Introduction to Transistor Amplifiers: Concept & Biasing

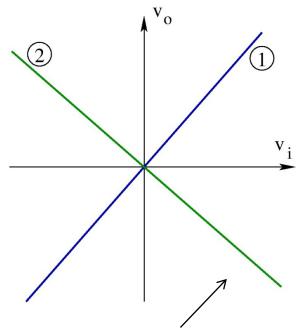
Lecture notes: Sec. 5

Sedra & Smith (6th Ed): Sec. 5.4-5.4.6 & 6.3-6.4

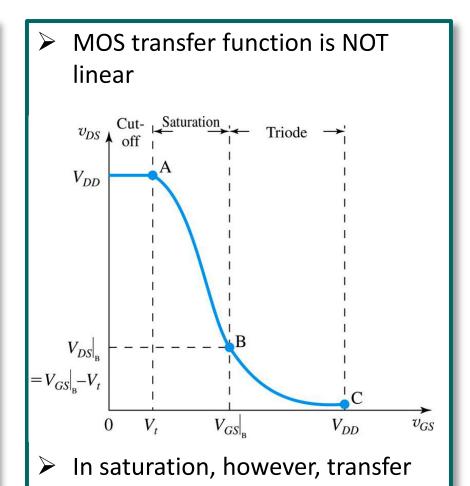
Sedra & Smith (5th Ed): Sec. 4.4-4.4.6 & 5.3-5.4

Foundation of Transistor Amplifiers (1)

A voltage amplifier requires $v_o/v_i = {\rm const.}$ (2 examples are shown below)



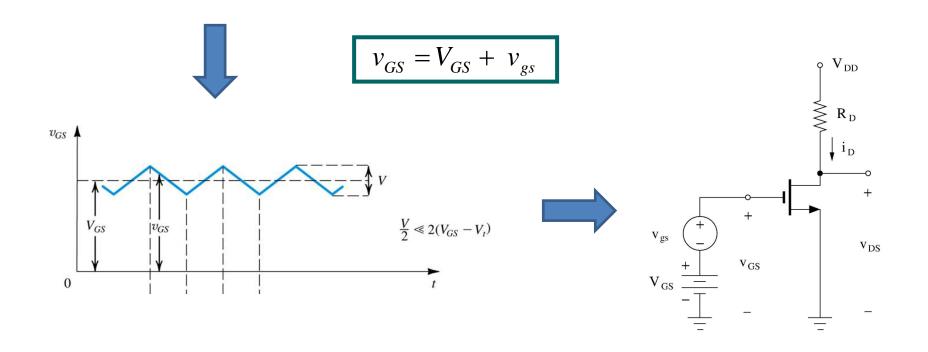
 $\sim v_o/v_i$ can be negative (minus sign represents a 180° phase shift)



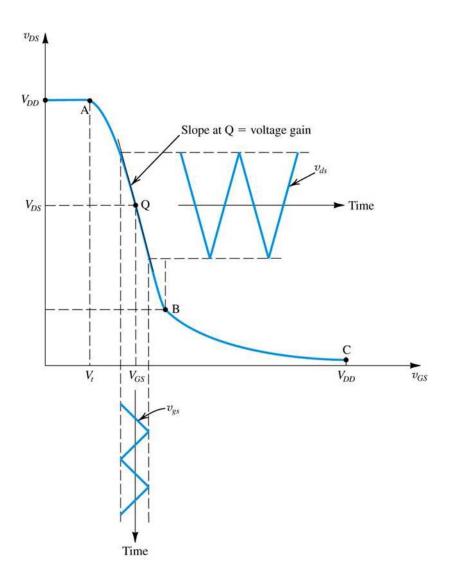
function looks linear (but shifted)

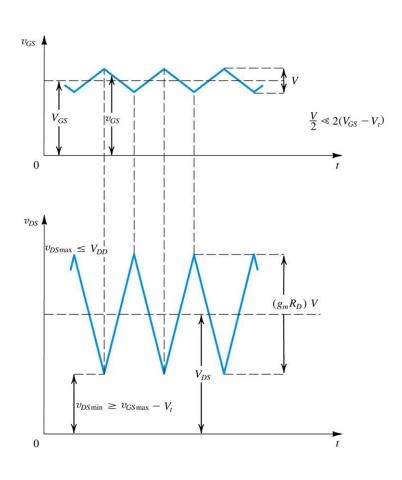
Foundation of Transistor Amplifiers (2)

Let us consider the response if NMOS remain in saturation at all times: $\circ v_{GS}$ should be a combination of a constant value (V_{GS}) and a signal (v_{gs}).

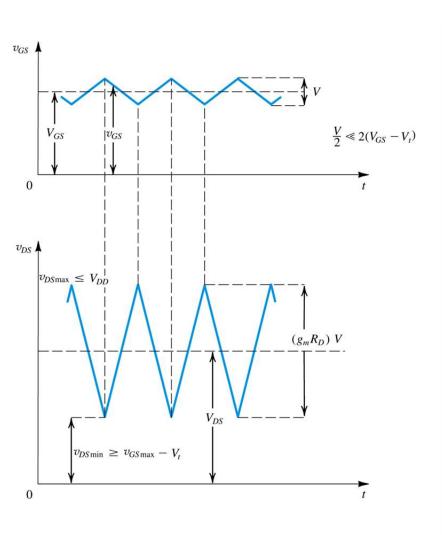


The response to a combination of v_{GS} = V_{GS} + v_{gs} can be found from the transfer function



