Lecture 20

Bipolar Junction Transistors (BJT): Part 4

Small Signal BJT Model

Reading:

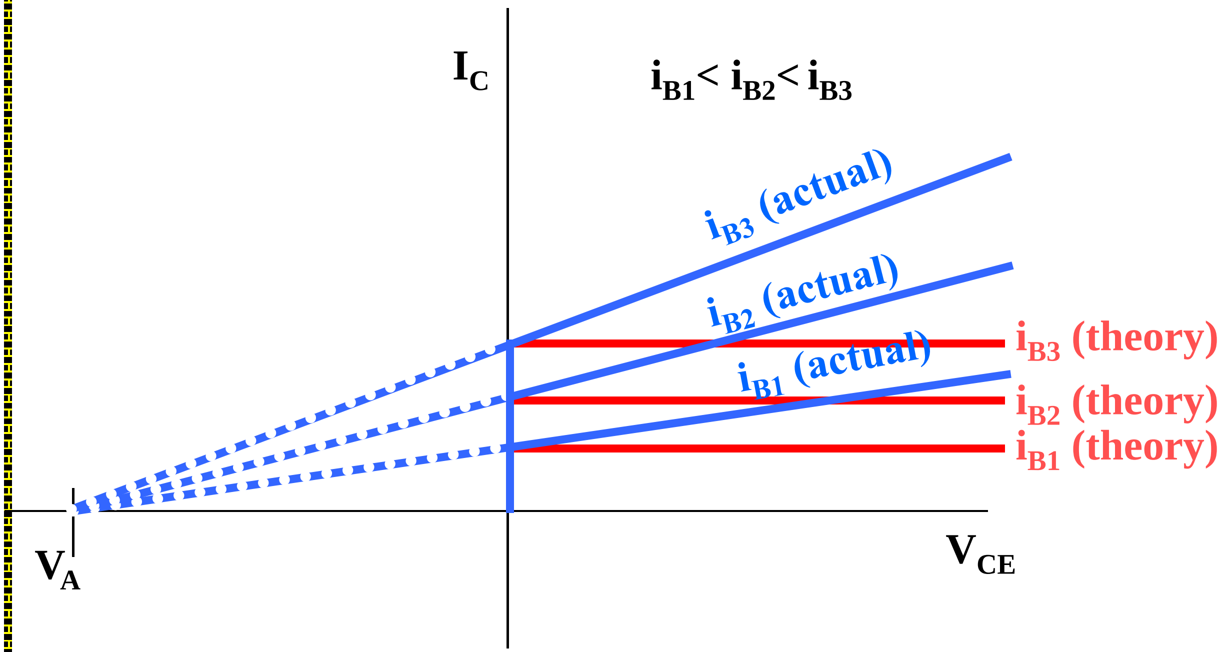
Jaeger 13.5-13.6, Notes

Further Model Simplifications

(useful for circuit analysis)

$$\underbrace{I_C = \alpha_F I_{F0}\left(e^{V_{EB}/V_T} - 1\right) - I_{R0}\left(e^{V_{CB}/V_T} - 1\right)}^{\textbf{Ebers-Moll}} \Rightarrow \overbrace{I_C = \alpha_F I_{F0}\left(e^{V_{EB}/V_T} - 1\right) + I_{R0}}^{\textbf{Forward Active Mode}} \Rightarrow \overbrace{I_C = I_S e^{V_{EB}/V_T}}^{\textbf{Neglect Small Terms}}$$

Modeling the "Early Effect" (non-zero slopes in IV curves)



- Base width changes due to changes in the base-collector depletion width with changes in V_{CB}.
- This changes α_T, which changes I_C, α_{DC} and B_F

Major BJT Circuit Relationships

$$i_C = I_S e^{V_{EB}/V_T} \Rightarrow i_C = I_S e^{V_{EB}/V_T}\left[1 + \frac{v_{CE}}{V_A}\right] \quad \beta_F = \beta_{FO}\left[1 + \frac{v_{CE}}{V_A}\right] \quad i_B = \frac{i_C}{\beta_F} = \frac{I_S}{\beta_{FO}} e^{V_{EB}/V_T}$$

Small Signal Model of a BJT

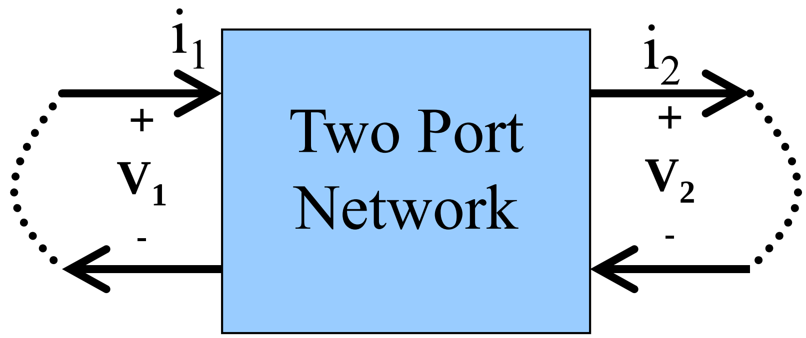
- Just as we did with a p-n diode, we can break the BJT up into a large signal analysis and a small signal analysis and "linearize" the non-linear behavior of the Ebers-Moll model.

- Small signal Models are only useful for Forward active mode and thus, are derived under this condition. (Saturation and cutoff are used for switches which involve very large voltage/current swings from the on to off states.)

- Small signal models are used to determine amplifier characteristics (Example: "Gain" = Increase in the magnitude of a signal at the output of a circuit relative to it's magnitude at the input of the circuit).

- Warning: Just like when a diode voltage exceeds a certain value, the non-linear behavior of the diode leads to distortion of the current/voltage curves (see previous lecture), if the inputs/outputs exceed certain limits, the full Ebers-Moll model must be used.

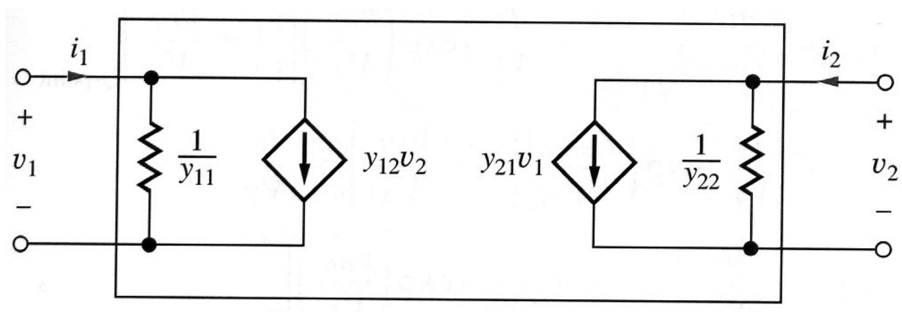
Consider the BJT as a two-port Network



General "y-parameter" Network

$$i_1 = y_{11}v_1 + y_{12}v_2$$

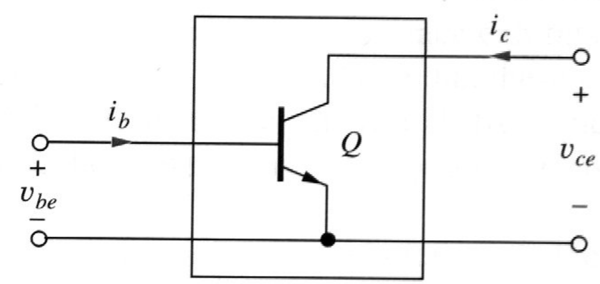
$$i_2 = y_{21}v_1 + y_{22}v_2$$



BJT "y-parameter" Network

$$i_b = y_{11}v_{be} + y_{12}v_{ce}$$

$$i_c = y_{21}v_{be} + y_{22}v_{ce}$$



Consider the BJT as a two-port Network

$i_b = y_{11} v_{be} + y_{12} v_{ce}$

$i_c = y_{21} v_{be} + y_{22} v_{ce}$

$$y_{11} = \left.\frac{\mathbf{i_b}}{\mathbf{v_{be}}}\right|_{\mathbf{v_{ce}}=0} = \left.\frac{\partial i_B}{\partial v_{BE}}\right|_{\text{Q-point}}$$

$$y_{12} = \left.\frac{\mathbf{i_b}}{\mathbf{v_{ce}}}\right|_{\mathbf{v_{be}}=0} = \left.\frac{\partial i_B}{\partial v_{CE}}\right|_{\text{Q-point}}$$

$$y_{21} = \left.\frac{\mathbf{i_c}}{\mathbf{v_{be}}}\right|_{\mathbf{v_{ce}}=0} = \left.\frac{\partial i_C}{\partial v_{BE}}\right|_{\text{Q-point}}$$

$$y_{22} = \left.\frac{\mathbf{i_c}}{\mathbf{v_{ce}}}\right|_{\mathbf{v_{be}}=0} = \left.\frac{\partial i_C}{\partial v_{CE}}\right|_{\text{Q-point}}$$

