The node voltage method

Needed: Circuit analysis methods that don't rely on recognizing patterns or transformation tricks. We want some that we can apply to any circuit with expectation of finding all the voltages and currents in the circuit.

There are two methods that fit this bill: the *node-voltage method* and the *mesh-current method*. In EE 442, we will emphasize the node-voltage method, which lends itself well to most classes of circuits, including electronics.

The method proceeds in a set fashion, almost like an algorithm. Following the method will lead to complete solutions in nearly every case.

The node-voltage method is not guaranteed to be faster than the other short-cut methods. But it will always work, and in same cases, it may well be faster.

The node voltage procedure

Here are the steps:

1. Identify all of the nodes in a circuit.

   Recall that a node connects the various components making up a circuit. Stated another way: the nodes define a circuit. Also, the voltage of a node is constant. Currents flowing in and out of a node must obey Kirchoff's Current Law.

2. Choose one node to serve as ground.

   Differences in voltage are important in a circuit. The difference in voltage from node to node is the driving force for current to flow through a circuit element connecting the two nodes. Since voltage is a relative quantity, we can choose one node to define as zero volts, which we will call ground. All other node voltages will be with respect to the ground node.

In principle, it doesn't matter which node is used as ground. But, from a practical point-of-view, the choice may reduce the amount of algebra required for a given circuit.

3. Assign variables naming the unknown voltages to each of the other nodes.

Give each node voltage a variable name. eg. v_A, v_B, v_C, etc

3a. Identify any nodes where the voltage is known due to a voltage source.

If there is an independent voltage source connected between ground and a node, the voltage at that node *is* known. It doesn't need a variable name. Note that this is might be a consideration in choosing which node will be ground.

The number of unknown node voltages defines the scale of the algebra that will be required to solve the circuit. If there is only one unknown node voltage, you will need only one equation to find it. If there are two unknown nodes, you will need equations. Three unknown nodes → three equations.

4. Assign currents to all the branches connected to each of the unknown nodes.

The directions that you choose for the currents in each branch are not important.

5. Use KCL to relate the currents flowing in and out of each node.

   There will be one equation for each node.

6. Use Ohm's law to relate the currents to the node voltages on either side the resistors.

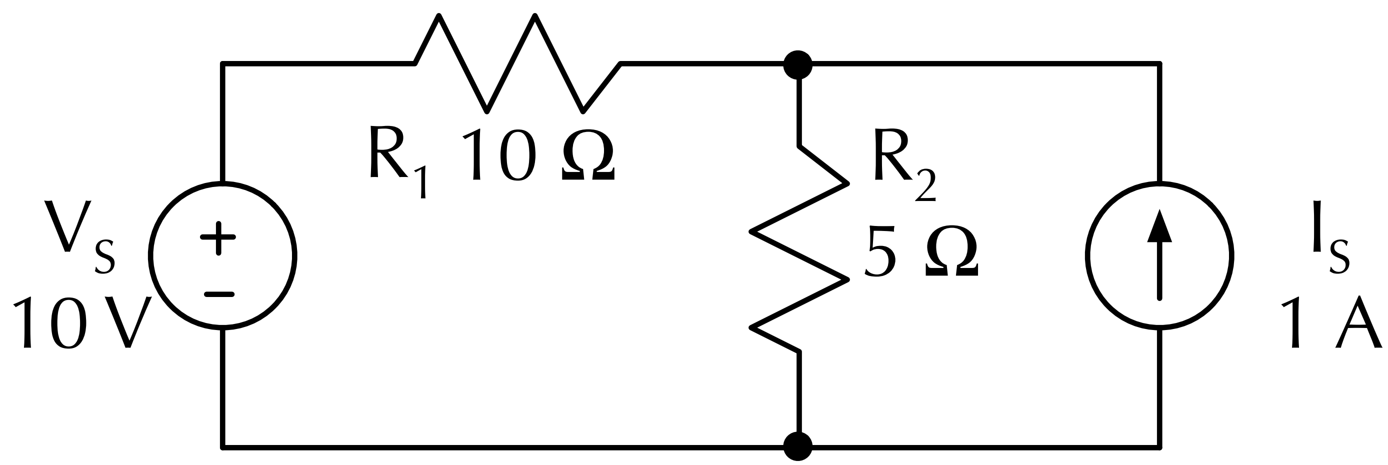
   At this point, it is absolutely essential that you follow the current direction - voltage polarity relationship for resistors — the current flows in the direction of the voltage drop. If you do this part wrong, there is no way that you can get the correct answers.