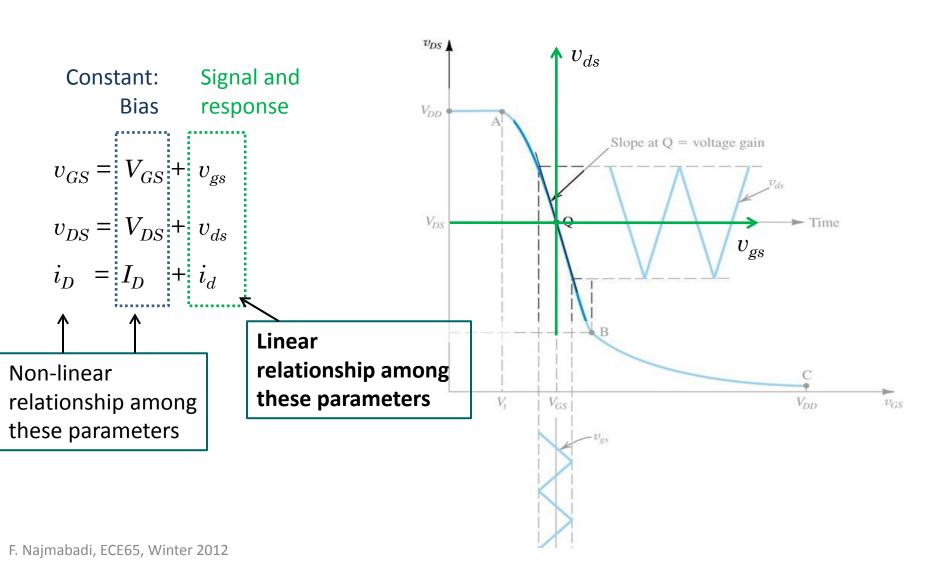


Response to the signal appears to be linear!

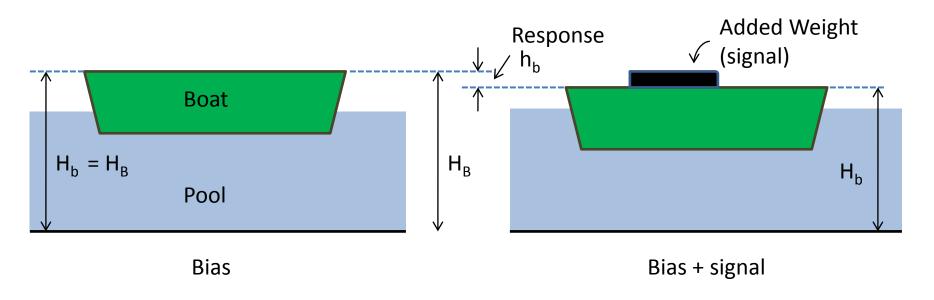


- Response ($v_o = v_{DS}$) is also made of a constant part (V_{DS}) and a signal response part (v_{ds}).
- ightharpoonup Constant part of the response, V_{DS} , is ONLY related to V_{GS} , the constant part of the input (Q point on the transfer function of previous slide).
 - o i.e., if $v_{gs} = 0$, then $v_{ds} = 0$
- The shape of the time varying portion of the response (v_{ds}) is similar to v_{gs} .
 - o i.e., v_{ds} is proportional to the input signal, v_{gs}
- While overall response is non-linear, response to the signal appears to be linear!

Although the overall response is non-linear, the transfer function for the signal is linear!

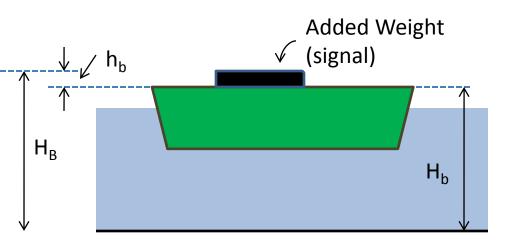


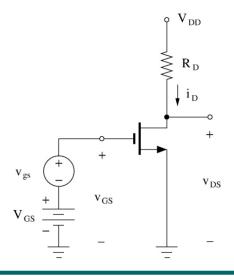
An Analogy: How much water is under the keel of a boat?



- Total Height, H_b = Bias (H_B) + response to the added weight (h_b)
- Complicated correlation between the total height, H_b, and the weight of the boat.
- Simple correlation between h_b and the added weight

An Analogy (2)





- Bias: zero added weight & H_B
- Bias + Signal: total weight & H_b
- Signal & response: added weight & h_b

- ightharpoonup Bias: V_{GS} , V_{DS} , I_{D}
- \blacktriangleright Bias + Signal: v_{GS} , v_{DS} , i_D
- ightharpoonup Signal & response: v_{gs} , v_{ds} , i_d
- ightharpoonup Non-linear correlations among Bias + Signal: v_{GS} , v_{DS} , i_D
- > Simple (and linear) correlation between signal and response to the signal: v_{gs} , v_{ds} , i_d

Important Observations!

- > Signal: We want the response of the circuit to this input.
- ➢ Bias: State of the system when there is no signal (current and voltages in all elements).
 - Bias is constant in time (may vary very slowly compared to the signal)
 - o Purpose of the bias is to ensure that MOS is in saturation at all times.
- ➤ **Response** of the circuit (and elements within) to the signal is different than the response of the circuit and its elements to Bias (or to Bias + signal):
 - o Different transfer function for the circuit
 - o <u>Different iv characteristics for elements</u>, i.e. relationships among v_{gs} , v_{ds} , i_d are different than relationships among v_{GS} , v_{DS} , i_D .

Above observations & conclusions equally apply to a BJT in the active mode!

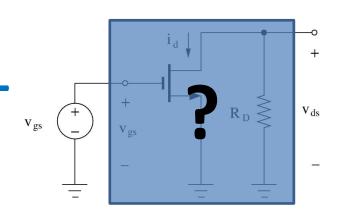
Transistor Amplifier Development

Bias



V_{DD} V_{DD} R_{D} R_{D} I_{D} V_{DS} V_{GS} V_{GS} V_{DS} V_{DS}

Signal only = (Bias + Signal) - Bias



$$MOS: v_{GS}, v_{DS}, i_D,$$

$$(v_{GS} = V_{GS} + v_{gs}, \dots)$$

$$R_D$$
: $v_R = V_R + v_r$
$$i_R = I_R + i_r$$

 $MOS: V_{GS}, V_{DS}, I_D,$

$$R_D: V_R, I_R$$

••••

 $MOS: v_{gs}, v_{ds}, i_d,$

 $R_D: v_r, i_r$

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Issues in developing a transistor amplifier:

- **1. Establish a Bias point** (bias is the state of the system when there is no signal).
 - Stable and robust bias point should be resilient to variations in eta, $\mu_n C_{ox} (W/L), V_t$, ... due to temperature and/or manufacturing variability.
- 2. Find the iv characteristics of the elements for the signal (which can be different than their characteristics equation for bias).
 - This will lead to different circuit configurations for bias versus signal:
 Signal circuit
- 3. Compute circuit response to the signal & develop transistor amplifier circuits

Transistor Biasing (Bias is the state of the circuit when there is no signal)

- 1. **Purpose:** BJT should be in active (or MOS should in saturation) at all times.
 - Bias point impacts the small-signal parameters.
 - Bias point impacts how large a signal can be amplified
- 2. Bias point should be resilient to variations in β , $\mu_n C_{ox}$ (W/L), V_t , ... due to temperature and/or manufacturing variability.

