BLG 335E – Analysis of Algorithms I
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Research Lab 2

Exercise 1

a) How many people must there be in a room before the probability that someone has the same birthday as you do is at least 1/2?

Solution 1.a

Probability that someone in the room has the same birthday as me, denoted by

P (B) is 1− probability
that no one in the room has the same birthday as me.

$$P(B) = 1 - \left(\frac{364}{365}\right)^n$$

Solution 1.a

We wish $P(B) \geq 1/2$

$$1-\left(\frac{364}{365}\right)^{n} \geq 1/2$$

$$\log(1/2) \geq \log\left(\frac{364}{365}\right)^{n}$$

$$-\log(2) \geq n \log\left(\frac{364}{365}\right)$$

$$\log(2) \leq n \log\left(\frac{364}{365}\right)$$

$$253 \leq n$$

Exercise 1

b) How many people must there be in a room before the probability that at least two people have a birthday on July 4 is greater than 1/2?

Solution 1.b

Probability that at least two people have a birthday on July 4, denoted by

P(J) is 1 **–** (probability that **exactly one person** in the room has a birthday on July 4) **-** (probability that **no one** in the room has a birthday on July 4).

$$P(J) = 1 - \binom{n}{1}\left(\frac{1}{365}\right)\left(\frac{364}{365}\right)^{n-1} - \binom{n}{0}\left(\frac{1}{365}\right)\left(\frac{364}{365}\right)^{n}$$

$P(J) \geq 1/2$

$n \geq 613$

Exercise 2

- Illustrate the operation of **Counting-Sort** on the array below.

  A = [6, 0, 2, 0, 1, 3, 4, 6, 1, 3, 2]

1. Max{A[i]}=6
2.

0	0	0	0	0	0	0

3.

2	2	2	2	1	0	2

4.

2	4	6	8	9	9	11

Exercise 2

6	0	2	0	1	3	4	6	1	3	2

5.

1	2	3	4	5	6	7	8	9	10	11
					2					
					2		3			
			1		2		3			
			1		2		3			6
			1		2		3	4		6
			1		2	3	3	4		6
		1	1		2	3	3	4		6
	0	1	1		2	3	3	4		6
	0	1	1	2	2	3	3	4		6
0	0	1	1	2	2	3	3	4		6
0	**0**	**1**	**1**	**2**	**2**	**3**	**3**	**4**	**6**	**6**

0	1	2	3	4	5	6
2	4	6	8	9	9	11
2	4	5	8	9	9	11
2	4	5	7	9	9	11
2	3	5	7	9	9	11
2	3	5	7	9	9	10
2	3	5	7	8	9	10
2	3	5	6	8	9	10
2	2	5	6	8	9	10
1	2	5	6	8	9	10
1	2	4	6	8	9	10
0	2	4	6	8	9	10
0	2	4	6	8	9	9

Exercise 3

- Illustrate the operation of **Radix-Sort** on the following list of English words:

  COW, DOG, SEA, RUG, ROW, MOB, BOX, TAB, BAR, EAR, TAR, DIG, BIG, TEA, NOW, FOX.

Exercise 3 – Solution

COW
DOG
SEA
RUG
ROW
MOB
BOX
TAB

BAR
EAR
TAR
DIG
BIG
TEA
NOW
FOX

Exercise 3 – Solution

SE**A**	BA**R**
TE**A**	EA**R**
MO**B**	TA**R**
TA**B**	FO**X**
DO**G**	BO**X**
RU**G**	CO**W**
DI**G**	RO**W**
BI**G**	NO**W**

Exercise 3 – Solution

T**A**B
B**A**R
E**A**R
T**A**R
S**E**A
T**E**A
D**I**G
B**I**G

M**O**B
D**O**G
F**O**X
B**O**X
C**O**W
R**O**W
N**O**W
R**U**G

Exercise 3 – Solution

BAR
BIG
BOX
COW
DIG
DOG
EAR
FOX
MOB
NOW
ROW
RUG
SEA
TAB
TAR
TEA

Exercise 4

Given input array A[0..9]. The array B[0..9] of sorted lists or buckets.

