Basic of Electrical Circuits
EHB 211E

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Basic course information

Day(s)/Time/Place/

Exams

Midterm I:
Midterm II:
Final Exam: The date of the exam will be announced on http://www.sis.itu.edu.tr/tr/sinav_programi/

Grading

%30 Midterm I +%30 Midterm II + %40 Final Exam

Web

http://www2.itu.edu.tr/~yalcinmust/dersler.html

Schedule

1. Week	Lecture				
2. Week	Lecture				
3. Week	Lecture				
4. Week	Lecture				
5. Week	Midterm I	02 11		10-12	
6. Week	Lecture	07 12		10-12	
7. Week	Lecture		Homework I		
8. Week	Exercise session				
9. Week	Lecture		Homework I Return		
10. Week	Midterm II				
11. Week	Lecture				
12. Week	Lecture		Homework II		
13. Week	Lecture				
14. Week	Exercise session		Homework II Return		

Handouts & Reading Materials

A copy of the handouts can be obtained from the faculty copy center.

Textbook:

Leon O. Chua, Charles A. Desoer, Ernest S. Kuh, "Linear and Nonlinear Circuits," McGraw-Hill, 1987.

Reading Materials:

- Yılmaz Tokat, "Devre Analizi Dersleri: Kısım I," Çağlayan Kitapevi, 1986.
- Cevdet Acar, "Elektrik Devrelerinin Analizi," İstanbul Teknik Üniversitesi, 1995.
- Müştak E. Yalçın, "Elektrik Devre Temelleri Ders Notları", 2011.

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Contents I

Contents II

Contents III

Contents IV

Video to watch

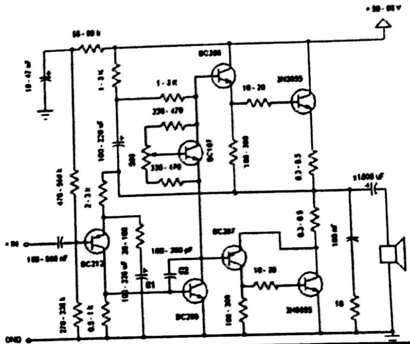
Life without Electrical Engineer
http://www.youtube.com/watch?v=F-ivig$_a$*VDYCurrentandVoltage*?
http://www.youtube.com/watch?v=1xPjES-sHwg
How to measure Voltage and Current ?
http://www.youtube.com/watch?v=bF3OyQ3HwfU

Introduction: Physical Circuit

Physical Circuit

Any interconnection of (physical) electric device.

Electric devices are resistors, diodes, transistors , ...



The Fundamental Variables

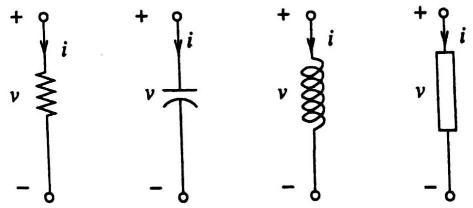
The variables are used to describe the behavior of a circuit.

- *Charge* (MKS units of charge is the coulomb) contained on 6.24 10^{18} electrons.
- *Current is the movement of charge* (the unit of current is the ampere *(amp or a)*).
- *Voltage is the ratio of the energy required to move charge between two points.*

Electric Circuit and Circuit Elements

Two-terminal element

Akım
+'dan
-'ye



(a) (b) (c) (d)

Typical examples of two-terminal element are resistor,inductor, ca
diode, voltage and current sources.

The instantaneous branch voltage (v) across a two-terminal a
instantaneous branch current (i) flows through a two-termi

Physical Circuit

The goal of circuit theory is to predict the electrical behavior of physical circuits.

Circuit theory focuses on the electrical behavior of circuits (... thermal, mechanical, chemical effects...).

An Electrical Circuit might be

Distributed circuit

A distributed circuit is one in which all dependent variables are functions of time and one or more spatial variables.

or

Lumped circuit

A lumped circuit is one in which the dependent variables of interest are a function of time alone.

Physical Circuit

Lumped circuit

Let l be the largest dimension of the circuit, λ the shortest wavelength of interest. If

$$\lambda >> l$$

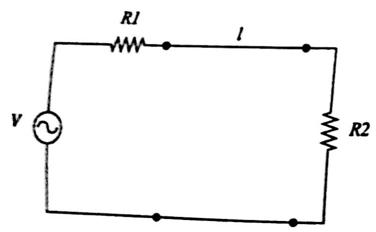
then the circuit may be considered to be lumped.

While Lumped circuit is analyzed by solving a set of ordinary differential equations (ODEs), Distributed circuit is analyzed by solving partial differential equations.

Typical examples of distributed circuits are circuits made of waveguides and transmission lines.

In this course we shall consider only lumped circuit.

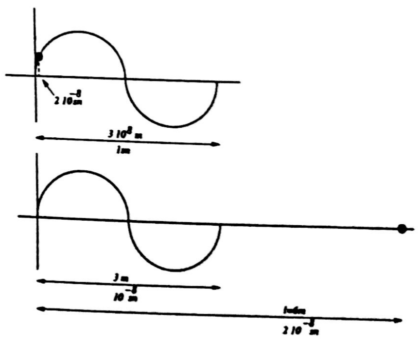
Physical Circuit



ow long does it take to reach end of line ?

Physical Circuit

Electromagnetic waves travel at the velocity of light $c = 3\,10^8$ meters second to travel l the time elapsed is $20ns$.



$f = 1Hz \rightarrow \frac{3\,10^8}{1} >> 6$ then the circuit may be considered to be lum
$f = 100MHz \rightarrow \frac{3\,10^8}{100\,10^6} < 6$ then the circuit may be considered to be disturbuted.

tric Circuit and Circuit Elements

The associated reference direction

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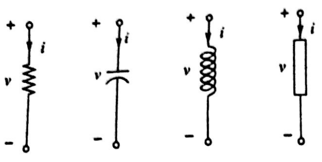
Electric Circuit and Circuit Elements



(a) (b) (c) (d)

The term $v(t)$ (or v) represent the instantaneous branch voltage, $i(t)$ the instantaneous branch current of the element. The voltage reference plus and mines symbol and the current reference arrow symbol. These symbols do not necessarily represent the actual direction of positive voltage drop or positive current flow.

The associated reference direction

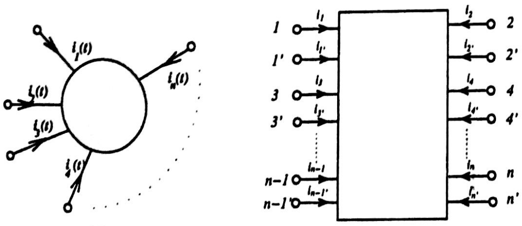
The direction of positive current flow coincides with the direction of positive voltage drop.

The associated reference direction

The instantaneous voltage and current have the same sign and power ($P = v\ i$) is being instantaneously delivered to the element.
When the instantaneous voltage and current have opposite sign, the element is the instantaneously delivering power to the remainder of the circuit.
A two-terminal circuit element is represented in the term of its associated reference directions just by an oriented branch. the direction of the arrow indicating both the voltage drop and the current flow reference polarities.