BJT biasing with Base Voltage (Fixed Bias)

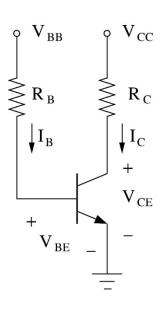
BE-KVL:
$$V_{BB} = I_B R_B + V_{BE}$$

$$I_B = \frac{V_{BB} - V_{D0}}{R_B}$$

$$I_C = \beta I_B = \beta \frac{V_{BB} - V_{D0}}{R_B}$$

CE-KVL:
$$V_{CC} = I_C R_C + V_{CE}$$

$$V_{CE} = V_{CC} - \frac{\beta R_C}{R_B} (V_{BB} - V_{D0})$$



* Typically $V_{BB} = V_{CC}$ in order to reduce the need for additional reference voltages.

Exercise 1: Find R_C and R_B such that BJT would be in active with I_C = 25 mA, V_{CE} = 5 V. (V_{CC} = 15 V, Si BJT with β = 100 and V_A = ∞).

BJT is in Active since $I_C > 0$ and $V_{CE} = 5 \ge V_{D0} = 0.7 \text{ V}$

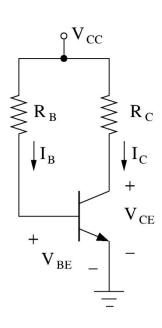
$$I_B = I_C / \beta = 0.25 \text{ mA}$$

BE-KVL:
$$15 = I_B R_B + V_{BE} = 0.25 \times 10^{-3} R_B + 0.7$$

 $R_B = 57.2 \text{ k}$

CE – KVL:
$$15 = I_C R_C + V_{CE} = 25 \times 10^{-3} R_C + 5$$

 $R_C = 400 \Omega$



Exercise 2: Consider the circuit designed in Exercise 1 (R_{C} = 400 , R_{B} = 57.2 k, V_{CC} = 15 V). Find the operating point of BJT if β = 200.

Assume BJT is in Active:

$$V_{BE} = 0.7 \text{ V}, I_C > 0 \text{ and } V_{CE} \ge 0.7 \text{ V}$$

BE-KVL:
$$15 = I_B R_B + V_{BE} = 57.2 \times 10^3 I_B + 0.7$$

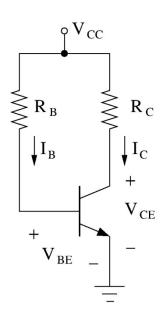
 $I_B = 0.25 \text{ mA}$

$$I_C = \beta I_B = 50 \text{ mA}$$

CE – KVL:
$$15 = I_C R_C + V_{CE} = 50 \times 10^{-3} \times 400 + V_{CE}$$

 $V_{CE} = -5 \text{ V}$

BJT in saturation!

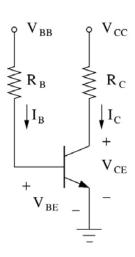


Note, compared to Exercise 1:

- $\succ I_B$ is the same.
- \succ I_{C}^{-} increased. \succ V_{CE} decreased.

Why biasing with base voltage (fixed bias) does not work?

- \triangleright Changes in BJT β changes the bias point drastically.
 - o BJT can end up in saturation or in cut-off easily.
- ightharpoonup In fixed bias, I_B is set through $I_B = \frac{V_{BB} V_{D0}}{R_{\rm B}}$
- > BJT β then sets I_C = βI_B (I_C changes with β). \circ CE circuit then sets V_{CE} .



- \blacktriangleright But, requirements for BJT in active are on I_C and V_{CE} and NOT on I_B o $I_C\!>\!0$, $V_{CE}\!>\!V_{D0}$
- \succ To make bias point independent of changes in eta, the bias circuit should "set" I_C and NOT I_B !

Biasing with Emitter Degeneration

Requires a resistor in the emitter circuit!

BE-KVL:
$$V_{BB} = I_B R_B + V_{BE} + I_E R_E$$

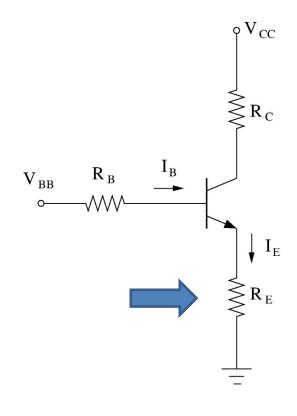
$$V_{BB} - V_{D0} = I_E \left[\frac{R_B}{\beta + 1} + R_E \right]$$

If:
$$R_B << (\beta+1)R_E$$

$$V_{BB} - V_{D0} \approx I_E R_E$$

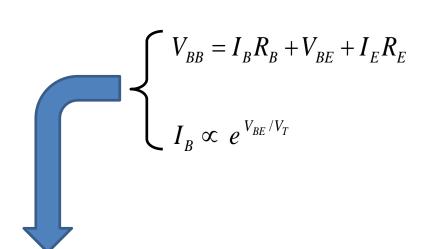
$$I_C \approx I_E \approx \frac{V_{BB} - V_{D0}}{R_E}$$
 Independent of β !

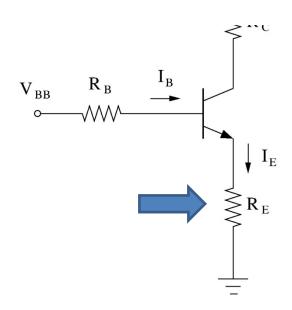




Condition of $\,R_{\!\scriptscriptstyle B} << (\beta + 1) R_{\!\scriptscriptstyle E}\,\,$ means that the voltage drop across $R_{\!\scriptscriptstyle B}$ is small and the bias voltage $V_{BB} - V_{D0}$ appears across R_E , setting $I_E pprox I_C$.

Emitter resistor provides negative feedback!