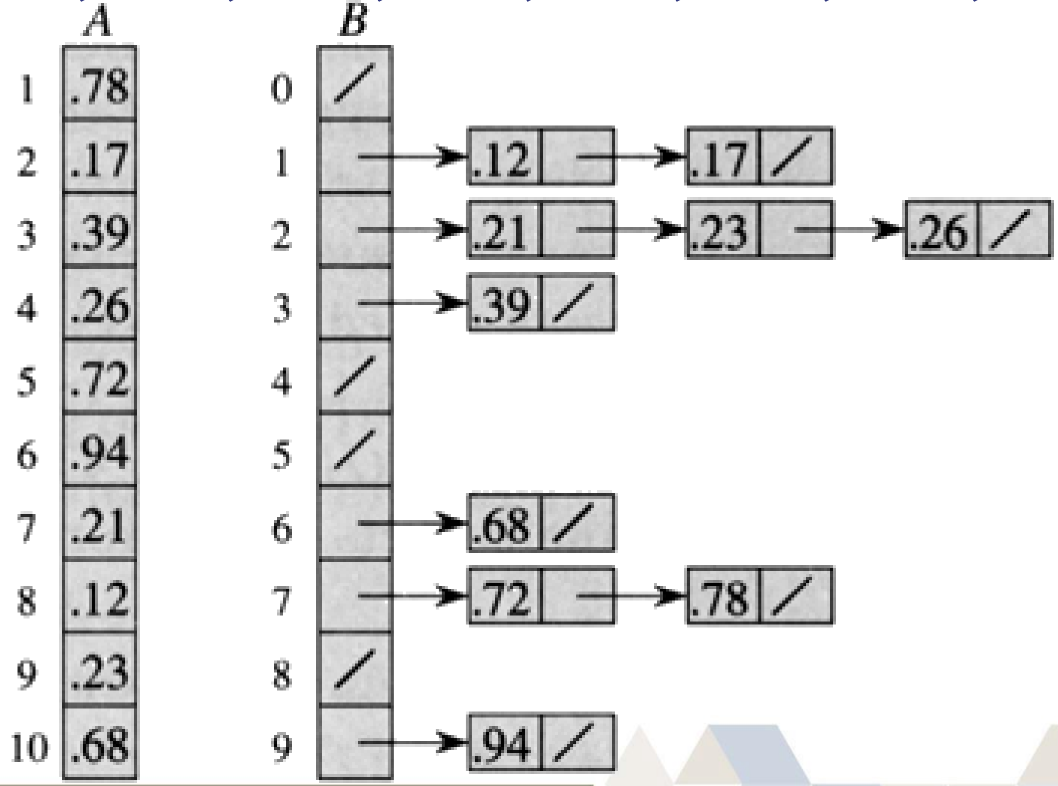
Bucket i holds values in the interval [i/10, (i +1)/10].

The sorted output consists of a concatenation in order of the lists first B[0] then B[1] then B[2] … and the last one is B[9].

Exercise 4

- Illustrate the operation of **Bucket-Sort** on the array below.

A = [.78, .17, .39, .26, .72, .94, .21, .12, .23, .68]



Exercise 5

Give tight asymptotic bounds for $T(n)$ in each of the following recurrences.

$$a.\ \ T(n) = 4T\left(\frac{n}{2}\right) + n^2\sqrt{n}$$

$$a = 4,\ \ b = 2,\ \ f(n) = n^2\sqrt{n} = \mathrm{n}^{5/2}$$

$$n^{log_b^a} = n^{log_2^4} = n^2$$

$$n^{5/2} = \Omega\left(n^{2+1/2}\right)$$

$$possibly\ case\ 3, let's\ check\ c$$

Master Method

$$T(n) = aT(n/b) + f(n)$$

1 $f(n) = O\left(n^{\log_b a-\varepsilon}\right) \Rightarrow T(n) = \Theta\left(n^{\log_b a}\right)$

2 $f(n) = \Theta\left(n^{\log_b a}\right) \Rightarrow T(n) = \Theta\left(n^{\log_b a}\log_2 n\right)$

3 $f(n) = \Omega\left(n^{\log_b a+\varepsilon}\right)$ and $af(n/b) \leq cf(n),$

$for \ \exists c \ \ c<1 \ \ and \ \ n>n_0$

$\Rightarrow T(n) = \Theta\left(f(n)\right)$

Exercise 5.a

$$a\,f\left(\frac{n}{b}\right) = 4\left(\frac{n}{2}\right)^2\sqrt{\frac{n}{2}} = \frac{n^{5/2}}{\sqrt{2}} \le cn^{\frac{5}{2}}$$

$$for\,\frac{1}{\sqrt{2}} \le c < 1$$

Case 3 applies $T(n) = \Theta(n^2\sqrt{n})$

Exercise 5 .b

$b.\ T(n) = 3T\left(\frac{n}{2}\right) + n \lg n$

$$a = 2 \ \ b = 3, \ \ f(n) = n \lg n$$

$$n^{log_b^a} = n^{\lg 3} \approx n^{1.585}$$

$$n\ lg\ n = O\left(n^{\lg 3 - \varepsilon}\right)$$

$$0 < \varepsilon \le 0.58 \qquad \text{case 1}$$

$$T(n) = \ \Theta(n^{\lg 3})$$