Consider the BJT as a two-port Network

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$$y_{21} = \left.\frac{\mathbf{i_c}}{\mathbf{v_{be}}}\right|_{\mathbf{v_{ce}}=0} = \left.\frac{\partial i_C}{\partial v_{BE}}\right|_{\text{Q-point}}$$

$$y_{22} = \left.\frac{\mathbf{i_c}}{\mathbf{v_{ce}}}\right|_{\mathbf{v_{be}}=0} = \left.\frac{\partial i_C}{\partial v_{CE}}\right|_{\text{Q-point}}$$

$$\boxed{y_{12}} = \left.\frac{\partial i_B}{\partial v_{CE}}\right|_{\text{Q-point}} = \boxed{0}$$

$$y_{21} = \left.\frac{\partial i_C}{\partial v_{BE}}\right|_{\text{Q-point}} = \frac{I_S}{V_T}\left[\exp\left(\frac{v_{BE}}{V_T}\right)\right]\left[1+\frac{v_{CE}}{V_A}\right]_{\text{Q-point}}$$

$$\boxed{y_{21}} = \frac{I_S}{V_T}\left[\exp\left(\frac{V_{BE}}{V_T}\right)\right]\left[1+\frac{V_{CE}}{V_A}\right] = \boxed{\frac{I_C}{V_T}}$$

$$y_{22} = \left.\frac{\partial i_C}{\partial v_{CE}}\right|_{\text{Q-point}} = \frac{I_S}{V_A}\left[\exp\left(\frac{v_{BE}}{V_T}\right)\right]_{\text{Q-point}}$$

$$\boxed{y_{22}} = \frac{I_S}{V_A}\left[\exp\left(\frac{V_{BE}}{V_T}\right)\right] = \boxed{\frac{I_C}{V_A+V_{CE}}}$$

$$y_{11} = \left.\frac{\partial i_B}{\partial v_{BE}}\right|_{\text{Q-point}} = \left[\frac{1}{\beta_F}\frac{\partial i_C}{\partial v_{BE}} - \frac{i_C}{\beta_F^2}\frac{\partial \beta_F}{\partial v_{BE}}\right]_{\text{Q-point}}$$

$$y_{11} = \frac{1}{\beta_F}\frac{\partial i_C}{\partial v_{BE}}\left[1-\frac{i_C}{\beta_F}\frac{\partial \beta_F}{\partial i_C}\right]_{\text{Q-point}} = \frac{I_C}{\beta_F V_T}\left[1-\left(\frac{i_C}{\beta_F}\frac{\partial \beta_F}{\partial i_C}\right)_{\text{Q-point}}\right]$$

$$y_{11} = \left[\frac{1}{\beta_F}\frac{\partial i_C}{\partial v_{BE}} - \frac{i_C}{\beta_F^2}\frac{\partial \beta_F}{\partial i_C}\frac{\partial i_C}{\partial v_{BE}}\right]_{\text{Q-point}}$$

$$\boxed{y_{11}} = \boxed{\frac{I_C}{\beta_o V_T}} \quad \text{where } \beta_o = \frac{\beta_F}{\left[1 - I_C\left(\frac{1}{\beta_F}\frac{\partial \beta_F}{\partial i_C}\right)_{\text{Q-point}}\right]}$$

β_o is most often taken as a constant, β_F

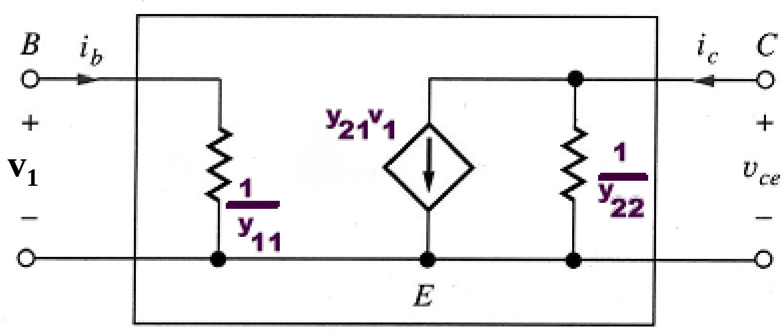
Alternative Representations

Transconductance $g_m = y_{21} = \dfrac{I_C}{V_T} \approx 40 I_C$

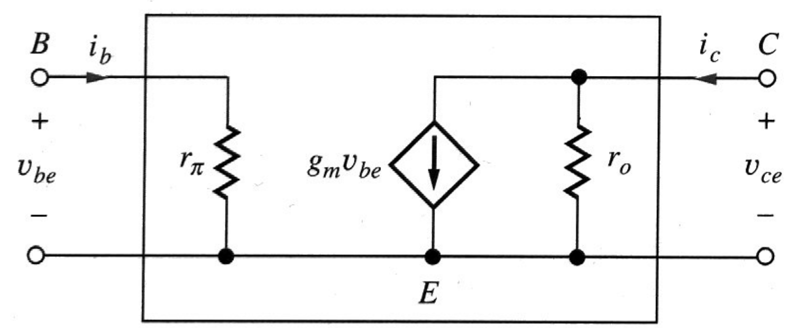
Input Resistance $r_\pi = \dfrac{1}{y_{11}} = \dfrac{\beta_o V_T}{I_C} = \dfrac{\beta_o}{g_m}$

Output Resistance $r_o = \dfrac{1}{y_{22}} = \dfrac{V_A + V_{CE}}{I_C}$

Y-parameter Model



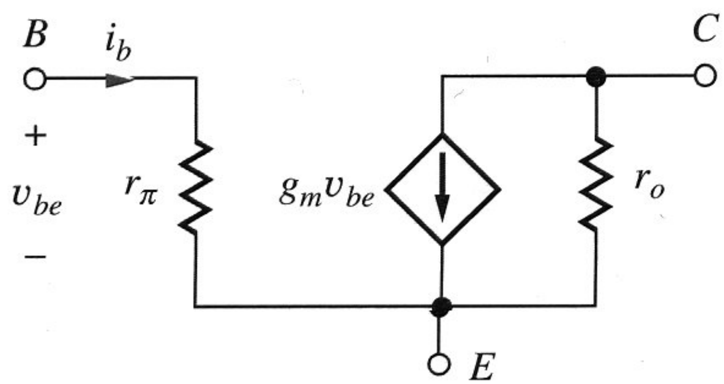
Hybrid-pi Model



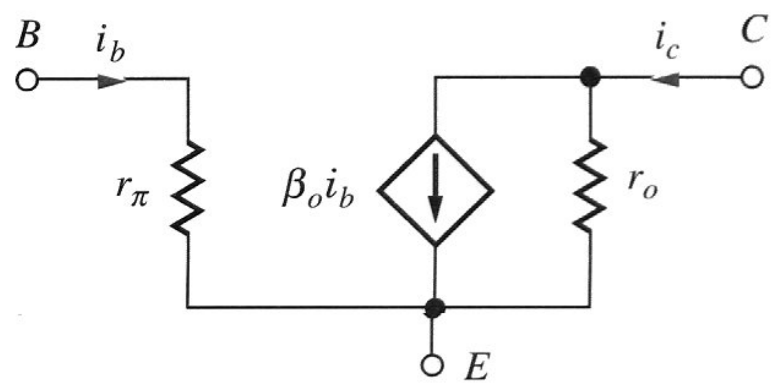
Alternative Representations

$$g_m v_{be} = g_m r_\pi i_b = \beta_o i_b$$

Voltage Controlled Current source version of Hybrid-pi Model



Current Controlled Current source version of Hybrid-pi Model



Single Transistor Amplifier Analysis: Summary of Procedure

Important!

Steps to Analyze a Transistor Amplifier

1.) Determine DC operating point and calculate small signal parameters (see next page)

2.) Convert to the AC only model.
- DC Voltage sources are shorts to ground
- DC Current sources are open circuits
- Large capacitors are short circuits
- Large inductors are open circuits

3.) Use a Thevenin circuit (sometimes a Norton) where necessary. Ideally the base should be a single resistor + a single source. **<u>Do not confuse this with the DC Thevenin you did in step 1.</u>**

4.) Replace transistor with small signal model

5.) Simplify the circuit as much as necessary.

6.) Calculate the small signal parameters (r_π, g_m, r_o etc…) and then gains etc…