Negative Feedback:

o If
$$I_C \approx I_E \uparrow$$
 (because $\beta \uparrow$), $\xrightarrow{\text{BE-KVL}} V_{BE} \downarrow \xrightarrow{\text{BE junction}} I_B \downarrow \xrightarrow{\beta} I_C \approx I_E \downarrow$
o If $I_C \approx I_E \downarrow$ (because $\beta \downarrow$), $\xrightarrow{\text{BE-KVL}} V_{BE} \uparrow \xrightarrow{\text{BE junction}} I_B \uparrow \xrightarrow{\beta} I_C \approx I_E \uparrow$

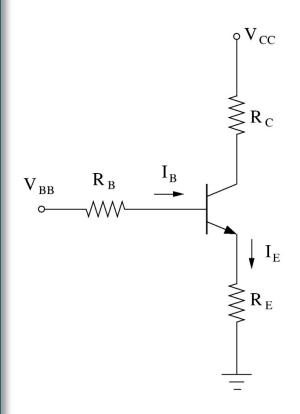
Requirements for Biasing with Emitter Degeneration

- Requires a resistor in the emitter circuit.

$$V_{BB} - V_{BE} = I_B R_B + I_E R_E$$

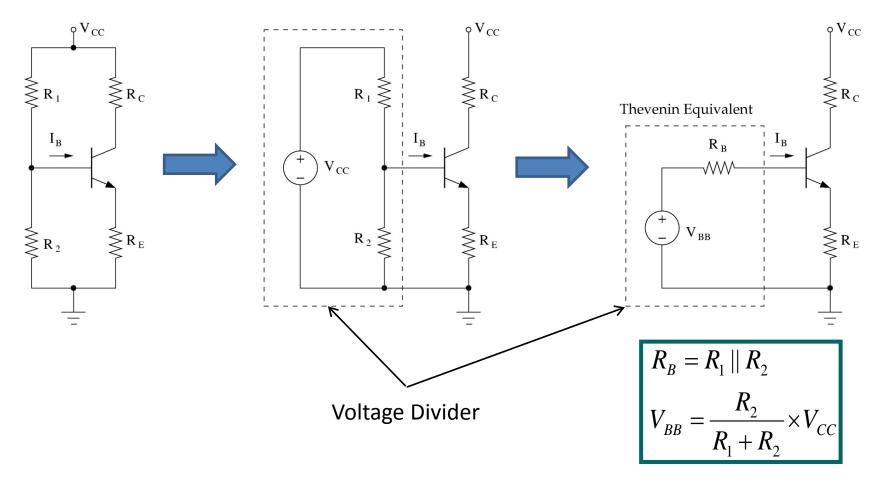
- 1. $I_B R_B \ll I_E R_E \implies R_B \ll (\beta + 1) R_E$
 - We need to set $R_B << (\beta_{\min} + 1)R_E$ to ensure that this condition is always satisfied!
- 2. $V_{BE} pprox V_{D0}$. In reality, $V_{BE} = V_{D0} \pm \Delta V_{BE}$ with $\Delta V_{BE} pprox 0.1~{
 m V}$
 - o We need to set $I_E R_E >> 0.1 \,\mathrm{V}$ or

$$I_E R_E \ge 1 \text{ V}$$



Emitter Degeneration Bias with a voltage divider

Real Circuit



Exercise 3: Find the bias point of the BJT (Si BJT with $\beta=200$ and $V_A=\infty$).

$$R_B = R_1 \parallel R_2 = 5.9 \text{ k} \parallel 34 \text{ k} = 5.03 \text{ k}$$

$$V_{BB} = \frac{R_2}{R_1 + R_2} \times V_{CC} = \frac{5.9 \text{ k}}{5.9 \text{ k} + 34 \text{ k}} \times 15 = 2.22 \text{ V}$$

Assume BJT is in Active:

$$V_{BE} = 0.7 \text{ V}, I_C > 0 \text{ and } V_{CE} \ge 0.7 \text{ V}$$

BE-KVL:
$$2.22 = I_B R_B + V_{BE} + I_E R_E$$

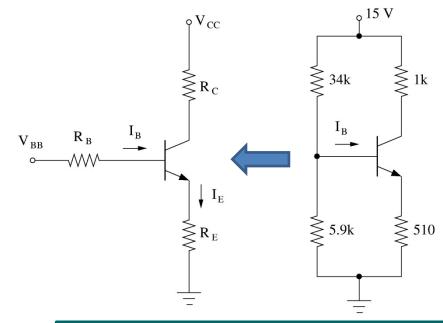
 $2.22 = 5.03 \times 10^3 I_E / (\beta + 1) + 0.7 + 510 I_E$
 $I_E = 2.84 \text{ mA}$

$$I_C = I_E \times \beta / (\beta + 1) = 2.82 \text{ mA}$$

 $I_B = I_E / (\beta + 1) = 14.1 \mu\text{A}$

CE-KVL:
$$15 = I_C R_C + V_{CE} + I_E R_E$$

 $15 = 2.82 \times 10^{-3} \times 10^3 + V_{CE} + 2.84 \times 10^{-3} \times 510$
 $V_{CE} = 10.7 \text{ V} > V_{D0} = 0.7 \text{ V}$



Notes:

- 1. We need to solve the complete BE-KVL as we do not know if $R_B << (\beta+1)R_E$
- 2. $\beta >> 1$ is a good approximation that reduces the amount of work. Answers using $\beta >> 1$ approximation:

$$I_C \approx I_E \approx 2.84$$
 mA, $I_B = 14.2$ μ A
 $V_{CE} = 10.7$ V

Exercise 4: Design a BJT bias circuit (emitter degeneration with voltage divider) such that I_C = 2.5 mA and V_{CE} = 7.5 V. (V_{CC} = 15 V Si BJT with β ranging from 50 to 200 and V_A = ∞).

