideal model (middle circuit above) and noting that $I_n \approx 0$ and $I_p \approx 0$, we get:

$$V_p = V_i$$

$$\text{Voltage divider: } V_n = V_o \frac{R_1}{R_1 + R_2}$$

$$V_d = V_p - V_n = V_i - V_o \frac{R_1}{R_1 + R_2}$$

Substituting for $V_d = V_o/A_0$ in the last equation above, we get:

$$\begin{aligned} \frac{V_o}{A_0} &= V_i - V_o \frac{R_1}{R_1 + R_2} \quad \rightarrow \quad V_o \left(\frac{1}{A_0} + \frac{R_1}{R_1 + R_2} \right) = V_i \\ \frac{V_o}{V_i} &= \frac{1}{\frac{1}{A_0} + \frac{R_1}{R_1 + R_2}} = \frac{R_1 + R_2}{R_1} \times \frac{1}{1 + \frac{R_1 + R_2}{A_0 R_1}} = \left(1 + \frac{R_2}{R_1} \right) \times \frac{1}{1 + \frac{1}{A_0} \left(1 + \frac{R_2}{R_1} \right)} \end{aligned}$$

If R_2 and R_1 are chosen such that their ratio is not very large, $1 + R_2/R_1 \ll A_0$, then the voltage transfer function of the OpAmp is

$$\frac{V_o}{V_i} = 1 + \frac{R_2}{R_1}$$

This circuit is called a non-inverting amplifier because its voltage transfer function is “positive.” (as opposed to the inverting amplifier we a negative voltage transfer function.) Note that the voltage transfer function is “independent” of the OpAmp gain, A_0 , and is only set by the values of the resistors R_1 and R_2 .

3.2.4 OpAmp Circuits with Negative Feedback

An important feature of OpAmp circuits with negative feedback is that because the OpAmp is NOT saturated, $V_d = V_o/A_0$ is very small (because A_0 is very large). As a result,

$$\text{Negative Feedback} \quad \rightarrow \quad V_d \approx 0 \quad \rightarrow \quad V_n \approx V_p$$

Second Golden Rule of OpAmps: For OpAmps circuits with negative feedback, the OpAmp adjusts its output voltage such that $V_d \approx 0$ or $V_n \approx V_p$ (also called “Virtual Short Principle”). This rule is derived by assuming $A \rightarrow \infty$. Thus, V_o cannot be found from

$V_o = A_0 V_d = \infty \times 0 = \text{indefinite value}$. The virtual short principle replace $V_o = A_0 v_d$ expression with $V_d \approx 0$.

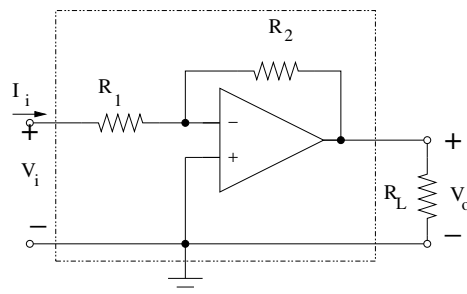
The second golden rule of OpAmps allows us to solve OpAmp circuits in a much simpler manner following the “recipe” below.

Recipe for solving OpAmp circuits:

- 1) Replace the OpAmp with its circuit model.
- 2) Check for negative feedback, if so, $V_p \approx V_n$.
- 3) Solve. Best method is usually node-voltage method. You can solve simple circuits with KVL and KCLs. Do not use mesh-current method.
- 4) Find problem unknowns in term of node voltages.

For example, for the inverting amplifier we will have:

$$\begin{aligned} \text{Negative Feedback} &\rightarrow V_n \approx V_p \approx 0 \\ \frac{V_n - V_i}{R_1} + \frac{V_n - V_o}{R_2} = 0 &\rightarrow \frac{V_i}{R_1} + \frac{V_o}{R_2} = 0 \\ A_v = \frac{V_o}{V_i} = -\frac{R_2}{R_1} \end{aligned}$$



Note that you should not write a node equation at OpAmp output as its a node attached to a voltage source. The value of V_o is AV_d and is indefinite. Instead of using this equation, we used $V_d \approx 0$.

Input and Output resistances of inverting amplifier configuration can now be found. From the circuit,

$$I_i = \frac{V_i - 0}{R_1} \rightarrow R_i = \frac{V_i}{I_i} = R_1$$

The input impedance of the inverting amplifier circuit is R_1 (although input impedance of OpAmp is very large).

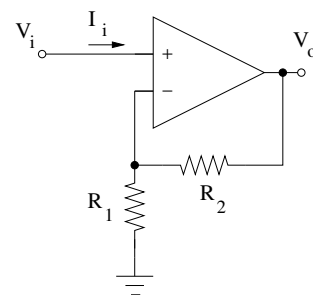
The output impedance of the circuit is “zero” because V_o is independent of R_L (V_o does not change when R_L is changed).

Similarly, for the non-inverting amplifier circuit above, we will have:

$$V_p = V_i$$

$$\text{Negative Feedback: } V_n \approx V_p = V_i$$

$$\frac{V_n - 0}{R_1} + \frac{V_n - V_o}{R_2} = 0$$



Note that again we do not write a node equation at OpAmp output as its a node attached to a voltage source. Substituting for $V_n = V_i$, we get

$$\frac{R_2}{R_1} V_i + V_i - V_o = 0 \quad \Rightarrow \quad \frac{V_o}{V_i} = 1 + \frac{R_2}{R_1}$$

Input Resistance: $I_i = I_p = 0$. Therefore, $R_i \rightarrow \infty$.

Output Resistance: V_o is independent of R_L , so $R_o = 0$.

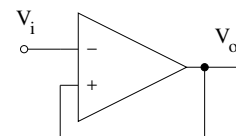
Note that $R_i \rightarrow \infty$ and $R_o = 0$ should be taken in the context that we are using an “ideal” OpAmp model. In reality, the above circuit will have input and output resistances equal to that of the OpAmp itself.

3.3 OpAmps as linear amplifiers

3.3.1 Voltage Follower

In some cases, we have two-terminal networks which do not match well, *i.e.*, the input impedance of the later stage is not very large, or the output impedance of preceding stage is not low enough. A “buffer” circuit is usually used in between these two circuits to solve the matching problem. A “buffer” circuits has a gain of 1 but has a very large input impedance and a very small output impedance. Because the gain of buffer is 1, it is also called a “voltage follower.”

The non-inverting amplifier above has $R_i \rightarrow \infty$ and $R_o = 0$ and, therefore, can be turned into a voltage follower (buffer) by adjusting R_1 and R_2 such that the gain is 1.



$$\frac{V_o}{V_i} = 1 + \frac{R_2}{R_1} = 1 \quad \rightarrow \quad R_2 = 0$$

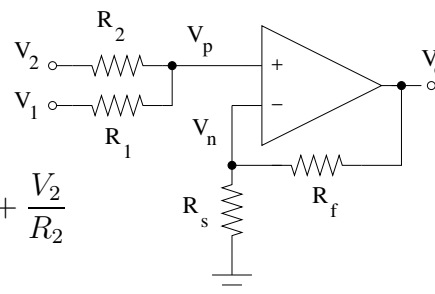
So by setting $R_2 = 0$, we have $V_o = V_i$ or a gain of unity. We note that this expression is valid for any value of R_1 . As we want to minimize the number of components in a circuit as a rule (cheaper circuits!) we set $R_1 = \infty$ (open circuit) and remove R_1 from the circuit.

3.3.2 Non-Inverting Summer

Negative Feedback: $V_n \approx V_p$

$$\frac{V_n - 0}{R_s} + \frac{V_n - V_o}{R_f} = 0 \quad \rightarrow \quad V_o = \left(1 + \frac{R_f}{R_s}\right) V_n$$

$$\frac{V_p - V_1}{R_1} + \frac{V_p - V_2}{R_2} = 0 \quad \rightarrow \quad V_p \left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{V_1}{R_1} + \frac{V_2}{R_2}$$



Substituting for V_n in the 1st equation from the second (noting $V_p = V_n$):

$$V_o = \frac{1 + R_f/R_s}{1/R_1 + 1/R_2} \left(\frac{V_1}{R_1} + \frac{V_2}{R_2}\right)$$

So, this circuit also signal adds (sums) two signals. It does not, however, invert the signals.

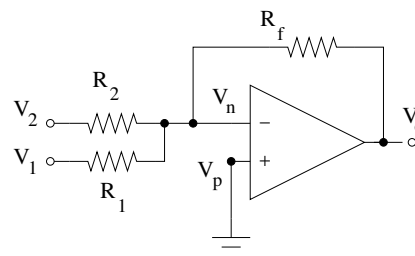
3.3.3 Inverting Summer

Negative Feedback: $V_n \approx V_p$

$$V_p = 0 \quad \rightarrow \quad V_n = 0$$

$$\frac{V_n - V_1}{R_1} + \frac{V_n - V_2}{R_2} + \frac{V_n - V_o}{R_f} = 0$$

$$V_o = -\frac{R_f}{R_1} V_1 - \frac{R_f}{R_2} V_2$$



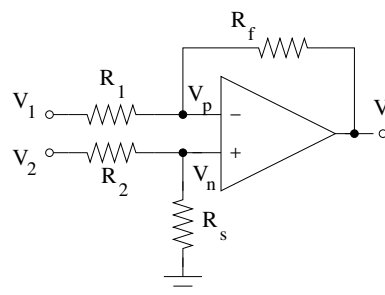
So, this circuit adds (sums) two signals and invert them.

