IMPORTANT: Besides your calculator and the sheets you use for calculations you are only allowed to have an A4 sized "copy sheet" during this exam. Notes, problems and alike are not permitted. Please submit your "copy sheet" along with your solutions. You may get your "copy sheet" back after your solutions have been graded. *Do not forget to write down units and convert units carefully!*

ELE222E INTRODUCTION TO ELECTRONICS (21727)

Midterm Exam #2 ✎ 24 April 2006 ⌚ 10.00-12.00

İnci ÇİLESİZ, PhD, Özgür ATEŞ, MSE

---

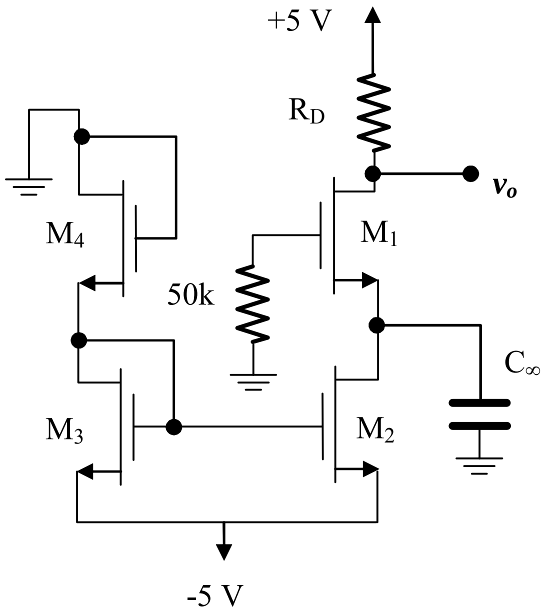
1. Below you see a constant current source implemented by MOS transistors. The transistors M_2, M_3, and M_4 form the constant current source feeding the common source amplifier built with M_1.

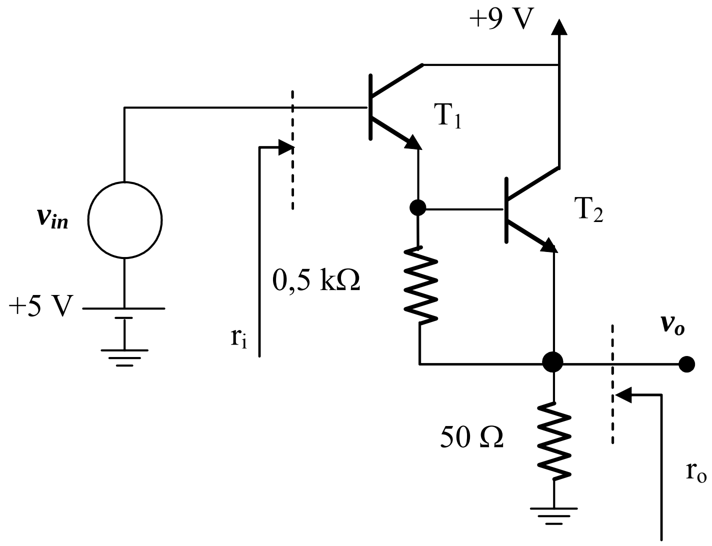
   M_2, M_3, and M_4 are all identical with the following parameters:
   $\frac{1}{2}\mu_n C_{ox}\frac{W}{L} = 0{,}1 mA/V^2$, and $V_{tn} = 1\ V$.
   The parameters for M_1 are:
   $\frac{1}{2}\mu_n C_{ox}\frac{W}{L} = 0{,}2 mA/V^2$, $V_{A1} = \infty$, and $V_{tn1} = 1\ V$.

   a. Determine the currents and voltages in the constant current source. (25 points)

   b. Determine the value of the drain resistance R_D. (10 points)

   c. Find the voltage gain from gate to drain with the AC signal source connected to the gate of M_1. (15 points)





2. For the circuit shown on the left, transistor parameters for T_1 and T_2 are $V_{A1} = V_{A2} = \infty$, $h_{fe1} = h_{fe2} = h_{FE1} = h_{FE2} = h_{fe} = h_{FE} = \beta = 100$, $V_T = 25$ mV and $|V_{BE}| = 0{,}6$ V.

   a. Determine the DC collector current for each BJT. (20 points)

   b. Find the small signal voltage gain v_o/v_{in}. (15 points)

   c. Determine the input and output resistances. (15 points)

GOOD LUCK!