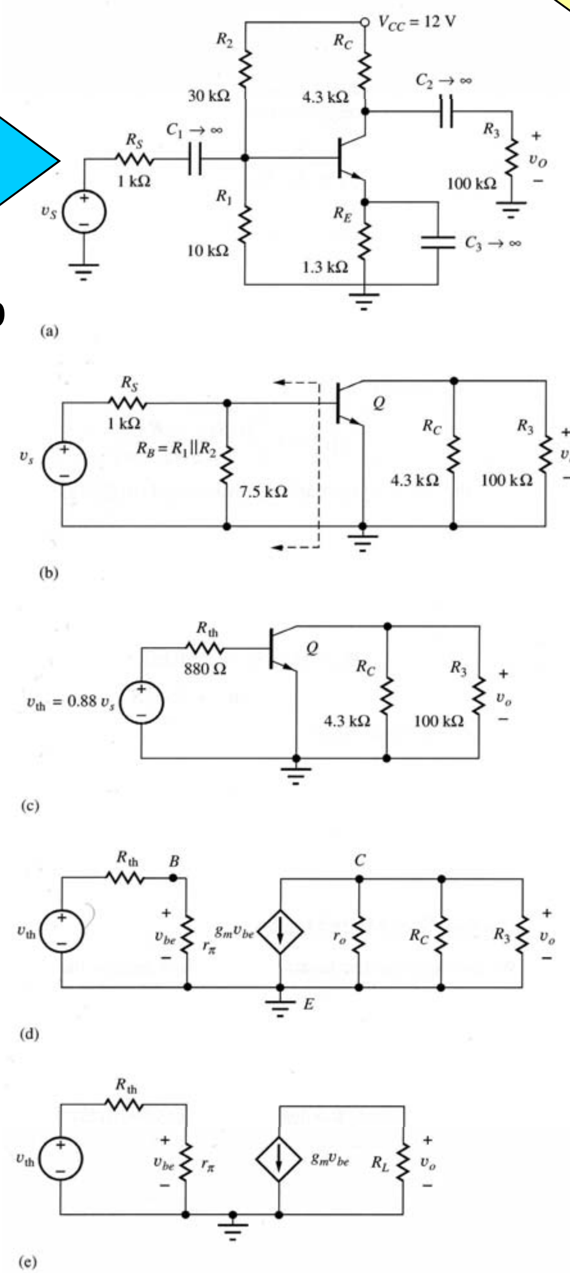
Step 1

Step 2

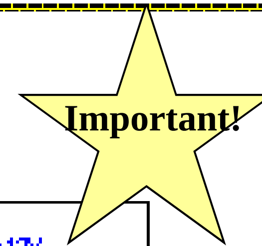
Step 3

Step 4

Step 5

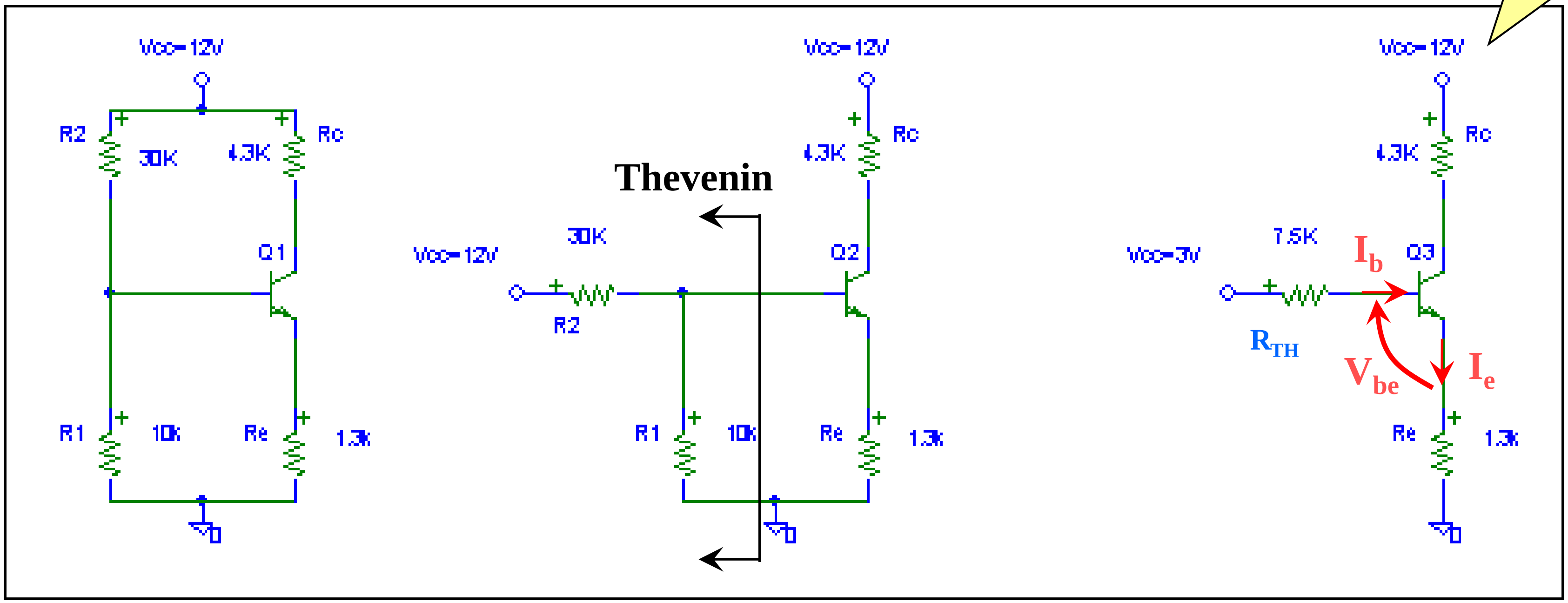


Single Transistor Amplifier Analysis



Step 1 detail

DC Bias Point



$$3V=I_E R_E+V_{be}+I_B R_{TH}$$

$$3V=I_C((\beta_o+1)/\beta_o)\ R_e+0.7V+I_B R_{TH}$$

$$3V=I_B\ \beta_o((\beta_o+1)/\beta_o)\ R_e+0.7V+I_B R_{TH}$$

$$3V=\ I_B(100+1)1300+0.7+\ I_B 7500$$

$$I_B=16.6\ uA,\ I_C=\ I_B\ \beta_o=1.66\ mA,\ I_E=(\beta_o+1)\ I_c/\ \beta_o=1.67\ mA$$

Single Transistor Amplifier Analysis

Important!

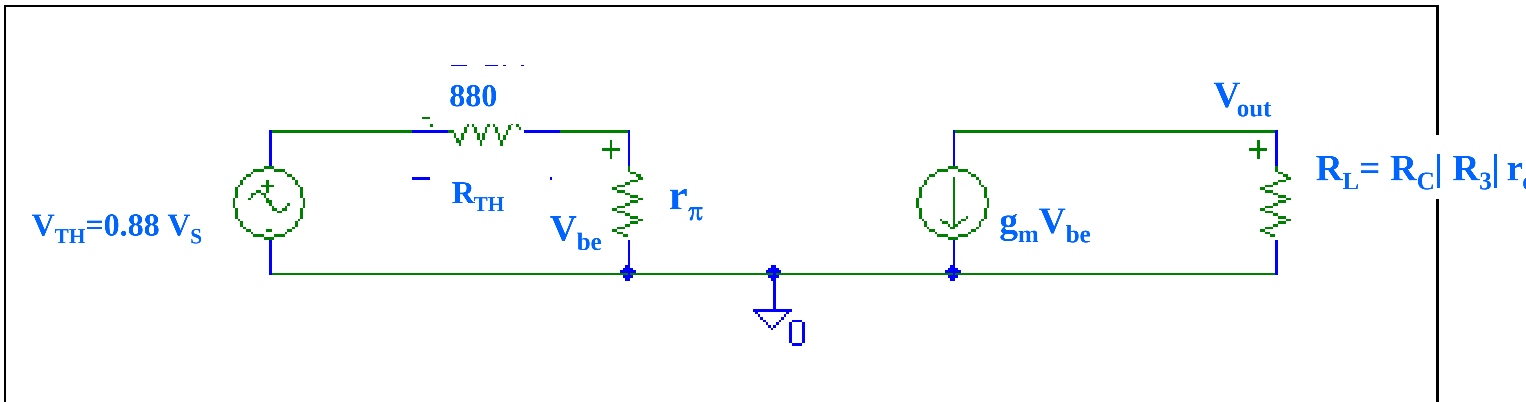
Step 6 detail

Calculate small signal parameters

Transconductance $g_m = y_{21} = \frac{I_C}{V_T} \approx 40\,I_C = 0.0664 \;\; S$

Input Resistance $r_\pi = \frac{1}{y_{11}} = \frac{\beta_o V_T}{I_C} = \frac{\beta_o}{g_m} = 1506 \;\; \Omega$

Output Resistance $r_o = \frac{1}{y_{22}} = \frac{V_A + V_{CE}}{I_C} \approx \frac{V_A}{I_C} = 45.2K \;\; \Omega$



$$v_{out} = -g_m v_{be} R_L \quad and \quad v_{be} = v_{Th}\frac{r_\pi}{R_{Th} + r_\pi} \quad and \quad v_{Th} = 0.88 v_S$$

$$\Downarrow$$

$$A_v \equiv Voltage\ Gain = \frac{v_{out}}{v_S} = \left(\frac{v_{out}}{v_{be}}\right)\left(\frac{v_{be}}{v_{th}}\right)\left(\frac{v_{th}}{v_S}\right) = \left(-g_m R_L\right)\left(\frac{r_\pi}{R_{Th} + r_\pi}\right)(0.88)$$

$$A_v = \left(-(0.0664)(45{,}200 \,||\, 4300 \,||\, 100{,}000)\right)\left(\frac{1506}{880 + 1506}\right)(0.88)$$

$$A_v = -139 \;\; V/V$$

For Extra Examples see: Jaeger section 13.6, and pages 627-630 (top of 630)

Important!