3.3.4 Difference Amplifier

Negative Feedback: $V_n \approx V_p$

$$\frac{V_p - V_2}{R_2} + \frac{V_p - 0}{R_3} = 0 \quad \rightarrow \quad V_n \approx V_p = \frac{R_3}{R_2 + R_3} V_2$$

$$\frac{V_n - V_1}{R_1} + \frac{V_n - V_o}{R_f} = 0$$



Substituting for V_n in the 2nd equation, one can get:

$$V_o = -\frac{R_f}{R_1} V_1 + \left(1 + \frac{R_f}{R_1}\right) \left(\frac{R_3}{R_2 + R_3}\right) V_2$$

If one choose the resistors such that $\frac{R_3}{R_2} = \frac{R_f}{R_1}$, then

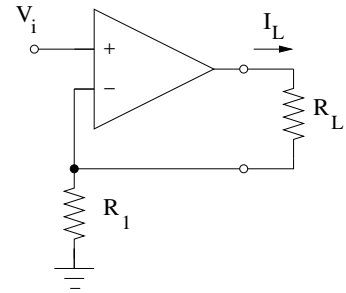
$$V_o = \frac{R_f}{R_1}(V_2 - V_1)$$

3.3.5 Transconductance Amplifier or Current Source

Negative Feedback: $V_n \approx V_p = V_i$

$$-I_o = I_L = \frac{V_n}{R_1} = \frac{V_i}{R_1}$$

$$\frac{V_i}{I_o} = -R_1 = G_m = \text{const}$$



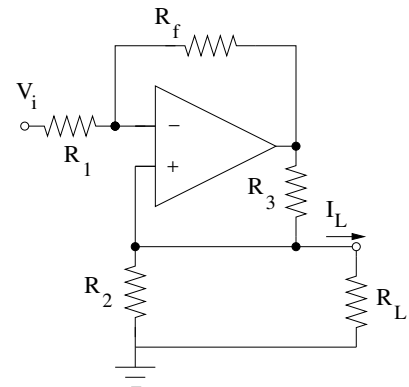
So, this circuit is a “transconductance” amplifier: the output current is proportional to the input voltage (see page 45). Alternatively, if a fixed value of V_i is applied to the circuit, the current I_L is independent of value of R_L and output voltage V_o . As such, this circuit is also an independent current source. Note that when the output of a circuit is the current, a load is necessary for the circuit to function properly.

This circuit is similar to the non-inverting amplifier circuit. It is an example of circuits which perform different functions depending on the location of the output terminals. For this circuit, if the output terminals are between the output of the OpAmp chip and the ground, it is a non-inverting amplifier while if the output terminals are taken across R_L it would be a current-source. Another example of this type of circuits is RC first-order filters that we examined before (see pages 27 and 31). If the output is taken across the resistor, it would be a high-pass filter while if the output is taken across the capacitor, it would be a low-pass filter.

3.3.6 Grounded Current Source

The problem with the above current source is that the load is not grounded. This may not be desirable in some cases. This circuit is also a current source with a grounded load if $R_f/R_1 = R_3/R_2$.

Exercise: Compute I_L and show that it is independent of R_L .

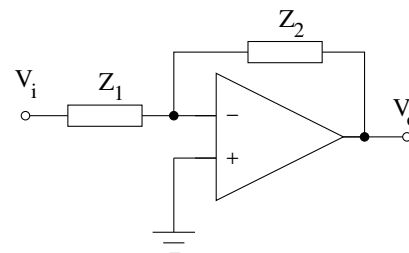


3.4 Active Filters, Integrators & Differentiators

Consider the circuit shown. This is an inverting amplifier with impedances instead of resistors. Following the inverting amplifier solution, we find:

$$H(j\omega) = \frac{V_o}{V_i} = -\frac{Z_2}{Z_1}$$

Various filter circuits can be made with different choices for Z_1 and Z_2 .

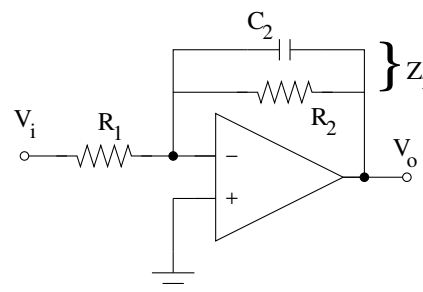


3.4.1 1st Order Low-Pass Filter:

$$Z_1 = R_1$$

$$Z_2 = R_2 \parallel C_2 = \frac{R_2}{1 + j\omega C_2 R_2}$$

$$H(j\omega) = -\frac{Z_2}{Z_1} = -\frac{R_2/R_1}{1 + j\omega R_2 C_2}$$



Comparing the above with the generalized transfer function of a 1st order low-pass filter:

$$H(j\omega) = \frac{K}{1 + j\omega/\omega_c}$$

We find that the above circuit is a low pass filter with

$$K = -\frac{R_2}{R_1} \quad \text{and} \quad \omega_c = \frac{1}{R_2 C_2}$$

The minus sign in front of K indicates an additional -180° phase shift. A low-pass RC or RL filter has a phase shift of 0° at low frequencies and -90° at high frequencies. The above amplifier has a phase shift of -180° at low frequencies and -270° at high frequencies (or alternatively $+180^\circ$ at low frequencies and $+90^\circ$ at high frequencies as we can add 360° to the phase angle). Another difference with passive RC or RL filters is that the gain, $|K| = R_2/R_1$ can be set to be larger than one (*i.e.*, amplify the signals in the pass band). As such this kind of filters are called “active filters.”

Input Resistance: $R_i = R_1$.

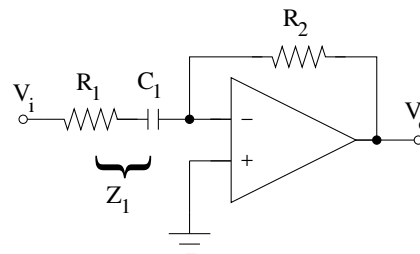
Output Resistance: $R_o = 0$ (OpAmp output resistance).

3.4.2 1st Order High-Pass Filter:

$$Z_1 = R_1 + \frac{1}{j\omega C_1} = R_1 \left(1 - j \frac{1}{\omega R_1 C_1} \right)$$

$$Z_2 = R_2$$

$$H(j\omega) = -\frac{Z_2}{Z_1} = -\frac{R_2/R_1}{1 - j \frac{1}{\omega R_1 C_1}}$$



Comparing the above with the generalized transfer function of a 1st order high-pass filter,

$$H(j\omega) = \frac{K}{1 - j\omega_c/\omega}$$

We find that the above circuit is a high-pass filter with $K = -R_2/R_1$ and $\omega_c = 1/R_1 C_1$. Again, the minus sign in K indicates an additional -180° phase shift. A high-pass RC or RL filter has a 90° at low frequencies and 0° at high frequencies. The above amplifier has a phase shift of -90° at low frequencies and -180° at high frequencies.

Input Resistance: $R_i = Z_1$ and $R_i|_{min} = R_1$.

Output Resistance: $R_o = 0$ (OpAmp output resistance).

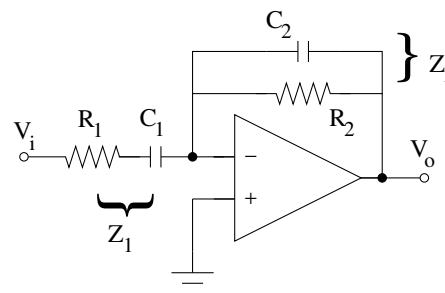
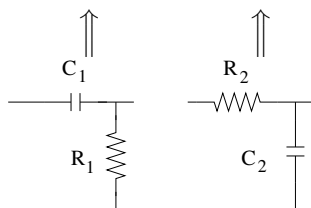
3.4.3 2nd Order Band-Pass Filter:

$$Z_1 = R_1 + \frac{1}{j\omega C_1} = R_1 \left(1 - j \frac{1}{\omega C_1 R_1} \right)$$

$$Z_2 = R_2 \parallel C_2 = \frac{R_2}{1 + j\omega C_2 R_2}$$

$$\text{Defining } \omega_{c1} = \frac{1}{R_1 C_1}, \quad \omega_{c2} = \frac{1}{R_2 C_2}$$

$$H(j\omega) = -\frac{Z_2}{Z_1} = -\frac{R_2}{R_1} \times \frac{1}{1 - j\omega_{c1}/\omega} \times \frac{1}{1 + j\omega/\omega_{c2}}$$



As can be seen, the voltage transfer function looks like a high-pass and a low-pass filter put together (a wide-band, band-pass filter).

To find the cut-off frequencies, bandwidth, *etc.* of this filter, it is simplest to write $H(j\omega)$ in the form similar to the general form for 2nd order band-pass filters:

$$H(j\omega) = \frac{K}{1 + jQ \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right)}$$

To do so, we rearrange the terms in the expression for $H(j\omega)$ of the above filter to get:

$$H(j\omega) = \frac{-R_2/R_1}{(1 + j\omega/\omega_{c2})(1 - j\omega_{c1}/\omega)} = \frac{-R_2/R_1}{(1 + \omega_{c1}/\omega_{c2}) + j(\omega/\omega_{c2} - \omega_{c1}/\omega)}$$

We compare the above expression with the general form for 2nd order band-pass filters to find K , Q , ω_0 . Note that $H(j\omega)$ above is very similar to $H(j\omega)$ of the wide-band band-pass filter of page 36 (with the exception of $-R_2/R_1$ in the nominator. Following the same procedure as in page 36, we get:

$$K = -\frac{R_2/R_1}{1 + \omega_{c1}/\omega_{c2}} \quad Q = \frac{\sqrt{\omega_{c1}/\omega_{c2}}}{1 + \omega_{c1}/\omega_{c2}} \quad \omega_0 = \sqrt{\omega_{c1}\omega_{c2}}$$

Values of band-width, B , and upper and lower cut-off frequencies can then be calculated from ω_0 and Q values above (see page 36-38).

As discussed in pages 37-38, this is a wide-band band-pass filter and would work properly only if $\omega_{c2} \gg \omega_{c1}$. In this limit:

$$K \approx -\frac{R_2}{R_1}, \quad Q \approx \sqrt{\frac{\omega_{c1}}{\omega_{c2}}}, \quad \omega_u \approx \omega_{c2} = \frac{1}{R_2 C_2}, \quad \omega_l \approx \omega_{c1} = \frac{1}{R_1 C_1}$$

Question: Computer R_i and R_o of the above filter.

3.4.4 Integrator

In the integrator and differentiator circuits below, the input is assumed to be an arbitrary function of time (it does not have to be a sinusoidal function). Voltages and currents are written in lower case to remind you of this fact.

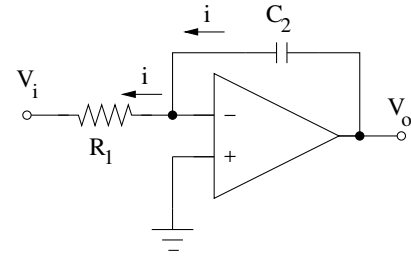
Negative feedback $\rightarrow v_p = v_n = 0$

$$i = C_2 \frac{dv_c}{dt} = C_2 \frac{d(v_o - v_n)}{dt} = C_2 \frac{dv_o}{dt}$$

$$\text{Also, } i = \frac{v_n - v_i}{R_1} = -\frac{v_i}{R_1}$$

$$\text{Thus: } -\frac{v_i}{R_1} = C_2 \frac{dv_o}{dt} \rightarrow \frac{dv_o}{dt} = -\frac{1}{R_1 C_2} v_i$$

$$v_o(t) - v_o(0) = -\frac{1}{R_1 C_2} \int_0^t v_i(t') dt'$$

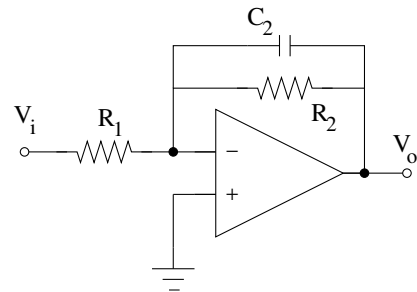


So the output of this circuit is proportional to the integral of the input. Examples of use of such circuit include making a triangular wave from a square wave (as is done in the function generator), charge amplifiers, and analog computers.