SOLUTIONS

1. Since the reference current is the same in transistors M_3 and M_4,

$$\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS3}-V_{tn3})^2=\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS4}-V_{tn4})^2$$

Additionally, we see from the sketch that M_2 and M_3 operate in saturation because for each transistor, the gate is shorted to the drain.

$V_{GS3}+V_{GS4}=\left[0-(-5V)\right]=5V$, thus for identical transistors $V_{GS3}=V_{GS4}=2{,}5V$

Since M_2 and M_3 are identical transistors with $V_{GS3}=V_{GS2}$, the bias current is

$$I_4=I_3=I_{REF}=I_2=I_1=I_Q=\left[\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\right]_2(V_{GS3}-V_{tn2})^2=0{,}225mA$$

To find V_{GS1} we use $I_Q=\left[\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\right]_1(V_{GS1}-V_{tn1})^2=0{,}225mA$ and we obtain

$$V_{GS1}=\pm\sqrt{\frac{0{,}225mA}{0{,}2mA/V^2}}+1V=\begin{cases}2{,}06V\\0{,}06V\end{cases}.$$

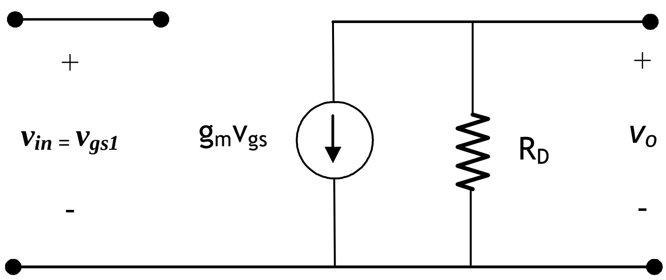
From these two solutions only $V_{GS1}=2{,}06$ V satisfies the condition for channel generation.

Since $V_{DS2}=-(-5V)-V_{GS1}=5-2{,}06=2{,}94V>V_{GS2}-V_{tn2}=2{,}5-1=1{,}5V$, M_2 is biased in saturation region.

For M1 to operate in saturation $V_{DS1}>V_{GS1}-V_{tn1}=2{,}06-1=1{,}06V$.

Since $V_{G1}=0$ V, $V_{S1}=-2{,}06$V, that is $V_{DS1}=V_{D1}-V_{S1}=V_{D1}+2{,}06V>1{,}06V\Rightarrow V_{D1}>-1V$.

Let's choose $V_{D1}=5V-I_QR_D=0V\Rightarrow R_D=\dfrac{5V}{0{,}225mA}=22k2$

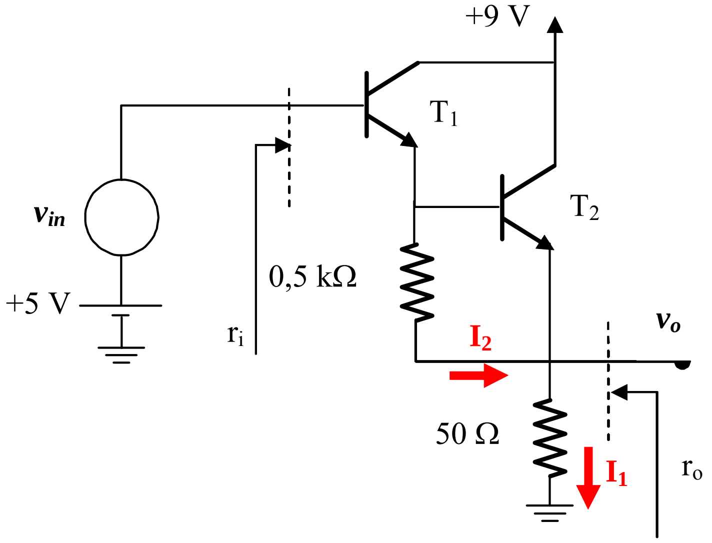


$$A_v=\frac{v_o}{v_{in}}=-g_{m1}R_D$$

$$g_{m1}=\sqrt{\left[2\mu_n C_{ox}\frac{W}{L}\right]_1 I_Q}=0{,}212mA/V$$

Thus

$$A_v=\frac{v_o}{v_{in}}=-g_{m1}R_D=-4{,}71(V/V)$$



2. Since $V_{B1} = 5$ V , $V_{E1} = V_{B2} = 4{,}4$V and $V_{E2} = 3{,}8$ V.

Using the notation as is on the left,

$$\underline{\underline{I_1}} = \frac{V_{E2}}{50\Omega} = \underline{\underline{76mA}}$$ and

$$\underline{\underline{I_2}} = \frac{V_{BE2}}{0{,}5k} = \frac{0{,}6V}{0{,}5k} = \underline{\underline{1{,}2mA}}$$