Supplement Added to describe more details of the Solution of this Problem

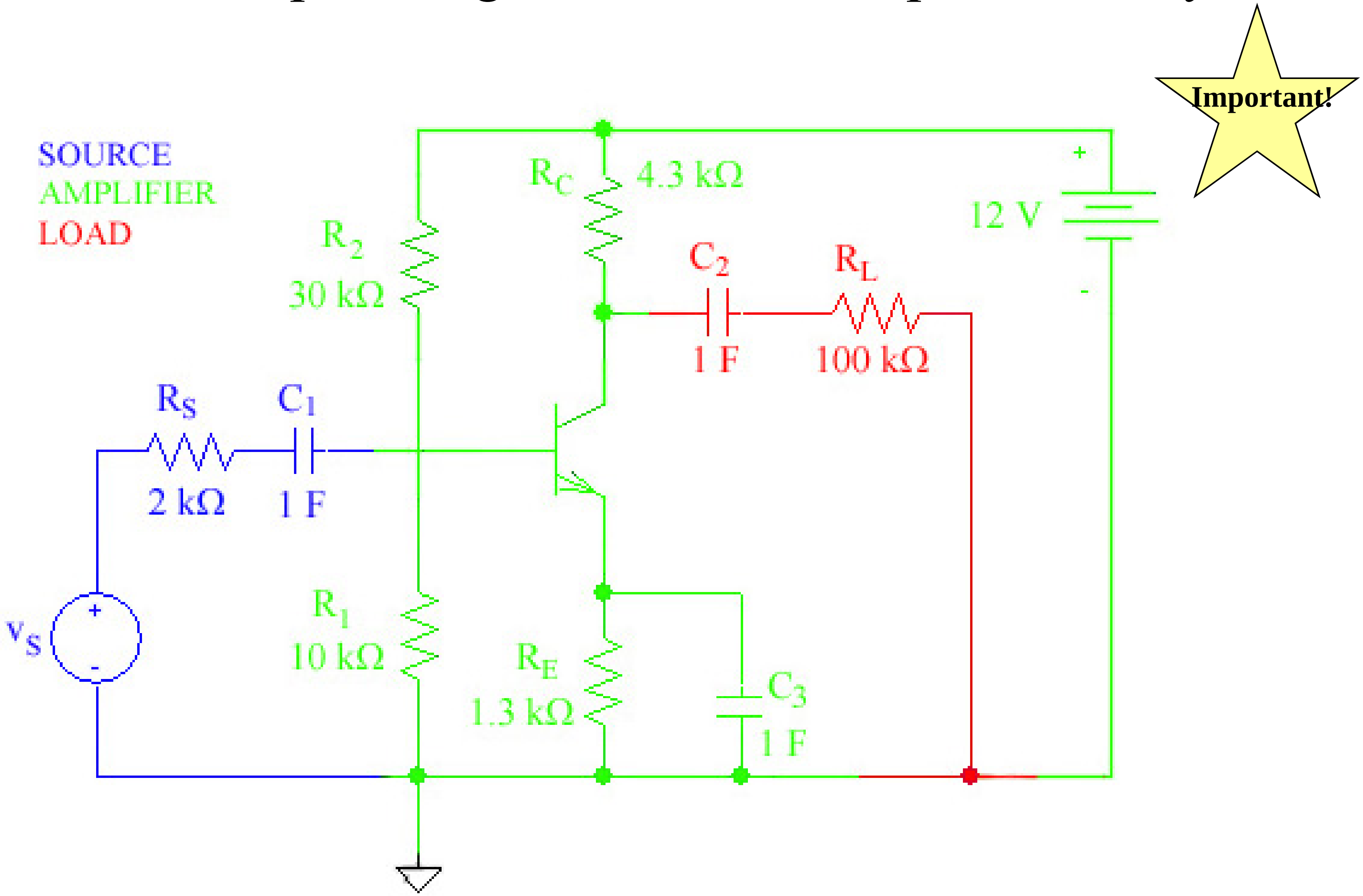
Bipolar Junction Transistors (BJT): Part 5

Details of Amplifier Analysis

Reading:

Jaeger 13.5-13.6, Notes

Detailed Example: Single Transistor Amplifier Analysis



Notes on slides 14-25 were prepared by a previous student.

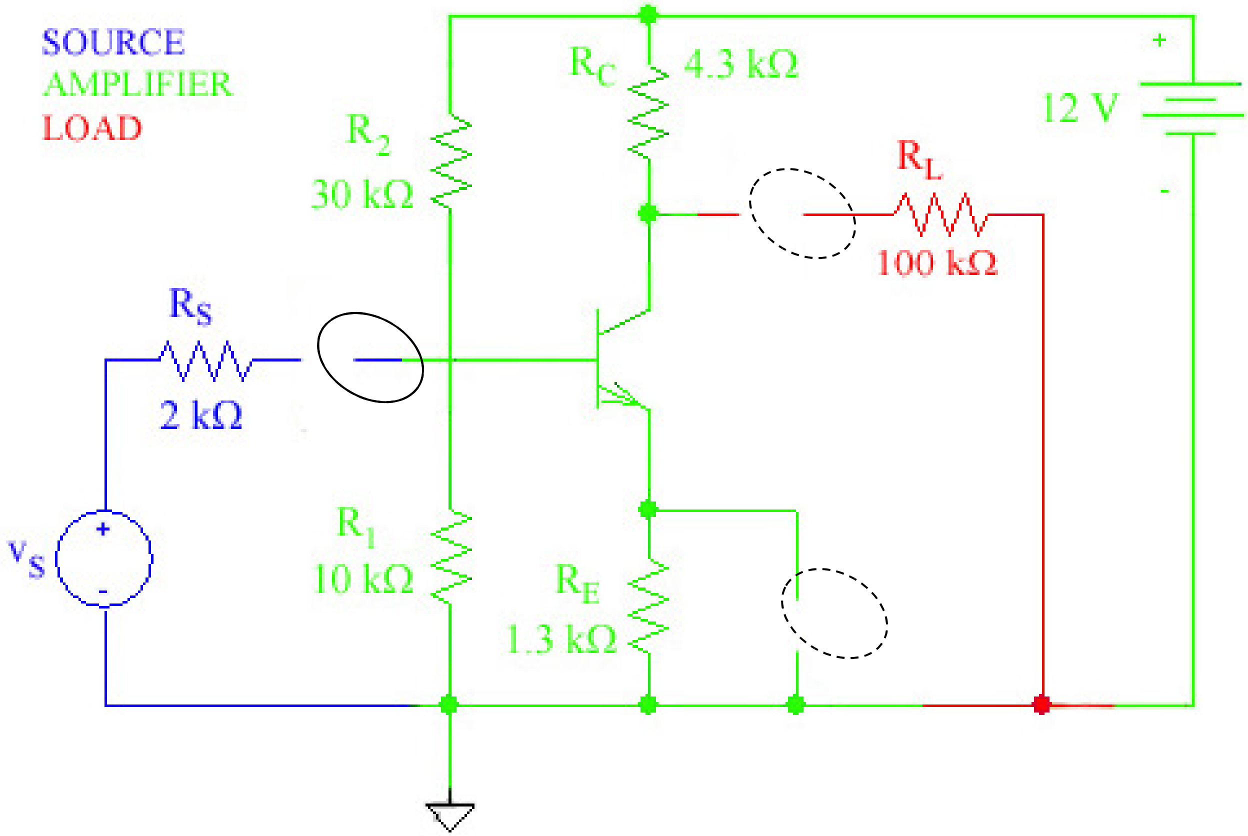
Step 1: Determine DC Operating Point

Remove the Capacitors

Important!

Because the impedance of a capacitor is $Z = 1/(j\omega C)$, capacitors have infinite impedance or are open circuits in DC ($\omega = 0$).

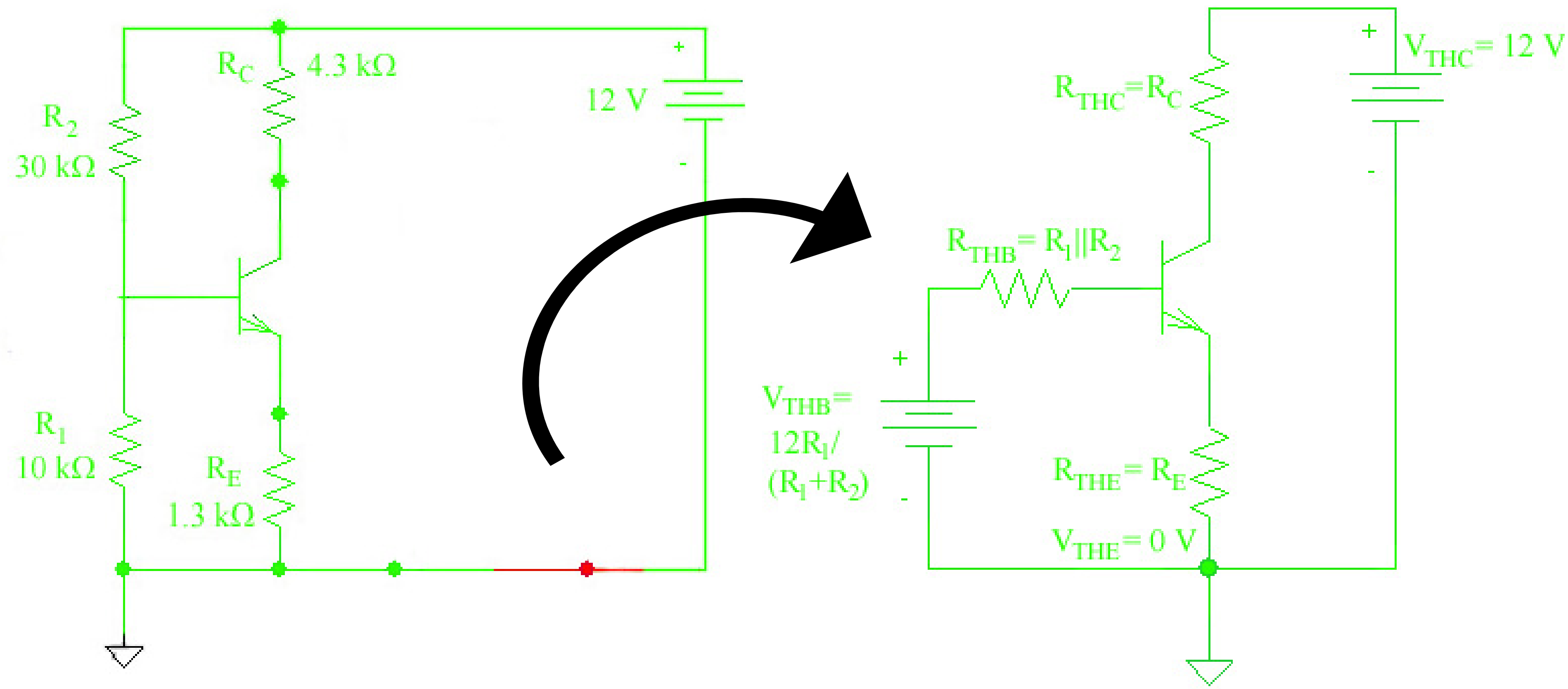
Inductors (not present in this circuit) have an impedance $Z = j\omega L$, and are shorts in DC.