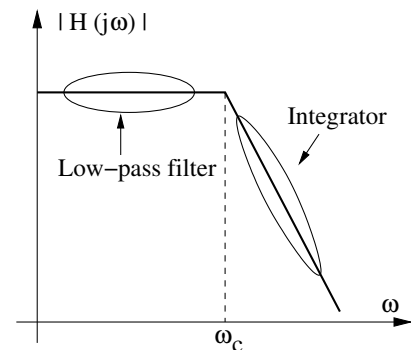
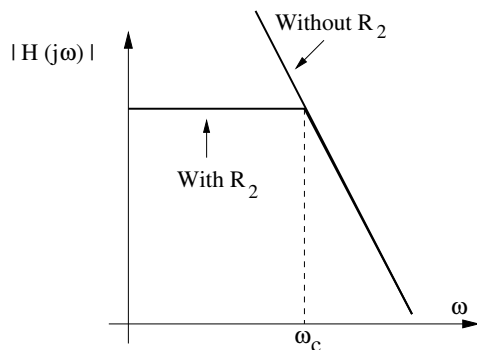
The problem with this circuit is that it is too good! It integrates everything including noise. Low frequency noise is a considerable problem because even small DC inputs are integrated rapidly, increasing v_o , and saturating the OpAmp. This can be seen by examining the frequency response of this circuit:

$$H(j\omega) = \frac{V_o}{V_i} = -\frac{1/(j\omega C_2)}{R_1} = j \frac{1}{\omega R_1 C_2}$$

which becomes infinite for $\omega = 0$. Solution is to add a resistor R_2 parallel to C_2 that discharges the capacitor in long times, getting rid of integrated DC noise. The value of the resistor is chosen such that the time constant of this RC circuit $\tau = R_2 C_2$ is about 10-100 times the period of the lowest frequency signal of interest.



Note that the addition of R_2 makes the circuit look like a low-pass filter. This is an example of a circuit that behaves differently depending on the frequency range of interest. If we are interested in this circuit as an integrator, we should set the cut-off frequency of the filter to be much smaller than the smallest frequency of interest (*i.e.*, signals that we want to be integrated).



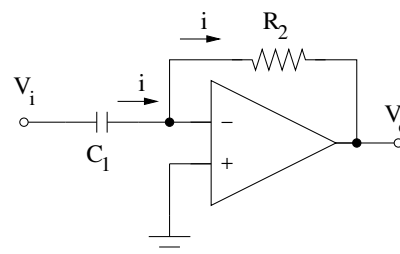
3.4.5 Differentiator

Negative feedback $\rightarrow v_p = v_n = 0$

$$i = C_1 \frac{dv_c}{dt} = C_1 \frac{d(v_i - v_n)}{dt} = C_1 \frac{dv_i}{dt}$$

$$\text{Also, } i = \frac{v_n - v_o}{R_2} = -\frac{v_o}{R_2}$$

$$\text{Thus: } v_o(t) = -R_2 C_1 \frac{dv_i}{dt}$$



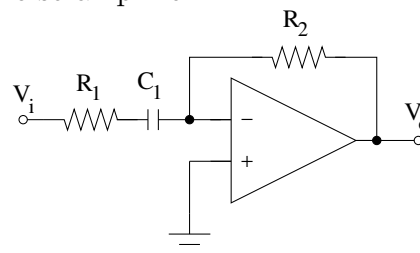
So the output of this circuit is proportional to the derivative of the input.

In practice, this circuit does not work as advertised. The problem can be seen by examining the transfer function for the circuit in the frequency domain:

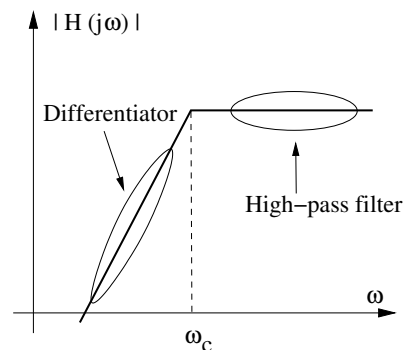
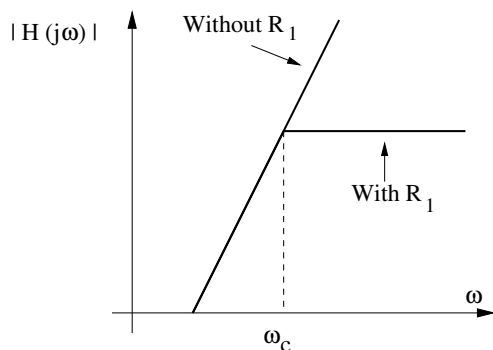
$$H(j\omega) = \frac{V_o}{V_i} = -\frac{R_2}{1/(j\omega C_1)} = -j\omega R_2 C_1$$

As can be seen when ω becomes large, circuit gain becomes infinite. So, practically, this circuit is not a differentiator, rather it is a “high-frequency-noise amplifier.”

The solution is to attenuate the amplitude of high-frequency signals by adding a resistance R_1 in series with C_1 . At low frequencies, C_1 dominates and the circuit is a differentiator. At high frequencies, R_1 dominates and the circuit becomes a simple amplifier.



Note that the addition of R_1 makes the circuit look like a high-pass filter. This is another example of a circuit that behaves differently depending on the frequency range of interest. If we are interested in this circuit as a differentiator, we should set the cut-off frequency of the filter to be much larger than the largest frequency of interest (*i.e.*, signals that we want to be differentiated).



3.5 Practical Amplifiers and OpAmp Limitations

Real voltage amplifiers differ from the ideal amplifiers. Not only, the input resistance is not infinite and the output resistance is not zero, but the amplifier works properly only in certain conditions. One should always be aware of the range where the circuit acts as a linear (ideal) amplifier, *i.e.*, the output is proportional to the input with the ratio of $A_v = V_o/V_i = \text{constant}$ (exactly the same waveform) .

3.5.1 Voltage-supply limit or Saturation:

Amplifiers do not create power. Rather, they act as a “valve” adjusting the power flow from the power supply into the load according to the input signal. As such, the output voltage amplifier cannot exceed the power supply voltage (it is usually lower because of voltage drop across some active elements). The fact that the output voltage of a practical amplifier cannot exceed certain threshold value is called saturation. A voltage amplifier behaves linearly, *i.e.*, $V_o/V_i = A_v = \text{constant}$ as long as the output voltage remains below the “saturation” voltage,

$$V_{sat}^- < v_o < V_{sat}^+$$

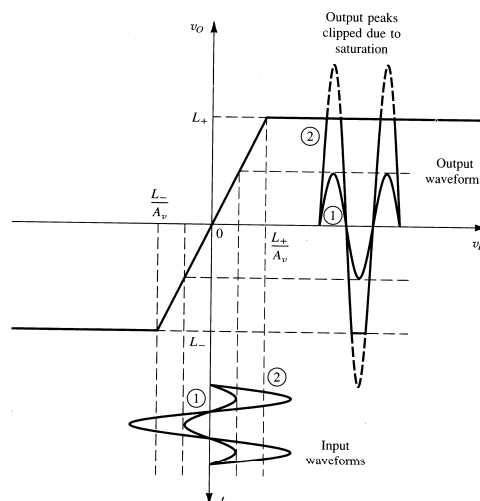
Note that the saturation voltage, in general, is not symmetric, *i.e.*, $V_{sat}^- \neq -V_{sat}^+$.

For an amplifier with a given gain, A_v , the above range of v_o translate into a certain range for v_i

$$\begin{aligned} V_{sat}^- &< v_o < V_{sat}^+ \\ V_{sat}^- &< A_v v_i < V_{sat}^+ \\ \frac{V_{sat}^-}{A_v} &< v_i < \frac{V_{sat}^+}{A_v} \end{aligned}$$

i.e., any amplifier will enter its saturation region if V_i is raised above certain limit. The figure shows how the amplifier output clips when amplifier is not in the linear region. Alternatively, if an amplifier is saturated, one can recover the linear response by reducing the input amplitude.

For OpAmps, the saturation voltages are close to power supply voltages that power the OpAmp chip, or $V_s^- < v_o < V_s^+$



3.5.2 Maximum Output Current:

A voltage amplifier model includes a controlled voltage source in its output circuit. This means that for a given input signal, this controlled source will have a fixed voltage independent of the current drawn from it. If we attach a load to the circuit and start reducing the load resistance, the output voltage remains a constant and load current will increase. Following this model, one could reduce the load resistance to a very small value and draw a very large current from the amplifier.

In reality, this does not happen. Each voltage amplifier has a limited capability in providing output current. This maximum output current limit is called the “Short-Circuit Output Current,” I_{SC} .

$$I_{SC}^- \leq I_o \leq I_{SC}^+$$

If the a fixed load resistance, R_L , is connected to the amplifier, the maximum output current means that the output voltage cannot exceed the $R_L I_{SC}$:

$$R_L I_{SC}^- \leq v_o \leq R_L I_{SC}^+$$

In this case, the maximum output current limit manifests itself in a form similar to amplifier saturation. The output voltage waveform will be clipped at value of $R_L I_{SC}$.

Similar limitation also applies to OpAmp chips and circuits. However, one should note that the maximum output current of the OpAmp amplifier configuration is NOT the same as the maximum output current of the OpAmp chip itself. For example in the inverting amplifier configuration, the maximum output current of the amplifier is smaller than I_{SC} of OpAmp because OpAmp has to supply current to both the load (maximum output current of amplifier configuration itself) and the feedback resistor R_2 .

