Task 1

zz: $a^T K a \geq 0 \quad \forall a \in \mathbb{R}^n$

Bew

Seien k_1, k_2 Kernels mit $K(x,y) = k_1(x,y) + k_2(x,y)$

Dann ist

$$a^T K a = \sum_{i,j}^{n} a_i k_1(x_i, x_j) a_j + \sum_{i,j}^{n} a_i k_2(x_i, x_j) a_j$$

$$\overset{\text{def.}}{=} \sum_{i,j}^{n} a_i \langle \phi(x_i), \phi(x_j) \rangle_{k_1} a_j + \sum_{i,j}^{n} a_i \langle \phi(x_i), \phi(x_j) \rangle_{k_2} a_j$$

$$= \left\langle \sum_{i=1}^{n} a_i \phi(x_i), \sum_{j=1}^{n} a_j \phi(x_j) \right\rangle_{k_1} + \left\langle \sum_{i=1}^{n} a_i \phi(x_i), \sum_{j=1}^{n} a_j \phi(x_j) \right\rangle_{k_2}$$

$$\overset{i=j}{=} \left\| \sum_{i=1}^{n} a_i \phi(x_i) \right\|_{k_1}^2 + \left\| \sum_{i=1}^{n} a_i \phi(x_i) \right\|_{k_2}^2 \geq 0 \qquad \square$$