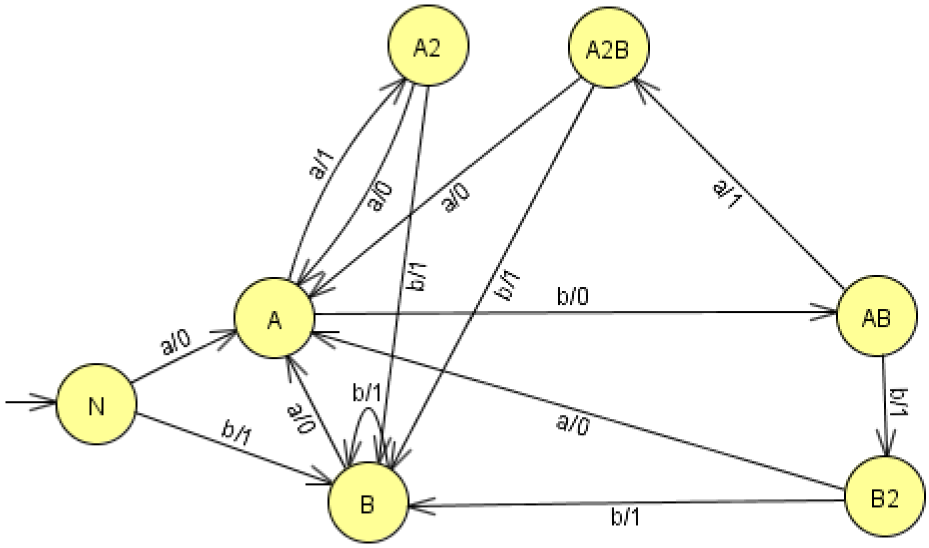
ITU Computer and Informatics Faculty
BLG311E - Formal Languages and Automata
20.03.2014
Midterm 1

1) Solution 1:

a) State transition diagram:



In this machine,

- N → the monkey has no fruits (initial state)
- A → the monkey has a single apple
- B → the monkey has a single banana (The monkey eats a banana any time he has a single banana)
- A2 → the monkey has two apples (If at any time monkey only has two apples he eats both of them)
- B2 → the monkey has two bananas and an apple (If at any time monkey has two bananas he eats the bananas and throws other fruits away)
- A2B → the monkey has two apples and a banana (If at any time monkey has two apples and a banana he eats all the fruits)
- AB → the monkey has an apple and a banana

State transition table:

	a	b
N	A/0	B/1
A	A2/1	AB/0
B	A/0	B/1
A2	A/0	B/1
B2	A/0	B/1
A2B	A/0	B/1
AB	A2B/1	B2/1

b) Dependency table:

N						
X	**A**					
OK	X	**B**				
OK	X	OK	**A2**			
OK	X	OK	OK	**B2**		
OK	X	OK	OK	OK	**A2B**	
X	X	X	X	X	X	**AB**

Reduced State transition table:

	a	b
S1={N,B,A2,B2,A2B}	S2/0	S1/1
S2={A}	S1/1	S3/0
S3={AB}	S1/1	S1/1

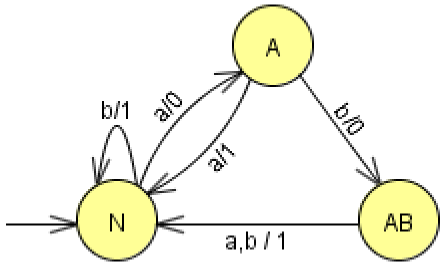
Solution 2:

a) The state transition diagram given on the right represents the machine.

N → the monkey has no fruits

A → the monkey has an apple

AB → the monkey has an apple and a banana



The state transition table can be drawn as follows:

	a	**b**
N	A/0	N/1
A	N/1	AB/0
AB	N/1	N/1

b)

	N	**A**
A	X	
AB	X	X

No equivalent states. The state machine cannot be reduced.