7. There is now a set of *n* equations relating the *n* unknown voltages in the circuit. Use your favorite algebra techniques to solve for the node voltages.

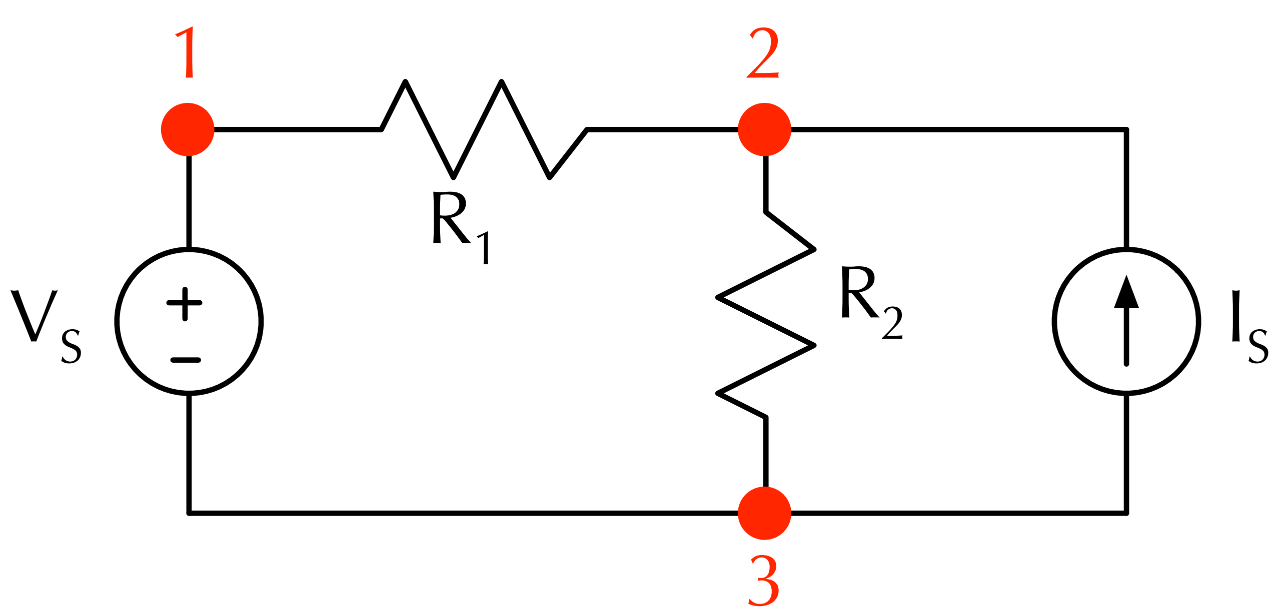
   The circuit analysis is done – the rest is math. If there is more than one variable and one equation, then you can use variable substitution, matrix algebra, or numerical techniques (via your calculator or matlab) to find the answers.

