DISCRETE MATHEMATICS

2006 – 2007 Spring Term

RECITATION 1

16.02.2007

Text book – Excercise 2.2 – Questions: 4,14

4. Simplify following statement

$p \rightarrow q = \neg p \lor q$

$[[[(p \land q) \land r] \lor [(p \land r) \land \neg r]] \lor \neg q] \rightarrow s$

$= [[[(p \land q) \land r] \lor [p \land \mathbf{(r \land \neg r)}]] \lor \neg q] \rightarrow s$
$= [[[(p \land q) \land r] \lor [p \land \mathbf{F}]] \lor \neg q] \rightarrow s$
$= [[[(p \land q) \land r] \lor \mathbf{F}] \lor \neg q] \rightarrow s$
$= [[(p \land q) \land r] \lor \neg q] \rightarrow s$
$= [(p \land q \land r) \lor \neg q] \rightarrow s$
$= [(p \lor \neg q) \land (q \lor \neg q) \land (r \lor \neg q)] \rightarrow s$
$= [(p \lor \neg q) \land \mathbf{T} \land (r \lor \neg q)] \rightarrow s$
$= [(p \lor \neg q) \land (r \lor \neg q)] \rightarrow s$
$= \neg [(p \lor \neg q) \land (r \lor \neg q)] \lor s$
$= \neg [(p \land r) \lor \neg q] \lor s$
$= [\neg (p \land r) \land q] \lor s$

14. a) Show that $p \rightarrow [q \rightarrow (p \land q)]$ is a tautology

$= \neg p \lor [\neg q \lor (p \land q)]$
$= \neg p \lor [(\neg q \lor p) \land (\neg q \lor q)]$
$= \neg p \lor [(\neg q \lor p) \land \mathbf{T}]$
$= \neg p \lor (\neg q \lor p)$
$= (\neg p \lor p) \lor \neg q$
$= \mathbf{T} \lor \neg q$
$= \mathbf{T}$

b) Show that (pVq) → [q→q] is a tautology using the result in the previous question.

Rewrite the last result we obtained like p' → [q' → (p' Λ q')]

In our current statement, we can apply substition like

p' = (pVq)
q' = q

If statement (p' Λ q') equals q, than we can deduce that our initial statement is a tautology

p V (pΛq) ↔ p p Λ (pVq) ↔ p	Law of Absorption

(p' Λ q') = (pVq) Λ q = (pΛq) V (qΛq) = (pΛq) V q

Using law of absorption

(pΛq) V q = q

c) Test if (pVq) → [q→(p Λ q)] is tautology.

= ¬(pVq) V [¬qV(p Λ q)]
= (¬pΛ¬q) V [(¬qVp) Λ (¬qVq)]
= (¬pΛ¬q) V [(¬qVp) Λ **T**]
= (¬pΛ¬q) V (¬qVp)
= (¬pΛ¬q) V p V ¬q
= [(¬pVp) Λ (¬qVp)] V ¬q
= [**T** Λ (¬qVp)] V ¬q
= (¬qVp) V ¬q
= ¬qV ¬q V p
= ¬q V p   NOT TAUTOLOGY !

Text book – Excercise 2.3 – Questions: 4,8

4. Deduce results from the following verbal statements using Modus Ponens or Modus Tollens

Modus Ponens

$$\begin{array}{l} p \rightarrow q \\ p \\ \hline \therefore\ q \end{array} \qquad (p \rightarrow q) \wedge p \Rightarrow q$$

a) → If Tolga can't fix the computer Tahir will arrive and check the computer
→ Tolga wasn't able to fix the computer.

$\therefore$ Tahir arrives and checks the computer. **Modus Ponens**

b) → If Ayça has solved the problem she should've get the answer as 137.
→ Ayça hasn't reached 137.

$\therefore$ Ayça wasn't able to solve the problem. **Modus Tollens**

Modus Tollens

$$\begin{array}{l} p \rightarrow q \\ \neg q \\ \hline \therefore\ \neg p \end{array} \qquad (p \rightarrow q) \wedge \neg q \Rightarrow \neg p$$

8. Explain the following inferences.

$$\begin{array}{l} (\neg p \vee q) \rightarrow r \\ r \rightarrow (s \vee t) \\ \neg s \wedge \neg u \\ \neg u \rightarrow \neg t \\ \hline \therefore\ p \end{array}$$