2) $A = abb^{+}ba \rightarrow ab^{n}a, n \geq 3$

$B = a(bb)^{+}ba \rightarrow ab^{2n+1}a, n > 0$

$C = a(bbb)^{*}a \rightarrow ab^{3n}a, n \geq 0$

i) An example string that is accepted by the all three languages: $abbba$

ii) An example string that is accepted by only A: $abbbba$

iii) An example string that is accepted by only A and B: $abbbbba$

iv) An example string that is accepted by only A and C: $abbbbbba$

v) An example string that is accepted by only C: aa

vi) $B \subset A$ as $(ab^{2n+1}a, n > 0) \subset (ab^{n}a, n \geq 3)$
C has no subset/superset relation with the others as it is the only language that accepts aa and it is more restrictive than A and B for b's.

3) $\alpha^{s+kp+i} = \alpha^{s+i};\ \forall k > 0\ \wedge \forall i (0 < i < p)$ and $\alpha^{s} = \alpha^{t}$ where $s < t$ and $p = t - s$

$\alpha^{s+kp}\alpha^{i} = \alpha^{s}\alpha^{i} \rightarrow \alpha^{s+kp} = \alpha^{s};\ \forall k > 0$ (We do not need to consider α^{i}, thus i.)

Proving $\alpha^{s+kp} = \alpha^{s};\ \forall k > 0$ by induction,

- Basis step ($k = 1$): $\alpha^{s+p} = \alpha^{s+t-s} = \alpha^{t} = \alpha^{s}$ as $\alpha^{s} = \alpha^{t}$ where $s < t$ and $p = t - s$
- Inductive step ($k = n$):
  Assume $\alpha^{s+np} = \alpha^{s}$
- For $k = n + 1$, checking if $\alpha^{s+(n+1)p}$ is equal to α^{s}:
  $\alpha^{s+(n+1)p} = \alpha^{s+np+p} = \alpha^{s+p+np}$
  As $\alpha^{s} = \alpha^{t}$ where $s < t$ and $p = t - s$:
  $$\alpha^{s+p+np} = \alpha^{s+t-s+np} = \alpha^{t+np} = \alpha^{t}\alpha^{np} = \alpha^{s}\alpha^{np}$$
  We assumed $\alpha^{s+np} = \alpha^{s}$:
  $$\alpha^{s}\alpha^{np} = \alpha^{s+np} = \alpha^{s}$$

We can also prove the complete expression ($\alpha^{s+kp+i} = \alpha^{s+i}$) without omitting i.

- Basis step ($i = 1$ and $k = 1$): $\alpha^{s+p+1} = \alpha^{s+t-s+1} = \alpha^{t+1} = \alpha^t\alpha = \alpha^s\alpha = \alpha^{s+1}$ as $\alpha^s = \alpha^t$ where $s < t$ and $p = t - s$
- Inductive step ($i = m$ and $k = n$):
  Assume $\alpha^{s+np+m} = \alpha^{s+m}$
- For $i = m + 1$ and $k = n + 1$, checking if $\alpha^{s+(n+1)p+m+1}$ is equal to α^{s+m+1}:
  $\alpha^{s+(n+1)p+m+1} = \alpha^{s+np+p+m+1} = \alpha^{s+p+np+m+1}$
  As $\alpha^s = \alpha^t$ where $s < t$ and $p = t - s$:
  $\alpha^{s+p+np+m+1} = \alpha^{s+t-s+np+m+1} = \alpha^{t+np+m+1} = \alpha^t\alpha^{np+m+1} = \alpha^s\alpha^{np+m+1}$
  We assumed $\alpha^{s+np+m} = \alpha^{s+m}$:
  $\alpha^s\alpha^{np+m+1} = \alpha^{s+np+m+1} = \alpha^{s+np+m}\alpha = \alpha^{s+m}\alpha = \alpha^{s+m+1}$

4) a) It is Type-2 since it suits the definition of Type-2 given as follows:

"Type-2 grammars are defined by rules of the form $A \rightarrow \gamma$ with a nonterminal(A) and a string of terminals and nonterminals(γ).**"**

It is not Type-3 as it does not satisfy the relevant definition stated in b (involving multiple nonterminals on the right-hand side (n_0A)).

b) A Type-3 grammar can be given as follows by taking the relevant definition (i.e., "A Type-3 grammar restricts its rules to a single nonterminal on the left-hand side and a right-hand side consisting of ***a number of terminals***, ***possibly*** followed by a single nonterminal.") into account.

$$< S >= ab \mid aab \mid abb \mid ab < S > \mid aab < S > \mid abb < S >$$

c) $n_0 = (ab \vee aab \vee abb)^+$