BIL 108E Intr. to Sci. & Eng.Computing

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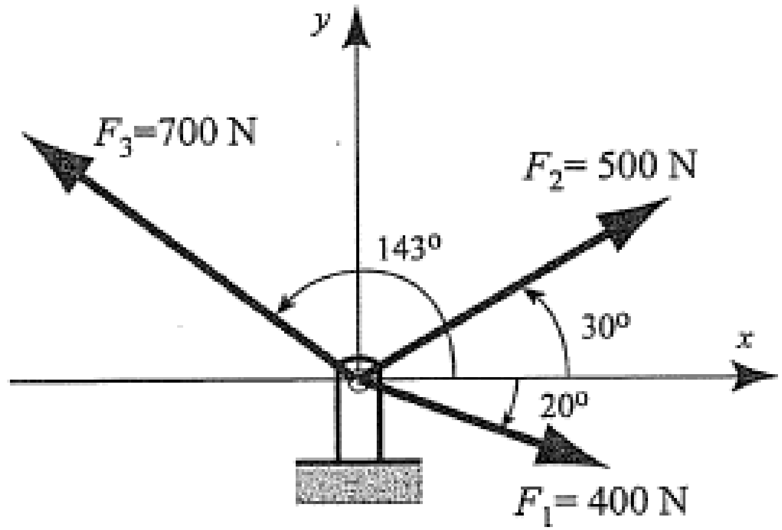
EXERCISES -1

Example-1



Three forces are applied to a bracket as shown. Determine the total (equivalent) force applied to the bracket.

Solution-1

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} = F\cos\theta\mathbf{i} + F\sin\theta\mathbf{j} = F(\cos\theta\mathbf{i} + \sin\theta\mathbf{j})$$

$$F = \sqrt{F_x^2 + F_y^2} \quad \text{and} \quad \tan\theta = \frac{F_y}{F_x}$$

```
>> F1M = 400;  F2M = 500;  F3M = 700;
```

Define variables with the magnitude of each vector.

```
>> Th1 = -20*pi/180;  Th2 = 30*pi/180;  Th3 = 143*pi/180;
```

Define variables with the angle (in radians) of each vector.

```
>> F1 = F1M*[cos(Th1)  sin(Th1)]
F1 =
  375.8770 -136.8081
>> F2 = F1M*[cos(Th2)  sin(Th2)]
F2 =
  346.4102  200.0000
>> F3 = F1M*[cos(Th3)  sin(Th3)]
F3 =
 -319.4542  240.7260
```

Define the three vectors.

```
>> Ftot = F1 + F2 + F3
Ftot =
  402.8330  303.9180
```

Calculate the total force vector.

```
>> FtotM = sqrt(Ftot(1)^2 + Ftot(2)^2)
FtotM =
  504.6192
```

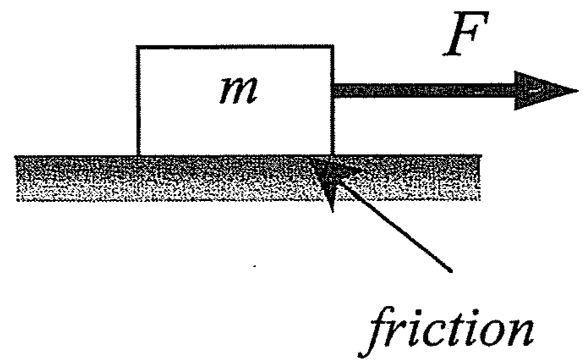
Calculate the magnitude of the total force vector.

```
>> Th = (180/pi)*atan(Ftot(2)/Ftot(1))
Th =
  37.0328
```

Calculate the angle (in degrees) of the total force vector.

Example-2

The coefficient of friction, μ, can be determined in an experiment by measuring the force F required to move a mass m. When F is measured and m is known, the coefficient of friction can be calculated by:



$$\mu = F/(mg) \quad (g = 9.81 \text{ m/s}^2).$$

Results from measuring F in six tests are given in the table below. Determine the coefficient of friction in each test, and the average from all tests.

Test #	1	2	3	4	5	6
Mass m (kg)	2	4	5	10	20	50
Force F (N)	12.5	23.5	30	61	117	294

Solution-2

```
>> m = [2 4 5 10 20 50];
>> F = [12.5 23.5 30 61 117 294];
>> mu = F./(m*9.81)

mu =

   0.6371   0.5989   0.6116   0.6218   0.5963   0.5994

>> mu_ave = mean(mu)

mu_ave =

   0.6109
```

Enter the values of m in a vector.

