Waveforms in Matlab

1 Sampled Waveforms

Signals like speech, music, sensor outputs, etc., are broadly classified as continuous-time (CT) or discrete-time (DT), depending on whether the times for which the signal is defined are continuous or discrete. Correspondingly, a CT waveform is referred to as $s(t)$ or $x(t)$, etc., where t is a (continuous) real number, and a DT waveform is denoted as s_n, $s[n]$ or x_n, $x[n]$, etc., where n is a (discrete) integer.
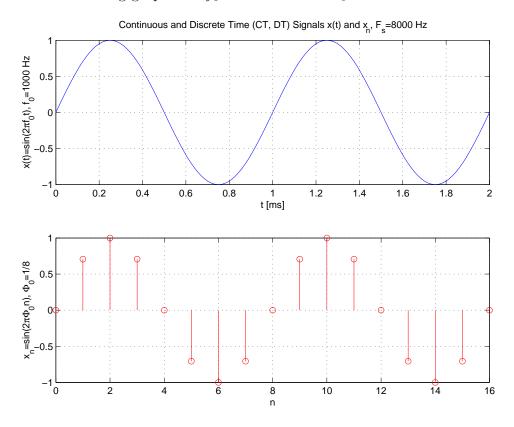
Example: CT and DT sine with frequency f_0. A CT sine waveform with amplitude 1 and frequency f_0 in Hz can be written as

$$x(t) = \sin(2\pi f_0 t) .$$

To obtain a DT sine waveform with the same parameters, $x(t)$ can be sampled at times $t = nT_s = n/F_s$, where $F_s = 1/T_s$ is the sampling frequency in Hz. This yields

$$x(nT_s) = x_n = \sin(2\pi f_0 n T_s) = \sin(2\pi f_0 n/F_s) = \sin(2\pi \Phi_0 n) ,$$

where $\Phi_0 = f_0/F_s$ is a normalized (with respect to F_s) frequency. The signals $x(t)$ and x_n are shown in the following graphs for $f_0 = 1000$ Hz and $F_s = 8000$ Hz.

Continuous and Discrete Time (CT, DT) Signals x(t) and x_n, F_s=8000 Hz

A soundcard in a computer, for example, performs the process of sampling for all signals that are applied to its input. The reverse process, which is called interpolation, is then performed by the soundcard when a DT signal is played back through a speaker or headphones, both of which require a CT signal.

2   Waveform Parameters

Waveforms are often characterized (partially) in terms of their amplitude parameters. Let $x(t)$ be a CT waveform. Then the **minimum** and the **maximum amplitude values** of $x(t)$ are defined as

$$X_{\min} = \min_t x(t) , \qquad \text{and} \qquad X_{\max} = \max_t x(t) .$$

The **peak-to-peak value** describes the maximum excursion of $x(t)$ and is defined as

$$X_{pp} = X_{\max} - X_{\min} .$$

Note that X_{pp} is always positive. The **peak value** is the maximum deviation of $x(t)$ from zero. It is defined as

$$X_p = \max\{|X_{\min}|, |X_{\max}|\} ,$$

and it is also always positive.

The **average value** of $x(t)$ is defined as

$$X_{avg} = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t) \, dt ,$$

if the limit exists. If $x(t)$ is periodic with period T_0 then this simplifies to
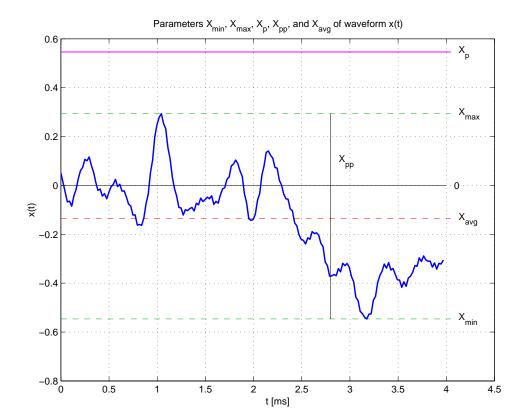
$$X_{avg} = \frac{1}{T_0} \int_{T_0} x(t) \, dt ,$$

where the integral can be taken over any convenient time interval of length T_0.

