Subsets

Definition: $A \subset B$ (A is a *subset* of B) if every element of A is an element of B.

Equivalently: $A \subset B$ if $\forall x \; (x \in A \Rightarrow x \in B)$

Equivalently: $A \subset B$ if $\forall x \in A \; (x \in B)$

Examples :

1. $\{1,2,3\} \subset \{1,2,3,4,5\}$
2. $\{1,2,3,4,5\} \not\subset \{1,2,3\}$
3. To show that $A \subset B$ you need to show that *every* element of A is an element of B. To show that $A \not\subset B$ you need only find *one* element of B that is not in S.
4. $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$
5. $\{1,2,3,4,5\} \subset \{1,2,3,4,5\}$
6. In In fact, for *any* set A, $A \subset A$ (Proof: class and/or text)
7. $\{\} \subset \{1,2,3,4,5\}$ (!)
8. In fact, for *any* set A, $\emptyset \subset A$ (Proof: class and/or text)
9. List all the subsets of $\{1,2,3\}$

Graphical representations often help our intuition (class)

Proper Subset: Say that A is a *proper* subset of B if $A \subset B$ and $A \neq B$.

Notation: Sometimes you'll see proper subset written as $A \subsetneq B$ or $A \subsetneqq B$ or $A \varsubsetneqq B$.

Similarly, regular subset might be written $A \subseteq B$ (emphasizing "subset of *or equal to*")

Examples :

1. $\mathbb{N} \subsetneq \mathbb{Z}$
2. $\mathbb{N} \subseteq \mathbb{N}$, but $\mathbb{N} \not\subsetneq \mathbb{N}$
3. List proper subsets of $\{1,2,3\}$

Some Useful Properties:

$\emptyset \subset A$ for any set A

$A \subset A$ for any set A

Transitivity: If $A \subset B$ and $B \subset C$ than $A \subset C$ (proof in class)

(Alternate definition of equality) If A and B are sets, then $A = B$ if and only if ($A \subset B$ and $B \subset A$)

If A is finite with N elements then A has 2^N subsets.

14.2 Set Operations (union, intersection, set difference, complement

Intersection: The *intersection* of two sets is the set of elements common to both of them.

Equivalently,

$$A \cap B =_{\text{def}} \{x \mid x \in A \text{ and } x \in B\}$$

Equivalently,

$$x \in (A \cap B) \text{ provided } (x \in A) \wedge (x \in B)$$

Union: The *union* of two sets is the set of elements appearing in either of them.

Equivalently,

$$A \cup B =_{\text{def}} \{x \mid x \in A \text{ or } x \in B\}$$

Equivalently,

$$x \in (A \cup B) \text{ provided } (x \in A) \vee (x \in B)$$

Examples :

1. $\{1, 2, 3, 4\} \cap \{2, 4, 6, 8\} =$

2. $\{1, 2, 3, 4\} \cup \{2, 4, 6, 8\} =?$

3. $\{2k | k \in \mathbb{N}\} \cap \{3k | k \in \mathbb{N}\} =?$

4. $\{2k | k \in \mathbb{N}\} \cup \{2k + 1 | k \in \mathbb{N}\} =?$