Step 1: Find $R_{\it C}$ and $R_{\it E}$

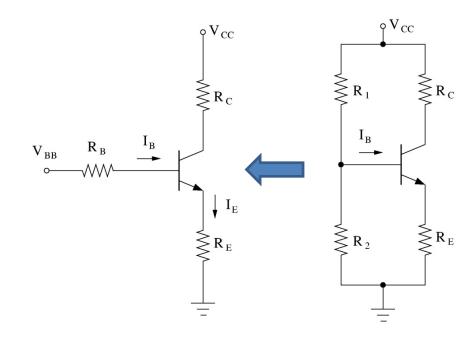
CE - KVL:
$$15 = I_C R_C + V_{CE} + I_E R_E$$

 $15 = 2.5 \times 10^{-3} \times (R_C + R_E) + 7.5$
 $R_C + R_E = 3.0 \text{ k}$

Free to choose individual values $R_E \& R_C$ (we will see later that amplifier parameters sets the individual values)

Choose:
$$R_E = 1.0 \text{ k}$$

 $R_C = 3.0 \text{ k} - R_E = 2.0 \text{ k}$



Circuit Prototype

Check:
$$I_E R_E \ge 1 \text{ V}$$

 $I_E R_E = 2.5 \times 10^{-3} \times 10^3 = 2.5 \ge 1 \text{ V}$

Exercise 4 (Cont'd): Design a BJT bias circuit (emitter degeneration with voltage divider) such that I_C = 25 mA and V_{CE} = 5 V. (V_{CC} = 15 V Si BJT with β ranging from 50 to 200 and V_A = ∞).

Step 2: Find R_{B} and V_{BB}

$$R_{B} << (\beta_{\min} + 1)R_{E} \quad \rightarrow \quad R_{B} \leq 0.1 \ (\beta_{\min} + 1)R_{E} = 5.1 \ \mathrm{k} \qquad \qquad \uparrow \qquad \qquad \uparrow$$
 Using relative error, $\epsilon = 10\%$ Use largest R_{B} (Will see later why)

$$V_{BB} = I_{B}R_{B} + V_{BE} + I_{E}R_{E}$$

$$V_{BB} \approx V_{D0} + I_{C}R_{E} = 0.7 + 2.5 \times 10^{-3} \times 10^{3} \rightarrow V_{BB} = 3.20 \text{ V}$$

Step 3: Find $oldsymbol{R}_1$ and $oldsymbol{R}_2$

$$R_B = R_1 || R_2 = \frac{R_1 R_2}{R_1 + R_2} = 5.10 \text{ k}$$

$$\frac{V_{BB}}{V_{CC}} = \frac{R_2}{R_1 + R_2} = \frac{3.20}{15} = 0.213$$
 $R_1 = \frac{5.10 \text{ k}}{0.213} = 23.9 \text{ k}$

$$R_2 = 6.4 \text{ k}$$

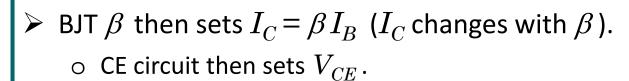
Step 4: Find commercial R values:

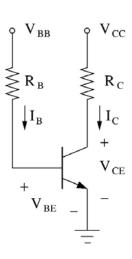
$$R_C = 2 \text{ k}$$

 $R_E = 1 \text{ k}$
 $R_1 = 24 \text{ k}$
 $R_2 = 6.4 \text{ k}$

Why biasing with base voltage (fixed bias) does not work?

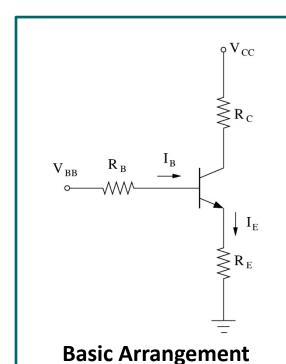
- \triangleright Changes in BJT β changes the bias point drastically.
 - o BJT can end up in saturation or in cut-off easily.
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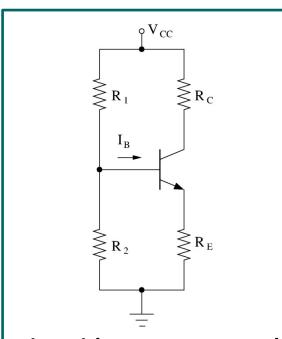


- \blacktriangleright But, requirements for BJT in active are on I_C and V_{CE} and NOT on I_B o $I_C\!>\!0$, $V_{CE}\!>\!V_{D0}$
- \succ To make bias point independent of changes in eta, the bias circuit should "set" I_C and NOT I_B !

Emitter-degeneration bias circuits

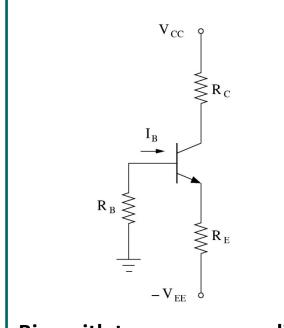


 $V_{RR} = I_R R_R + V_{RE} + I_E R_E$



Bias with <u>one</u> power supply (voltage divider)

$$V_{BB} = I_B R_B + V_{BE} + I_E R_E$$



Bias with two power supplies

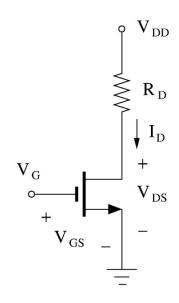
$$V_{EE} = I_B R_B + V_{BE} + I_E R_E$$

$$0 = I_B R_B + V_{BE} + I_E R_E - V_{EE}$$

MOS bias with Gate Voltage

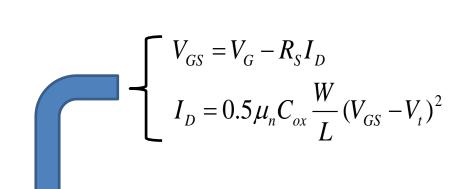
$$I_{D} = 0.5 \mu_{n} C_{ox} \frac{W}{L} (V_{GS} - V_{t})^{2}$$

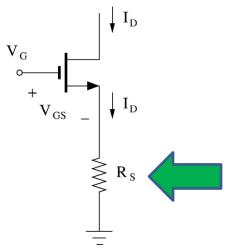
$$V_{DS} = V_{DD} - I_{D} R_{D}$$



- This method is NOT desirable as $\mu_n C_{ox} (W/L)$ and V_t are not "well-defined." Bias point (i.e., I_D and V_{DS}) can change drastically due to temperature and/or manufacturing variability.
 - \circ See S&S Exercise 5.33 (S&S 5th Ed: Exercise 4.19): Changing V_t from 1 to 1.5 V leads to a 75% change in I_D .