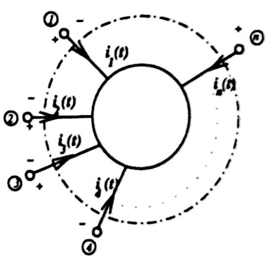
Electric Circuit and Circuit Elements

n-terminal element



(a) (b)

we assign arbitrarily a reference direction to each current variable by an arrow, and a reference polarity to each voltage variable by a pair of plus (+) and minus (-) sign.



If $i_1 = 3A$ it means that a current of 3A flows into the n-terminal element by node 2. If $i_1 = -0.3A$ it means that a current of 3A flows out of the n-terminal element by node 2.
If $V_1 = 3V$ it means that the electrical potential of terminal 1 is 3V larger than the electrical potential of terminal 2. If $V_1 = -3V$ it means that the electrical potential of terminal 1 is 3V smaller than the electrical potential of terminal 2.

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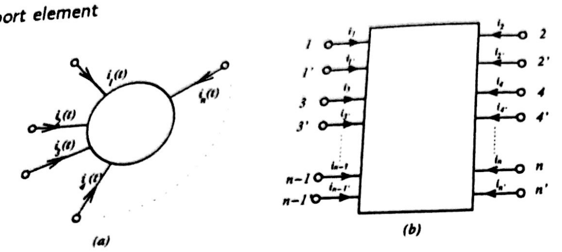
Electric Circuit and Circuit Elements

n-port element

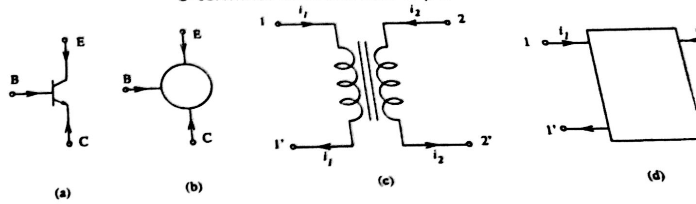


(a) (b)

port currents $i_1 = -i_1', \ldots$ and port voltages $V_{1,1'}, \ldots$

Electric Circuit and Circuit Elements

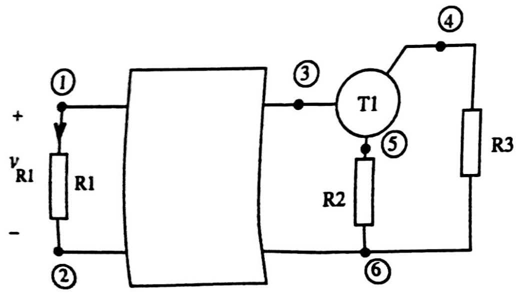
3-terminal element and 2-port element



(a) (b) (c) (d)

Electric Circuit and Circuit Elements

We use conducting wires to tie the terminals together as shown in Figure...



A node is any junction in a circuit where terminals are joined together or any isolated terminal of a circuit element, which is not connected.

Modelling Circuit Element

A mathematical model can be develop for each circuit element mathematical model is obtained after performing certain tests element.

Table : Electrical measurements for 2-terminal circuit ele

Measurements	Terminal variables i	v
1	-	-
2	-	-
3	-	-

The relation between the terminal variables is called termina

$$f(v, i) = 0$$

or

$$f(v, i, \frac{dv}{dt}, \frac{di}{dt}) = 0$$

Modelling Circuit Element

Modelling Circuit Element

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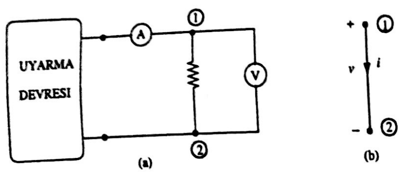
A node is any junction in a circuit where terminals are joined together or any isolated terminal of a circuit element, which is not connected.

or

$$f(v, i, \frac{dv}{dt}, \frac{di}{dt}) = 0$$

Modelling Circuit Element



Terminal graph with two nodes and one branch (the arrow on the branch indicating the reference direction of the current).

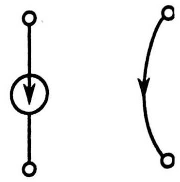
Mathematical Model

The terminal graph and the terminal equation are the mathematical model of the circuit element.

Modelling Circuit Element

Mathematical Model of Current Source :

Terminal graph:



and the terminal equation:

$$i = i_k$$

Modelling Circuit Element

Mathematical Model of Voltage Source :

Terminal graph:

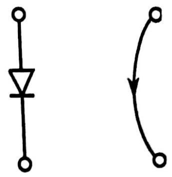
and the terminal equation:

$$v = v_k$$

Modelling Circuit Element

Mathematical Model of Diode:

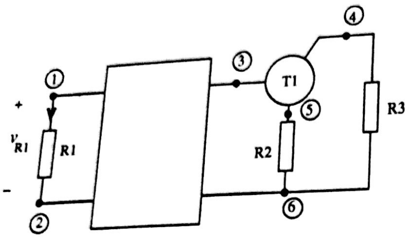
Terminal graph of the diode:



and the terminal equation:

$$i = I_0 e^{(v/v_T - 1)}$$

From Circuit to Graph



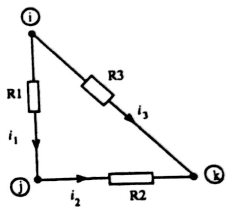
Node voltages: $e_1, e_2, ...e_n$.
Let $v_{k,l}$ *denote the voltage difference between node k and node l.*

$$v_{k,l} = e_k - e_l$$

Kirchhoff's Law

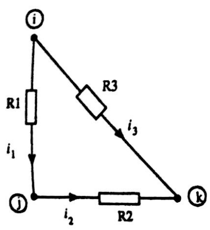
Second Postulate of Circuit Theory: Kirchhoff Voltage Law (KVL)

For all lumped connected circuits, for all closed node sequences, for all times t, the algebraic sum of all node-to-node voltages around the chosen closed node sequence is equal to zero.



Let us consider the closed node sequence $i - j - k - i$.

Kirchhoff Voltage Law (KVL)



Let us consider the closed node sequence $i - j - k - i$.

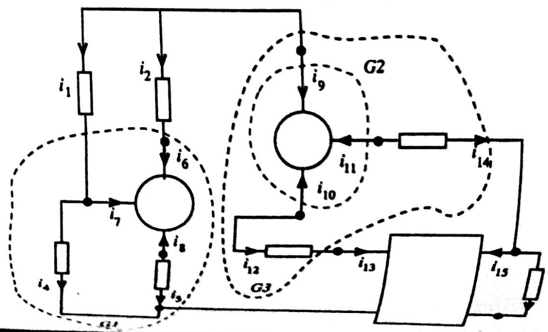
$$V_{i,j} + V_{j,k} + V_{k,i} = 0$$

$$e_i - e_j + e_j - e_k + e_k - e_i = 0$$

Kirchhoff Current Law (KCL)

Gaussian Surface

It is a closed surface such that it cuts only the connecting wires which connect the circuit elements.