Enter the values of F in a vector.

A value for mu is calculated for each test, using element-by-element calculations.

The average of the elements in the vector mu is determined by using the function mean.

Example-3

An object with an initial temperature of T_0 that is placed at time $t = 0$ inside a chamber that has a constant temperature of T_s, will experience a temperature change according to the equation:

$$T = T_s + (T_0 - T_s)e^{-kt}$$

where T is the temperature of the object at time t, and k is a constant. A soda can at a temperature of 120°F (was left in the car) is placed inside a refrigerator where the temperature is 38°F. Determine, to the nearest degree, the temperature of the can after three hours. Assume $k = 0.45$. First define all the variables and then calculate the temperature using one MATLAB command.

Solution-3

```
>> Ts = 38;  T0 = 120; k = 0.45; t = 3;

>> T = round(Ts + (T0 - Ts)*exp(-k*t))

T =
   59
>>
```

Round to the nearest integer.

Example-4

The balance B of a savings account after t years when a principal P is invested at an annual interest rate r and the interest is compounded n times a year is given by:

$$B = P\left(1 + \frac{r}{n}\right)^{nt} \qquad (1)$$

If the interest is compounded yearly, the balance is given by:

$$B = P(1 + r)^t \qquad (2)$$

In one account \$5,000 is invested for 17 years in an account where the interest is compounded yearly. In a second account \$5,000 is invested in an account in which the interest is compounded monthly. In both accounts the interest rate is 8.5%. Use MATLAB to determine how long (in years and months) it would take for the balance in the second account to be the same as the balance of the first account after 17 years.

Solution-4

```
>> P = 5000;  r = 0.085;  ta = 17; n = 12;
>> B = P*(1 + r)^ta
B =
  2.0011e+004
>> t = log(B/P)/(n*log(1 + r/n))
t =
  16.3737
>> years = fix(t)
years =
    16
>> months = ceil((t - years)*12)
months =
     5
```

Step (*a*): Calculate `B` from Eq. (2).

Step (*b*): Solve Eq. (1) for *t*, and calculate `t`.

Step (*c*): Determine the number of years.

Determine the number of months.

Example-5

Define two variables: *alpha* = 5π/9, *beta* = π/7. Using these variables, show that the following trigonometric identity is correct by calculating the value of the left and right sides of the equation.

$$\cos\alpha - \cos\beta = 2\sin\frac{1}{2}(\alpha+\beta)\sin\frac{1}{2}(\beta-\alpha)$$

Solution-5

```
>> alpha=5*pi/9

alpha =

    1.7453

>> beta=pi/7

beta =

    0.4488

>> LHS=cos(alpha)-cos(beta)

LHS =

   -1.0746

>> x=alpha+beta

x =

    2.1941

>> y=beta-alpha

y =

   -1.2965

>> RHS=2*sin(x/2)*sin(y/2)

RHS =

   -1.0746
```

Example-6

Create the following three matrices:

$$A = \begin{bmatrix} 5 & 2 & 4 \\ 1 & 7 & -3 \\ 6 & -10 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 11 & 5 & -3 \\ 0 & -12 & 4 \\ 2 & 6 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 7 & 14 & 1 \\ 10 & 3 & -2 \\ 8 & -5 & 9 \end{bmatrix}$$

a) Calculate $A+B$ and $B+A$ to show that addition of matrices is commutative.

b) Calculate $A+(B+C)$ and $(A+B)+C$ to show that addition of matrices is associative.

c) Calculate $5(A+C)$ and $5A+5C$ to show that, when matrices are multiplied by a scalar, the multiplication is distributive.

d) Calculate $A*(B+C)$ and $A*B+A*C$ to show that matrix multiplication is distributive.