3.5.3 Frequency Response limit or Amplifier Bandwidth:

Typically, amplifiers work in a certain range of frequencies. Their gain, $A_v = V_o/V_i$ drops outside this range. The voltage transfer function (gain) of an amplifier is plotted similar to that of filters (Bode plots). It looks similar to those of a band-pass filter or a low-pass filter. Cut-off frequencies and bandwidth of the amplifier is defined similar to those of filters (3 dB drop from the maximum value). In terms of frequency response, voltage amplifiers are divided into two categories. (1) AC amplifiers which only amplify AC signals. Their Bode plots look like a band-pass filter. (2) DC amplifiers which amplify both DC signals and AC

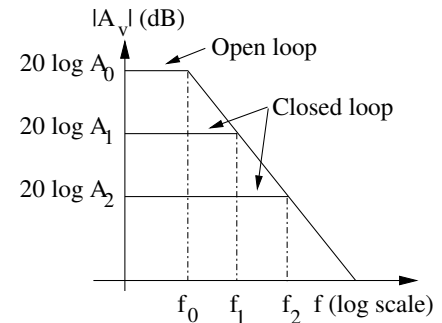
signals. Their Bode plots look like a low-pass filter. For this class amplifiers, the bandwidth is equal to the cut-off frequency (lower cut-off frequency is set to zero!).

A major concern for any amplifier circuit is its stability (which you will study in depth in junior and senior courses). A requirement for stability is that the circuit gain should be less than unity at high frequencies for stable operation. To reduce the gain at high frequencies and avoid instability, the voltage gain of a practical OpAmp looks like a low-pass filter as shown (marked by open loop meaning no feedback). This is achieved by an adding of a relatively large capacitor in the OpAmp circuit chip (internally compensated OpAmps) or by providing for connection of such a capacitor outside the chip (uncompensated OpAmp). In order for the OpAmp gain to become smaller than 1 at high frequencies, the cut-off frequency of the OpAmp chip itself, f_0 , is usually small (10 to 100 Hz, typically).

The open-loop gain of the OpAmp chip, A_o , is given by:

$$A_o = \frac{A_0}{1 + j\omega/\omega_0}$$

$$|A_o| = \frac{A_0}{\sqrt{1 + (\omega/\omega_0)^2}}$$



Where ω_0 is the cut-off frequency of the chip. For frequencies much larger than $\omega_0 = 2\pi f_0$ (sloped line in the figure), $\omega \gg \omega_0$, the open-loop gain of the chip scales as

$$|A_o| \approx \frac{A_0}{\sqrt{(\omega/\omega_0)^2}} = \frac{A_0\omega_0}{\omega}$$

$$A_0\omega_0 = |A_o|\omega \quad \text{or} \quad A_0f_0 = |A_o|f = f_u$$

Therefore, the product of the open-loop gain and bandwidth (which the same as the cut-off frequency) of the OpAmp chip is a constant. This product is given in manufacturer spec sheet for each OpAmp and sometimes is denoted as “unity gain bandwidth,” f_u . Note that gain in the expression above is NOT in dB.

Recall that we found the voltage gain for the inverting amplifier to be $-R_2/R_1$. This gain was independent of frequency: same gain for a DC signal ($\omega \rightarrow 0$) as for high frequencies. However, this voltage gain was found using an ideal OpAmp model (ideal OpAmp parameters are independent of frequency) while in a practical OpAmp chip, the open-loop voltage gain is reduced significantly at high frequencies.

To examine this effect, recall the inverting amplifier circuit (pages 47) that we examined in detail. In that circuit, the voltage transfer function was independent of A_0 as long as

$(1 + R_2/R_1) \ll A_0$. Replacing A_0 with $A_o = A_0/(1 + j\omega/\omega_0)$ in the voltage transfer function of the inverting amplifier in page 47 from the formula above, we get:

$$\begin{aligned} A = \frac{V_o}{V_i} &= - \frac{R_2/R_1}{1 + \frac{1}{A_o} \left(1 + \frac{R_2}{R_1}\right)} = - \frac{R_2/R_1}{1 + (1 + j\omega/\omega_0) \frac{1}{A_0} \left(1 + \frac{R_2}{R_1}\right)} \\ &= - \frac{R_2/R_1}{1 + \frac{1}{A_0} \left(1 + \frac{R_2}{R_1}\right) + j \frac{\omega}{\omega_0} \frac{1}{A_0} \left(1 + \frac{R_2}{R_1}\right)} \approx - \frac{R_2/R_1}{1 + j \frac{\omega}{\omega_0} \frac{1}{A_0} \left(1 + \frac{R_2}{R_1}\right)} \end{aligned}$$

which is similar to the transfer function of a 1st-order low-pass filter, $H = K/(1 + j\omega/\omega_c)$. Therefore, the inverting amplifier has a cut-off frequency given by:

$$\omega_c = \frac{A_0\omega_0}{1 + R_2/R_1} \quad \rightarrow \quad (1 + A)\omega_c = A_0\omega_0 \quad \rightarrow \quad (1 + A)f_c = A_0f_0 = f_u$$

in which we have replaced $R_2/R_1 = A$, the gain of the inverting amplifier. As can be seen, the feedback has caused the cut-off frequency of the amplifier circuit to increase drastically from the value of f_0 to $[A_0/(1 + A)]f_0$. Also note that if $A \gg 1$, we recover the gain-bandwidth formulas of the OpAmp chip itself: $Af = \text{const.}$

A similar analysis can be formed for the non-inverting amplifier of page 49. We will find:

$$A = \frac{V_o}{V_i} = \left(1 + \frac{R_2}{R_1}\right) \frac{1}{1 + j \frac{\omega}{\omega_0} \frac{1}{A_0} \left(1 + \frac{R_2}{R_1}\right)}$$

Therefore, the non-inverting amplifier has a cut-off frequency given by:

$$\omega_c = \frac{A_0\omega_0}{1 + R_2/R_1} \quad \rightarrow \quad A\omega_c = A_0\omega_0 \quad \rightarrow \quad Af_c = A_0f_0 = f_u$$

In general, the bandwidth (or the cut-off frequency) of OpAmp circuits should be calculated individually. However, as most circuit containing one OpAmp chip is configured in either inverting or non-inverting configuration, simple estimates of the circuit bandwidth can be derived:

Inverting Amplifier: $(1 + A)f_c = A_0f_0 = f_u$

Non-inverting Amplifier: $Af_c = A_0f_0 = f_u$

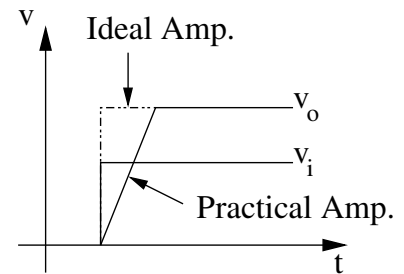
Inverting and Non-inverting Summers: There is a cut-off frequency associated with each input and is given by the above formulas for the inverting or non-inverting amplifier. The smallest of these cut-off frequencies sets the band-width of the summer.

Difference Amplifier: is a combination of an inverting and non-inverting amplifiers. Find the cut-off frequency for each input from the above formulas for the inverting and non-inverting amplifier. The smallest of these cut-off frequencies sets the band-width of the difference amplifier.

Active filters: The three active filters we worked on are all in inverting amplifier configuration. In order to ensure proper operation, we should set the “effective” cut-off frequency set by the chip (*i.e.*, cut-off frequency based on the gain of the filter) to be at least 5 times larger than the upper cut-off frequency of the filter.

3.5.4 Rise Time

In an ideal amplifier, if the input voltage is a unit step function, the output voltage will also be a unit step function as shown. A practical amplifier cannot change its output instantaneously if the input changes suddenly. It takes some time (a short but finite time) for the amplifier output voltage to reach its nominal level. The maximum rate of change in the output voltage is called the rise time.



The maximum rate of change of the output voltage of an OpAmp is called the “slew rate” (given usually in the units of $V/\mu s$):

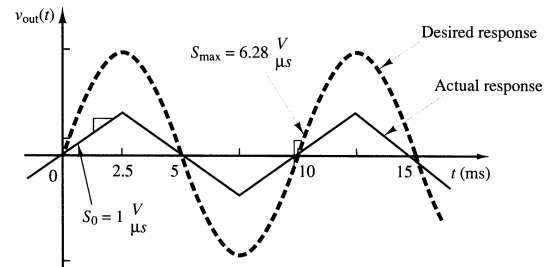
$$S_0 \equiv \left. \frac{dv_o}{dt} \right|_{Max}$$

Slew rate (or rise time) affects all signals (not limited to square waves). For example, at a high enough frequency and/or at a high enough amplitude, a sinusoidal input turns into a triangular output signal. As an example, consider an inverting amplifier with a gain of A , build with an OpAmp with a slew rate of S_0 . The input is a sinusoidal wave with an amplitude of V_i and frequency of ω .

$$v_i = V_i \cos(\omega t) \rightarrow v_o = -AV_i \cos(\omega t)$$

$$\frac{dv_o}{dt} = +AV_i \omega \sin(\omega t)$$

$$\left. \frac{dv_o}{dt} \right|_{Max} = AV_i \omega \leq S_0$$



For example, for $V_i = 1$ V, $A = 10$, and $S_0 = 1$ V/ μ S, we have

$$\left. \frac{dv_o}{dt} \right|_{Max} = 10\omega \leq 10^6 \rightarrow \omega \leq 10^5$$

Which means that at frequencies above 10^5 rad/s, the output will depart from a sinusoidal signal due to the slew rate limit. Because for the sinusoidal wave, the slew rate limit is in the form $AV_i\omega \leq S_0$, one can avoid this nonlinear behavior by either decreasing the frequency, or by lowering the amplifier gain, or reducing the input signal amplitude.

Other limits: OpAmps have other physical limitations such as Input offset voltage, Input bias current, and Common-mode reject ratio (CMMR) that you will see in junior and senior courses.

Note: In order to ensure that an OpAmp circuit would operate properly, we should consider all of the physical limitations of OpAmps. For example, for an inverting amplifier, the maximum operating frequency may be limited by the frequency response limit or by the slew rate. Alternatively, the output voltage of an amplifier may be limited by saturation and/or maximum current and/or slew rate limits. In general, we should consider all relevant limits and use the most restrictive as the operating condition for the circuit.

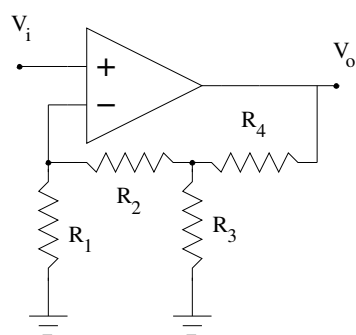
3.6 Exercise Problems

Use the following information in designing circuits:

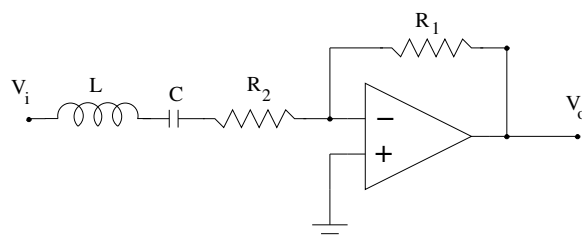
- 1) OpAmps have a unity-gain bandwidth of 10^6 Hz, a maximum output current limit of 100 mA, and a slew rate of $1 \text{ V}/\mu\text{s}$. OpAmps are powered by $\pm 15 \text{ V}$ power supplies (power supplies not shown).
- 2) In circuit design, use commercial resistor and capacitor values of 1, 1.1, 1.2, 1.3, 1.5, 1.6, 1.8, 2, 2.2, 2.4, 2.7, 3., 3.3, 3.6, 3.9, 4.3, 4.7, 5.1, 5.6, 6.2, 6.8, 7.5, 8.2, 9.1 ($\times 10^n$ where n is an integer). You can also use 5 mH inductors.