Step 1: Determine DC Operating Point
Determine the DC Thevenin Equivalent

Important!

Replace all connections to the transistor with their Thevenin equivalents.



Step 1: Determine DC Operating Point
Calculate Small Signal Parameters

Important!

Identify the type of transistor (npn in this example) and draw the base, collector, and emitter currents in their proper direction and their corresponding voltage polarities.

Applying KVL to the controlling loop (loop 1):

$$V_{THB} - I_B R_{THB} - V_{BE} - I_E R_E = 0$$

Applying KCL to the transistor:

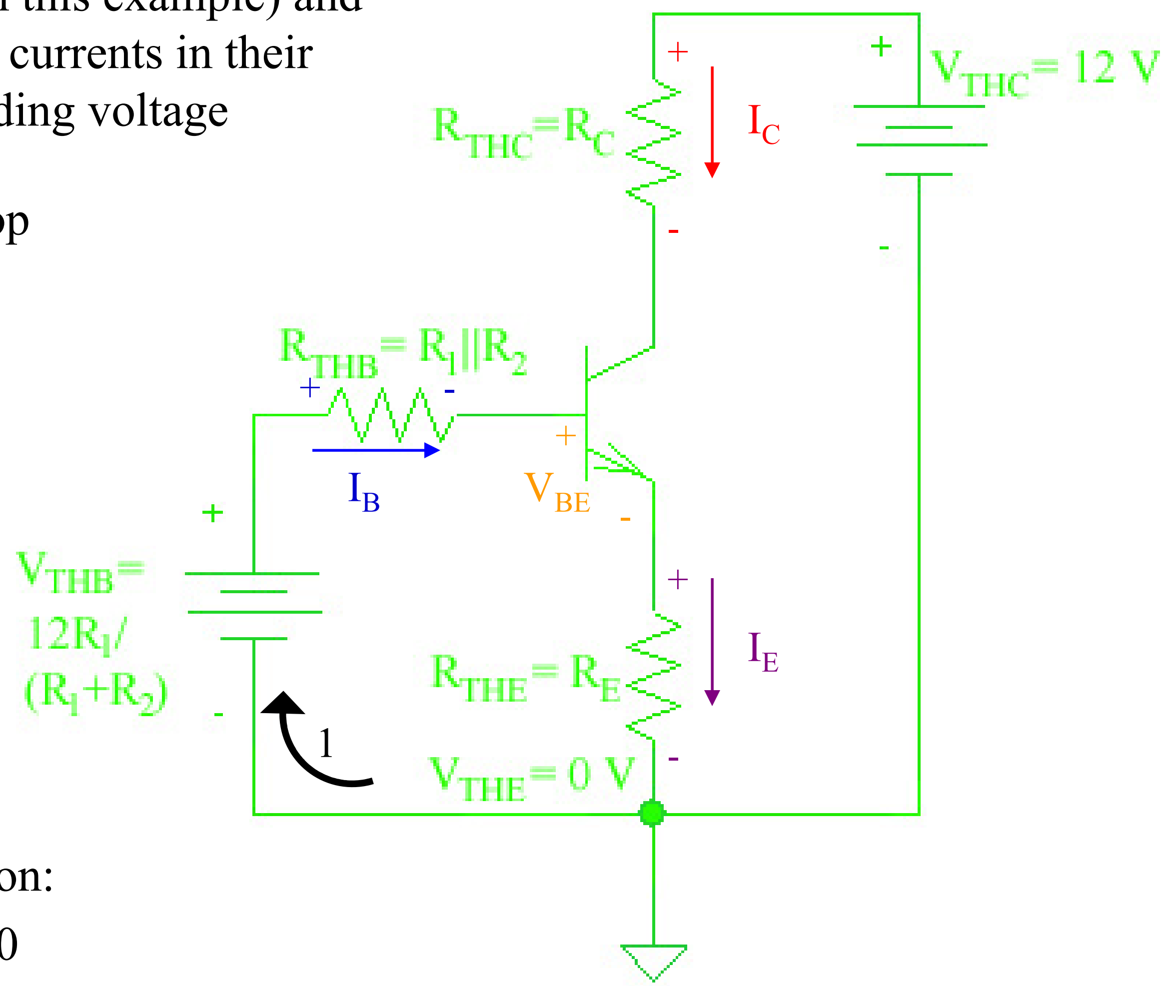
$$I_E = I_B + I_C$$

Because $I_C = \beta I_B$,

$$I_E = I_B + I_C = I_B + \beta I_B = I_B(1+\beta)$$

Substituting for I_E in the loop equation:

$$V_{THB} - I_B R_{THB} - V_{BE} - I_B(1+\beta)R_E = 0$$



Step 1: Determine DC Operating Point
Plug in the Numbers

Important!

$V_{THB} - I_B R_{THB} - V_{BE} - I_B(1+\beta)R_E = 0$

$V_{THB} - V_{BE} - I_B(R_{THB} + (1+\beta)R_E) = 0$

$V_{THB} = 12R_1/(R_1+R_2) = 3\ V$

$R_{THB} = R_1 \parallel R_2 = 7.5\ k\Omega$

Assume $V_{BE} = 0.7\ V$

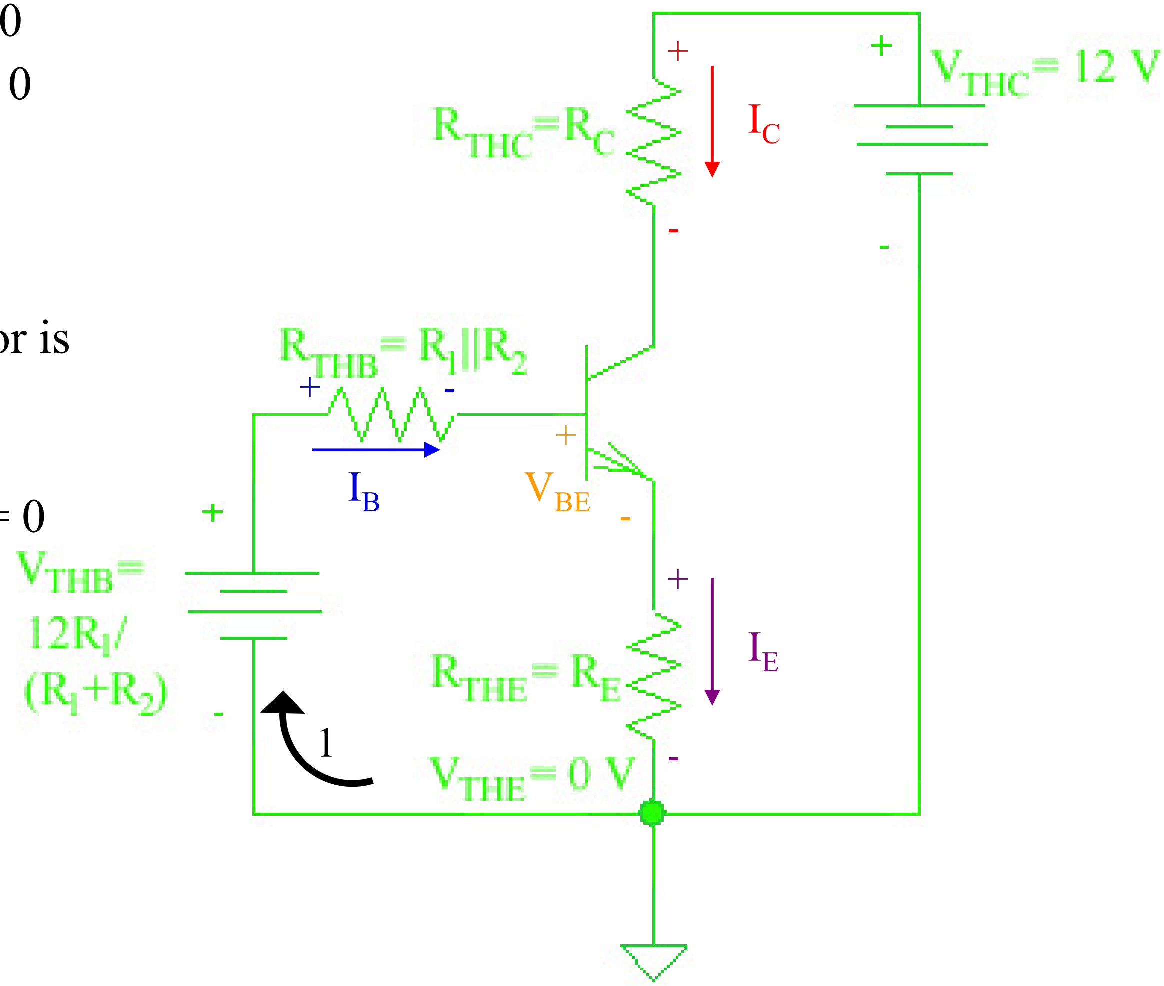
Assume β for this particular transistor is given to be 100.

$3 - 0.7 - I_B(7500 + (1+100)*1300) = 0$

$I_B = 16.6\ \mu A$

$I_C = \beta I_B = 1.66\ mA$

$I_E = I_B + I_C = 1.676\ mA$



Step 1: Determine DC Operating Point

Check Assumptions: Forward Active?

Important!