example of the method

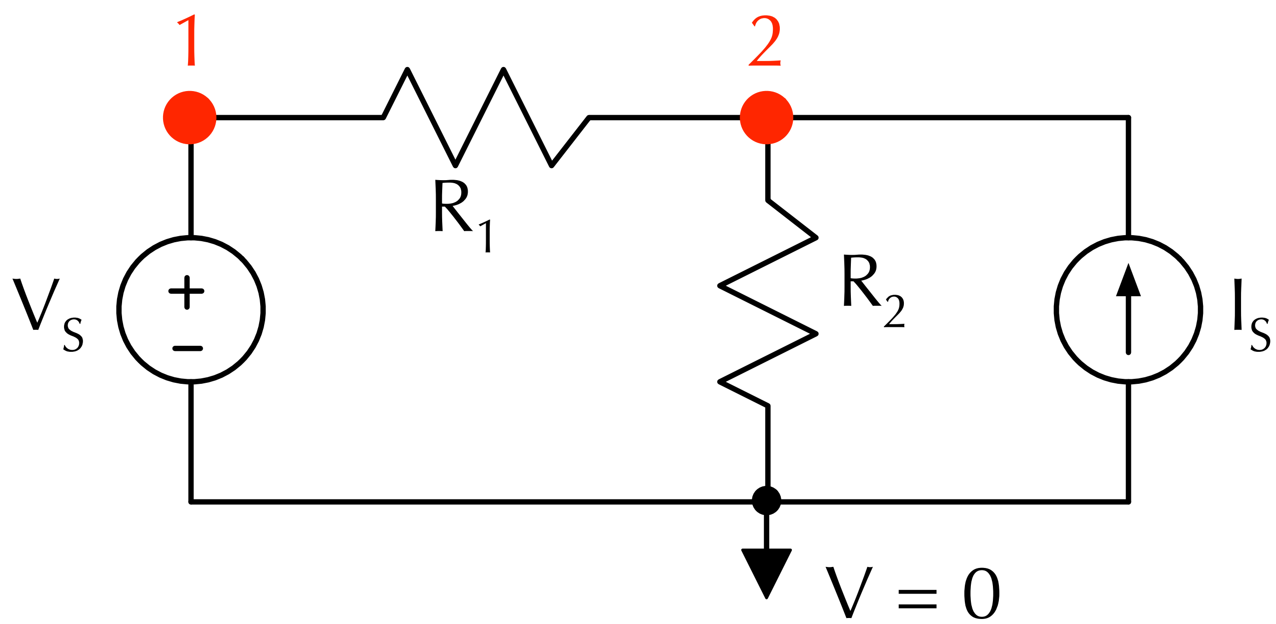
As always, the best way to understand a new concept is to look at some examples. Consider the circuit below.