Problem 1. Find V_o if $R_1 = R_2 = R$ and $R_3 = R_4 = 2R$. (Assume OpAmp is ideal).

Problem 2. What is the transfer function of OpAmp circuit below. What is its function? (Assume OpAmp is ideal).



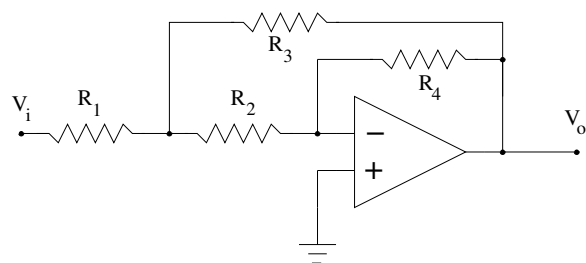
Problem 1



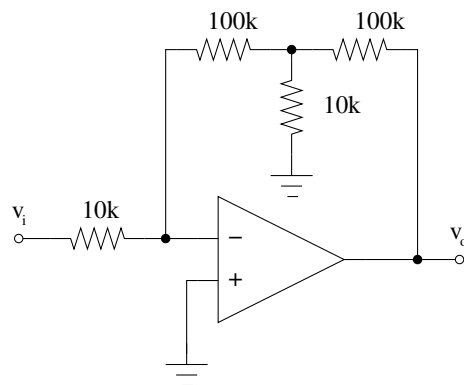
Problem 2

Problem 3. Find the gain of the amplifier circuit shown below. (Assume OpAmp is ideal and $R_1 = R_2 = R_3 = 3R$ and $R_4 = R$).

Problem 4. Find v_o/v_i . (Assume OpAmp is ideal).



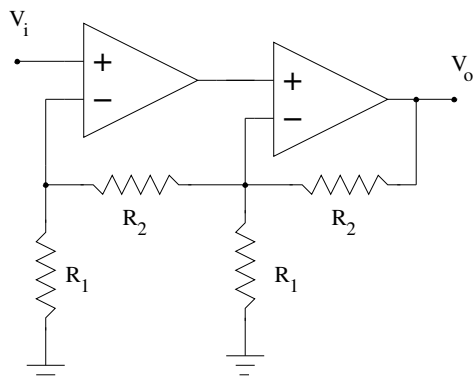
Problem 3



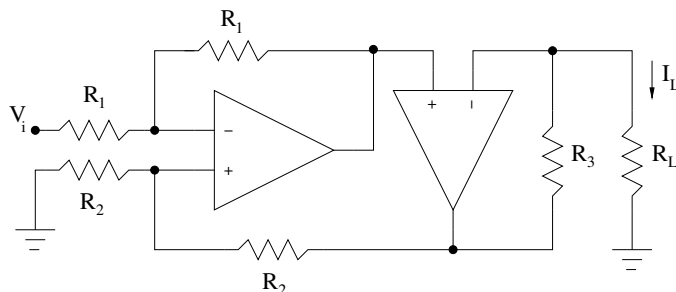
Problem 4

Problem 5. Find v_o/v_i . (Assume OpAmp is ideal).

Problem 6. The circuit below is a voltage-to-current converter with $I_L = gV_i$. Find the value of g and show that it is independent of R_L . (Assume OpAmp is ideal).



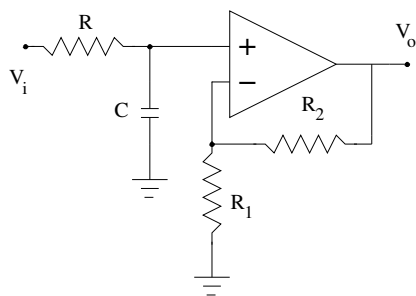
Problem 5



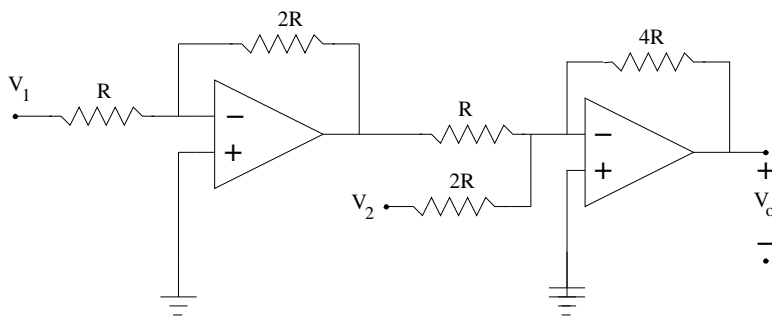
Problem 6

Problem 7. Assume the OpAmp in the circuit below is ideal. Find the voltage transfer function $H(j\omega) = V_o/V_i$.

Problem 8. Find v_o in terms of v_1 and v_2 . (Assume OpAmp is ideal).



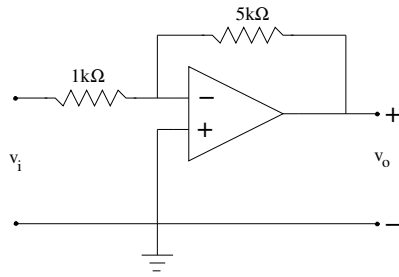
Problem 7



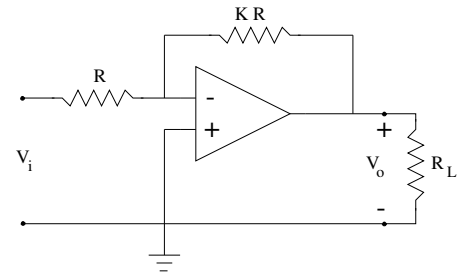
Problem 8

Problem 9. Consider the amplifier circuit below with $v_i = 0.5 \cos(\omega t)$. a) what is the amplifier gain in dB? b) For what range of frequencies, does this amplifier behave as a linear amplifier? (Assume a unity-gain bandwidth of 10^6 Hz, $I_{SC} = 100$ mA, $S_0 = 1$ V/ μ s and OpAmps are powered by ± 15 V power supplies.)

Problem 10. Consider the inverting amplifier below with $R_L = 500 \Omega$. The input signal is $v_i = 1 \cos(10^6 t)$ V. We want the amplifier to behave linearly (*i.e.*, output signal to be sinusoidal) and $V_o/V_i = -K$. What is the maximum value of K we can choose? (Assume a unity-gain bandwidth of 10^6 Hz, $I_{SC} = 10$ mA, $S_0 = 4$ V/ μ s and OpAmps are powered by ± 15 V power supplies.)



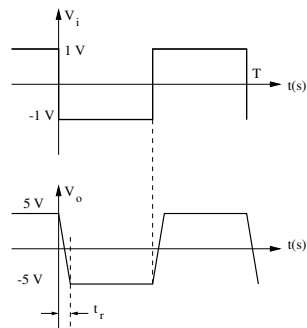
Problem 9



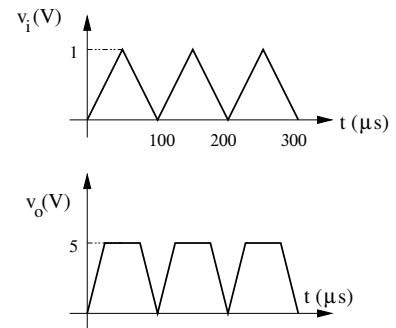
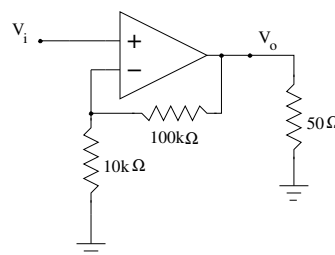
Problem 10

Problem 11. The input and output signals to an OpAmp circuit are given below. A) Draw the circuit (assume $t_r = 0$ for this part only). B) If the slew-rate of the OpAmp is $1 \text{ V}/\mu\text{s}$, calculate t_r . C) Suppose that we consider a signal as a square wave as long as $t_r/(T/2) < 0.1$. What is the frequency range that this amplifier produce a square wave? (Assume a unity-gain bandwidth of 10^6 Hz , $I_{SC} = 100 \text{ mA}$, $S_0 = 1 \text{ V}/\mu\text{s}$ and OpAmps are powered by $\pm 15 \text{ V}$ power supplies.)

Problem 12. Consider the amplifier circuit below. The input and output waveforms are shown. Explain why the output is not a triangular waveform. (Assume a unity-gain bandwidth of 10^6 Hz , $I_{SC} = 100 \text{ mA}$, $S_0 = 1 \text{ V}/\mu\text{s}$ and OpAmps are powered by $\pm 15 \text{ V}$ power supplies.)



Problem 11



Problem 12

Problem 13. Design a non-inverting amplifier with a gain of 20 dB to drive a $10 \text{ k}\Omega$ load. What is the bandwidth of the circuit you have designed? What is its input impedance?

Problem 14. Design an active wide-band filter with $f_l = 20 \text{ Hz}$, $f_u = 1 \text{ kHz}$, a gain of 4 and an input impedance larger than $5 \text{ k}\Omega$.

Problem 15. Design a filter with the transfer function of $H(j\omega) = \frac{-3}{1 + j\omega/5000}$ and a minimum input impedance of $50 \text{ k}\Omega$.

Problem 16. We have an oscillator that makes a square wave with a frequency of 1 kHz , a peak-to-peak amplitude of 10 V , and a DC-offset of 0 V . Design a circuit that converts this square wave to a triangular wave with a peak-to-peak amplitude of 10 V , and a DC-offset of 0 V . What would be the frequency of the triangular wave?

3.7 Solution to Exercise Problems

Problem 1. Find V_o if $R_1 = R_2 = R$ and $R_3 = R_4 = 2R$. (Assume OpAmp is ideal).

