RECITATION 1
ANALYSIS OF ALGORITHMS II

2016 SPRING

PROBLEM 1

Suppose you have algorithms with the five running times listed below. (Assume these are the exact running times.) How much slower do each of these algorithms get when you (a) double the input size, or (b) increase the input size by one?

(a) n^2

(b) n^3

(c) $100n^2$

(d) $n \log n$

(e) 2^n

PROBLEM 1

- When the input size is doubled, the algorithms gets slower by

$(2n)^2$ → the factor of 4

$(2n)^3$ → the factor of 8

$100x(2n)^2$ → the factor of 4

2n log2n → a factor of 2, plus an additive 2n

2^{2n} → the square of previous running time

PROBLEM 1

- When the input size is increased by additive one, the algorithms get slower by

$(n+1)^2 \rightarrow$ an additive $2n+1$

$(n+1)^3 \rightarrow$ an additive $3n^2+3n+1$

$100x(n+1)^2 \rightarrow$ an additive $200n+100$

$(n+1)\log(n+1) \rightarrow$ an additive

$\log(n+1)+ n[\log(n+1) - \log n]$

$2^{n+1} \rightarrow$ a factor of 2

PROBLEM 2- GALE SHAPLEY

There are 3 men, called A, B, C and 3 women, called X, Y, Z, with the following preference lists:

$$\begin{array}{llcll} \text{For } A: & X > Y > Z & \qquad & \text{For } X: & C > A > B \\ \text{For } B: & X > Y > Z & & \text{For } Y: & A > C > B \\ \text{For } C: & Y > X > Z & & \text{For } Z: & A > B > C \end{array}$$

Find the matchings given by the men proposing and women proposing Gale-Shapley algorithm for these preferences.

(ii) Find a set of preference list for 3 men and women such that in the men-proposing Gale-Shapley algorithm no man gets his first preference.

PROBLEM 2- GALE SHAPLEY

For A: $X > Y > Z$
For B: $X > Y > Z$
For C: $Y > X > Z$

For X: $C > A > B$
For Y: $A > C > B$
For Z: $A > B > C$

Men-proposing algorithm

- A proposes X, A is engaged with X
- B proposes X, B is rejected
- B proposes Y. B is engaged with Y
- C proposes Y. Y prefers C to her fiancee. C is engaged Y
- B proposes Z. B is engaged with Z.

STABLE MATCHES:

A-X B-Z C-Y

PROBLEM 2- GALE SHAPLEY

For A: $X > Y > Z$	For X: $C > A > B$
For B: $X > Y > Z$	For Y: $A > C > B$
For C: $Y > X > Z$	For Z: $A > B > C$

- Women-proposing

- X proposes C. X is engaged with C.
- Y proposes A. Y is engaged with A
- Z proposes A. A prefers Y instead of Z
- Z proposes B. Z is engaged with B

STABLE MATCHES

X-C Y-A Z-B

PROBLEM 2-GALE SHAPLEY

(ii) Find a set of preference list for 3 men and women such that in the men-proposing Gale-Shapley algorithm no man gets his first preference.

For A : $X > Y > Z$	For X : $C > A > B$
For B : $X > Y > Z$	For Y : $B > C > A$
For C : $Y > X > Z$	For Z : $A > B > C$

PROBLEM 2-GALE SHAPLEY

- Men-proposing

- A proposes X. A is engaged with X.
- B proposes X. X prefers A to B.
- B proposes Y. B is engaged with Y.
- C proposes Y. Y prefers B to C.
- C proposes X. X prefers C to A. C is engaged with X.
- A proposes Y. Y prefers B to A.
- A proposes Z . A is engaged with Z.

STABLE MATCHES

A-Z B- Y C-X

PROBLEM 3 – GS Contradiction

Consider a town with n men and n women seeking to get married to one another. Each man has a preference list that ranks all the women, and each woman has a preference list that ranks all the men.

The set of all $2n$ people is divided into two categories: *good* people and *bad* people. Suppose that for some number k, $1 \leq k \leq n - 1$, there are k good men and k good women; thus there are $n - k$ bad men and $n - k$ bad women.

Everyone would rather marry any good person than any bad person. Formally, each preference list has the property that it ranks each good person of the opposite gender higher than each bad person of the opposite gender: its first k entries are the good people (of the opposite gender) in some order, and its next $n - k$ are the bad people (of the opposite gender) in some order.

PROBLEM 3 – GS Contradiction

- Show that in every stable matching, every good man is married to a good woman.
- Use contradiction

ASSUMPTION:

There would exist a match that a good woman is married to a bad man.

✓ Such w’ is good woman who is married to a bad man. Furthermore m is good man that is marries to bad woman.

PROBLEM 3 – GS Contradiction

- m and w' prefers the other to their current partner.
- So (m,w') causes unstable match.
- They prefer each other instead of their current partner.
- This contradict the assumption

M is stable, when m is married to a bad woman and w' is married to bad man.

PROBLEM 4- COMPLEXITY ANALYSIS

Order the following functions by growth rate. Explain in detail how you decided on the ordering.

$$n^{\log n} \quad e^{\log n} \quad \log(n+n^2) \quad n^{5n} \quad 2^{3n} \quad 3^{2n} \quad n^{1000}\log n$$

Logarithms < Polynomials < Exponential with constant base < Exponentials with base n

→ $\log(n+n^2) < n < n^{1000}\log n,\ n^{\log n},\ 8^n,\ 9^n < n^{5n}$

→ $8^n < 9^n$ (Base 8 is smaller than 9)

PROBLEM 4- COMPLEXITY ANALYSIS

Order the following functions by growth rate. Explain in detail how you decided on the ordering.

$$n^{\log n} \quad e^{\log n} \quad \log(n+n^2) \quad n^{5n} \quad 2^{3n} \quad 3^{2n} \quad n^{1000}\log n$$

In order to compare some items we can use their limits

$$\lim_{n\to\infty}\frac{n^{1000}\log n}{n^{\log n}} < \lim_{n\to\infty}\frac{n^{1001}}{n^{\log n}} = \lim_{n\to\infty}\frac{1}{n^{\log n - 1001}} = \frac{1}{\infty} = 0$$

Therefore $n^{1000}\log n < n^{\log n}$

PROBLEM 4- COMPLEXITY ANALYSIS

Order the following functions by growth rate. Explain in detail how you decided on the ordering.

$n^{\log n}$ $e^{\log n}$ $\log(n + n^2)$ n^{5n} 2^{3n} 3^{2n} $n^{1000} \log n$

In order to compare some items we can use their logs

$\log(n^{\log n}) = \log^2 n \rightarrow \log(\log^2 n) = 2 \log \log n = O(\log \log n)$

$\log(8^n) = n \log 8 \rightarrow \log(n \log 8) = \log(n)$

Therefore $n^{\log n} < 8^n$

$\log(n+n^2) < n < n^{1000}\log n < n^{\log n} < 8^n < 9^n < n^{5n}$