1. Identify all of the nodes

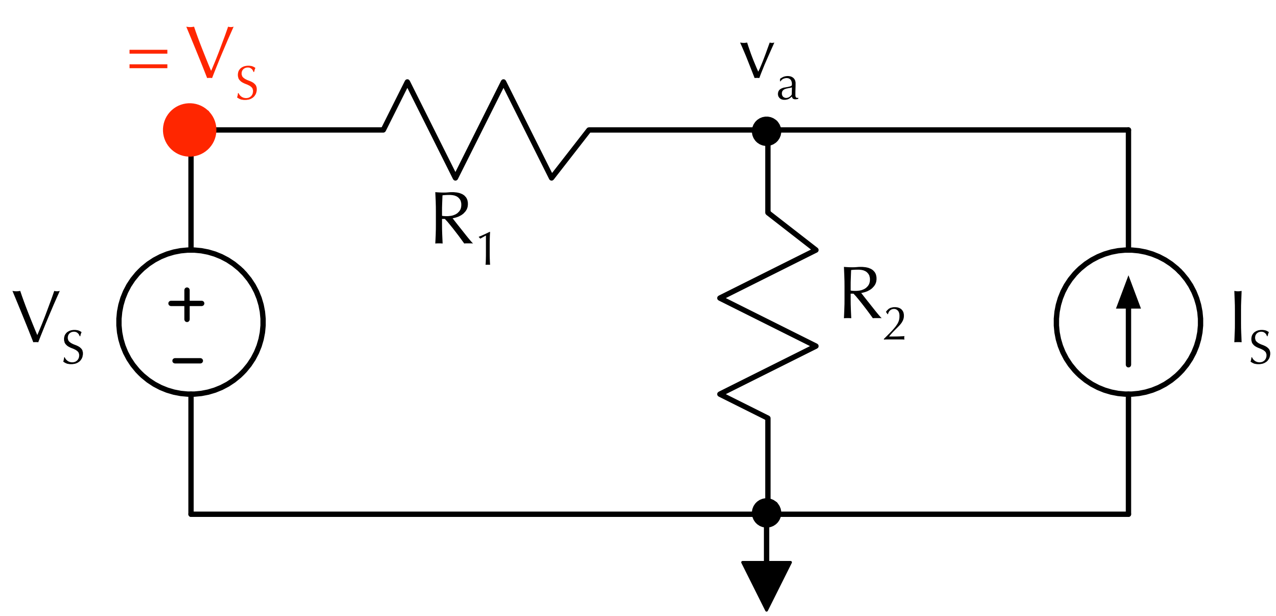


2. Choose one node to serve as ground.

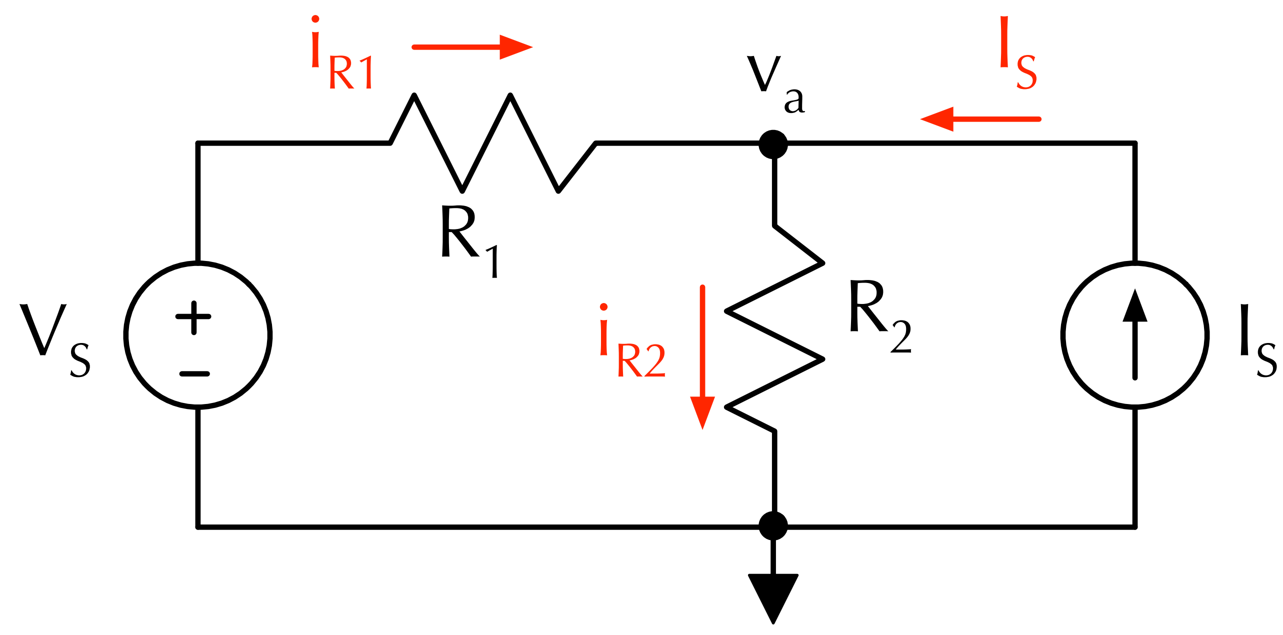


3. Assign variables naming the unknown voltages to each of the other nodes.

3a. Identify any nodes where the voltage is known due to a voltage source.