MOS bias with Source Degeneration (Resistor R_s provides negative feedback!)





Negative Feedback:

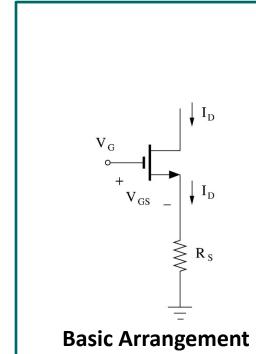
$$\circ \text{ If } I_D \uparrow \text{ (because } \mu_n C_{ox} \left(W/L \right) \uparrow \text{ or } V_t \downarrow \text{)} \xrightarrow{\text{GS KVL}} V_{GS} \downarrow \xrightarrow{I_D \text{ Eq.}} I_D \downarrow$$

$$\circ \text{ If } I_D \downarrow \text{ (because } \mu_n C_{ox} \left(W/L \right) \downarrow \text{ or } V_t \uparrow \text{)} \xrightarrow{\text{GS KVL}} V_{GS} \uparrow \xrightarrow{I_D \text{ Eq.}} I_D \uparrow$$

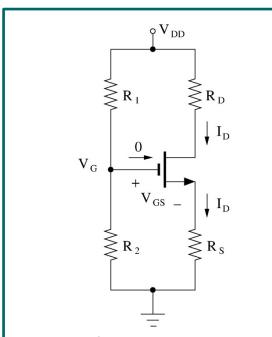
Feedback is most effective if
$$R_S I_D >> V_{GS}$$

$$V_{GS} - V_G + R_S I_D = 0 \implies I_D \approx V_G / R_S$$

Source-degeneration bias circuits

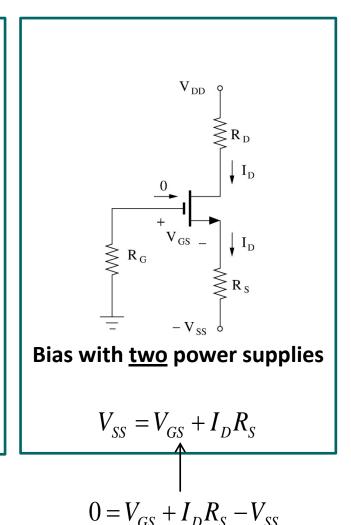


$$V_G = V_{GS} + I_D R_S$$



Bias with <u>one</u> power supply (voltage divider)

$$V_G = V_{GS} + I_D R_S$$



Exercise 5: Find the bias point for $V_t = 1$ V and $\mu_n C_{ox} (W/L) = 1.0$ mA/V² (Ignore channel-width modulation).

Voltage divider (
$$I_G = 0$$
)

$$V_G = (7)/(7+8) \times 15 = 7 \text{ V}$$

$$I_{D} = 0.5 \mu_{n} C_{ox} \frac{W}{L} V_{OV}^{2}$$

$$GS - KVL : V_{G} = 7 = V_{GS} + R_{S} I_{D}$$

$$V_{OV} + V_{t} + R_{S} I_{D} = 7$$

$$V_{OV} + 1 + 10^{4} \times (0.5 \times 10^{-3} V_{OV}^{2}) = 7$$

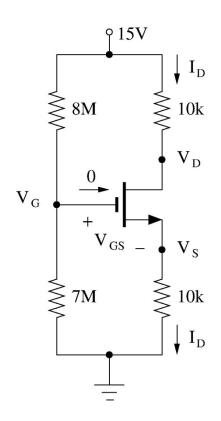
$$5V_{OV}^{2} + V_{OV} - 6 = 0 \rightarrow V_{OV} = 1 \text{ V}$$

$$V_{GS} = V_{OV} + 1 = 2 \text{ V}$$

 $V_S = V_G - V_{GS} = 7 - 2 = 5 \text{ V}$
 $I_D = V_S / R_S = 0.5 \text{ mA}$

DS-KVL:
$$15 = R_D I_D + V_D$$

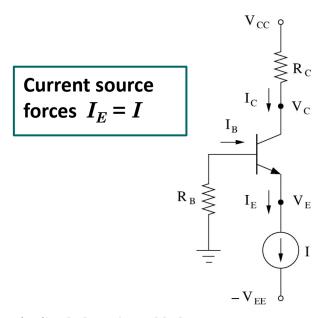
 $V_D = 15 - R_D I_D = 10 \text{ V}$
 $V_{DS} = V_D - V_S = 5 \text{ V}$

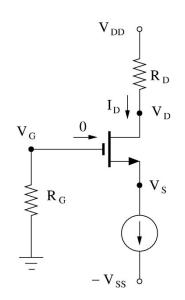


Exercise (impact of R_{\rm S}): Prove that if $V_t=1.5$ V (50% change), $I_D=0.455{\rm mA}$ (9% change)

Biasing in ICs

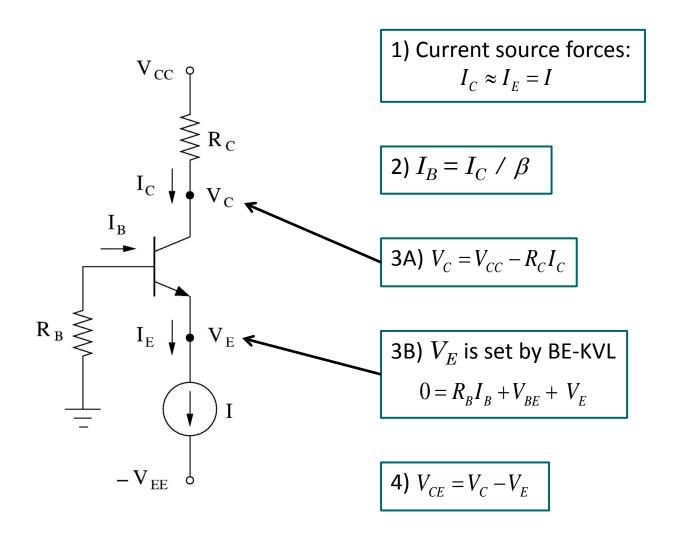
- ➤ Resistors take too much space on the chip. So, biasing with emitter or source degeneration are NOT implemented in ICs.
- \blacktriangleright Recall that the goal of a good bias is to ensure that I_C and V_{CE} (or I_D and V_{DS} for MOS) do not change. One can force I_C (or I_D for MOS) to be constant using a current source.