We have negative feedback: $V_n \approx V_p = V_i$. Using node-voltage method:

$$\text{Node } V_n \quad \frac{V_n - 0}{R_1} + \frac{V_n - V_1}{R_2} = 0$$

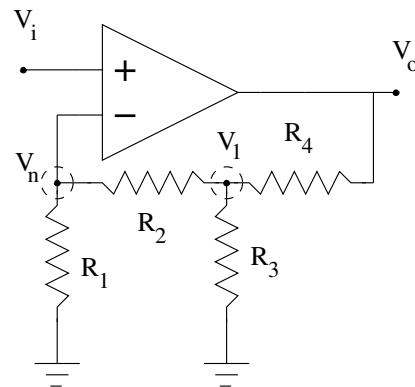
$$\text{Node } V_1 \quad \frac{V_1 - V_n}{R_2} + \frac{V_1 - 0}{R_3} + \frac{V_1 - V_o}{R_4} = 0$$

Letting $V_n = V_i$ and using $R_1 = R_2 = R$ and $R_3 = R_4 = 2R$, the above two equations become:

$$\frac{V_i - 0}{R} + \frac{V_i - V_1}{R} = 0 \quad \rightarrow \quad V_1 = 2V_i$$

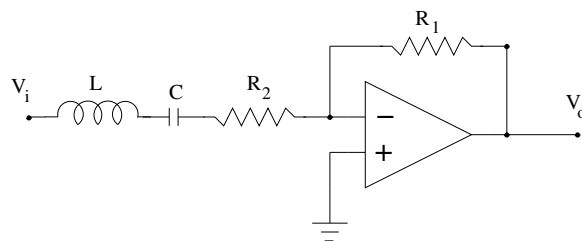
$$\frac{V_1 - V_i}{R} + \frac{V_1 - 0}{2R} + \frac{V_1 - V_o}{2R} = 0 \quad \rightarrow \quad 2V_1 - 2V_i + V_1 + V_1 - V_o = 0$$

$$V_o = 4V_1 - 2V_i = 4(2V_i) - 2V_i = 6V_i$$



Problem 2. What is the transfer function of OpAmp circuit below. What is its function? (Assume OpAmp is ideal).

The circuit is in general form of an inverting amplifier with impedances. So:



$$H(j\omega) = \frac{V_o}{V_i} = -\frac{Z_2}{Z_1} = -\frac{R_1}{R_2 + j\omega L + \frac{1}{j\omega C}}$$

$$H(j\omega) = \frac{-R_1/R_2}{1 + j\left(\frac{\omega L}{R_2} - \frac{1}{\omega R_2 C}\right)}$$

This is a band-pass active filter as its transfer function is in the general form of transfer function of a band-pass filter:

$$H(j\omega) = \frac{K}{1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$$

The center frequency and bandwidth can be found from:

$$K = -\frac{R_1}{R_2}$$

$$\frac{Q\omega}{\omega_0} = \frac{\omega L}{R_2} \quad \text{and} \quad \frac{Q\omega_0}{\omega} = \frac{1}{\omega R_2 C} \quad \rightarrow \quad Q = \sqrt{\frac{L}{R^2 C}} \quad \text{and} \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

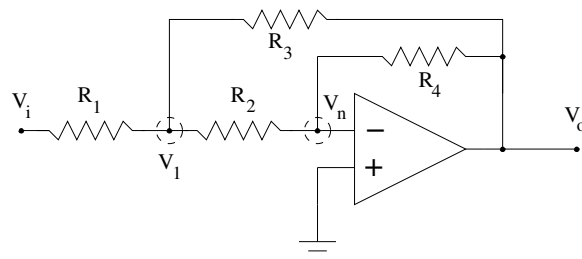
Problem 3. Find the gain of the amplifier circuit shown below. (Assume OpAmp is ideal and $R_1 = R_2 = R_3 = 3R$ and $R_4 = R$).

Negative feedback: $V_n \approx V_p = 0$

Using Node-voltage Method:

$$\text{Node } V_n \quad \frac{V_n - V_1}{R_2} + \frac{V_n - V_o}{R_4} = 0$$

$$\text{Node } V_1 \quad \frac{V_1 - V_i}{R_1} + \frac{V_1 - V_o}{R_3} + \frac{V_1 - V_n}{R_2} = 0$$



where we have utilized the first golden rule of OpAmp ($I_n = I_p = 0$). Substituting for $V_n \approx 0$, $R_1 = R_2 = R_3 = 3R$ and $R_4 = R$, and multiplying both equations by $3R$ we get:

$$\frac{0 - V_1}{3R} + \frac{0 - V_o}{R} = 0 \quad \rightarrow \quad V_1 = -3V_o$$

$$\frac{V_1 - V_i}{3R} + \frac{V_1 - V_o}{3R} + \frac{V_1 - 0}{3R} = 0 \quad \rightarrow \quad 3V_1 - V_o - V_i = 0$$

$$3(-3V_o) - V_o - V_i = 0 \quad \rightarrow \quad -10V_o = V_i$$

$$A_v = \frac{V_o}{V_i} = -0.1$$

Problem 4. Find v_o/v_i . (Assume OpAmp is ideal).

Replace OpAmp with its Ideal model.

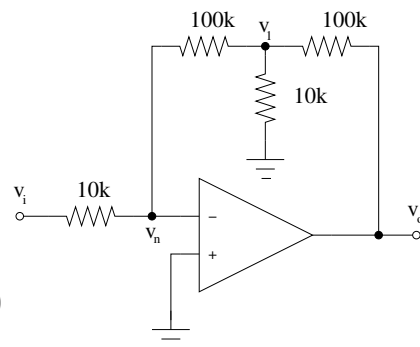
Negative feedback: $v_n \approx v_p = 0$

Using node-voltage method:

$$\frac{v_n - v_i}{10,000} + \frac{v_n - v_1}{100,000} = 0 \quad \rightarrow \quad v_1 = -10v_i$$

$$\frac{v_1 - v_n}{100,000} + \frac{v_1 - v_o}{100,000} + \frac{v_1}{10,000} = 0 \quad \rightarrow \quad 12v_1 - v_o = 0$$

$$v_o = 12v_1 = -120v_i$$

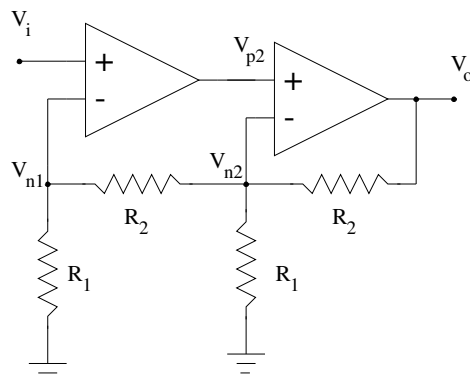


Problem 5. Find v_o/v_i . (Assume OpAmp is ideal).

Both OpAmps have negative feedback, so $v_i = v_{p1} = v_{n1}$ and $v_{p2} = v_{n2}$. Then, by node-voltage method:

$$\frac{v_{n1} - 0}{R_1} + \frac{v_{n1} - v_{n2}}{R_2} = 0$$

$$\frac{v_{n2} - v_{n1}}{R_2} + \frac{v_{n2} - 0}{R_1} + \frac{v_{n2} - v_o}{R_2} = 0$$



Substitute for $v_{n1} = v_i$ and eliminate v_{n2} between the two equations to get the voltage transfer function.

$$\frac{v_{n2} - 0}{R_2} = \frac{v_{n1}}{R_1} + \frac{v_{n1}}{R_2} = v_i \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \rightarrow v_{n2} = v_i \left(\frac{R_2}{R_1} + 1 \right)$$

$$\frac{v_o}{R_2} = v_{n2} \left(\frac{1}{R_2} + \frac{1}{R_1} + \frac{1}{R_2} \right) - \frac{v_{n1}}{R_2} = v_i \left(\frac{R_2}{R_1} + 1 \right) \left(\frac{2}{R_2} + \frac{1}{R_1} \right) - \frac{v_i}{R_2}$$

$$\frac{v_o}{v_i} \times \frac{1}{R_2} = \frac{2}{R_1} + \frac{2}{R_2} + \frac{R_2}{R_1^2} + \frac{1}{R_1} - \frac{1}{R_2}$$

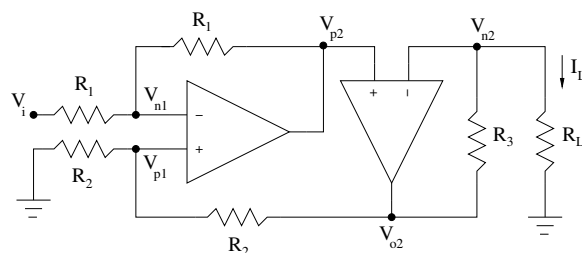
$$\frac{v_o}{v_i} = \left(\frac{R_2}{R_1} \right)^2 + \frac{3R_2}{R_1} + 1$$

Problem 6. The circuit below is a voltage-to-current converter with $I_L = gV_i$. Find the value of g and show that it is independent of R_L . (Assume OpAmp is ideal).

Both OpAmps have negative feedback:

$$V_{n1} \approx V_{p1} \text{ and } V_{n2} \approx V_{p2}$$

Using Node-voltage Method and substituting for $V_{p1} = V_{n1}$ and $V_{p2} = V_{n2}$:



$$\text{Node } V_{n1} \quad \frac{V_{n1} - V_i}{R_1} + \frac{V_{n1} - V_{p2}}{R_2} = 0 \rightarrow V_{n1} - V_i + V_{n1} - V_{n2} = 0 \rightarrow 2V_{n1} = V_i + V_{n2}$$

$$\text{Node } V_{p1} \quad \frac{V_{p1} - 0}{R_2} + \frac{V_{p1} - V_{o2}}{R_2} = 0 \rightarrow V_{n1} + V_{n1} - V_{o2} = 0 \rightarrow 2V_{n1} = V_{o2}$$

$$\text{Node } V_{p2} \quad \frac{V_{n2} - 0}{R_L} + \frac{V_{n2} - V_{o2}}{R_3} = 0 \rightarrow i_L = \frac{V_{n2}}{R_L} = -\frac{V_{n2} - V_{o2}}{R_3}$$

where we have utilized the first golden rule of OpAmp ($I_n = I_p = 0$).

Eliminating V_{n1} between the first two equations give:

$$2V_{n1} = V_i + V_{n2} = V_{o2}$$

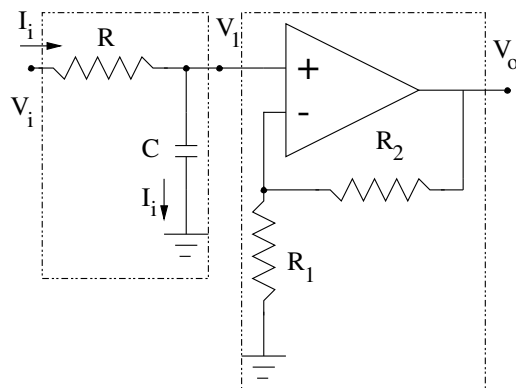
and substituting for V_{o2} in the third equation, we get:

$$i_L = -\frac{V_{n2} - V_{o2}}{R_3} = -\frac{V_{n2} - V_i - V_{n2}}{R_3} = \frac{V_i}{R_3}$$

Thus $g = 1/R_3$ and is independent of R_L .