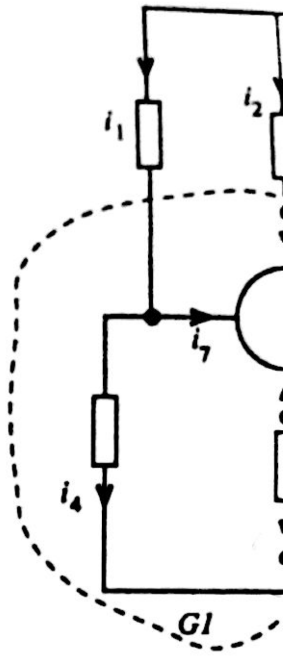


Kirchhoff Current Law (KCL)

$e_i - e_j + e_j - e_k + e_k - e_i = 0$

Kirchhoff Current Law (KCL)

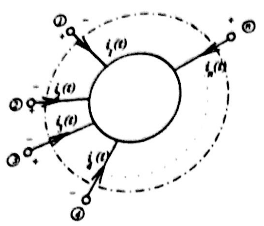
Kirchhoff Current La



Examples

For Gaussian surface;

$n-1$ currents can be



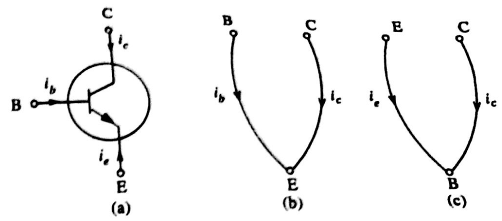
Let us consised the closed node sequence 1-2-3-...-n-1 and apply KVL (the sum of the voltages is equal to zero)

$$V_{1,2} + V_{2,3} + V_{3,4} + .. + V_{n-1,n} + V_{n,1} = 0$$

$n-1$ voltages can be specified independently! Why ?

$$-V_{n,1} = V_{1,2} + V_{2,3} + V3, 4 + .. + V_{n-1,n}$$

Remember : First Postulate of Circuit Theory: All the properties of an n-terminal (or $n-1$-port) electrical element can be described by a mathematical relation between a set of $(n-1)$ voltage and a set of $(n-1)$ current variables



Mathematical model is given by the terminal equation

$$\begin{bmatrix} v_{bc} \\ i_c \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} i_b \\ V_{ce} \end{bmatrix}$$

and terminal graph (b). Find the terminal equation in the form

$$\begin{bmatrix} V_{eb} \\ i_c \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} i_e \\ V_{cb} \end{bmatrix}$$

if (c) is the terminal graph.

Terminal equations

$$\begin{aligned} v_{bc} &= h_{11} i_b + h_{12} v_{ce} \\ i_c &= h_{21} i_b + h_{22} v_{ce} \end{aligned}$$

KCL and KVL for the circuit element

$$\begin{aligned} i_c + i_e + i_b &= 0 \\ v_{ce} + v_{eb} + v_{bc} &= 0. \end{aligned}$$

New terminal variables are i_e and V_{eb} (additional to i_c and V_{cb}). Substituting KVL and KCL Eqs. into above Eqs. we obtain

$$\begin{aligned} v_{bc} &= h_{11}(-i_c - i_e) + h_{12}(-v_{eb} + v_{cb}) \\ i_c &= h_{21}(-i_c - i_e) + h_{22}(-v_{eb} + v_{cb}) \end{aligned}$$

$$\begin{aligned} h_{12} v_{eb} + h_{11} i_c &= -h_{11}(i_e) + (1 + h_{12}) v_{cb} \\ (1 + h_{21}) i_c + h_{22} v_{eb} &= -h_{21} i_e + h_{22} v_{cb} \end{aligned}$$

New terminal equations

$$\begin{bmatrix} h_{12} & h_{11} \\ h_{22} & 1 + h_{21} \end{bmatrix} \begin{bmatrix} v_{eb} \\ i_c \end{bmatrix} = \begin{bmatrix} -h_{11} & (1 + h_{12}) \\ -h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} i_e \\ V_{cb} \end{bmatrix}$$

and terminal graph (c) will be the new mathematical model !

Tellegen Theorem

Tellegen's theorem is based on the fundamental law of conservation of energy!

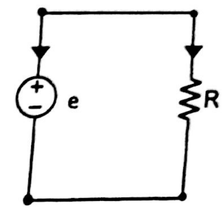
Tellegen Theorem

It states that the algebraic sum of power absorbed by all elements in a circuit is zero at any instant.

Tellegen's theorem asserts that

$$\sum_{k=1}^{n_e} v_k i_k = 0$$

Tellegen Theorem



$R = 2\Omega$ and $e = 2V$, from KCL

$$i_e = -i_R = \frac{2}{2} = 1A$$

Lets apply Tellegen Theorem:

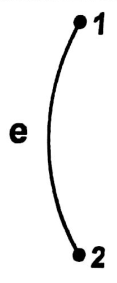
$$P = i_e \cdot e + V_R \cdot i_R = 2 \cdot (-1) +$$

Power absorbed by a resistor is always positive, wher deliver power. Then in this case, the power associate negative.

Fundamentals of Graph Theory

Edge: A line segment together with its two distinct end points is called an edge.

Node: An end point of an edge is called a node (vertex).

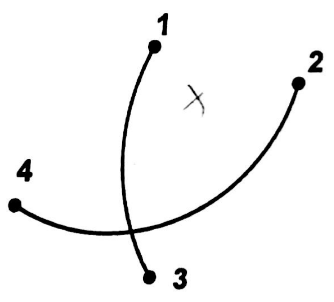


A node n_i and an edge e_i are incident with each other if n_i is one of the two end points of the edge e_i

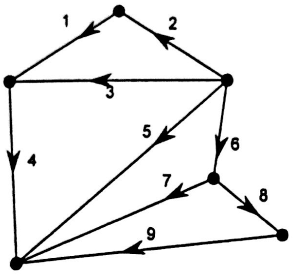
Fundamentals of Graph Theory

Graph: A graph $G = (N, E)$ is defined to be a set N c of edges with a prescribed edge-node incidence relatior incident with two nodes in V.

A planar graph is a graph which can be drawn on a pla that no two branches intersect at a point which is not a

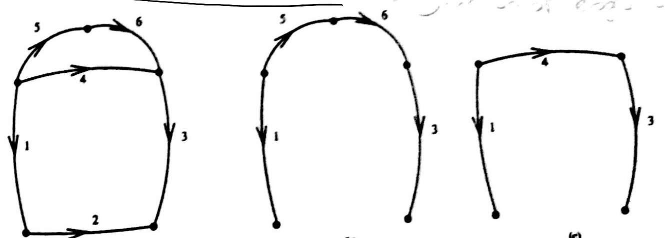


Graph: A graph $G = (N, E)$ is defined to be a set N of nodes and a set E of edges with a prescribed edge-node incidence relation, i.e., each edge is incident with two nodes in V.



Subgraph: G_1 is called a subgraph of G iff G_1 itself is a graph, N_1 is a subset of N, and E_1 is a subset of E.

Path: A path graph is a graph that can be drawn so that all of its nodes and edges lie on a single straight line.

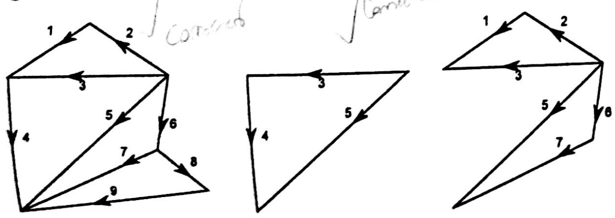


Connected graph : A graph which is connected If there is a path from any point to any other point in the graph. A graph that is not connected is said to be disconnected.