4. Assign currents to all the branches connected to each of the unknown nodes.



5. Use KCL to relate the currents flowing in and out of each node.

$$i_{R1} + I_S = i_{R2}$$

6. Use Ohm's law to relate the currents to the node voltages on either side the resistors.

$$i_{R1} = \frac{V_S - v_a}{R_1} \qquad i_{R2} = \frac{v_a - 0}{R_2}$$

$$\frac{V_S - v_a}{R_1} + I_S = \frac{v_a}{R_2}$$

7. Solve it.

$$\frac{V_S - v_a}{R_1} + I_S = \frac{v_a}{R_2}$$

$$V_S - v_a + R_1 I_S = \frac{R_1}{R_2} v_a$$

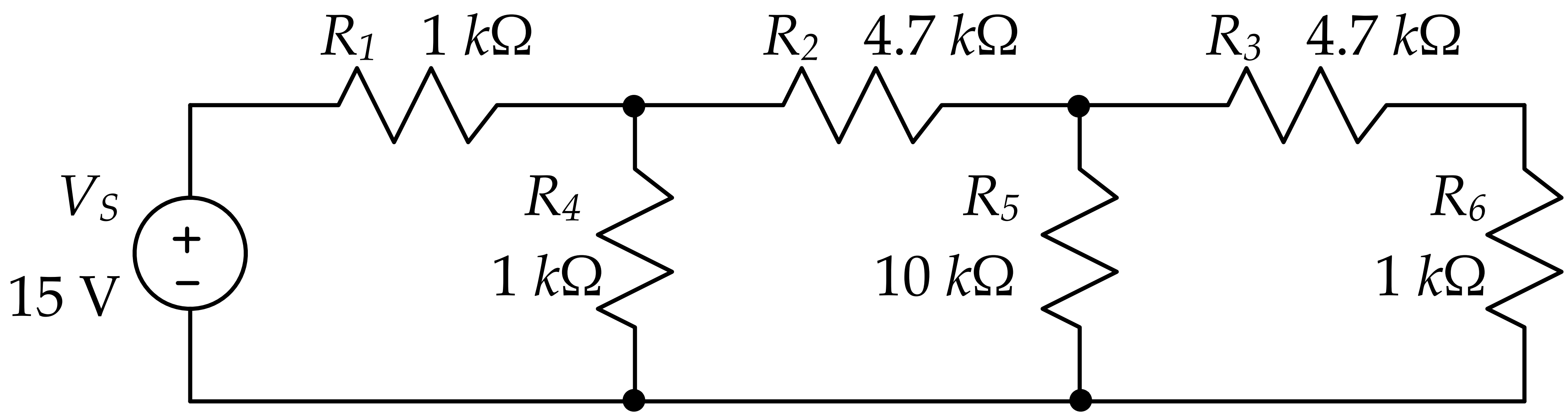
$$V_S + R_1 I_S = \left(1 + \frac{R_1}{R_2}\right) v_a$$