Problem 7. Assume the OpAmp in the circuit below is ideal. Find the voltage transfer function $H(j\omega) = V_o/V_i$.

This is the same circuit as problem 2 of sample quiz. The circuit is made of two “two-port” networks: A low-pass RC filter and a non-inverting amplifier. As the input impedance of the non-inverting amplifier is infinite, it will not “load” the RC filter and each two-port network can be analyzed independently. We know:



$$\frac{V_1}{V_i} = \frac{(1/j\omega C)}{R + (1/j\omega C)} = \frac{1}{1 + j\omega RC}$$

$$\frac{V_o}{V_1} = 1 + \frac{R_2}{R_1}$$

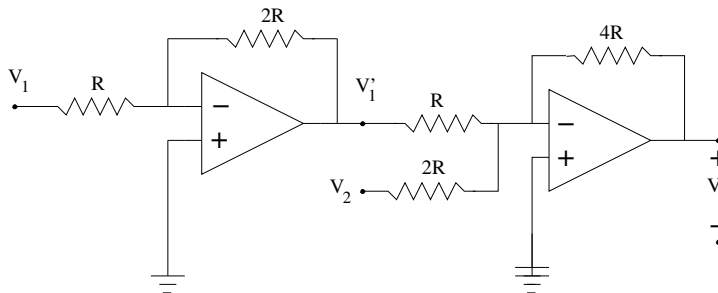
$$H(j\omega) = \frac{V_o}{V_i} = \frac{1 + R_2/R_1}{1 + j\omega RC}$$

So the circuit is a 1st order low pass filter with a gain of $K = 1 + R_2/R_1$ and a cut-off frequency of $\omega_c = 1/(RC)$.

Problem 8. Find v_o in terms of v_1 and v_2 . (Assume OpAmp is ideal).

The OpAmp to the left is configured as an inverting amplifier. Therefore,

$$v'_1 = -\frac{2R}{R}v_1 = -2v_1$$



The OpAmp to the right is configured as an inverting summer. Thus,

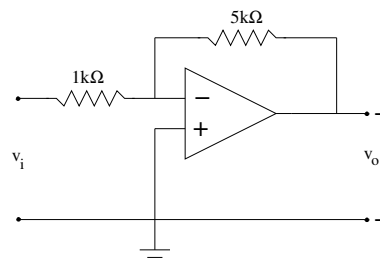
$$v_o = -\frac{4R}{R}v'_1 - \frac{4R}{2R}v_2 = -4v'_1 - 2v_2 = -4(-2v_1) - 2v_2 = 8v_1 - 2v_2$$

Problem 9. Consider the amplifier circuit below with $v_i = 0.5 \cos(\omega t)$. a) what is the amplifier gain in dB? b) For what range of frequencies, does this amplifier behave as a linear amplifier? (Assume a unity-gain bandwidth of 10^6 Hz, $I_{SC} = 100$ mA, $S_0 = 1$ V/ μ s and OpAmps are powered by ± 15 V power supplies.)

part a: This is an inverting amplifier:

$$A = \frac{V_o}{V_i} = \frac{5k}{1k} = 5$$

$$A_{dB} = 20 \log_{10} A = 20 \log_{10}(5) = 14 \text{ dB}$$



Two limits impact the bandwidth of this amplifier:

Bandwidth of OpAmp itself:

$$A_0 f_0 = f_u = (1 + A)f \quad \rightarrow \quad f = \frac{f_u}{1 + A} = \frac{10^6}{6} = 1.66 \times 10^5 = 166 \text{ kHz}$$

Slew Rate:

$$AV_i \omega \leq S_0 \quad \rightarrow \quad 5 \times 0.5 \times \omega \leq 1 \text{ V}/\mu\text{s} = 10^6 \text{ V/s}$$

$$\omega \leq \frac{10^6}{2.5} = 4 \times 10^5 \text{ rad/s}$$

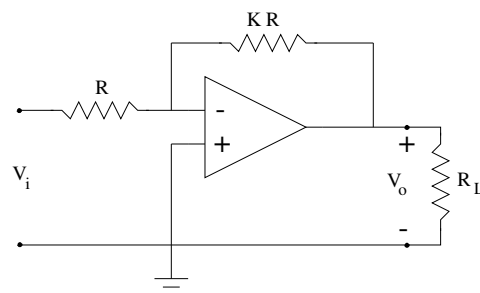
$$f = \frac{\omega}{2\pi} = 64 \text{ kHz}$$

The slew rate is the most restrictive, so this OpAmp circuit behaves linearly in the frequency range $0 \leq f \leq 64$ kHz.

Problem 10. Consider the inverting amplifier below with $R_L = 500 \Omega$. The input signal is $v_i = 1 \cos(10^6 t)$ V. We want the amplifier to behave linearly (*i.e.*, output signal to be sinusoidal) and $V_o/V_i = -K$. What is the maximum value of K we can choose? (Assume a unity-gain bandwidth of 10^6 Hz, $I_{SC} = 10$ mA, $S_0 = 4$ V/ μ s and OpAmps are powered by ± 15 V power supplies.)

$$\omega = 10^6 \text{ rad/s}, \quad f = 10^6/(2\pi) \text{ Hz}$$

$$V_i = 1 \text{ V}, \quad |V_o| = KV_i = K \text{ V}$$



1) Voltage supply limit (Saturation):

$$v_s^- < v_o < v_s^+ \rightarrow K \leq 15$$

2) Frequency response limit:

$$A_0 f_0 = f_u = (1 + A)f = 10^7 \rightarrow (1 + K) \times \frac{10^6}{2\pi} \leq 10^7 \rightarrow K \leq 20\pi - 1 = 62$$

3) Maximum output current limit:

$$I_L = \frac{V_o}{R_L} \leq I_{SC} \rightarrow \frac{K}{500} \leq 10 \times 10^{-3} \rightarrow K \leq 5$$

4) Slew Rate:

$$\frac{dv_o}{dt} = AV_{im}\omega \leq S_0 \rightarrow K \times 1 \times 10^6 \leq 4 \times 10^6 \rightarrow K \leq 4$$

Slew rate limit is most restrictive with $K \leq 4$.

Problem 11. The input and output signals to an OpAmp circuit are given below. A) Draw the circuit (assume $t_r = 0$ for this part only). B) If the slew-rate of the OpAmp is $1 \text{ V}/\mu\text{s}$, calculate t_r . C) Suppose that we consider a signal as a square wave as long as $t_r/(T/2) < 0.1$. What is the frequency range that this amplifier produce a square wave? (Assume a unity-gain bandwidth of 10^6 Hz , $I_{SC} = 100 \text{ mA}$, $S_0 = 1 \text{ V}/\mu\text{s}$ and OpAmps are powered by $\pm 15 \text{ V}$ power supplies.)

Part A: For $t_r = 0$, $v_o = 5v_i$. So, the OpAmp circuit is a non-inverting amplifier with a gain of 5. Since:

$$A = 1 + \frac{R_2}{R_1} = 5 \quad \rightarrow \quad R_2 = 4R_1$$

Part B: The departure of the output from a square wave is due to the slew rate:

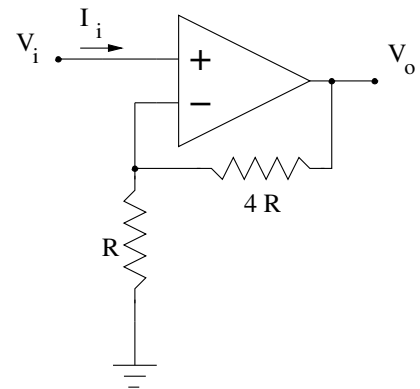
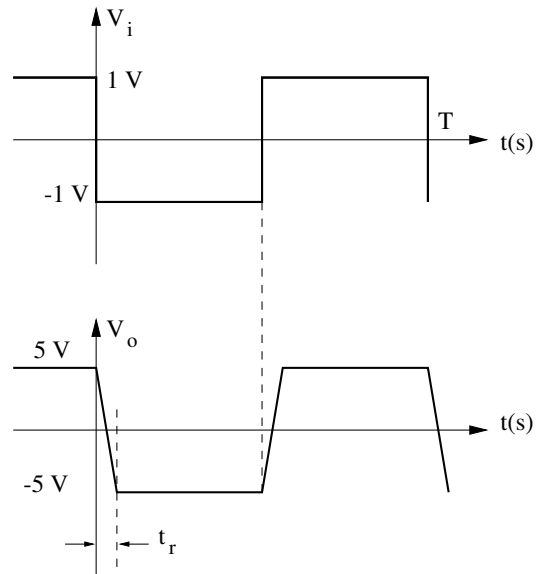
$$S_0 = 1 \text{ V}/\mu\text{s} = \left| \frac{dv_o}{dt} \right| = \left| \frac{-5 - 5}{t_r} \right| = \frac{10}{t_r}$$

$$t_r = \frac{10 \text{ V}}{1 \text{ V}/\mu\text{s}} = 10 \mu\text{s}$$

Part C:

$$\frac{t_r}{T/2} < 0.1 \quad \rightarrow \quad T > \frac{2t_r}{0.1}$$

$$f = \frac{1}{T} < \frac{0.05}{t_r} = \frac{0.05}{10 \times 10^{-6}} = 5 \text{ kHz}$$



Problem 12. Consider the amplifier circuit below. The input and output waveforms are shown. Explain why the output is not a triangular waveform. (Assume a unity-gain bandwidth of 10^6 Hz, $I_{SC} = 100$ mA, $S_0 = 1$ V/ μ s and OpAmps are powered by ± 15 V power supplies.)

The circuit is a non-inverting amplifier with a gain of $1 + R_2/R_1 = 11$. Input is a triangular signal with a peak-to-peak amplitude of 1 V. If the OpAmp was ideal, we would expect that output to be a triangular signal with a peak-to-peak amplitude of $11 \times 1 = 11$ V.