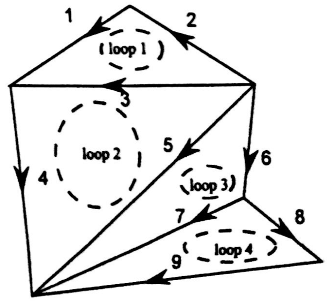
Loop: A loop G_L is defined to be a connected subgraph of G is defined to be a set of a closed node sequence and a set of edges between these nodes.

A loop G_L is defined to be a connected subgraph of G in which precisely two edges are incident with each node.

√ Connected √ Connected √ Conn



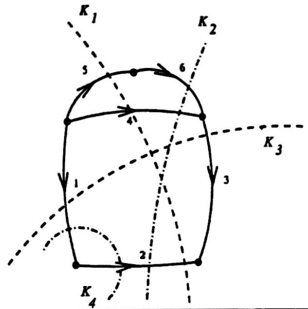
Mesh : A mesh is a loop of a graph drawn on a plane, which encircle nothing in its interior.



Fundamentals of Graph Theory

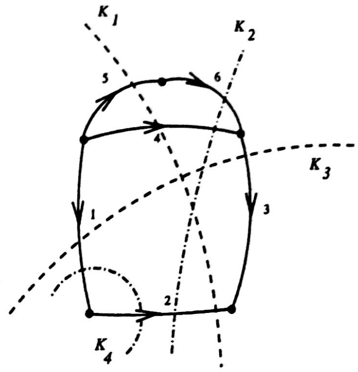
Cut set: Given a connected graph G a set of edges G_C of G is called a cut set iff

- the removal of all the edges of the cut set results in an unconnected graph.
- the removal of all but any one edge of G leaves the graph connected.



Fundamentals of Graph Theory

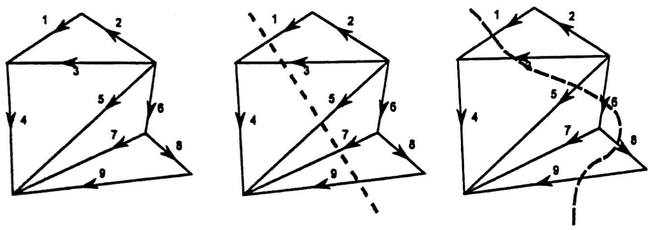
$G_{K1} = \{2,4,5\}$, $G_{K2} = \{2,4,6\}$, $G_{K3} = \{1,3\}$, $G_{K4} = \{1,2\}$.



To any cut set corresponds a gaussian surface which cuts precisely the same edges.

Fundamentals of Graph Theory

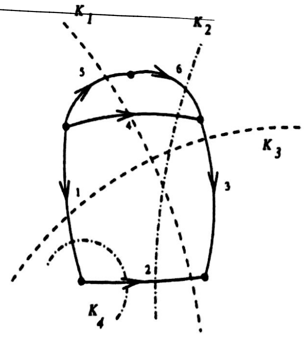
3'ü bölüyor



Fundamentals of Graph Theory

Cut set for the node: A gaussian surface that surrounds only the node in question.

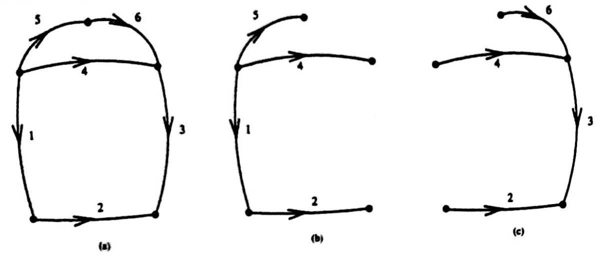
$G_{K4} = \{1,2\}$ is the cut set of the node 4.



Fundamentals of Graph Theory

Tree: Let G be a connected graph and T is a connected subgraph of G. We say that T is a tree of the connected graph G if

- T is a connected subgraph,
- it contains all the nodes of G,
- it contains no loops.



$T_{A1} = \{1,2,4,5\}$ and $T_{A2} = \{2,3,4,6\}$ subgraphs are tree.

Fundamentals of Graph Theory

Theory: Given a connected graph G of n_n nodes and n_b edges, and a tree T of G, there is a unique path along the tree between any given pair of nodes.

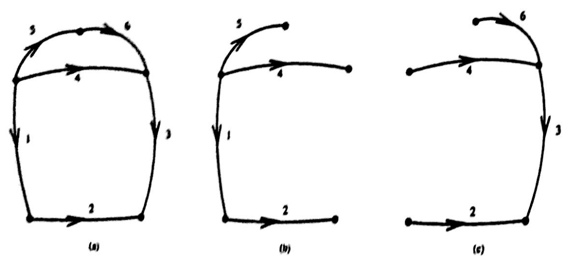
Branch (twig): Given a connected graph G and a tree T, the edges of T are called branches.

Link: The edges of G not in T are called links (chords).

co-tree: The complement of tree (The edges of the co-tree are links).

There are $n_n - 1$ tree branches and $n_b - n_n + 1$ links.

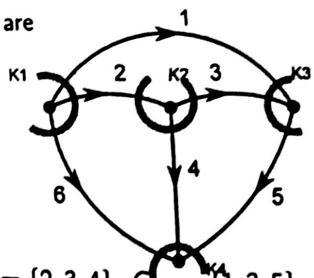
Fundamentals of Graph Theory



Links of the trees G_{T1} and G_{T2} are $G_{L1} = \{3,6\}$ and $G_{L2} = \{1,5\}$, respectively.

Independent KCL Equations

Cut sets for the nodes are



$G_{K1} = \{1,2,6\}$, $G_{K2} = \{2,3,4\}$, $G_{K3} = \{1,3,5\}$, $G_{K4} = \{4,5,6\}$.
If we apply KCL to the cut sets, we obtain

$$
\begin{aligned}
i_1 + i_2 + i_6 &= 0 \\
i_4 + i_3 - i_2 &= 0 \\
i_5 - i_1 - i_3 &= 0 \\
-i_5 - i_4 - i_6 &= 0
\end{aligned}
$$

which are node equations. *current reference direction [+ if branch leaves the node]*

Independent KCL Equations

which are node equations. current reference direction [+ if branch le... the node]

Independent KCL Equations

In matrix form

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ -1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} = 0$$

For an n_n-node n_b-branch connected graph G, node equations are given by

$$A_a i = 0$$

where $i = [i_1\, i_2 \ldots i_{n_b}]^T$ is called the branch current vector. A_a is called incidence matrix of the graph G and $A_a \in \{-1, 0, 1\}^{n_n \times n_b}$.

Independent KCL Equations

$$\begin{aligned} -i_1 - i_2 - i_6 &= 0 \\ -i_4 - i_3 + i_2 &= 0 \\ -i_5 + i_1 + i_3 &= 0 \\ + - - - - - & \\ -i_5 - i_4 - i_6 &= 0 \end{aligned}$$

The four equations are linearly dependent. Any three of the four equations are linearly independent.

Independence property of KCL equations

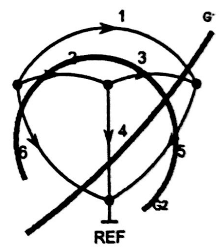
For any connected grahp with n_n nodes, the KCL equations for any $n_n - 1$ of these nodes form a set of $n_n - 1$ linearly independent equations.