Steps	Rules
1) $\neg s \wedge \neg u$	Given
2) $\neg u$	(1) and simplification
3) $\neg u \rightarrow \neg t$	Given
4) $\neg t$	(2), (3) and Modus Ponens
5) $\neg s$	(1) and simplification
6) $\neg s \wedge \neg t$	(4), (5) (Rule of Conjunction)
7) $r \rightarrow (s \vee t)$	Given
8) $\neg(s \vee t) \rightarrow \neg r$	(1) and contrapositivness
9) $(\neg s \wedge \neg t) \rightarrow \neg r$	De Morgan
10) $\neg r$	(6),(9)and Modus Ponens
11) $(\neg p \vee q) \rightarrow r$	Given
12) $\neg r \rightarrow \neg(\neg p \vee q)$	(11) and contrapositivness
13) $\neg r \rightarrow (p \wedge \neg q)$	De Morgan
14) $p \wedge \neg q$	(10),(13) and Modus Ponens
15) $\therefore$ **p**	**(14) and simplification**

Basitleştirme

$$\begin{array}{l} p \wedge q \\ \hline \therefore\ p \end{array} \qquad p \wedge q \Rightarrow p$$

Rule of Conjunctive Simplification

$$\begin{array}{l} p \\ q \\ \hline \therefore\ p \wedge q \end{array}$$

Rule of Conjunction

$p \rightarrow r \leftrightarrow \neg r \rightarrow \neg p \leftrightarrow \neg p \vee r$

Contrapositive

10. Show the correctness of the following statements

b) $[p \wedge (p \rightarrow q) \wedge (\neg q \vee r)] \rightarrow r$

$$\begin{array}{l} p \\ (p \rightarrow q) \\ \underline{(\neg q \vee r)} \\ \therefore r \end{array}$$

	Steps	**Rules**
1)	$p \rightarrow q$	Given
2)	q	Modus Ponens
3)	$\neg q \vee r$	Given
4)	r	Disjunctive Syllogism

Ayırıcı Kıyas

$$\begin{array}{l} p \vee q \\ \underline{\neg p} \\ \therefore q \end{array} \qquad (p \vee q) \wedge \neg p \Rightarrow q$$

Rule of Disjunctive Syllogism

Varsayımlı Kıyas

$$\begin{array}{l} p - q \\ \underline{q - r} \\ \therefore p - r \end{array} \qquad (p - q) \wedge (q - r) \Rightarrow (p - r)$$

Law of the Syllogism

c)

$$\begin{array}{l} p \rightarrow q \\ \neg q \\ \underline{\neg r} \\ \therefore \neg(p \vee r) \end{array}$$

	Steps	**Rules**
1)	$p \rightarrow q$	Given
2)	$\neg q$	Given
3)	$\neg p$	Modus Tollens
4)	$\neg r$	Given
5)	$\neg p \wedge \neg r$	Rule of Conjunction
6)	$\neg (p \vee r)$	DeMorgan

d)

$$\begin{array}{l} p \rightarrow q \\ r \rightarrow \neg q \\ \underline{r} \\ \therefore \neg p \end{array}$$

	Steps	**Rules**
1)	$r \rightarrow \neg q$	Given
2)	$q \rightarrow \neg r$	Contrapositive (1)
3)	$p \rightarrow q$	Given
4)	$p \rightarrow \neg r$	Law of the Syllogism
5)	r	Given
6)	$\neg p$	Modus Tollens

f)

$$\begin{array}{l} p \wedge q \\ p \rightarrow (r \wedge q) \\ r \rightarrow (s \vee t) \\ \underline{\neg s \qquad\qquad} \\ \therefore t \end{array}$$

	Steps	**Rules**
1)	$p \wedge q$	Given
2)	p	Simplification
3)	$r \wedge q$	Modus Ponens
4)	r	Simplification
5)	$s \vee t$	Modus Ponens
6)	$\neg s$	Given
7)	t	Rule of Disjunctive Syllogism

g)

$$\begin{array}{l} p \rightarrow (q \rightarrow r) \\ p \vee s \\ t \rightarrow q \\ \underline{\neg s \qquad\qquad} \\ \therefore \neg r \rightarrow \neg t \end{array}$$

	Steps	**Rules**
1)	$p \vee s$	Given
2)	$\neg s$	Given
3)	p	Rule of Disjunctive Syllogism
4)	$q \rightarrow r$	Modus Ponens
5)	$t \rightarrow q$	Given
6)	$t \rightarrow r$	Law of the Syllogism
7)	$\neg r \rightarrow \neg t$	Contrapositive (6)