Solution-6

```
>> A=[5 2 4; 1 7 -3; 6 -10 0]

A =

     5     2     4
     1     7    -3
     6   -10     0

>> B=[11 5 -3; 0 -12 4; 2 6 1]

B =

    11     5    -3
     0   -12     4
     2     6     1

>> C=[7 17 1; 10 3 -2; 8 -5 9]

C =

     7    17     1
    10     3    -2
     8    -5     9

>> 
```

```
>> D=A+B

D =

    16     7     1
     1    -5     1
     8    -4     1

>> E=B+A

E =

    16     7     1
     1    -5     1
     8    -4     1
```

Solution-6

```
>> F=A+(B+C)

F =

    23    24     2
    11    -2    -1
    16    -9    10

>> G=(A+B)+C

G =

    23    24     2
    11    -2    -1
    16    -9    10
```

```
>> H=5*(A+B)

H =

    80    35     5
     5   -25     5
    40   -20     5

>> G=5*A+5*B

G =

    80    35     5
     5   -25     5
    40   -20     5
```

```
>> K=A*(B+C)

K =

   150    96    34
    58   -44   -18
     8   222   -32

>> L=A*B+B*C

L =

   166   242   -29
   -83  -153    82
   148   197   -59

>> M=A*B+A*C

M =

   150    96    34
    58   -44   -18
     8   222   -32
```

Example-7

Solve the following system of four linear equations:

$$
\begin{aligned}
5x + 4y - 2z + 6w &= 4 \\
3x + 6y + 6z + 4.5w &= 13.5 \\
6x + 12y - 2z + 16w &= 20 \\
4x - 2y + 2z - 4w &= 6
\end{aligned}
$$

Solution-7

```
>> A=[5 4 -2 6; 3 6 6 9/2; 6 12 -2 16; 4 -2 2 -4]

A =

    5.0000    4.0000   -2.0000    6.0000
    3.0000    6.0000    6.0000    4.5000
    6.0000   12.0000   -2.0000   16.0000
    4.0000   -2.0000    2.0000   -4.0000
```

```
>> B=[4; 26/2; 20; 6]

B =

     4
    13
    20
     6
```

```
>> X=A\B

X =

   -0.6984
   32.3651
  -11.6825
  -24.2222
```

Example-8

Define x and y as the vectors $x = 2, 4, 6, 8, 10$ and $y = 3, 6, 9, 12, 15$. Then use them in the following expression to calculate z using element-by-element calculations.

$$z = \frac{xy + \frac{y}{x}}{(x+y)^{(y-x)}} + 12^{x/y}$$

Solution-8

```
>> X=[2 4 6 8 10]

X =

     2     4     6     8    10

>> Y=[3 6 9 12 15]'

Y =

     3
     6
     9
    12
    15

>> A=X*Y

A =

   330
```

```
X =

     2     4     6     8    10

>> Y=[3 6 9 12 15]

Y =

     3     6     9    12    15

>> B=Y/X

B =

    1.5000

>> C=X+Y

C =

     5    10    15    20    25
```

```
>> D=(Y-X)

D =

     1     2     3     4     5

>> E=X/Y

E =

    0.6667
```

Solution-8

```
>> Z=(A+B)\C.^D+12^E

Z =

   1.0e+04 *

    0.0005    0.0006    0.0015    0.0488    2.9464
```

Example-9

For the function $y = (x^2 + 1)^3 x^3$, calculate the value of y for the following values of x: -2.5 -2 -1.5 -1 -0.5 0 0.5 1 1.5 2 2.5 3. Solve the problem by first creating a vector x, and then creating a vector y, using element-by-element calculations.

Example-9

```
>> X=[-5/2 -2 -3/2 -1 -1/2 0 1/2 1 3/2 2 5/2 3]

X =

  Columns 1 through 11

   -2.5000   -2.0000   -1.5000   -1.0000   -0.5000         0    0.5000    1.0000    1.5000    2.0000    2.5000

  Column 12

    3.0000
```

```
>> Y=(X.^2+1)

Y =

  Columns 1 through 11

    7.2500    5.0000    3.2500    2.0000    1.2500    1.0000    1.2500    2.0000    3.2500    5.0000    7.2500

  Column 12

   10.0000
```

Example-9

```
>> K=Y.^3

K =

   1.0e+03 *

  Columns 1 through 11

    0.3811    0.1250    0.0343    0.0080    0.0020    0.0010    0.0020    0.0080    0.0343    0.1250    0.3811

  Column 12

    1.0000
```

```
>> L=X.^3'

L =

  -15.6250
   -8.0000
   -3.3750
   -1.0000
   -0.1250
         0
    0.1250
    1.0000
    3.3750
    8.0000
   15.6250
   27.0000
```

```
>> M=K*L

M =

       27000
```