Independent KCL Equations

If in A_a, we delete the row corresponding to the datum node (reference node), we obtain the *reduced incidence matrix A which is of dimension* $n_n - 1 \times n_b$. *The KCL equations is given by*

$$A i = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ -1 & 0 & -1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} = 0$$

Independent KCL Equations

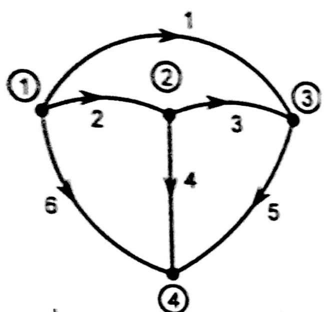


$$i_1 + i_3 + i_4 + i_6 = 0$$

The first one is obtained by first and second node equations

$$\begin{aligned} i_1 + i_2 + i_6 &= 0 \\ i_4 + i_3 - i_2 &= 0 \\ + - - - - - & \\ i_1 + i_3 + i_4 + i_6 &= 0 \end{aligned}$$

Independent KVL Equations



All 6 edges voltages can be expressed in terms of 3 node-to-datum voltages as follows:

$$
\begin{aligned}
V_1 &= V_{n1} - V_{n3}\\
V_2 &= V_{n1} - V_{n2}\\
V_3 &= V_{n2} - V_{n3}\\
V_4 &= V_{n2}\\
V_5 &= V_{n3}\\
V_6 &= V_{n1}
\end{aligned}
$$

Independent KVL Equations

Independent KVL Equations

For an n_n-node n_b-branch connected graph G, independent KVL equations are given by

$$V = MV_n$$

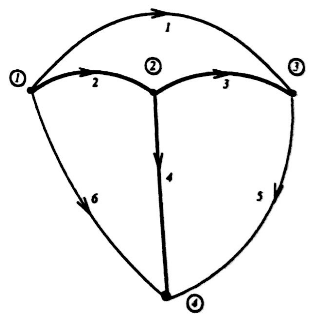
where $V = [V_1\ V_2 \ldots V_{n_b}]^T$ and $V_n = [V_{n1}\ V_{n2} \ldots V_{nn_n-1}]^T$ are called the branch voltage vector and node-to-datum voltage vector, respectively. M is a $n_b \times n_n - 1$ matrix.

Independent KVL Equations

$$
\begin{aligned}
V_1 &= V_{n1} - V_{n3}\\
V_2 &= V_{n1} - V_{n2}\\
V_3 &= V_{n2} - V_{n3}\\
V_4 &= V_{n2}\\
V_5 &= V_{n3}\\
V_6 &= V_{n1}
\end{aligned}
$$

$$
V = \begin{bmatrix} 1 & 0 & -1\\ 1 & -1 & 0\\ 0 & 1 & -1\\ 0 & 1 & 0\\ 0 & 0 & 1\\ 1 & 0 & 0 \end{bmatrix} V_n
$$

Independent KVL Equations



KVL equation for the closed node sequence 1-2-3-1

$$V_2 + V_3 - V_1 = 0$$

This equation can be obtained *from the first three equations*.

Tellegen's Theorem

... Let the graph have n_b branches. Let us use

Tellegen's Theorem

Consider an arbitrary circuit. Let the graph have n_b branches. Let us use associated reference directions. Let $i = [i_1\, i_2 \ldots i_{n_b}]^T$ be any set of branch currents satisfying KCL for G and let $V = [V_1\, V_2 \ldots V_{n_b}]^T$ be any set of branch voltages satisfying KVL for G, then

$$\sum_{k=1}^{n_b} v_k i_k = 0.$$

Proof Since i satisfies KCL, we have

$$Ai = 0$$

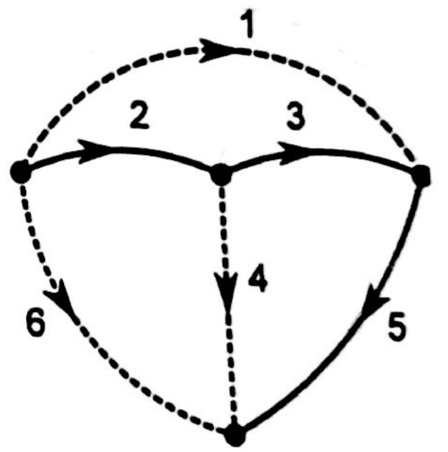
and since V satisfies KVL, we have

$$V = A^T V_n.$$

Using these two equations, we obtain

$$V^T i = V_n^T A i = V_n^T (Ai) = 0.$$

Fundamental Loop Analysis



The links $G_L = \{1, 4, 6\}$ for the chosen tree $G_T = \{2, 3, 5\}$. The Fundamental loop sets are $G_{L1} = \{1, 2, 3\}$ $G_{L4} = \{4, 5, 3\}$ $G_{L6} = \{6, 2, 3, 5\}$.
If we apply KVL to the Fundamental loops, we obtain:

$$\begin{aligned} V_1 - V_3 - V_2 &= 0 \\ V_4 - V_5 - V_3 &= 0 \\ V_6 - V_5 - V_2 - V_3 &= 0 \end{aligned}$$

voltage vector. In matrix

$$\begin{bmatrix} V_1 \\ V_4 \\ V_6 \\ -- \\ V_2 \\ V_3 \\ V_5 \end{bmatrix} = 0$$

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

B is an $n_b - n_n + 1 \times n_b$ matrix calle

$$BV = [\ I | F\ ]$$

$$V_l = -$$

The number of Fundamental loop equat links).

Mesh Analysis

... meshes. Corresponding loop sets and mesh currents (loop currents) $G_{M1}=\{1,2,3\}$ and i_{m1}: $G_{M2}=\{3,4,5\}$ and i_{m2}; $G_{M3}=\{2,4,6\}$ and i_{m3}.

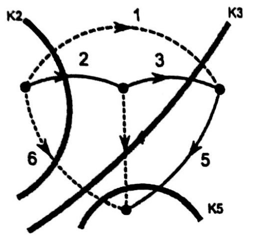
$$\begin{bmatrix} 0 & 0 & 1 & -1 & 0 & -1 \end{bmatrix} = B_l^T i_c$$

where $i=[i_1\ i_5\ i_6\ i_2\ i_3\ i_4]^T$ is branch current vector and $i_m=[i_{m1}\ i_{m2}\ i_{m3}]^T$ is mesh current vector.

Fundamental Cut-set

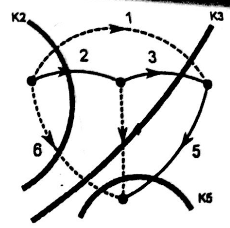
Cut-set

is made up of links and of one tree branch, namely the tree branch which defines the cut set. Every tree branch defines a unique Fundamental cut set.



Cut sets of the tree of $G_T=\{2,3,5\}$ are $G_{C2}=\{2,1,6\}$ $G_{C3}=\{3,1,4,5\}$ $G_{C5}=\{5,4,6\}$.

Fundamental Cut-set



If we apply KCL to the three cut sets, we obtain

$$\begin{aligned} i_2+i_1+i_6 &= 0\\ \cancel{i_3+i_1+i_4}+i_6 &= 0\\ i_4+i_5+i_6 &= 0 \end{aligned}$$

which are called Fundamental cut-set equations. Reference direction for the cut set which agrees with that of the tree branch defining the cut set.

