#### **DISCRETE MATHEMATICS**

### 2006 - 2007 Spring Term

#### **RECITATION 1**

#### 16.02.2007

Text book – Excercise 2.2 – Questions: 4,14

#### 4. Simplify following statement

## $p \rightarrow q = \neg p V q$

#### 14. a) Show that $p \rightarrow [q \rightarrow (p \land q)]$ is a tautology

```
= \neg p \ V \ [\neg q \ V \ (p \Lambda q)]
= \neg p \ V \ [(\neg q V p) \Lambda (\neg q V q)]
= \neg p \ V \ [(\neg q V p) \ \Lambda \ T]
= \neg p \ V \ (\neg q V p)
= (\neg p \ V \ p) \ V \ \neg q
= T \ V \ \neg q
= T
```

# b) Show that $(pVq) \rightarrow [q \rightarrow q]$ is a tautology using the result in the previous question.

Rewrite the last result we obtained like  $p' \rightarrow [q' \rightarrow (p' \land q')]$ 

In our current statement, we can apply substition like

$$p' = (pVq)$$
$$q' = q$$

If statement (p'  $\Lambda$  q') equals q, than we can deduce that our initial statement is a tautology

$$(p' \Lambda q') = (pVq) \Lambda q = (p\Lambda q) V (q\Lambda q) = (p\Lambda q) V q$$

 $\begin{array}{ll} p \; V \; (p \Lambda q) \leftrightarrow p & \quad Law \; of \\ p \; \Lambda \; (p V q) \leftrightarrow p & \quad Absorption \end{array}$ 

Using law of absorption

$$(p\Lambda q) V q = q$$

#### c) Test if $(pVq) \rightarrow [q \rightarrow (p \land q)]$ is tautology.

$$= \neg (pVq) \ V \ [\neg qV(p \ \Lambda \ q)]$$

$$= (\neg p\Lambda \neg q) \ V \ [(\neg qVp) \ \Lambda \ (\neg qVq)]$$

$$= (\neg p\Lambda \neg q) \ V \ [(\neg qVp) \ \Lambda \ T]$$

$$= (\neg p\Lambda \neg q) \ V \ (\neg qVp)$$

$$= (\neg p\Lambda \neg q) \ V \ p \ V \ \neg q$$

$$= [(\neg pVp) \ \Lambda \ (\neg qVp)] \ V \ \neg q$$

$$= [\ T \ \Lambda \ (\neg qVp)] \ V \ \neg q$$

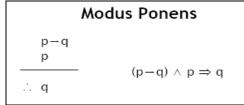
$$= (\neg qVp) \ V \ \neg q$$

$$= \neg qV \ \neg q \ V \ p$$
NOT TAUTOLOGY!

**Text book – Excercise 2.3 – Questions: 4,8** 

# 4. Deduce results from the following verbal statements using Modus Ponens or Modus Tollens

- a) → If Tolga can't fix the computer Tahir will arrive and check the computer
  - → Tolga wasn't able to fix the computer.



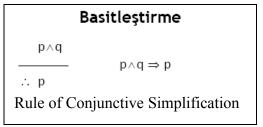
· Tahir arrives and checks the computer. Modus Ponens

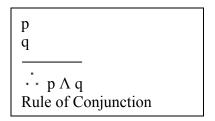
- **b)** → If Ayça has solved the problem she should've get the answer as 137.
  - → Ayça hasn't reached 137.

· Ayça wasn't able to solve the problem. Modus Tollens

# Modus Tollens $\begin{array}{c} p-q \\ \neg q \\ \hline \vdots \quad \neg p \end{array}$ $(p-q) \land \neg q \Rightarrow \neg p$

#### 8. Explain the following inferences.





$$p \rightarrow r \leftrightarrow \neg r \rightarrow \neg p \leftrightarrow \neg p V r$$
  
Contrapositive

10. Show the correctness of the following statements

b) 
$$[p \land (p \rightarrow q) \land (\neg q \lor r)] \rightarrow r$$

 $p \\ (p \to q) \\ \underline{(\neg q \lor r)} \\ \therefore r$ 

## Steps Rules

- 1)  $p \rightarrow q$  Given
- 2) *9* Modus Ponens
- 3)  $\neg q \lor r$  Given
- 4) P Disjunctive Syllogism

#### Ayırıcı Kıyas

 $\frac{\neg p}{\neg p} \qquad (p \lor q) \land \neg p \Rightarrow q$   $\vdots q$ 

Rule of Disjunctive Syllogism

#### Varsayımlı Kıyas

$$\frac{p-q}{q-r}$$

$$(p-q) \land (q-r) \Rightarrow (p-r)$$
Law of the Syllogism

c)

$$p \to q$$

$$\neg q$$

$$\frac{\neg r}{\therefore \neg (p \lor r)}$$

- $\begin{array}{ccc} \textbf{Steps} & \textbf{Rules} \\ p \rightarrow q & \textbf{Given} \end{array}$
- 1)  $P \rightarrow Q$  Given 2)  $\neg Q$  Given
- 3)  $\neg P$  Modus Tollens
- 4)  $\neg r$  Given
- 5)  $\neg p \land \neg r$  Rule of Conjunction
- 6)  $\neg (p \lor r)$  DeMorgan

d)

$p \rightarrow q$	
$r \rightarrow \neg q$	
r	
$\therefore \neg p$	

- Steps Rules
- 1)  $r \rightarrow \neg q$  Given 2)  $q \rightarrow \neg r$  Contrapositive (1)
- 3)  $p \rightarrow q$  Given
- 4)  $p \rightarrow \neg r$  Law of the Syllogism
- $\vec{5}$ )  $\vec{r}$  Given
- **6)**  $\neg P$  Modus Tollens

$$p \wedge q$$

$$p \to (r \wedge q)$$

$$r \to (s \vee t)$$

$$\frac{\neg s}{\therefore t}$$

StepsRules1) 
$$P \wedge q$$
Given2)  $P$ Simplification3)  $r \wedge q$ Modus Ponens4)  $r$ Simplification5)  $S \vee t$ Modus Ponens6)  $\neg S$ Given7)  $t$ Rule of Disjunctive Syllogism

g)

$$p \to (q \to r)$$

$$p \lor s$$

$$t \to q$$

$$\frac{\neg s}{\therefore \neg r \to \neg t}$$

	Steps	Rules
1)	$p \vee s$	Given
2)	$\neg S$	Given
3)	p	Rule of Disjunctive Syllogism
4)	$q \rightarrow r$	Modus Ponens
<b>5</b> )	$t \rightarrow q$	Given
6)	$t \rightarrow r$	Law of the Syllogism
7)	$\neg r \rightarrow \neg t$	Contrapositive (6)