

Let $X_{i,j}$ be an indicator random variable equal to 1 if elements i and j collide, and equal to 0 otherwise. Simple uniform hashing means that the probability of element i hashing to slot k is $1/m$. Therefore, the probability that i and j both hash to the same slot $\Pr(X_{i,j}) = 1/m$. Hence, $E[X_{i,j}] = 1/m$. We now use linearity of expectation to sum over all possible pairs i and j :

$$\begin{aligned}
 E[\text{number of colliding pairs}] &= E\left[\sum_{i=1}^n \sum_{j=i+1}^n X_{i,j}\right] \\
 &= \sum_{i=1}^n \sum_{j=i+1}^n E[X_{i,j}] \\
 &= \sum_{i=1}^n \sum_{j=i+1}^n 1/m \\
 &= \frac{n(n+1)}{2m} \\
 &= \Theta(n^2/m)
 \end{aligned}$$

When $m=c*n$, you get $\theta(n/c)$.