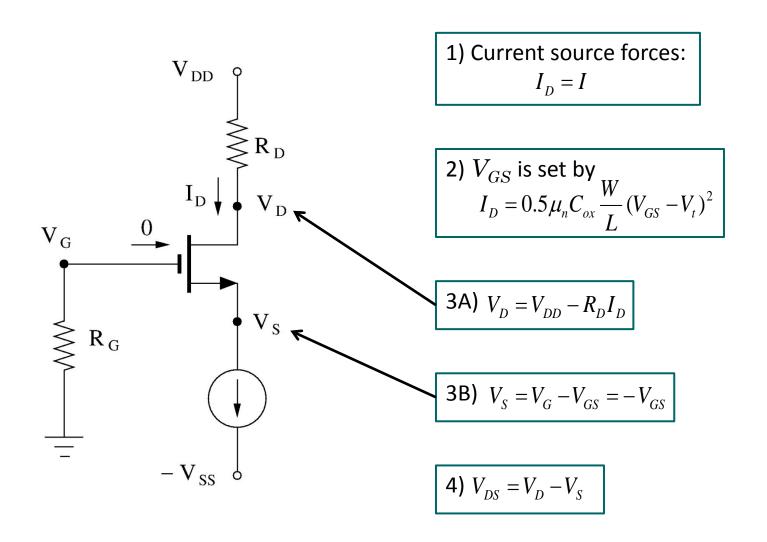


Current source forces $I_D = I$

BJT response to a current source



MOS response to a current source



Current Mirrors (or Current Steering circuits) are used as current sources for biasing ICs

Q_{ref} is <u>always</u> in active since

$$i_{C,ref} > 0$$

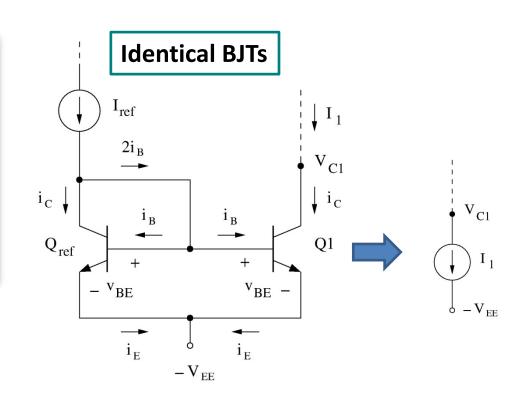
$$V_{CE,ref} = V_{BE,ref} = V_{D0}$$

- \blacktriangleright Identical BJTs and $v_{BE,ref} = v_{BE1}$
 - o BJTs will have the same i_B and the same i_C (ignoring Early effect)

KCL:
$$I_{ref} = i_{C,ref} + 2i_B = i_C + 2\frac{i_C}{\beta}$$

$$I_1 = i_C = \frac{I_{ref}}{1 + 2/\beta} \approx I_{ref}$$

ightharpoonup Since $I_1 = const.$ regardless of V_{C1} , this is a current source!



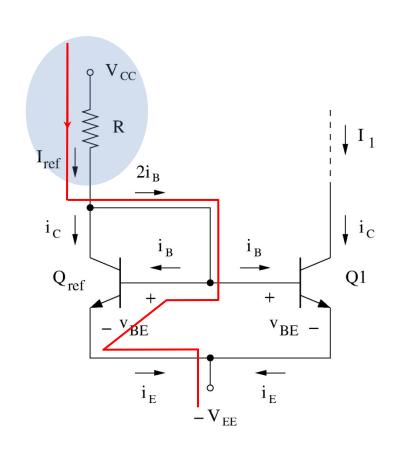
➤ For the current mirror to work, Q1 should be in active:

$$V_{CE1} = V_{C1} + V_{EE} \ge V_{D0}$$

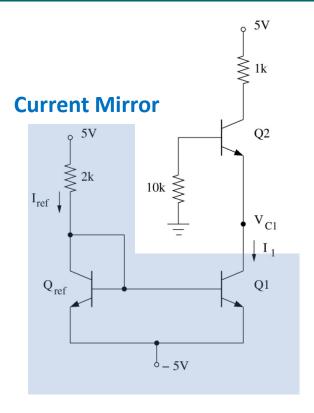
An implementation of a BJT Current Mirror

BE-KVL (Q_{ref}):
$$V_{CC} = R I_{ref} + V_{BE} - V_{EE}$$

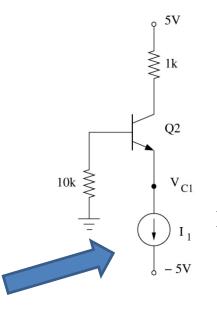
$$I_1 \approx I_{ref} = \frac{V_{CC} + V_{EE} - V_{D0}}{R}$$



Exercise 6: Find the bias point of Q2 (Si BJT with $\beta=100$ and $V_A=\infty$).



$$5 = 2 \times 10^{3} I_{ref} + V_{BE} - 5$$
$$I_{ref} = 4.65 \text{ mA}$$
$$I_{1} \approx I_{ref} = 4.65 \text{ mA}$$



Assume Q2 in active:

$$I_{C2} \approx I_{E2} = I_1 \approx 4.65 \text{ mA}$$

 $I_{B2} = I_{C2} / \beta \approx 46.5 \mu\text{A}$

BE2-KVL:
$$0 = 10 \times 10^{3} I_{B2} + V_{BE2} + V_{E2}$$

 $0 = 10 \times 10^{3} \times 46.5 \times 10^{-6} + 0.7 + V_{E2}$
 $V_{C1} = V_{E2} = -1.165 \text{ V}$

CE2-KVL:
$$5 = 10^3 I_{C2} + V_{CE2} + V_{E2}$$

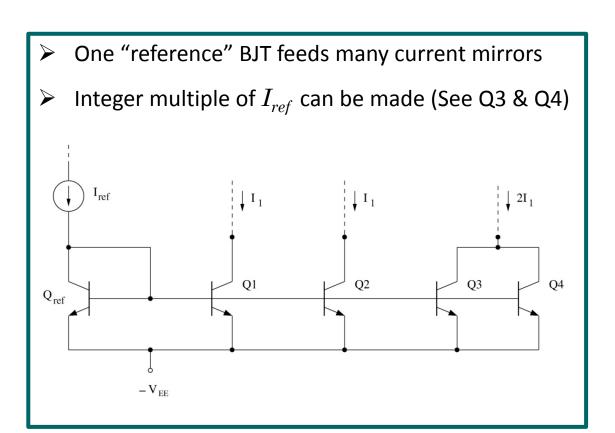
 $V_{CE2} = 5 - 10^3 \times 4.65 \times 10^{-3} - 1.165$
 $V_{CE2} = 1.56 \text{ V}$
 $V_{CE2} = 1.56 > V_{D0} = 0.7 \text{ V}$ Q2 in active!