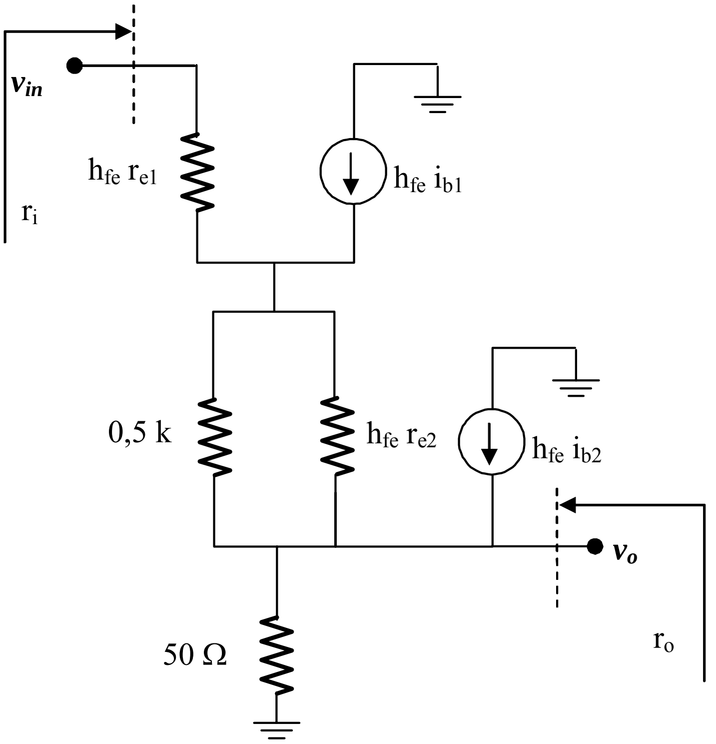
Since $I_{E2} + I_2 = I_1$,

$$I_{E2} = I_1 - I_2 = 74{,}8mA$$

$$\Rightarrow \underline{\underline{I_{C2}}} = I_{E2}\frac{h_{FE}}{h_{FE}+1} = \underline{\underline{74{,}1mA}}$$

Similarly, $I_{E1} - I_{B2} = I_2$, leading to $I_{E1} = I_{B2} + I_2 = 1{,}94mA \Rightarrow \underline{\underline{I_{C1}}} = I_{E1}\frac{h_{FE}}{h_{FE}+1} = \underline{\underline{1{,}92mA}}$

Thus, $\underline{\underline{r_{e1}}} = \frac{V_T}{I_{C1}} = \frac{25mV}{1{,}92mA} = \underline{\underline{13{,}02\Omega}}$ and $\underline{\underline{r_{e2}}} = \frac{V_T}{I_{C2}} = \frac{25mV}{74{,}1mA} = \underline{\underline{0{,}34\Omega}}$



Looking at the small-signal equivalent circuit on the left,

$h_{fe}\, r_{e1} = 1301\ \Omega$

$h_{fe}\, r_{e2} = 34\ \Omega$

I the middle part of the circuit

$R_{eq} = 0{,}5\text{ k} \parallel h_{fe}\, r_{e2} = 31{,}6\ \Omega \approx h_{fe}\, r_{e2}$.

This means most of the emitter current, i.e., $i_{e1} = (h_{fe} + 1)i_{b1}$ flows over $h_{fe}\, r_{e2}$.

That in return means $i_{b2} \approx (h_{fe} + 1)i_{b1}$.

