BLG311E – FORMAL LANGUAGES AND AUTOMATA

2014 SPRING

RECITEMENT 2

1. Let A and B be languages defined over Σ and $\Lambda \notin B$,

a) Propose a solution to the equation $A \cup XB = X$.

b) Show that your solution is correct.

c) Let A={a, b} and B={aa, ab, ba, bb}. Write the solution set and give 5 example words from it.

Solution:

a) Solution is X=AB*

b) When we replace X in the equation:

$$A \cup (AB^*)B = A \cup AB^+ = A(\{\Lambda\} \cup B^+) = AB^*$$

Solution is correct.

c) AB* = {a, b}{aa, ab, ba, bb}*

Examples: {a}, {aaa}, {baa}, {bbbaa}, {aabbabb}

2. Let A and B be languages defined over Σ

Show that equation $A^*B^* \cap B^*A^* = A^* \cup B^*$ holds.

Solution:

$A^*B^* = (\{\Lambda\} \cup A^+)(\{\Lambda\} \cup B^+) = (\{\Lambda\} \cup A^+ \cup B^+ \cup A^+B^+)$

Same for B*A* → $B^*A^* = (\{\Lambda\} \cup A^+ \cup B^+ \cup B^+A^+)$

Intersection of 2 sets:

$A^*B^* \cap B^*A^* = (\{\Lambda\} \cup A^+ \cup B^+)$

$A^* \cup B^* = (\{\Lambda\} \cup A^+) \cup (\{\Lambda\} \cup B^+) = (\{\Lambda\} \cup A^+ \cup B^+)$

It holds.

3. Show that following expressions hold. If they do not hold give a counterexample.

a) $A^+A^+ = A^+$

Does not hold.

Let A = {1} :

$A^+ = \{1, 11, 111, 1111, ... 1^n, ...\}$

$A^+A^+ = \{11, 111, 1111, ... 1^n, ...\}$

b) $(A^*B^*)^* = (B^*A^*)^*$

By using Theorem 13 on page 30 of the slides:

$(A^*B^*)^* = (A \cup B)^* \rightarrow (B^*A^*)^* = (B \cup A)^* \rightarrow (B \cup A)^* = (A \cup B)^*$

Holds

c) $(AB)^* = (BA)^*$

Does not hold

Let A ={0} and B ={1}

$(AB)^* = \{\Lambda, 01, 0101, 010101, ... (01)^n, ...\}$

$(BA)^* = \{\Lambda, 10, 1010, 101010, ... (10)^n, ...\}$

4. Matrix below is a relation defined on the set {a, b, c}. Draw the relation graph of the relation itself, its powers, reflexive, symmetric, transitive closures as well as reflexive closure of its symmetric closure.

	a	**b**	**c**
a	0	1	0
b	1	0	1
c	0	0	0

Solution:

a) $R = \{(a, b), (b, a), (b, c)\}$

Relation graph R:

b) Powers of the relation will be found in (e).

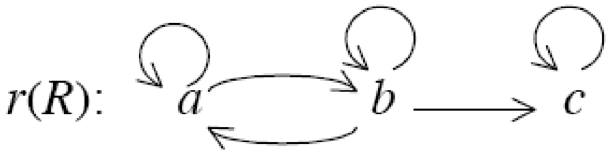
c) Reflexive closure:

$r(R) = R \cup R^0 = R \cup$ E, E $= R^0$ (E is the unit relation)

R = {(a,b), (b,a), (b,c)}

E = {(a,a), (b,b), (c,c)}

$r(R)$ = {(a,b), (b,a), (b,c), (a,a), (b,b), (c,c)}



d) Symmetric closure:

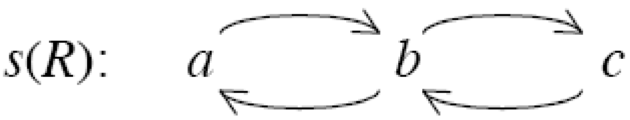
$s(R) = R \cup R^{-1}$

R = {(a,b), (b,a), (b,c)}

R^{-1} = {(b,a) | (a,b) Є R}

R^{-1} = {(a,b), (b,a), (c,b)}

$R \cup R^{-1} = s(R)$ = {(a,b), (b,a), (b,c), (c,b)}



e) Transitive closure:

$$t(R) = \bigcup_{i=1}^{\infty} R^i$$

We need to find the powers of the relation for the transitive closure.

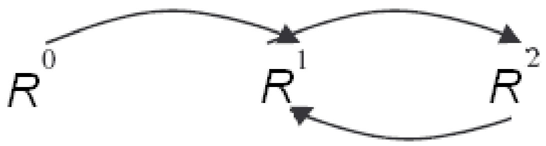
R = {(a,b), (b,a), (b,c)}

$R^2 = RR$ = {(a,b), (b,a), (b,c)}{(a,b), (b,a), (b,c)} = {(a,a), (b,b), (a,c)}

$R^3 = R^2R$ = {(a,a), (b,b), (a,c)}{(a,b), (b,a), (b,c)} = {(a,b), (b,a), (b,c)}

$R^1=R^3$
$RR= R^3R \rightarrow R^2=R^4$
$R^{2n+1}=R^1$ and $R^{2n}=R^2$ (n>0)

Powers of the relation graph:



Transitive closure $\rightarrow t(R) = R \cup R^2$:

$t(R)$ = {(a,b), (b,a), (b,c)} $\cup$ {(a,a), (b,b), (a,c)}
= {(a,b), (b,a), (b,c), (a,a), (b,b), (a,c)}

f) Reflexive closure of the symmetric closure

rs(*R*) = ?   Let *P* = *s*(*R*)
We know that: *s*(*R*) = {(a,b), (b,a), (b,c), (c,b)}
We need to find *r*(*P*).
r(*P*) = {(a,b), (b,a), (b,c), (c, b), (a,a), (b,b), (c,c)}