$V_C = 12 - I_C R_C = 12 - (1.66\text{ mA})(4300) = 4.86\text{ V}$

$V_E = I_E R_E = (1.67\text{ mA})(1300) = 2.18\text{ V}$

$V_B = V_{THB} - I_B R_{THB} = 3 - (16.6\mu\text{A})(7500) = 2.88\text{ V}$

Check:

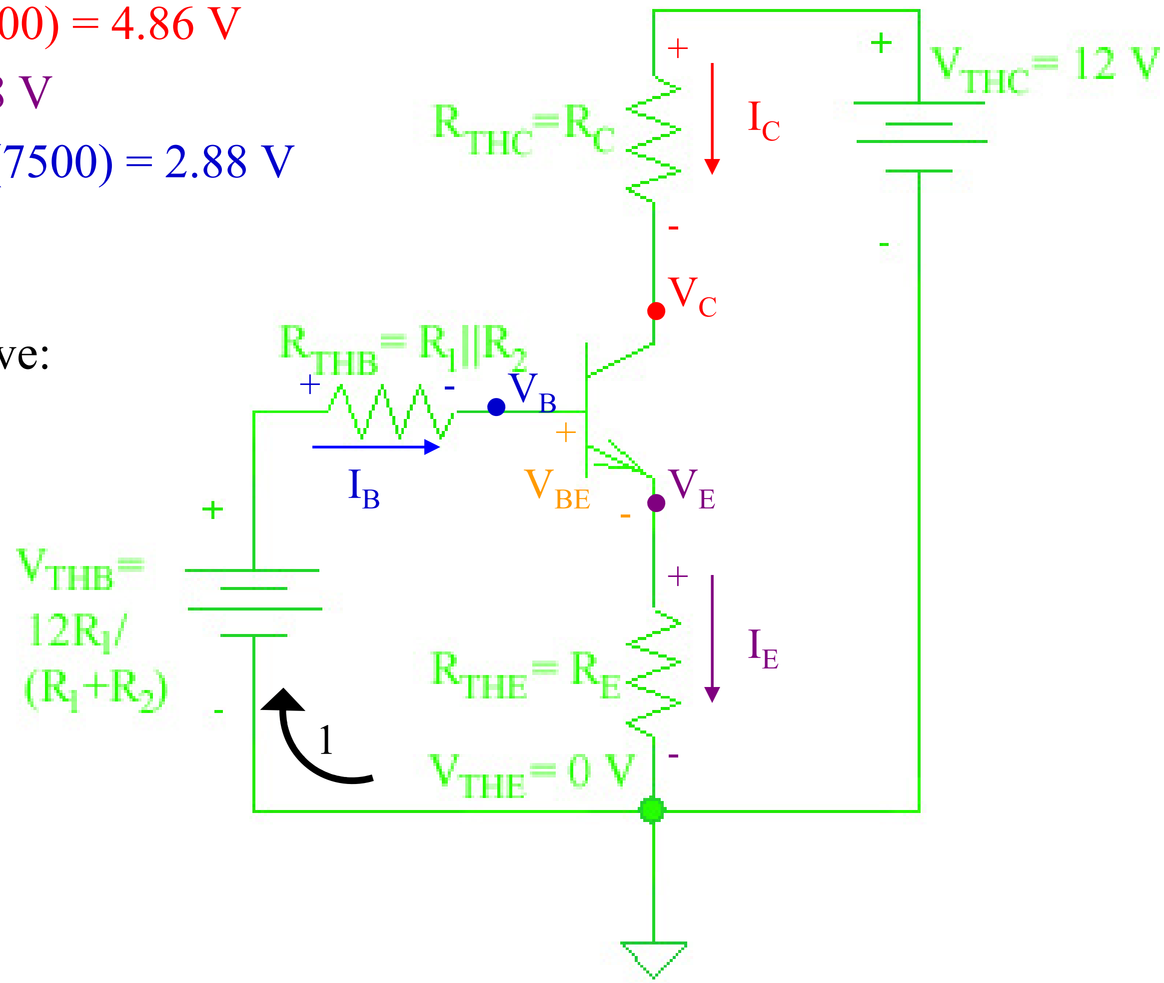
For an npn transistor in forward active:

$$\boxed{V_C > V_B}$$

$4.86\text{ V} > 2.88\text{ V}$

$V_B - V_E = V_{BE} = 0.7\text{ V}$

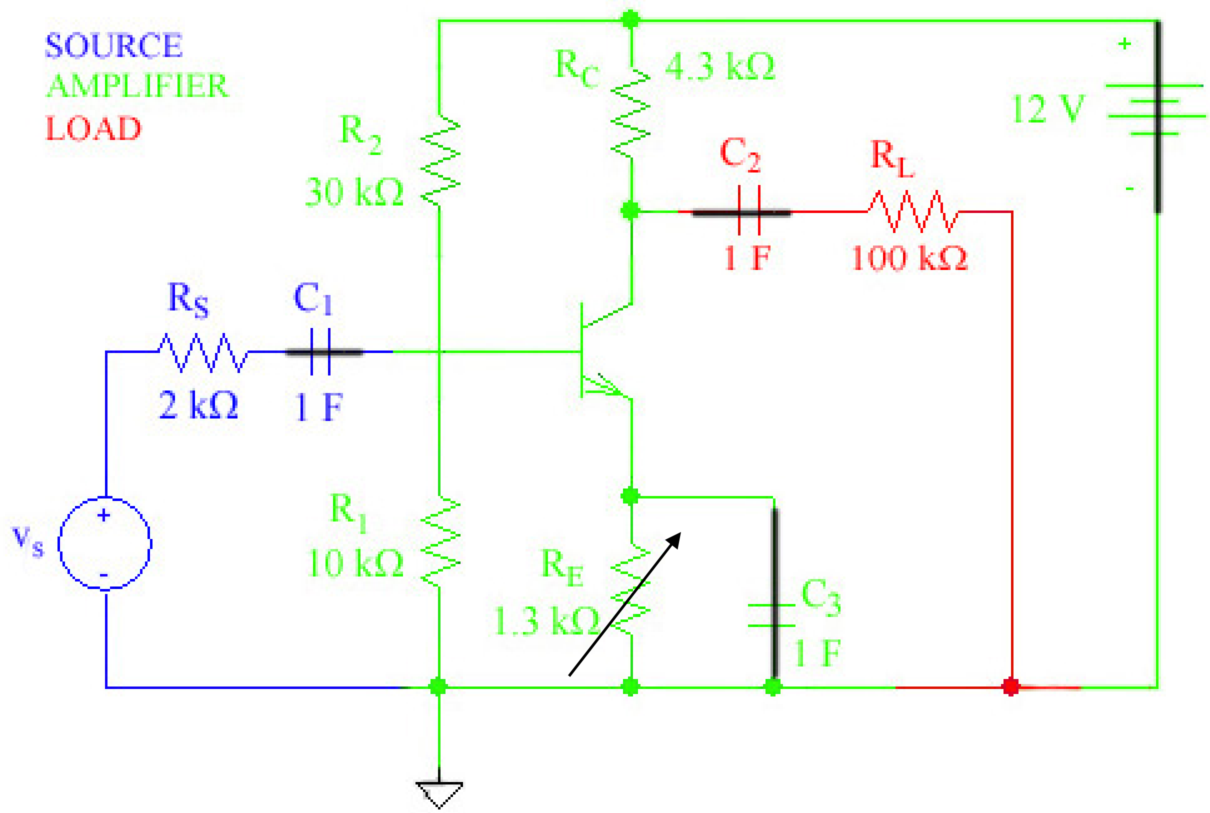
$2.88\text{ V} - 2.18\text{ V} = 0.7\text{ V}$



Step 2: Convert to AC-Only Model

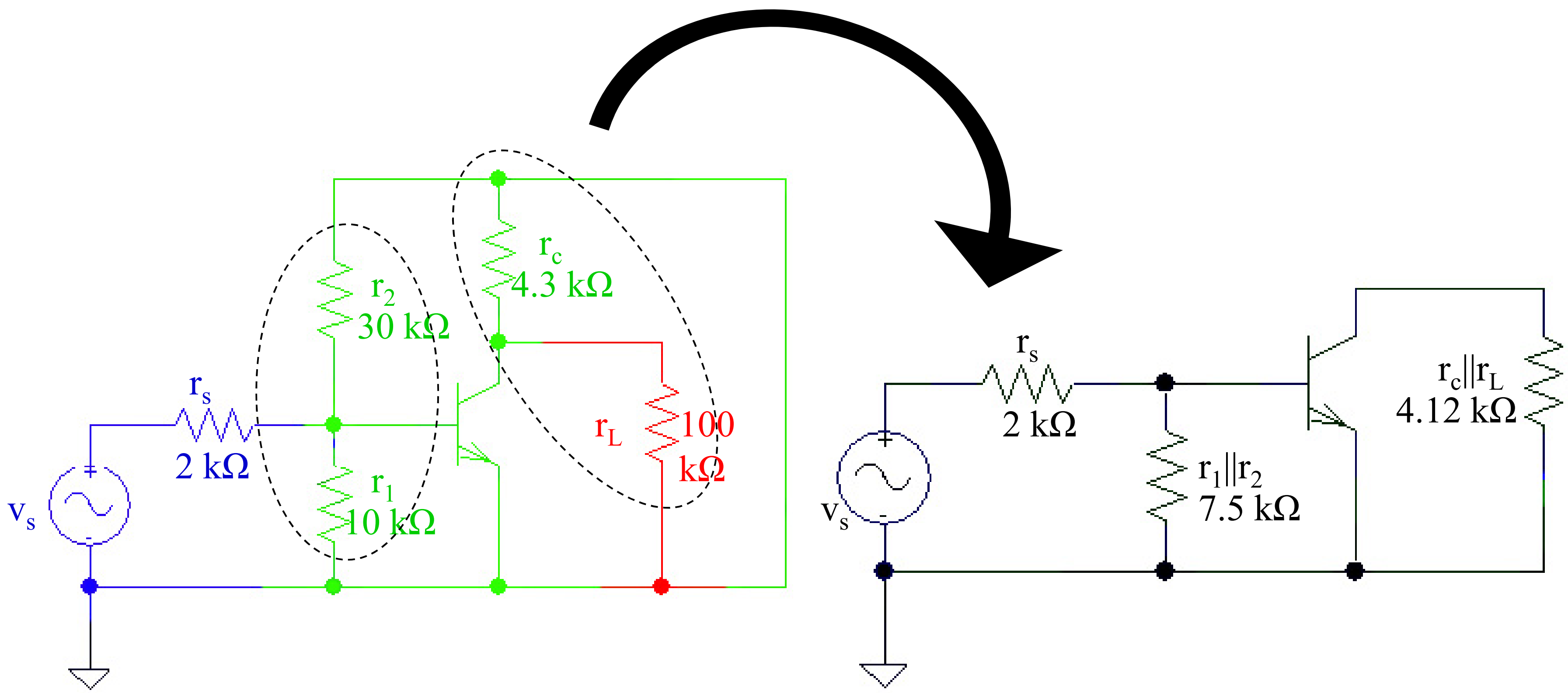
Short the Capacitors and DC Current Sources