Fundamental Cut-set

$$i=\begin{bmatrix} i_l \\ \hline i_b \end{bmatrix} = \begin{bmatrix} i_1\\ i_4\\ i_6\\ --\\ i_2\\ i_3\\ i_5 \end{bmatrix}$$

where i_l is link current vector and i_b is tree branch current vector. In matrix form:

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0\\ 1 & 1 & 1 & 0 & 1 & 0\\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \begin{bmatrix} i_1\\ i_4\\ i_6\\ --\\ i_2\\ i_3\\ i_5 \end{bmatrix} = 0$$

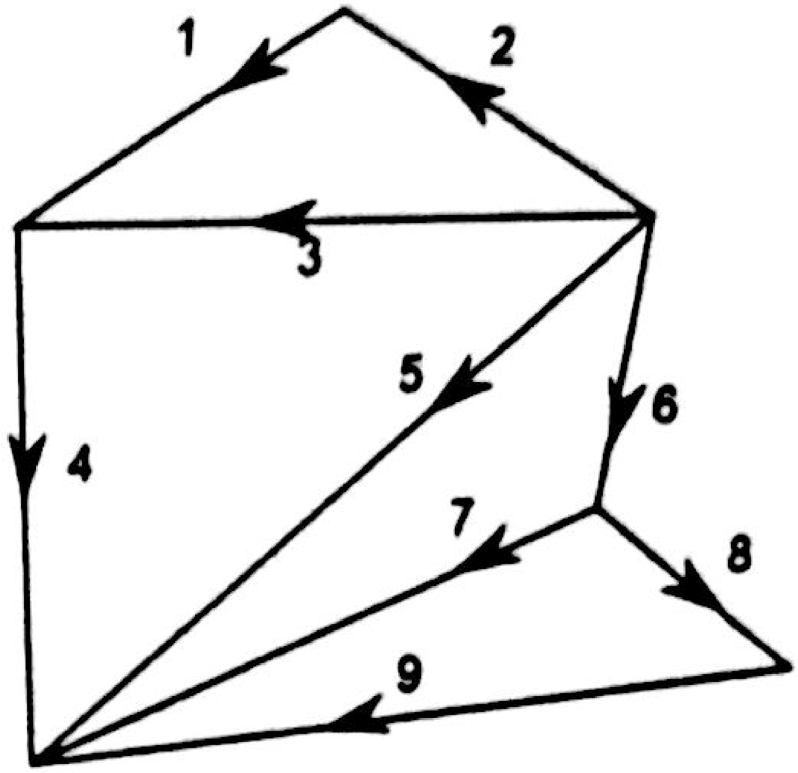
Fundamental Cut-set

$$Qi = [\ E|I\ ]\begin{bmatrix} i_l \\ -- \\ i_b \end{bmatrix} = 0$$

Q is called the Fundamental cut-set matrix. Q is an $n_b - n_n + 1 \times n_n -$

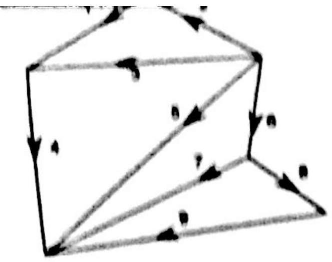
$$i_b = -Ei_l$$

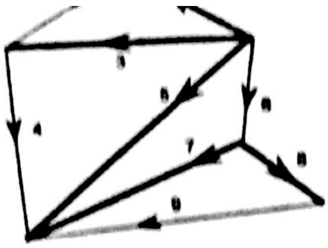
Q has a rank n_n-1 it includes the unit matrix. Hence the linear alge equations obtained by applying KCL to each Fundamental cut set constitute a set of n_n-1 linearly independent equations.



Fundamental cut sets of the tree $G_T = \{2$
$G_{C3} = \{3, 1, 4\}$, $G_{C5} = \{5, 4, 6\}$, $G_{C7} = \{7$

Example





ons based on the Fundamental loops

$$
\begin{bmatrix}
1 & 0 & 0 & 0 & | & 1 & -1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & | & 0 & 1 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 & | & 0 & 0 & -1 & 1 & 0 \\
0 & 0 & 0 & 1 & | & 0 & 0 & 0 & -1 & 1
\end{bmatrix}
\begin{bmatrix}
V_1 \\ V_4 \\ V_6 \\ V_9 \\ -- \\ V_2 \\ V_3 \\ V_5 \\ V_7 \\ V_8
\end{bmatrix} = 0
$$

KVL equations for the nc

$$
\begin{bmatrix}
V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ V_9
\end{bmatrix} =
$$

Two-terminal Elements

Two-terminal elements play a major role in electric circuits!

Two-terminal circuit elements are defined by the between basic variables which are current ($i(t)$), voltage ($v(t)$), charge ($q(t)$) and flux ($\phi(t)$). The units of them are Amperes, Volts, Coulomb and Weber, respectively.

Two pairs of the basic variables

$$i(t) = \frac{dq}{dt},$$

and

$$v(t) = \frac{d\phi}{dt},$$

are the definition.

Two-terminal Elements

Controlled circuit element (Dependent element)

If the relation between the terminal variable is given by the equation $x = h(y,t)$, this two-terminal element is called as a y controlled element e.g. voltage controlled voltage sources,...

Time-invariant two-terminal element

A two-terminal element whose variables x and y fall on some fixed curve in the $x-y$ plane at any time t is called a time-invariant circuit element e.g. Linear resistor $Vv = Ri$.

$x-y$ characteristic

The curve on the $x-y$ plane at any time t is called $x-y$ characteristic e.g. $v-i$ characteristic of linear resistor.

Two-terminal Elements

Bilateral property

A element has a $x-y$ characteristics which is not symmetric with respect to the origin of the $x-y$ plane.

Linear element

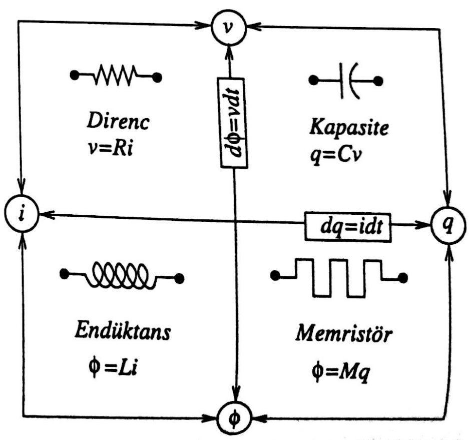
A linear element is an element with a linear relationship between its variables x and y.

Linear

$f(x)$ is a function which satisfies the following two properties:

- Additivity (superposition): $f(x+y) = f(x) + f(y)$.
- Homogeneity : $f(\alpha x) = \alpha f(x)$ for all α.

