BLG 477E Multimedia Computing

HOMEWORK #2 – Optimal Rate allocation

In this homework, you will learn about the concept of optimal rate allocation to multiple sources. The different sources might be different macroblocks, transform coefficients, frames depending on the application.

Optimal rate allocation

Suppose there are n sources {i=1,...,n} being the source index. Each source has a distortion vs. rate function given as $D_i(R_i)$.

In optimal rate allocation, the objective is to minimize the total distortion $D_T(R_T) = \sum_{i=1}^{n} D_i(R_i)$ subject to a constraint on average rate of n sources $R_T = \frac{1}{n}\sum_{i=1}^{n} R_i \leq R_{budget}$.

This problem is equivalent to the unconstrained problem of minimization of the Lagrangian cost function

$$J_T = D_T + \lambda R_T,$$

at $R_T = R_{budget}$.

When the sources are statistically independent, the problem is simplified to the minimization of

$J_i = D_i(R_i) + \lambda R_i$ for {i=1,...,n}. As λ grows from 0 to ∞, R_i diminishes from ∞ to 0 and D_i grows from 0 to $D_{i,\max}$ At the optimal solution, λ should satisfy the constraint $R_T = \frac{1}{n}\sum_{i=1}^{n} R_i = R_{budget}$.

How to locate the best (R_i, D_i) satisfying $\boldsymbol{R_T = \frac{1}{n}\sum_{i=1}^{n} R_i = R_{budget}}$

Method A: For a given λ : For each {i=1,...,n} apply bisection or golden section search (look up from Wikipedia if you do not know) by starting with two initial points (0, $D_{i,\max}$) and $(R_{i,\max}, D_i(R_{i,\max}))$. Let $R_{i,\max} = R_{\max} = 10$ {i=1,...,n}. The search should yield $(R_i^*(\lambda), D_i^*(\lambda))$, the best rate distortion pair for each source for the given λ .

Iterative determination of best λ : Start with the initial $\lambda = \lambda^1 = \min_i \left\{ -\left.\frac{dD_i(R)}{dR}\right|_{R=R_{\max}} \right\}$ and apply Method A. Now, if $R_T^* = \frac{1}{n}\sum_{i=1}^{n} R_i^* < R_{budget}$ then reduce constraint on rate by increasing λ ($\lambda^j > \lambda^{j-1}$) and repeat Method A. Otherwise if $R_T^* = \frac{1}{n}\sum_{i=1}^{n} R_i^* > R_{budget}$increase constraint on rate by decreasing λ ($\lambda^j < \lambda^{j-1}$) and repeat Method A. Again an efficient search method could be used to change λ and eventually locate the best λ . Continue the iterations until $\left|\frac{\lambda^j - \lambda^{j-1}}{\lambda^j}\right| < 0.05$

Let

$$D_1(R) = \frac{5}{10R - 9}$$

$$D_2(R) = \frac{10}{R^2 + 1}$$

$$D_3(R) = \frac{250}{R^4 + 50}$$

$$D_4(R) = 3e^{-(-0.5R+1)}$$

$$D_5(R) = \frac{5}{50\ln(x) + 1}$$

$$D_6(R) = \frac{0.00025\cosh(0.01R)}{\cosh(0.01R) - 1}$$

Each of you should solve the bit allocation problem for $R_{budget1} = 1$ $R_{budget2} = 2,$ $R_{budget3} = 5$ for three of the above six sources specified next to your student number below. You may write a short C/C++ code to facilitate the solution. However, in iteration i for at least i=1,2,3, you have to show all the calculated and tested rate and distortion pairs in Method A as well as the Lagrange parameter λ_i.

040080193,150110033: $D_1(R)$, $D_2(R)$, $D_4(R)$

150120251,040100031: $D_2(R)$, $D_3(R)$, $D_4(R)$

040090534,150110124: $D_1(R)$, $D_3(R)$, $D_4(R)$

040090523,150110047: $D_1(R)$, $D_2(R)$, $D_5(R)$

040100022,150120108: $D_2(R)$, $D_3(R)$, $D_5(R)$

040100009,040090806: $D_1(R)$, $D_3(R)$, $D_5(R)$

040050260,150110020: $D_1(R)$, $D_2(R)$, $D_6(R)$

040100041,150120066: $D_2(R)$, $D_3(R)$, $D_6(R)$

040050203,150110042: $D_1(R)$, $D_3(R)$, $D_6(R)$

040100048,150110108: $D_1(R)$, $D_4(R)$, $D_6(R)$

150120106,150130702: $D_2(R)$, $D_4(R)$, $D_6(R)$

150120134,150110032: $D_3(R)$, $D_4(R)$, $D_6(R)$

040100232: $D_1(R)$, $D_5(R)$, $D_6(R)$

Tell me and I forget. Teach me and I remember. Involve me and I learn.
-Benjamin Franklin