The graph on the next page shows $X_{\min}$, $X_{\max}$, X_{pp}, X_p, and X_{avg} for an example waveform $x(t)$.

Another quantity of interest is the average power carried by a waveform $x(t)$. **Instantaneous power** is defined as $p(t) = v(t)\, i(t)$. Using Ohm's law, $v(t) = R\,i(t)$ or $i(t) = v(t)/R$. Thus, the instantaneous power delivered to a resistor R is

$$p(t) = \frac{v^2(t)}{R} = R\,i^2(t).$$

Note that $p(t)$ is proportional to the square of the voltage and current waveforms.

Parameters X_{min}, X_{max}, X_p, X_{pp}, and X_{avg} of waveform x(t)

The average power delivered to R is

$$P_{avg} = \lim_{T \to} \frac{1}{2T} \int_{-T}^{T} p(t) \, dt \, ,$$

if the limit exists. If $p(t)$ is periodic with period T_0, then

$$P_{avg} = \frac{1}{T_0} \int_{T_0} p(t) \, dt \, .$$

This motivates the definition of the **root-mean-square value** as (if the limit exists)

$$X_{rms} = \sqrt{\lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x^2(t) \, dt} \, ,$$

to characterize the average power carried by $x(t)$. If $x(t)$ is periodic with period T_0, then the definition of X_{rms} simplifies to

$$X_{rms} = \sqrt{\frac{1}{T_0} \int_{T_0} x^2(t) \, dt} \, ,$$

where the integral is taken over any interval of lengh T_0.

3

3 Waveforms in Matlab

Whenever a digital computer is used to record, process, and/or generate CT waveforms, the internal representation of these waveforms will be as DT waveforms by necessity. The sampling theorem asserts that "as long as the sampling frequency F_s is at least twice the highest frequency contained in the original CT waveform $x(t)$, $x(t)$ can be recovered uniquely and without loss from its samples $x_n = x(n/F_s)$." As a result, it is very common to use digital signal processing (DSP) for CT waveforms that have been sampled at a high enough sampling rate. A typical example are modern cell phones that generate and process all signals using DSP and only the shifting to and from the carrier frequency of about 1-2 GHz is done using analog electronics and CT waveforms. The rest of this section describes ways to handle CT waveforms in Matlab by working with their DT representations, assuming that the sampling rate has been chosen large enough.

To generate a DT approximation to the CT waveform

$$x(t) = X_0 + A \, \cos(2\pi f_0 t + \theta) \,,$$

in Matlab with sampling frequency F_s, the following Matlab commands can be used (with $F_s = 44100$ Hz, $X_0 = -0.2$, $A = 0.6$, $f_0 = 1000$ Hz, $\theta = -90°$):

```
Fs = 44100;                      %Sampling frequency in Hz
Ts = 1/Fs;                       %Sampling time interval in sec
tlen = 0.5;                      %Signal duration in sec
A = 0.6;                         %Amplitude of sinusoid
f0 = 1000;                       %Frequency of sinusoid in Hz
theta = -90;                     %Phase of sinusoid in degrees
X0 = -0.2;                       %DC offset of sinusoid

tt = [0:round(tlen*Fs)-1]/Fs;  %Time axis
xt = X0 + A*cos(2*pi*f0*tt+(pi/180)*theta);
                                 %DT approximation for x(t)
```
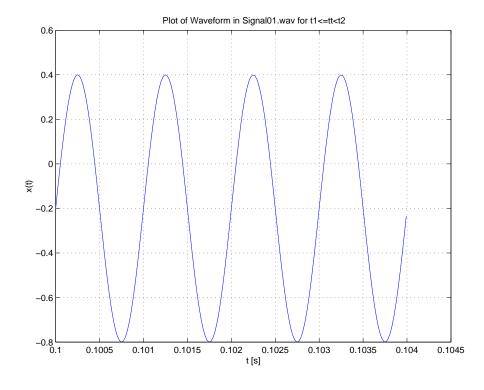
This creates a signal `xt` of 0.5 sec duration. You can listen to this signal and/or write it to a `wav` file using the Matlab commands:

```
sound(xt,Fs)                     %Play x(t) through soundcard
wavwrite(xt,Fs,16,'Signal01.wav');
                                 %Write x(t) to 'Signal01.wav'
```