Basic circuit element diagram



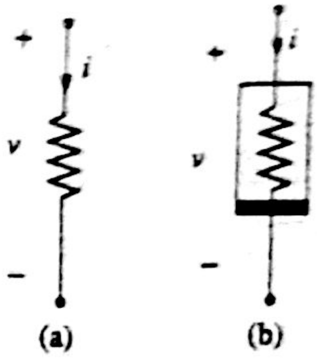
Linear and Nonlinear Resistor

Ohm's law states

$$v = Ri \quad or \quad i = Gv$$

where the constant R is the resistance of the linear resistor measured in the unit of ohms (Ω), and G is the conductance measured in the unit of Siemens (S). A resistor which is not linear is called nonlinear.

$$G = \frac{1}{R}$$



Time-varying and Nonlinear Capacitor

If the $q - v$ characteristic changes with time, the capacitor is said to be time-varying. Then the mathematical model becomes

$$q = C(t)v$$

and

$$i = \frac{dC}{dt}v + C(t)\frac{dv}{dt}$$

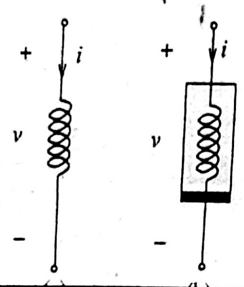
The most general case, a time-varying nonlinear capacitor is defined by a family of time-dependent and nonlinear $q - v$ characteristics

$$f(q, v, t) = 0$$

A two-terminal element whose flux $\phi(t)$ and current $i(t)$ fall on some fixed curve in the $\phi - i$ plane at any time t is called a time-invariant inductor. The mathematical model of LTI inductor is

$$\phi = Li \text{ veya } v = L\frac{di}{dt}$$

Values of inductors are specified in ranges of Henry (H).



Memristor

If the $\phi - i$ characteristic changes with time, the inductor is said to be time-varying. Then the mathematical model becomes

$$v = L(t)i$$

and

$$v = \frac{dL}{dt}i + L(t)\frac{di}{dt}$$

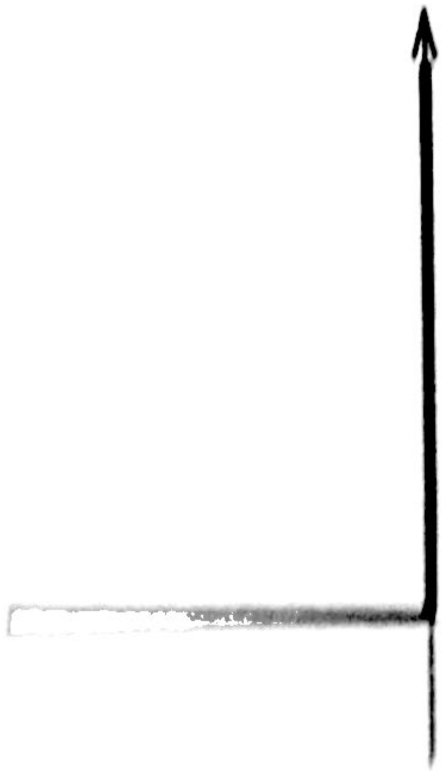
The most general case, a time-varying nonlinear capacitor is defined by a family of time-dependent and nonlinear $\phi - v$ characteristics

$$f(\phi, v, t) = 0$$

Memristor

ner diode

where $v_T = 0.026V$ ve I_0

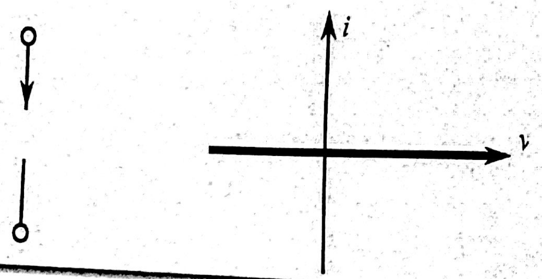


(a

onlinear resistor

Open circuit

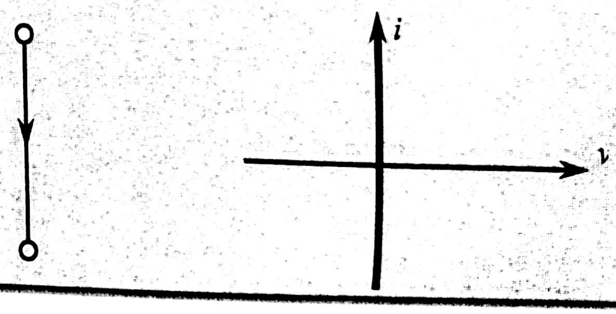
$i(t) = 0$



Nonlinear resistor

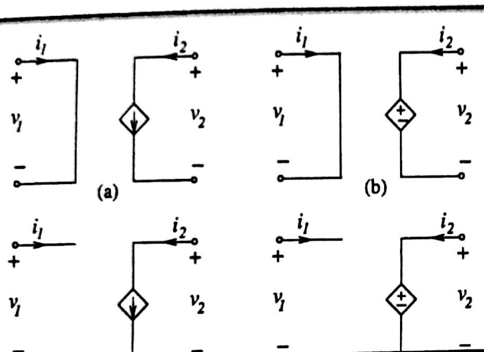
Short circuit

$v(t) = 0$



Linear Controlled Sources

Dependent Source: A controlled sources is a resistive two-port element consisting of two branches: a primary branch which is either an open circuit or a short circuit and a secondary branch which is either a voltage source or a current source. Diamond-shaped symbol to denote controlled sources.



Linear Controlled Sources

Current-Controlled Current Source (CCCS)

CCCS is characterized by two linear equations

$$\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \alpha & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix}$$

where α is called the current transfer ratio.

Current-Controlled Voltage Source (CCVS)

CCVS is characterized by two linear equations

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ r & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

where r is called the transresistance.

Linear Controlled Sources

Voltage-Controlled Current Source (VCCS)

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ g & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

where g is called the transconductance.

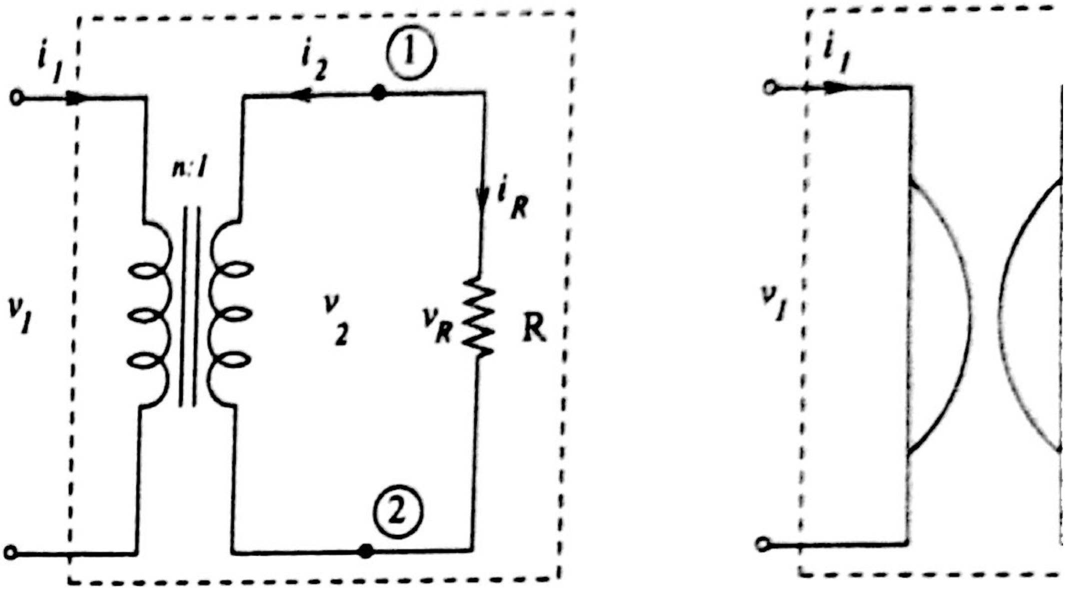
Two-port Elements

Two-port Elements

The ideal transformer is an ideal two-port resistive circuit element which is characterized by the following two equations:

$$\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & n \\ -n & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix}$$

where n is a real number called the turns ratio.