- DC voltage sources are shorts (no voltage drop/gain through a short circuit).
- DC current sources are open (no current flow through an open circuit).
- Large capacitors are shorts (if C is large, $1/j\omega C$ is small).
- Large inductors are open (if L is large, $j\omega L$ is large).



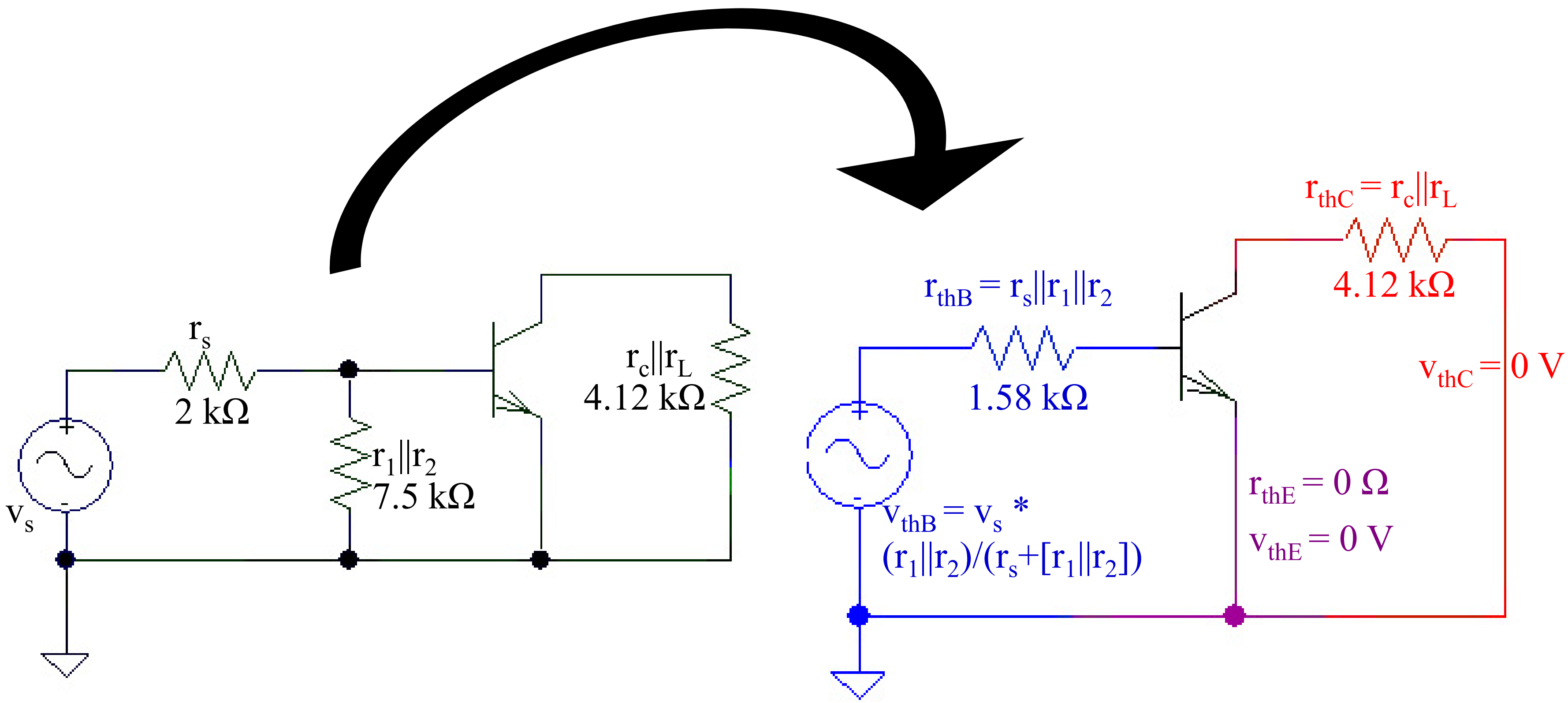
Step 2: Convert to AC-Only Model
(Optional) Simplify Before Thevenizing

Important!



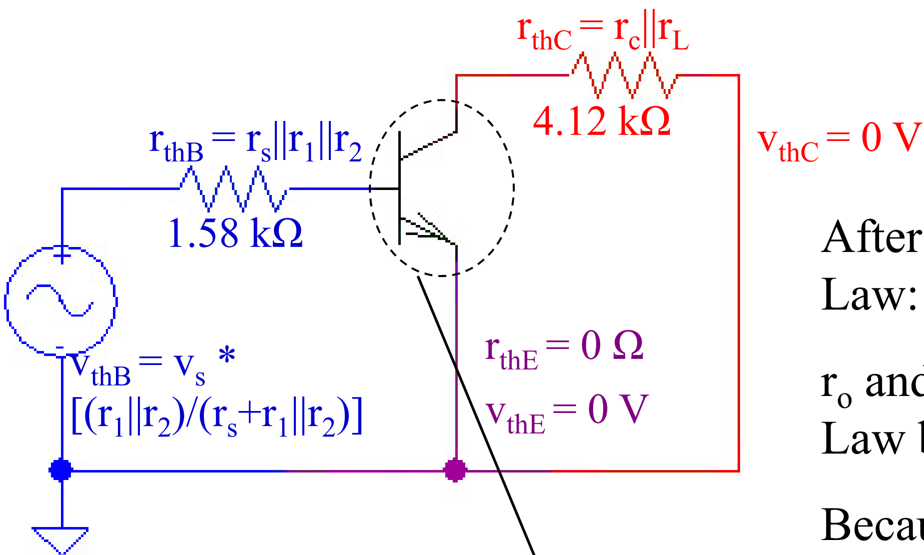
Step 3: Thevenize the AC-Only Model

Important!



Step 4: Replace Transistor With Small Signal Model

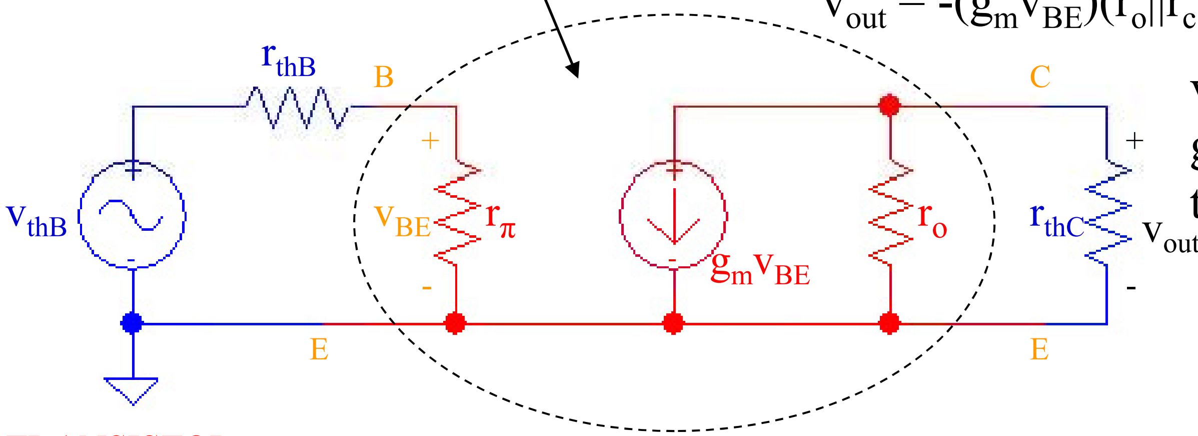
Important!



