BLG311E – FORMAL LANGUAGES AND AUTOMATA

2014 SPRING

RECITEMENT 3

1. State the type of each grammar given below with respect to Chomsky hierarchy and explain the reason why that grammar belongs to the type you stated. Write the regular expression for each of these grammars (Give the general form of productions for the grammars (e.g., $a^n b^n c^n$) for which it is not possible to write a regular expression).

Solution:

Grammar	Type and reason	Regular expression (or general form)
<S> ::= <A><B> <A> ::= a<A>b | ab <B> ::= b<B>a | Λ	**Type 0:** If there is an empty string on the right-hand side, it is definitely Type 0. The only exception is the language involving an empty string in the production rule for S (i.e., <S> ::= ... | Λ) which is allowed by all the grammars except Type 1.	This grammar does not have a regular expression as it is not Type 3. General structure is in the following form: $a^i b^{i+j} a^j,\ i > 0, j \geq 0.$
S → aS | bS | aba	**Type 3:** A single nonterminal on the left-hand side and a right-hand side consisting of a number of terminals followed by a single nonterminal.	$L(G) = (a \vee b)^* aba$
S → aAbc | abc A → aAbC | abC Cb → bC Cc → cc	**Type 1:** Multiple symbols on the left-hand side and the length of the left-hand side can not exceed the length of the right-hand side.	This grammar does not have a regular expression as it is not Type 3. General structure is in the following form: $a^n b^n c^n,\ n > 0.$
<S> ::= a<S>a | b<S>b | c a<S>a ::= ac	**Type 0:** Multiple symbols on the left-hand side and the length of the left-hand side exceeds the length of the right-hand side.	This grammar does not have a regular expression as it is not Type 3. General structure is in the following form: $x(c \vee ac)x^R,\ x = (a \vee b)^*.$

2. **[Midterm 2013]** If S is the initial non-terminal, which of the words ***aaab***, ***aabbaab***, and ***abaaabb*** can be derived using the following grammar. Draw the parse tree(s) of the derivable one(s).

$$S \to aT \mid baS$$
$$T \to bU \mid b$$
$$U \to S \mid aUb \mid a$$

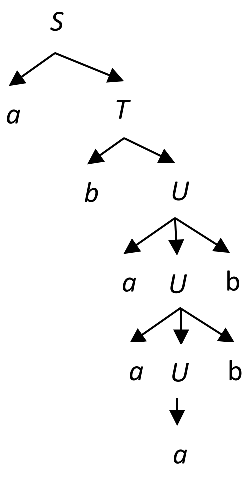
Provide an arithmetic rule between the number of a's ($\#a$) and b's ($\#b$) in the words derived using the grammar above.

Solution:

aaab and *aabbaab* cannot be derived using the given grammar.

abaaabb can be derived as

$$S \to aT \to abU \to abaUb$$
$$\to abaaUbb \to abaaabb.$$



Starting from the initial non-terminal S and eliminating parts where $\#a = \#b$:

$$S \to aT \vee baS \to (ba)^{*}aT \to aT$$
$$S \to a(bU \vee b) \to abU \vee ab \to U \vee ab$$
$$S \to a^{n}(S \vee a)b^{n} \vee ab \to S \vee a \vee ab$$
$$S \to a \vee ab$$

So $\#a = (\#b + 1) \vee \#b$

3. Consider the following grammar.

$$A \to AaA$$
$$A \to b$$

a. What is the type of the grammar according to Chomsky hierarchy? Why?

b. Find the language generated by the grammar.

c. Design another grammar with a more restrictive type that generates the same language you found in (b). (e.g. if the given grammar is Type-1 then design a Type-2 or Type-3 grammar.)

d. Is it possible to write a regular expression for the grammar you designed in (c)?

Solution:

a. It is type-2 since (1) length of the left side is always smaller than or equal to the length of the right side, and (2) left side always consists of a single nonterminal. It is not type-3 because there are multiple nonterminals in the right side (*AaA*).

b. (ba)*b or b(ab)*

c. New grammar can be:
A ::= bB | b
B ::= aA

d. Yes, it is possible because it is a regular (Type-3) grammar.
It is type-3 since (1) length of the left side is always smaller than or equal to the length of the right side, (2) left side always consists of a single nonterminal, and (3) right side consists of a number of terminals, possibly followed by a single nonterminal.

4. For the language defined over alphabet Σ = {a,b} that contains aaa or bbb;

a. Give generation rules of a grammar.

b. Identify the Chomsky hierarchy type of your grammar.

c. If possible, write a regular expression for your grammar.

Solution:

a. Words will be in form: {a,b}*{aaa,bbb}{a,b}*

$S \rightarrow aS \mid bS \mid aA \mid bB$
$A \rightarrow aD$
$B \rightarrow bE$
$C \rightarrow aC \mid bC \mid a \mid b$
$D \rightarrow aC \mid a$
$E \rightarrow bC \mid b$

b. Type-3 (Q3.d for explanation)

c. $L(M) = (a \vee b)^*(aaa \vee bbb)(a \vee b)^*$

5. L = { w | w $\in$ {a, b}* $\wedge$ |w| is odd $\wedge$ first, middle and last characters of w are the same. }

a. Give generation rules of a grammar for this language.

b. Identify the Chomsky hierarchy type of your grammar.

Solution:

a.

$S \rightarrow a \mid b \mid aAa \mid bBb$
$A \rightarrow CAC \mid a$
$B \rightarrow CBC \mid b$
$C \rightarrow a \mid b$

b. Type-2 since (1) length of the left side is always smaller than or equal to the length of the right side, (2) left side always consists of a single nonterminal. But right side contains multiple nonterminals.