

Analysis of Algorithms 1 (Fall 2015) Istanbul Technical University Computer Eng. Dept.

Chapter 9 Medians and Order Statistics

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Order Statistics

- The **i th order statistic** of a set of n elements is the **i th smallest element**.
- Minimum: first order statistic ($i = 1$),
- Maximum: n th order statistic ($i = n$).
- **Median**,: "halfway point" of the set.
 - Odd n : median occurs at $i = (n + 1)/2$
 - Even n :
 - lower median at $\text{floor}((n + 1)/2)$
 - Upper median at $\text{ceil}((n + 1)/2)$

Selection Problem

- **Input:** A set A of n (distinct) numbers and a number i , with $1 \leq i \leq n$.
- **Output:** The element $x \in A$ that is larger than exactly $i - 1$ other elements of A .
- can be solved in $O(n \lg n)$ time, since we can sort the numbers using heapsort or merge sort and then simply index the i th element in the output array
- But there are faster methods.

Minimum and Maximum

The best algorithm has complexity of $\theta(n)$

Easiest algorithm:

- MINIMUM(A)
- 1 $\text{min} \leftarrow A[1]$
- 2 for $i \leftarrow 2$ to $\text{length}[A]$
- 3 do if $\text{min} > A[i]$
- 4 then $\text{min} \leftarrow A[i]$
- 5 return min

Randomized Select

- $\text{RANDOMIZED-SELECT}(A, p, r, i)$
- 1 **if** $p = r$
- 2 **then return** $A[p]$
- 3 $q \leftarrow \text{RANDOMIZED-PARTITION}(A, p, r)$
- 4 $k \leftarrow q - p + 1$
- 5 **if** $i = k$ *▷ the pivot value is the answer*
- 6 **then return** $A[q]$
- 7 **elseif** $i < k$
- 8 **then return** $\text{RANDOMIZED-SELECT}(A, p, q - 1, i)$
- 9 **else return** $\text{RANDOMIZED-SELECT}(A, q + 1, r, i - k)$

expected time of RANDOMIZED-SELECT is $\Theta(n)$.

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$\Theta(n)$

$$T(n) = T(n/b) + \Theta(n)$$

$$3. f(n) = W\left(n^{\log_b a + \epsilon}\right) \quad \text{and} \quad af(n/b) \leq cf(n),$$

$$T(n) = \Theta(n) \quad // \text{Master Thm, Case 3}$$

$$\text{for } c < 1 \quad \text{and} \quad n > n_0$$

$$\square T(n) = O(f(n))$$