$$v_a = \frac{V_S + R_1 I_S}{1 + \frac{R_1}{R_2}}$$

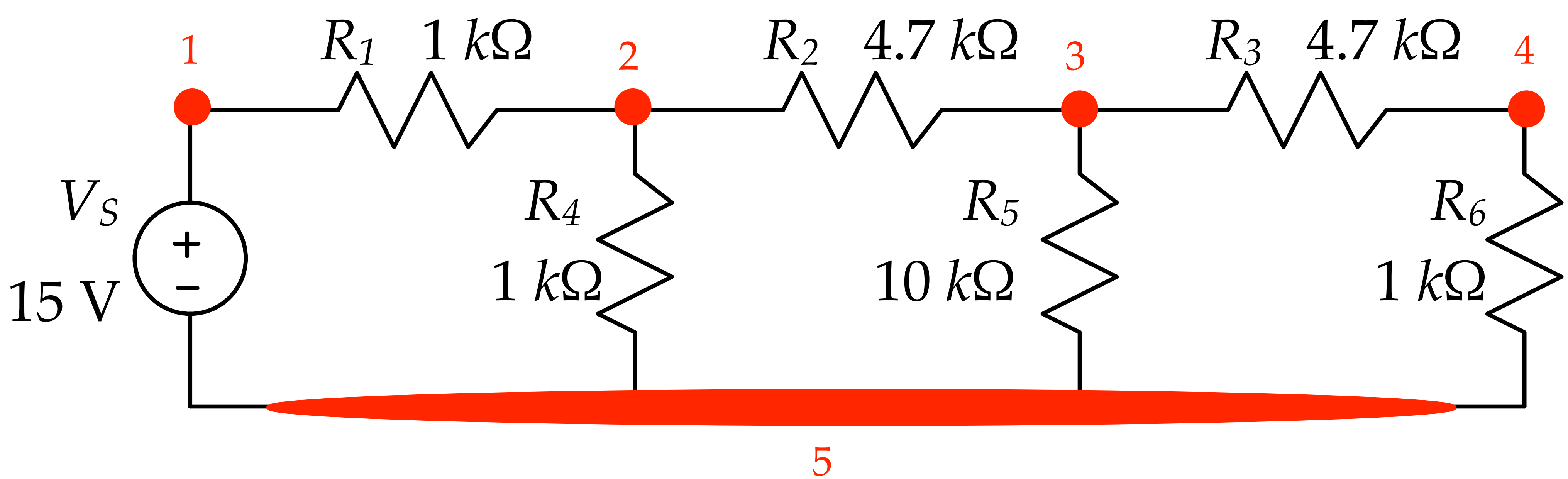
$$v_a = \frac{10V + (10\Omega)(1A)}{1 + \frac{10\Omega}{5\Omega}} = 6.67V$$