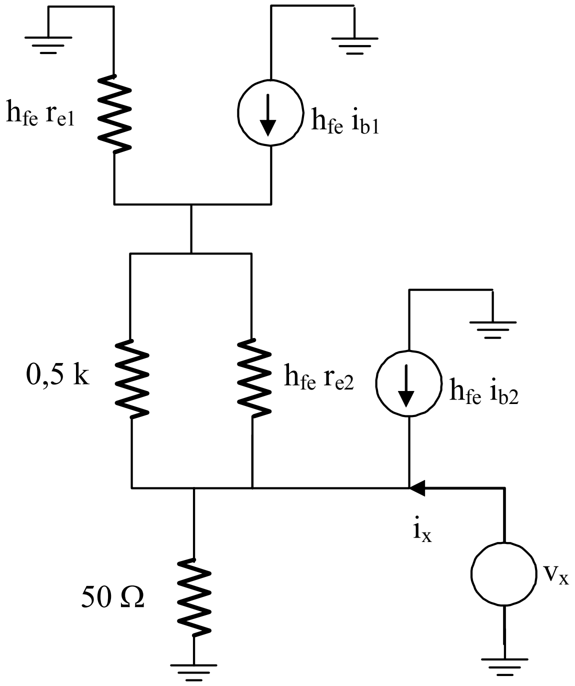
$$v_o = \left[(h_{fe}+1)i_{b1} + (h_{fe}+1)^2 i_{b1}\right]50\ \Omega$$

$$v_{in} = \left[h_{fe}r_{e1} + h_{fe}r_{e2}(h_{fe}+1) + 50\Omega(h_{fe}+1)^2\right]i_{b1}$$

$$\underline{\underline{A_v = \frac{v_o}{v_{in}}}} = \frac{\left[(h_{fe}+1)+(h_{fe}+1)^2\right]50\Omega}{h_{fe}r_{e1} + R_{eq}(h_{fe}+1) + 50\Omega(h_{fe}+1)^2} \simeq \frac{101^2 \cdot 50}{1301 + 31{,}6 \cdot 101 + 50(101)^2} = \underline{\underline{0{,}991}}$$

$$\underline{\underline{r_{in}}} = \frac{v_{in}}{i_{b1}} = h_{fe}r_{e1} + R_{eq}(h_{fe}+1) + 50\Omega(h_{fe}+1)^2 = 1301 + 3192 \cdot 101 + 50 \cdot 101^2 = \underline{\underline{511k35}}$$

To find the output resistance you have to remember how we calculated it for the emitter follower circuits using v_x and i_x. In fact, assume you connect a voltage source v_x in parallel to 50 Ω resistor. The current flowing from v_x is i_x. All independent voltage sources are shorted all independent current



sources are opened. Thus our equivalent small-signal circuit becomes the one shown on the left.

$$v_x = \left[(h_{fe}+1)i_{b1} + (h_{fe}+1)^2 i_{b1} + i_x\right]50\Omega$$

and therefore

$$i_x = \frac{v_x}{50\Omega} - (h_{fe}+1)i_{b1} - (h_{fe}+1)^2 i_{b1}$$

At the same time

$$v_x = -i_{b1}\left[1301 + (h_{fe}+1)R_{eq}\right]$$

Inserting v_x into i_x and reorganization yields

$$\frac{i_x}{v_x} = \frac{1}{50\Omega} + \frac{(h_{fe}+1)}{1301+(h_{fe}+1)R_{eq}} + \frac{(h_{fe}+1)^2}{1301+(h_{fe}+1)R_{eq}}$$

Since $r_o = \dfrac{v_x}{i_x}$, r_o = $50\Omega \;||\; \dfrac{1301+(h_{fe}+1)R_{eq}}{h_{fe}+1} \;||\; \dfrac{1301+(h_{fe}+1)R_{eq}}{\left(h_{fe}+1\right)^2}$

Since $h_{fe}+1 \cong h_{fe}$,

$$r_o \cong 50\Omega \,||\, \frac{h_{fe}r_{e1}+h_{fe}R_{eq}}{h_{fe}} \,||\, \frac{h_{fe}r_{e1}+h_{fe}R_{eq}}{h_{fe}{}^2}$$

$$\Rightarrow \underline{\underline{r_o}} \cong 50\Omega \,||\, (r_{e1}+R_{eq}) \,||\, \frac{r_{e1}+R_{eq}}{h_{fe}} \underline{\underline{= 0{,}44\Omega}}$$