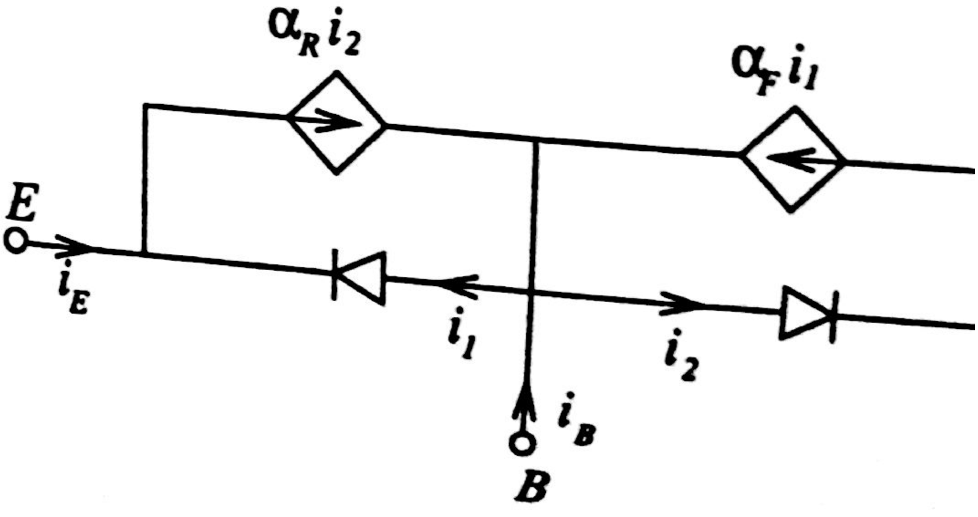
(a)

Modelling of the common emitter transistor configuratic dependent sources. One can obtain

$$i_1 = I_{es}(e^{-v_{eb}/v_T - 1})$$

and

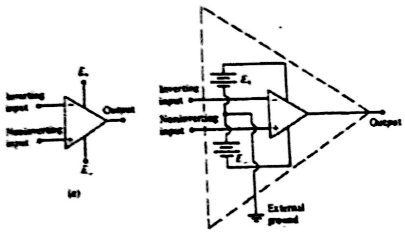
$$i_2 = I_{CS}(e^{-v_{cb}/v_T - 1})$$



(a) (b)

Operational amplifier

Operational amplifiers (op amps) are multi-terminal devices. Terminals are labeled inverting input, non-inverting input, output, E_+ , E_- and external ground. A "biased" op amp can be considered as a 4-terminal device.

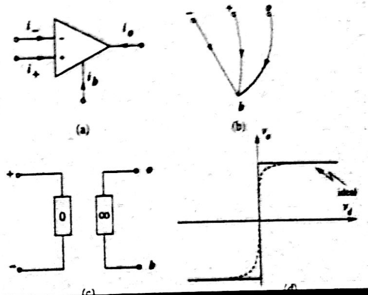


The variable $v_d = v_+ - v_-$ is called the differential input voltage and will play an important role in op-amp circuit analysis.

terminal currents and voltages obey the following approximate relation- ships:

$$i_+ \approx 0,\ i_- \approx 0 \text{ and } v_o = G(v_+ - v_-)$$

where G called the open-loop voltage gain. v_o saturates at $v_o = E_{sat}$ where E_{sat} is less than the power supply voltage.



Operational amplifier

In a small interval $-\epsilon < v_d < \epsilon$, we have

$$G(v_+ - v_-) = v_o$$

which is called linear region.
In ideal op amp

$$i_+ = i_- = \epsilon = 0$$

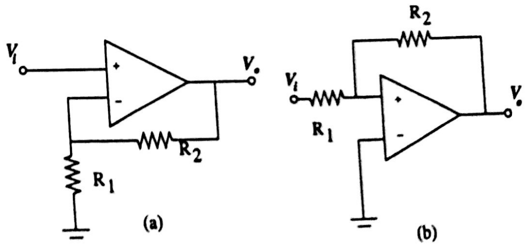
and

$$G = \infty$$

Using the gain formula, we will have

$$\begin{aligned}(v_+ - v_-) &= \tfrac{v_o}{G}\\ (v_+ - v_-) &\approx 0\end{aligned}$$

Examples



Examples

Applying KCL at the node where the inverting input is connected and using the definition of op amp, we obtain

$$\begin{aligned} -G_1V_- + G_2(V_o - V_-) &= 0 \\ G(V_i + V_-) &= V_o \end{aligned}$$

Substituting the first eqn. into the second eqn.

$$V_o = \frac{G}{1+KG}V_i$$

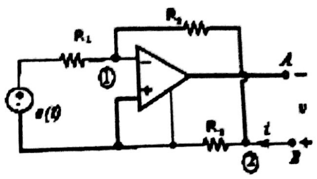
where $K = \frac{R_1}{R_1+R_2} \cdot \frac{G}{1+KG}$ is the gain. G is too big therefore

$$V_o = \frac{1}{K}V_i$$

!

The same result can be obtained using the relation $V_+ = V_-$.

Examples



What is the $v - i$ characteristic ?

$$\frac{e}{R_1} = -\frac{V_{d2}}{R_2}$$

$$i_3 = -\frac{V_{d2}}{R_3} = -\frac{R_2 e}{R_3 R_1}$$

$$i = i_2 + i_3 = -\frac{e}{R_1} + \frac{R_2 e}{R_3 R_1} = -\frac{e}{R_1}(1 + \frac{R_2}{R_3})$$

Elementary Function

Unit step function $u(t)$

It is defined by

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

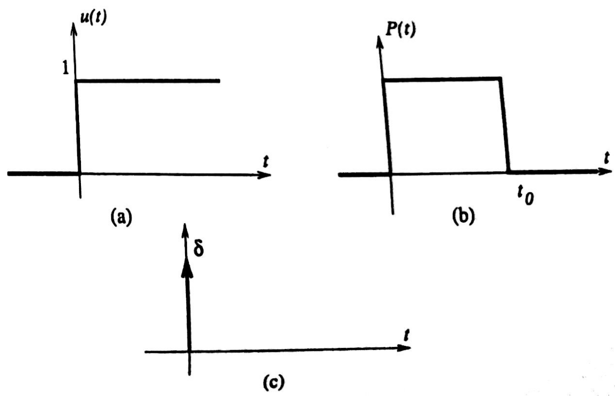
Rectangular pulse

$$P(t) = \begin{cases} 1 & 0 \leq t \leq t_0 \\ 0 & t < 0, t > t_0 \end{cases}$$

unit impulse (or delta function)

$$\delta(t) = \lim_{\Delta \to \infty} P_\Delta(t) = \begin{cases} \infty & t = 0 \\ 0 & t \neq 0 \end{cases}$$

Elementary Function



Elementary Function