another example

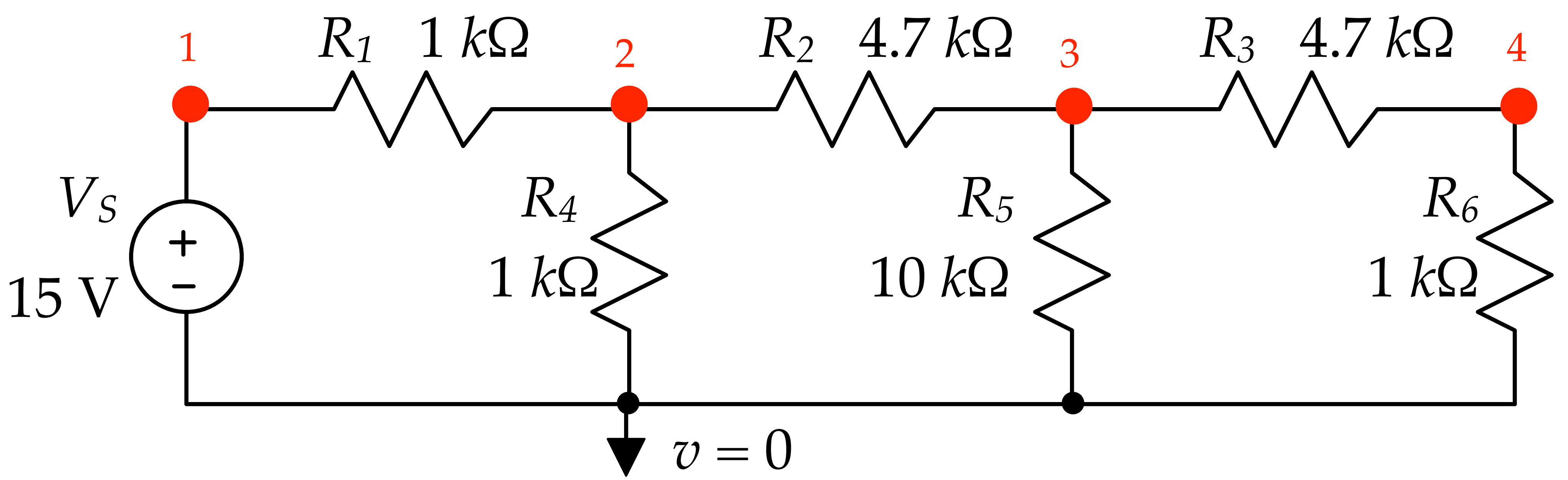
This circuit may be somewhat familiar



1. Identify all of the nodes

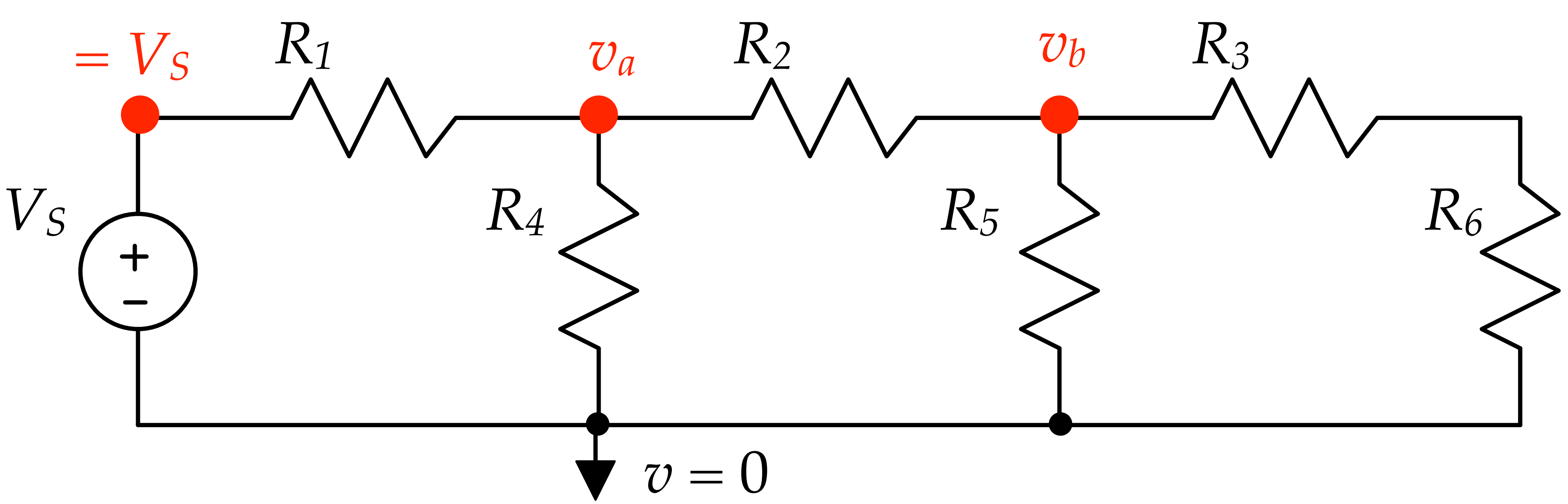


2. Choose one node to serve as ground.



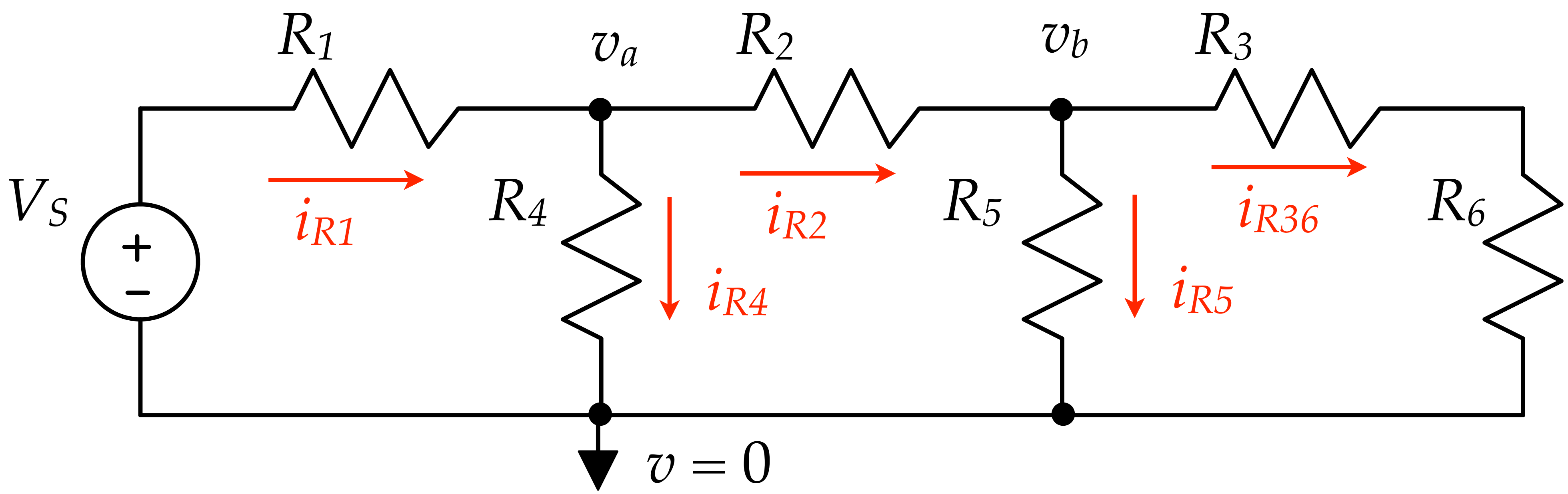
3. Assign variables naming the unknown voltages to each of the other nodes.

3a. Identify any nodes where the voltage is known due to a voltage source.



Don't really need to distinguish this node, since it is in the middle of a single branch.

4. Assign currents to all the branches connected to each of the unknown nodes.



5. Use KCL to relate the currents flowing in and out of each node.

$$i_{R1} = i_{R2} + i_{R4} \qquad i_{R2} = i_{R5} + i_{R36}$$

6. Use Ohm's law to relate the currents to the node voltages on either side the resistors.