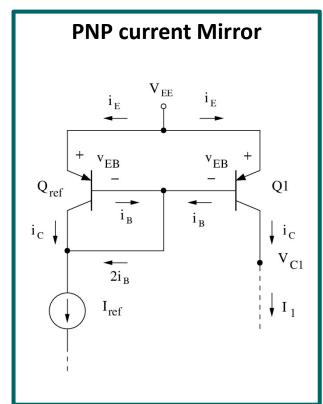
Check Q1 in active:

$$V_{CE1} = V_{C1} - (-5) = -1.165 + 5 = 3.835 \text{ V}$$

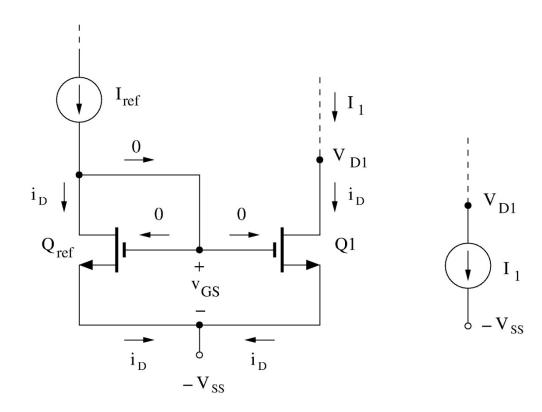
 $V_{CE1} = 3.835 > V_{D0} = 0.7 \text{ V}$

Examples of BJT current mirrors





MOS Current-Steering Circuit



An implementation of a MOS current steering circuit

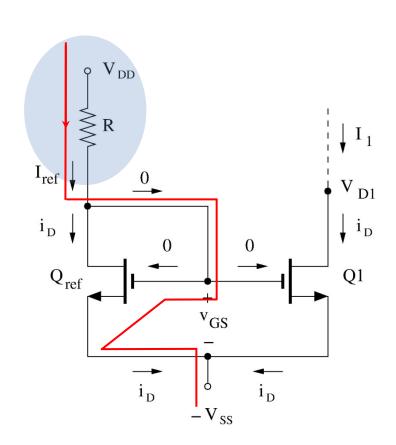
$$I_{ref} = i_D = 0.5 \mu_n C_{ox} (W/L)_{ref} V_{OV}^2$$

GS-KVL (Q_{ref}): $V_{DD} = Ri_D + v_{GS} - V_{SS}$

$$Ri_{D} + V_{OV} - V_{SS} - V_{DD} + V_{t} = 0$$

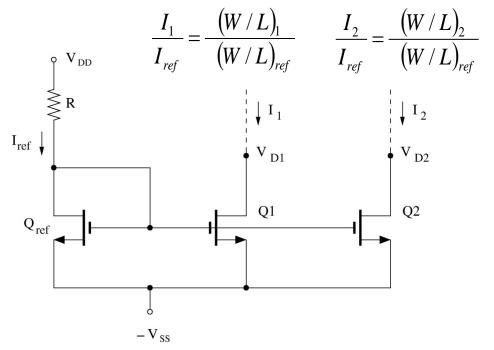
$$[0.5\mu_{n}C_{ox}(W/L)_{ref} R]V_{OV}^{2} + V_{OV} + [-V_{SS} - V_{DD} + V_{t}] = 0$$

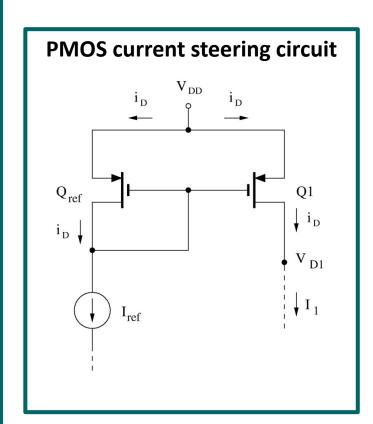
The above quadratic equation gives V_{OV} . I_1 is then found from the MOS i_D equation.



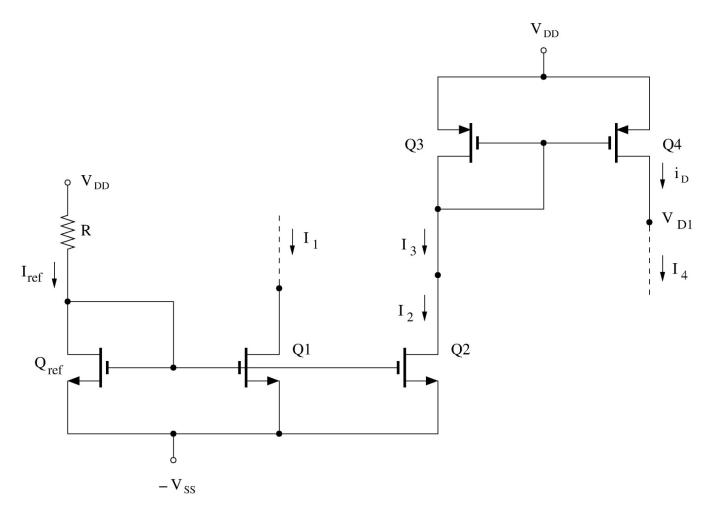
Examples of MOS current steering circuits

- One "reference" MOS feeds many current steering circuits.
- Any value of I_{ref} can be made (thus, currentsteering circuit instead of current-mirror)





An implementation of current steering circuit to bias several transistors in an IC



Exercise: Compute I_4/I_{ref}