Note that the amplitude range for 16-bit `wav` files is limited to the range $-1 \le x(t) < +1$. To read a `wav` file and plot the waveform in a graph you can use the Matlab commands:

```
[xt Fs] = wavread('Signal01.wav');
                                %Read wav file 'Signal01.wav'
tt = [0:length(xt)-1]/Fs;       %Create time axis
plot(tt,xt,'-b')                %Plot x(t) versus tt
grid                            %Add a grid
xlabel('t [s]')                 %Label x-axis
ylabel('x(t)')                  %Label y-axis
title('Plot of Waveform in Signal01.wav')
```

If you run these commands you will obtain a graph that is all blue for amplitude values between -0.8 and 0.4 (try it!). In general, if a signal is long enough to listen to, then it is too long for a detailed graph. A useful command to reduce the length of the waveform before it is printed is find as shown below:

```
td1 = 0.1;                      %Start of display time in sec
td2 = 0.1+4e-3;                 %End of display time in sec
ix = find(tt>=td1&tt<td2);      %Indexes for display
plot(tt(ix),xt(ix),'-b')        %Plot x(t) versus t1<=tt<t2
grid                            %Add a grid
xlabel('t [s]')                 %Label x-axis
ylabel('x(t)')                  %Label y-axis
title('Plot of Waveform in Signal01.wav for t1<=tt<t2')
```

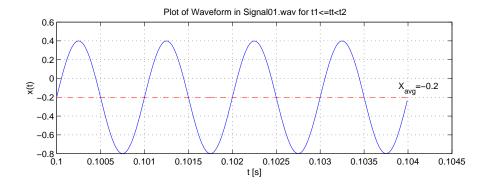The resulting graph is shown in the following figure.

Finding $X_{\min}$ and $X_{\max}$ for a waveform like `Signal01.wav` in Matlab is fairly straightforward. Just use the Matlab commands `min(xt)` and `max(xt)`. The quantities X_{pp} and X_p can then be readily computed. To computation of X_{avg} and X_{rms} requires a little more explanation because of the integrations which need to be approximated by sums because only a sampled version of $x(t)$ is available. If $x(t)$ is available from t_1 to t_2, the average value for that time interval is computed as

$$X_{avg} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} x(t)\, dt \approx \frac{1}{t_2 - t_1} \sum_{n=n_1}^{n_2-1} x(nT_s)\, T_s = \frac{1}{n_2 - n_1} \sum_{n=n_1}^{n_2-1} x(nT_s) \, ,$$

where $n_1\, T_s = t_1$, $n_2\, T_s = t_2$, $x(nT_s) = x_n$, and $T_s = 1/F_s$. In Matlab the following command can be used, assuming $x(t)$ is represented by the vector `xt` of length `length(xt)` (corresponding to $n_2 - n_1$):

```
Xavg = sum(xt)/length(xt);     %Average value of xt
```

The following labeled graph that shows $x(t)$ for $t_1 \leq t < t_2$ and X_{avg}

can be obtained using the Matlab commands:

```
subplot(211)                        %Sub-plot in upper half
plot(tt(ix),xt(ix),'-b',tt(ix),Xavg*ones(size(xt(ix))),'--r')
                                    %Plot x(t) and Xavg vs t
grid                                %Add a grid
xlabel('t [s]'),ylabel('x(t)')      %Label x,y-axis
title('Plot of Waveform in Signal01.wav for t1<=tt<t2')
text(tt(ix(end))-1e-4,Xavg+0.1,['X_{avg}=' num2str(Xavg)])
                                    %Label Xavg line
```

As always, if there is a *command* in Matlab which you do not know or for which you need to review the syntax, type `help` *command* at Matlab's command prompt.

The computation of X_{rms} from `xt` in Matlab is very similar to the computation of X_{avg} and is left as an exercise.

Last revised: 09-08-11, PM.