1

z.z. $\mathbb{E}\left[\alpha Z_1 + \beta Z_2\right] = \alpha \mathbb{E}\left[Z_1\right] + \beta \mathbb{E}\left[Z_2\right]$

bew.

$$
\begin{aligned}
\mathbb{E}\left[\alpha Z_1 + \beta Z_2\right] &= \sum_{\omega \in \Omega} \left(\alpha Z_1 + \beta Z_2\right)(\omega) \, \mathbb{P}\left[\{\omega\}\right] \\
&= \sum_{\omega \in \Omega} \left(\alpha Z_1(\omega) + \beta Z_2(\omega)\right) \mathbb{P}\left[\{\omega\}\right] \\
&= \sum_{\omega \in \Omega} \left(\alpha Z_1(\omega) \, \mathbb{P}\left[\{\omega\}\right] + \beta Z_2(\omega) \, \mathbb{P}\left[\{\omega\}\right]\right) \\
&= \sum_{\omega \in \Omega} \alpha Z_1(\omega) \, \mathbb{P}\left[\{\omega\}\right] + \sum_{\omega \in \Omega} \beta Z_2(\omega) \, \mathbb{P}\left[\{\omega\}\right] \\
&= \alpha \sum_{\omega \in \Omega} Z_1(\omega) \, \mathbb{P}\left[\{\omega\}\right] + \beta \sum_{\omega \in \Omega} Z_2(\omega) \, \mathbb{P}\left[\{\omega\}\right] \\
&= \alpha \mathbb{E}\left[Z_1\right] + \beta \mathbb{E}\left[Z_2\right]
\end{aligned}
$$

QED

2

z.z. $\mathbb{E}\left[(Z - c)^2\right] = Var\left[Z\right] + \mathbb{E}\left[Z - c\right]^2$

bew.

$$
\begin{aligned}
\mathbb{E}\left[(Z - c)^2\right] &= \mathbb{E}\left[Z^2 - 2Zc + c^2\right] \\
&= \mathbb{E}\left[Z^2\right] - \mathbb{E}\left[2Zc\right] + \mathbb{E}\left[c^2\right] \\
&= \mathbb{E}\left[Z^2\right] - 2c\,\mathbb{E}\left[Z\right] + c^2 \\
&= \mathbb{E}\left[Z^2\right] - \mathbb{E}\left[Z\right]^2 + \mathbb{E}\left[Z\right]^2 - 2\,\mathbb{E}\left[c\right]\mathbb{E}\left[Z\right] + E\left[c\right]^2 \\
&= Var\left[Z\right] + \mathbb{E}\left[Z - c\right]^2
\end{aligned}
$$

QED

3

z.z. $MSE\left(f, r_T, x_0\right) = Var\left[r_T(x_0)\right] + \text{Bias}(f, r_T, x_0)^2$

bew.

$$
\begin{aligned}
MSE\left(f, r_T, x_0\right) &= \mathbb{E}\left[\left(r_T(x_0) - f(x_0)\right)^2\right] \\
&= Var\left[r_T(x_0)\right] + \mathbb{E}\left[r_T(x_0) - f(x_0)\right]^2 \\
&= Var\left[r_T(x_0)\right] + \text{Bias}(f, r_T, x_0)^2
\end{aligned}
$$