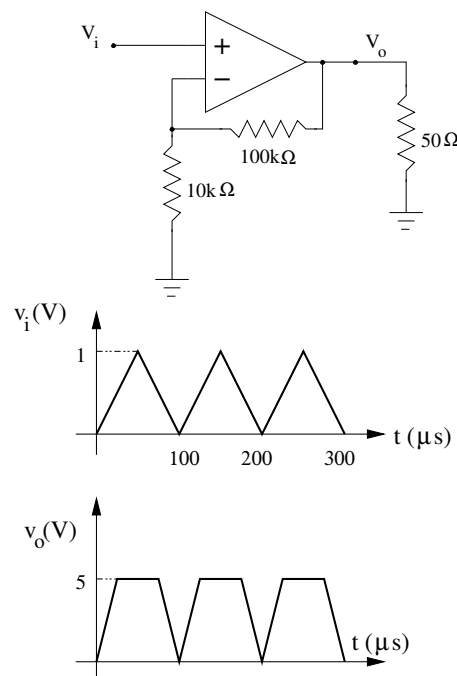
The output signal, however, is clipped at 5 V. Output can be clipped by either 1) amplifier saturation and/or 2) maximum output current limit. We need to examine both limits:

Saturation: As the OpAmp is powered by ± 15 V supplies, the saturation voltage for the OpAmp should be close to ± 15 V. As the output signal is clipped at 5 V, clipping is NOT due to the OpAmp saturation.

Maximum output current limit: As the load resistor ($50\ \Omega$) is much smaller than the feedback resistor ($100\ \text{k}\Omega$), almost all of OpAmp output current will flow in the load. Then:

$$i_L \leq 100\ \text{mA} \quad \rightarrow \quad v_o = 50i_L \leq 50 \times 0.1 = 5\ \text{V}$$

Therefore, maximum output current limit will force the output signal to be clipped at 5 V. This explains the shape of output signal.



Problem 13. Design a non-inverting amplifier with a gain of 20 dB to drive a 10 kΩ load. What is the bandwidth of the circuit you have designed? What is its input impedance?

Prototype of a non-inverting amplifier is shown with $A = 1 + R_2/R_1$. 20 dB gain translates to:

$$20 \text{ dB} = 20 \log(A) \rightarrow \log(A) = 1 \rightarrow A = 10$$

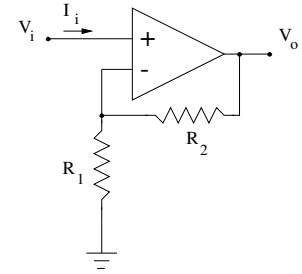
$$A = \left(1 + \frac{R_2}{R_1}\right) = 10 \rightarrow \frac{R_2}{R_1} = 9$$

As the output impedance of a non-inverting amplifier is “zero,” we can choose R_1 and R_2 arbitrarily. A reasonable choice is:

$$R_1 = 10 \text{ k}\Omega, \quad R_2 = 91 \text{ k}\Omega$$

Bandwidth: $A \times f_c = f_u = 10^6$. Thus, $f_c = 10^5 \text{ Hz} = 100 \text{ kHz}$.

Input impedance of the amplifier is infinity as $I_i = 0$.



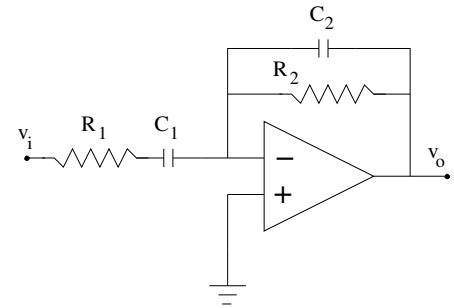
Problem 14. Design an active wide-band filter with $f_l = 20 \text{ Hz}$, $f_u = 1 \text{ kHz}$, a gain of 4 and an input impedance larger than 5 kΩ.

The prototype of this circuit is shown with:

$$\frac{\omega_{c1}}{\omega_{c2}} = 20 \times 10^{-3} \ll 1$$

$$\omega_u \approx \omega_{c2} = \frac{1}{R_2 C_2} \quad \omega_l \approx \omega_{c1} = \frac{1}{R_1 C_1}$$

$$K \approx \frac{R_2}{R_1} \quad Z_i|_{min} = R_1$$



Then, from design condition of $Z_i \geq 5 \text{ k}\Omega$, $R_1 > 5 \text{ k}\Omega$. To make capacitors small (and also to make input impedance large) choose R_1 and R_2 to be large with $K = R_2/R_1 = 4$. A reasonable set is $R_1 = 100 \text{ k}\Omega$ and $R_2 = 390 \text{ k}\Omega$ (commercial values). Then:

$$\omega_u \approx 2\pi 10^3 = \frac{1}{R_2 C_2} \rightarrow C_2 = 4 \times 10^{-10} \text{ F}$$

$$\omega_l \approx 2\pi 20 = \frac{1}{R_1 C_1} \rightarrow C_1 = 8 \times 10^{-8} \text{ F}$$

Commercial values are: $C_1 = 82 \text{ nF}$ and $C_2 = 390 \text{ pF}$.

We need to consider the impact of the bandwidth of OpAmp chip. $A \times f_c = f_u = 10^6$ leads to $f_c = 10^6 / (4 + 1) = 200 \gg f_u = 1 \text{ kHz}$. So the circuit should work properly.

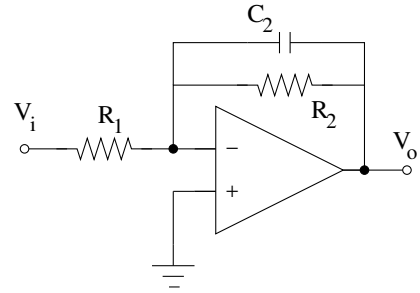
Problem 15. Design a filter with the transfer function of $H(j\omega) = \frac{-3}{1 + j\omega/5000}$ and a minimum input impedance of 50 k Ω .

The transfer function is in the general form for first-order low-pass filters:

$$H(j\omega) = \frac{K}{1 + j\omega/\omega_c}$$

with $K = -3$ and $\omega_c = 5000$ rad/s. As $|K| > 1$, we need to use an active filter. The prototype of the circuit is shown below with

$$H(j\omega) = -\frac{R_2/R_1}{1 + j\omega/\omega_c} \quad \omega_c = R_2C_2$$



The input impedance of this filter is $Z_i|_{min} = R_1$.

Comparing the transfer function of this prototype circuit with the desired one, we get:

$$\frac{R_2}{R_1} = 3$$

$$\omega_c = \frac{1}{R_2C_2} = 5,000$$

$$Z_i|_{min} = R_1 \geq 50 \text{ k}\Omega$$

Choosing commercial value of $R_1 = 51 \text{ k}\Omega$, we get:

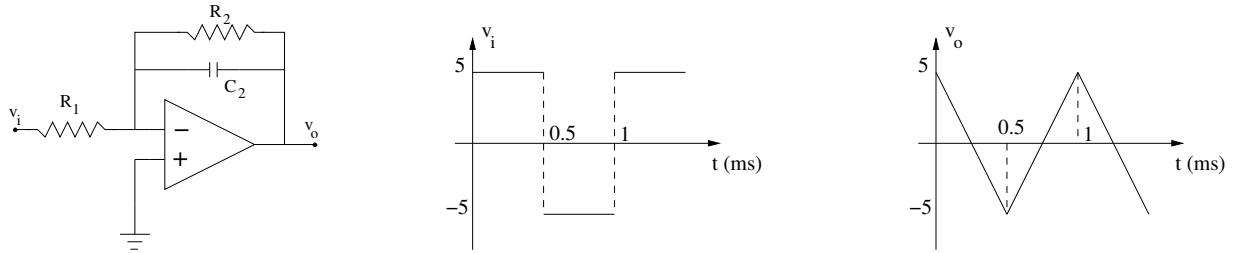
$$R_2 = 3R_1 = 153 \text{ k}\Omega \rightarrow 150 \text{ k}\Omega \text{ (Commercial)}$$

$$C_2 = \frac{1}{5,000R_2} = \frac{1}{5 \times 10^3 \times 150 \times 10^3} = 1.33 \times 10^{-9} \rightarrow 1.5 \text{ nF (Commercial)}$$

We need to consider the impact of the bandwidth of OpAmp chip. $A \times f_c = f_u = 10^6$ leads to $f_c = 10^6/(1 + 3) = 250 \gg f_u = 5/(2\pi) = 0.8 \text{ kHz}$. So the circuit should work properly.

Problem 16. We have an oscillator that makes a square wave with a frequency of 1 kHz, a peak-to-peak amplitude of 10 V, and a DC-offset of 0 V. Design a circuit that converts this square wave to a triangular wave with a peak-to-peak amplitude of 10 V, and a DC-offset of 0 V. What would be the frequency of the triangular wave?

Since a triangular wave is the integral of a square wave, we need to use an integrator circuit. The prototype, the input signal and the desired output signal are shown below. The output will have the same frequency as input (1 kHz) and same period (1 ms).



Because the input signal is symmetric (0 DC offset), we can consider only the half period $0 < t < T/2 = 0.5$ ms. In this range, $v_i = 5$ V and we want $v_o = +5 - 2 \times 10^4 t$ (found from $v_o(0) = +5$ and $v_o(t = 0.5 \text{ ms}) = -5$ V). The output of the integrator is given by:

$$v_o(t) - v_o(0) = -\frac{1}{R_1 C_2} \int_0^t v_i(t') dt'$$

$$5 - 2 \times 10^4 t - 5 = -\frac{1}{R_1 C_2} \int_0^t 5 dt'$$

$$-2 \times 10^4 t = -\frac{1}{R_1 C_2} t \quad \rightarrow \quad R_1 C_2 = 2.5 \times 10^{-4}$$

Resistor R_2 is needed to discharge the capacitor in long time scale (so that small DC input does not add up and saturate the OpAmp). This capacitor also ensure that the output has no DC off-set. Setting:

$$\tau = R_2 C_2 = 100T = 100 \times 10^{-3} = 0.1$$

We have two equations in three unknowns R_1 , R_2 , and C_2 and we can choose value of one arbitrarily. One has to be careful as if we divide the two equations, we get:

$$\frac{R_2 C_2}{R_1 C_2} = \frac{R_2}{R_1} = \frac{0.1}{2.5 \times 10^{-4}} = 400$$

Reasonable choices to keep resistors between 100 and 1 M Ω are $R_1 = 1$ k Ω and $R_2 = 400$ k Ω :

$$R_1 C_2 = 2.5 \times 10^{-4} \quad \rightarrow \quad C_2 = \frac{2.5 \times 10^{-4}}{1000} = 2.5 \times 10^{-7} = 250 \text{ nF}$$

Thus, reasonable commercial choices are $R_1 = 1$ k Ω , $R_2 = 390$ k Ω , and $C_2 = 0.24$ μ F.