After replacing the transistor, apply Ohm's Law: V = IR to find v_{out}.

r_o and r_{thC} are in parallel, so that Ohm's Law becomes: $v_{out} = -IR = -(g_m v_{BE})(r_o||r_{thC})$

Because $r_{thC} = r_c||r_L$
$v_{out} = -(g_m v_{BE})(r_o||r_c||r_L)$

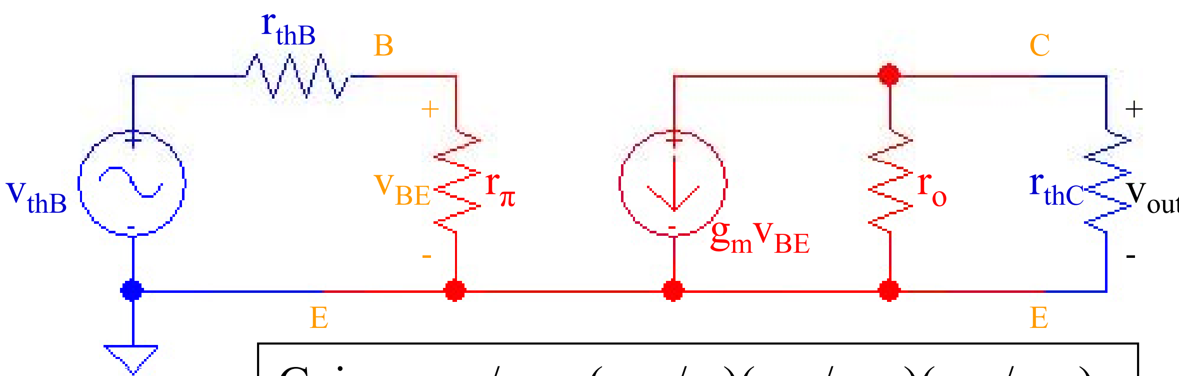


$v_{out}/v_{BE} = -g_m(r_o||r_c||r_L)$ is the gain from transitor input (v_{BE}) to transistor/circuit output v_{out})

TRANSISTOR
EXTERNAL

Important!

Step 5: Calculate Gain and Small Signal Parameters



TRANSISTOR
EXTERNAL

$$\boxed{\text{Gain} = v_{out}/v_s = (v_{thB}/v_s)(v_{BE}/v_{thB})(v_{out}/v_{BE})}$$

As previously determined:
$v_{thB}/v_s = (r_1||r_2)/([r_1||r_2] + r_s)$

Applying a voltage divider:
$v_{BE}/v_{thB} = r_\pi/(r_\pi+r_{thB})$

Gain factor:
$v_{out}/v_{BE} = -g_m(r_o||r_c||r_L)$

Because calculating the DC operating point was done *first*, we have equations for g_m, r_π, and r_o in terms of previously calculated DC currents and voltages.

Transconductance	$g_m = y_{21} = \dfrac{I_C}{V_T} \approx 40 I_C$
Input Resistance	$r_\pi = \dfrac{1}{y_{11}} = \dfrac{\beta_o V_T}{I_C} = \dfrac{\beta_o}{g_m}$
Output Resistance	$r_o = \dfrac{1}{y_{22}} = \dfrac{V_A + V_{CE}}{I_C}$

Plugging in the numbers:

Gain = v_{out}/v_s = -139 V/V

Interpretation/Analysis of Results

Important!

Gain = $v_{out}/v_s = (v_{thB}/v_s)(v_{BE}/v_{thB})(v_{out}/v_{BE}) = -139$ V/V

$v_{thB}/v_s = (r_1||r_2)/([r_1||r_2] + r_s)$

$v_{BE}/v_{thB} = r_\pi/(r_\pi+r_{thB})$

$v_{out}/v_{BE} = -g_m(r_o||r_c||r_L)$

Both terms are *loss factors*, i.e. they can never be greater than 1 in magnitude and thus cause the gain to decrease.

This term is the *gain factor* and is responsible for amplifying the signal.

The AC input signal has been amplified 139 times in magnitude. The negative sign indicates there has been a phase shift of 180°.

Completing the Small Signal Model of the BJT

Base Charging Capacitance (Diffusion Capacitance)

In active mode when the emitter-base is forward biased, the capacitance of the emitter-base junction is dominated by the diffusion capacitance (not depletion capacitance).

Recall for a diode we started out by saying:

Sum up all the minority carrier charges on either side of the junction

$$C_{Diffusion} = \frac{dQ_D}{dv_D^{'}}$$

$$= \frac{dQ_D}{dt}\frac{dt}{dv_D^{'}}$$

Neglect charge injected from the base into the emitter due to p+ emitter in pnp

$$Q_D = qA\int_0^{\infty} p_{no}\left(e^{v_D^{'}/V_T} - 1\right)e^{-x/L_p}\,dx + qA\int_0^{\infty} n_{po}\left(e^{v_D^{'}/V_T} - 1\right)e^{-x/L_n}\,dx$$

Excess charge stored is due almost entirely to the charge injected from the emitter.

Completing the Small Signal Model of the BJT

Base Charging Capacitance (Diffusion Capacitance)

•The BJT acts like a very efficient "siphon": As majority carriers from the emitter are injected into the base and become "excess minority carriers", the Collector "siphons them" out of the base.

•We can view the collector current as the amount of excess charge in the base collected by the collector per unit time.

•Thus, we can express the charge due to the excess hole concentration in the base as:

$$Q_B = i_C \tau_F$$

or the excess charge in the base depends on the magnitude of current flowing and the "forward" base transport time, τ_F, the average time the carriers spend in the base.

•It can be shown (see Pierret section 12.2.2) that:

$$\tau_F = \frac{W^2}{2D_B} \quad where,$$

$$W \equiv Base\ Quasi-neutral\ region\ width$$

$$D_B \equiv Minority\ carrier\ diffusion\ coefficient$$

Completing the Small Signal Model of the BJT

Base Charging Capacitance (Diffusion Capacitance)

Thus, the diffusion capacitance is,

$$C_B = \frac{\partial Q_B}{\partial v_{BE}}\bigg|_{Q-po\,\text{int}} = \left(\frac{W^2}{2D_B}\right)\frac{\partial i_C}{\partial v_{BE}}\bigg|_{Q-po\,\text{int}}$$

$$\boxed{C_B = \tau_F \frac{I_C}{V_T} = \tau_F g_m}$$

The upper operational frequency of the transistor is limited by the forward base transport time: $f \leq \dfrac{1}{2\pi\tau_F}$

Note the similarity to the Diode Diffusion capacitance we found previously:

$$C_{Diffusion} = g_d \tau_t \quad where\ \tau_t = \frac{\lfloor p_{no}L_p + n_{po}L_n \rfloor qA}{I_S}\ \ is\ the\ transit\ \ time$$

Completing the Small Signal Model of the BJT

Base Charging Capacitance (Total Capacitance)

In active mode for small forward biases the depletion capacitance of the base-emitter junction can contribute to the total capacitance

$$C_{jE} = \frac{C_{jEo}}{\sqrt{1 + \dfrac{V_{EB}}{V_{bi\ for\ emitter-base}}}}$$

$$where,$$

$$C_{jEo} \equiv zero\ bias\ depletion\ capaci\tan ce$$

$$V_{bi\ for\ emitter-base} \equiv built\ in\ voltage\ \ for\ the\ E-B\ \ junction$$

Thus, the total emitter-base capacitance is:

$$\boxed{C_{\pi} = C_{B} + C_{jE}}$$

Completing the Small Signal Model of the BJT
Base Charging Capacitance (Depletion Capacitance)

In active mode when the collector-base is reverse biased, the capacitance of the collector-base junction is dominated by the depletion capacitance (not diffusion capacitance).

$$C_{\mu} = \frac{C_{\mu o}}{\sqrt{1 + \dfrac{V_{CB}}{V_{bi\ \ for\ collector-base}}}}$$

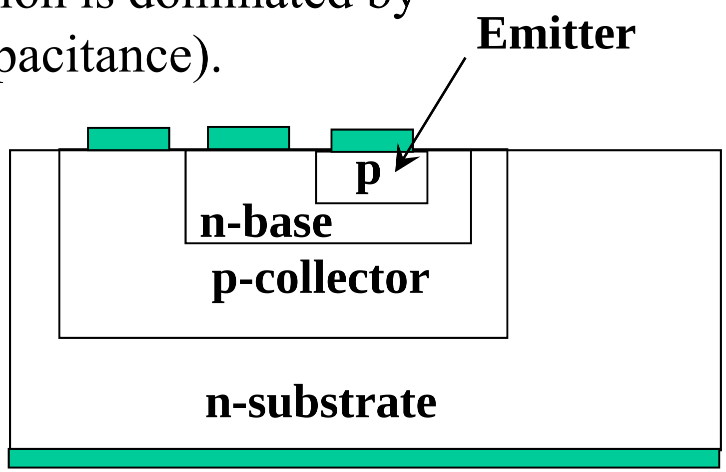
where,

$C_{\mu o} \equiv zero\ bias\ depletion\ capacitance$

$V_{bi\ \ for\ collector-base} \equiv built\ in\ voltage\ \ for\ the\ B-C\ \ junction$

Completing the Small Signal Model of the BJT
Collector to Substrate Capacitance (Depletion Capacitance)

In some integrated circuit BJTs (lateral BJTs in particular) the device has a capacitance to the substrate wafer it is fabricated in. This results from a "buried" reverse biased junction. Thus, the collector-substrate junction is reverse biased and the capacitance of the collector-substrate junction is dominated by the depletion capacitance (not diffusion capacitance).



$$C_{CS} = \frac{C_{CS}}{\sqrt{1+\dfrac{V_{CS}}{V_{bi\ \ for\ collector-substrate}}}}$$

where,

$$C_{CS} \equiv zero\ bias\ depletion\ capaci\tan ce$$

$$V_{bi\ \ for\ collector-substrate} \equiv built\ in\ voltage\ \ for\ the\ C-substrate\ \ junction$$

Completing the Small Signal Model of the BJT
Parasitic Resistances

•r_b = base resistance between metal interconnect and B- E junction
•r_c = parasitic collector resistance
•r_{ex} = emitter resistance due to polysilicon contact
•These resistance's can be included in SPICE simulations, but are usually ignored in hand calculations.

Completing the Small Signal Model of the BJT

Complete Small Signal Model