$$\frac{V_S - v_a}{R_1} = \frac{v_a - v_b}{R_2} + \frac{v_a - 0}{R_4} \qquad \frac{v_a - v_b}{R_2} = \frac{v_b - 0}{R_5} + \frac{v_b - 0}{R_3 + R_6}$$

7. Solve it. First rewrite the two equations.

$$\left[1+\frac{R_1}{R_2}+\frac{R_1}{R_4}\right]v_a-\frac{R_1}{R_2}v_b=V_S$$

$$-v_a+\left[1+\frac{R_2}{R_5}+\frac{R_2}{R_3+R_6}\right]v_b=0$$

Then insert the numbers:

$$\left[1+\frac{1k\Omega}{4.7k\Omega}+\frac{1k\Omega}{1k\Omega}\right]v_a-\frac{1k\Omega}{4.7k\Omega}v_b=15V \longrightarrow 2.21v_a-0.213v_b=15V$$

$$-v_a+\left[1+\frac{4.7k\Omega}{10k\Omega}+\frac{4.7k\Omega}{5.7k\Omega}\right]v_b=0 \longrightarrow -v_a+2.29v_b=0$$

Solve to give the node voltages:

v_a =7.07 V and v_b = 3.09 V

Once the voltages are known, all of the currents can be determined. (Compare these results to what you obtained in Lab 1.)