BLG 381E ADVANCED DATA STRUCTURES
FINAL EXAM – JANUARY 9, 2012 12:00-2:00 PM (PART I)

1 (20pt)	2 (10 pt)	3 (10 pt)	4 (10 pt)	5 (15 pt)	6 (15 pt)	7 (20 pt)	Total (100 pt)

On my honor, I declare that I neither give nor receive any unauthorized help on this exam

Student Signature:_____

Duration: 120 minutes. *Write your name on each sheet.Write your answers neatly in the space provided for them. You must show all your work for credit. Books and notes are closed. No questions are allowed during the exam. Good Luck!*

Q1) [20 pts] Red-black trees

a) [4 pts]What are the four properties of red-black trees (RBT)?

b) [2 pts] What are the two modifying operations in RBT to update the tree after insert or delete operation?

c) [6pts] Build a RBT by inserting the following numbers in the given order: [9,5,20,12,24,10,13,28]. (Label the node as R(red) or B(black) to represent the color)

Q1) Continued

Q1d) [8 pts] Insert 17 to the tree which you build in **(b)**. Explain clearly the processing steps.

Q2) [10 pts] Augmenting data structures

a) [2 pts]What is the purpose of augmenting a data structure?

b) [4 pts]What are the four steps of augmenting data structure?

c) [4 pts]Tell briefly how you would augment a data structure to perform fast order-statistic operations following the steps in **(b)**.

Q3) [10 pts] B-trees

a) [2 pts]Why do we need/use B-trees?

b) [5 pts]What are the properties of a B-tree?
(please use the following notation: x is a node, $n[x]$, the number of keys currently stored in node x, $key_i[x]$, ith key of node x, $leaf[x]$, leaf of node x, $c_i[x]$,ith children of node x)

c) [3 pts]How do you insert a key into a full node?

Q4) [10 pts] The Hiring Problem: Suppose that you need to hire a new office assistant. The employment agency will send you one candidate each day. You will interview that person and then decide to either hire that person or not. You must pay the employment agency a small fee to interview an applicant. To actually hire an applicant is more costly, however, since you must fire your current office assistant and pay a large hiring fee to the employment agency. You are committed to having, at all times, the best possible person for the job. Therefore, you decide that, after interviewing each applicant, if that applicant is better qualified than the current office assistant, you will fire the current office assistant and hire the new applicant. You are willing to pay the resulting price of this strategy, but you wish to estimate what that price will be.

It assumes that the candidates for the office assistant job are numbered 1 through *n*. Interviewing has a low cost, say *ci* , whereas hiring is expensive, costing *ch*.

a) [2 pts]What is the worst-case hiring cost?

b) [8 pts]What is the expected number of times of hiring a new office assistant?
Hint: Use indicator random variables

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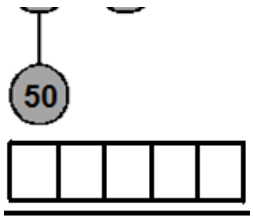
1 (20pt)	2 (10 pt)	3 (10 pt)	4 (10 pt)	5 (15 pt)	6 (15 pt)	7 (20 pt)	Total (100 pt)

Q5. [15 Points] Binomial and Fibonacci Heaps
Q5a) [5pts]
What are the main advantages of Binomial and Fibonacci Heaps compared to the ordinary Heap? How do Binomial and Fibonacci Heaps achieve that advantage?

State one difference between Binomial and Fibonacci Heaps
---in terms their complexity of operations.

---in terms of their data structure properties.

Q5c) [10pts] Consider the following min-heap ordered Binomial Heap. Show (graphically) how you would insert an item with key 35 into this Binomial Heap.



Q6)15pts [Amortized Analysis]
An array *A[0..k−1]* of bits (each array element is 0 or 1) stores a binary number $x = \sum_{i=0}^{k-1} A[i]2^i$. To add 1 (modulo 2^k) to *x*, we use the following procedure:

INCREMENT(A,k)
1. i ← 0
2. while i<k and A[i]=1 **do**
3. A[i] ← 0
4. i ← i+1
5. if i<k then
6. A[i] ← 1

Given a number x, define the potential Φ(x) of x to be the number of 1's in the binary representation of x. For example, Φ(19) = 3, because 19 = 100112. Use a potential-function argument to prove that the amortized cost of an increment is O(1), where the initial value in the counter is x = 0.

Hint:
If cost of operation i is c_i, Amortized cost $\hat{c}_i$ with respect to Φ is: $\hat{c}_i = c_i + \Phi(D_i) - \Phi(D_{i-1})$

Q7) [20points] Hashing
Insert numbers B={6, 1, 13, 18} into an empty hash table of size 6. Clearly indicate your hash functions. Show the hash table for all insertions.
Q7a) [5 points] Use open addressing and linear probing. What is the number of collisions?

Q7b) [7 points] Use open addressing and double hashing. What is the number of collisions?

Q7b) [8pts] Suppose we use a hash function ***h()*** to hash *n* distinct keys into an array *T* of length *m=c*n,* where *c>2* is an integer constant.
Assuming simple uniform hashing (i.e. the probability of element *i* hashing to any slot *k* is 1/m), show that the expected number of colliding pairs of elements is ***θ(n/c).***