Task 2

(1)

zz: $\dfrac{\partial A_{ij}}{\partial W_{kl}} = D_{ik}\,\delta_{jl}$

<u>Bew</u>

$$\frac{\partial A_{ij}}{\partial W_{kl}} = \frac{\partial}{\partial W_{kl}} \sum_{q=1}^{n} D_{iq} W_{qj} = \sum_{q=1}^{n} D_{iq} \frac{\partial W_{qj}}{\partial W_{kl}}$$

$$\overset{q=k}{=} \sum_{q=1}^{n} D_{iq}\, \delta_{qk}\, \delta_{jl} = D_{ik}\, \delta_{jl} \quad \square$$

zz: $\dfrac{\partial \mathcal{E}}{\partial W_{kl}} = (D^T B)_{kl}$

<u>Bew</u>

$$\frac{\partial \mathcal{E}}{\partial W_{kl}} = \sum_{i,j} \underbrace{\frac{\partial \mathcal{E}}{\partial A_{ij}}}_{=B_{ij}} \frac{\partial A_{ij}}{\partial W_{kl}} = \sum_{i,j} B_{ij}\, D_{ik}\, \delta_{jl}$$

$$\overset{j=l}{=} \sum_{i} B_{il} D_{ik} = \sum_{i} D^T_{ki} B_{il} = (D^T B)_{kl} \quad \square$$

zz: $\dfrac{\partial A_{ij}}{\partial D_{kl}} = S_{ik} W_{lj}$

Bew

$$\frac{\partial A_{ij}}{\partial D_{kl}} = \frac{\partial}{\partial D_{kl}} \sum_{q=1}^{n} D_{iq} W_{qj} = \sum_{q=1}^{n} \frac{\partial D_{iq}}{\partial D_{kl}} W_{qj}$$

$$= \sum_{q=1}^{n} S_{ik} S_{ql} W_{qj} = S_{ik} S_{ll} W_{lj} = S_{ik} W_{lj}$$

$\square$

zz: $\dfrac{\partial E}{\partial D_{kl}} = (BW^T)_{kl}$

Bew

$$\frac{\partial E}{\partial D_{kl}} = \sum_{i,j} \frac{\partial E}{\partial A_{ij}} \frac{\partial A_{ij}}{\partial D_{kl}} = \sum_{i,j} B_{ij} W_{lj} S_{ik}$$

$$\overset{i=k}{=} \sum_{j} B_{kj} W_{lj} = \sum_{j} B_{kj} W^T_{jl} = (BW^T)_{kl}$$

$\square$

(2)

zz: $$\frac{\partial A_{ij}}{\partial D_{kl}} = \delta_{ik}\,\delta_{jl}$$

<u>Bew</u>

$$\frac{\partial A_{ij}}{\partial D_{kl}} = \frac{\partial(D_{ij} + \overbrace{b_j}^{=0})}{\partial D_{kl}} = \delta_{ik}\,\delta_{jl}$$

$\square$

zz: $$\frac{\partial E}{\partial D_{kl}} = B_{kl}$$

<u>Bew</u>

$$\frac{\partial E}{\partial D_{kl}} = \sum_{i,j} \frac{\partial E}{\partial A_{ij}} \frac{\partial A_{ij}}{\partial D_{kl}} = \sum_{i,j} B_{ij}\,\delta_{ik}\,\delta_{jl} \overset{\substack{i=k \\ j=l}}{=} B_{kl}$$

$\square$

zz: $$\frac{\partial A_{ij}}{\partial b} = \delta_{kj}$$

<u>Bew</u>

$$\frac{\partial A_{ij}}{\partial b_k} = \frac{\partial(D_{ij} + \overbrace{b_j}^{1 \text{ wenn } j=k})}{\partial b_k} = \delta_{kj}$$

zz: $\frac{\partial E}{\partial b_k} = \sum_i B_{ik}$

<u>Bew</u>

$$\frac{\partial E}{\partial b_k} = \sum_{i,j} \frac{\partial E}{\partial A_{ij}} \frac{\partial A_{ij}}{\partial b_k} = \sum_{i,j} B_{ij} \delta_{kj} \overset{j=k}{=} \sum_i B_{ik}$$

(3)

zz: $\sigma'(t) = \sigma(t)(1-\sigma(t))$

<u>Bew</u>

$$\sigma'(t) = \left(\frac{1}{1+e^{-t}}\right)' = -\frac{1}{(1+e^{-t})^2}(-e^{-t})$$

$$= \frac{1}{1+e^{-t}} \frac{e^{-t}}{1+e^{-t}} = \sigma(t)(1-\sigma(t)) \quad \square$$

zz: $\frac{\partial E}{\partial D_{kl}} = B_{kl} A_{kl}(1-A_{kl})$

<u>Bew</u>

$$\frac{\partial E}{\partial D_{kl}} = \sum_{i,j} \frac{\partial E}{\partial A_{ij}} \frac{\partial A_{ij}}{\partial D_{kl}} = \sum_{i,j} B_{ij} \frac{\partial \sigma(D_{ij})}{\partial D_{kl}} \delta_{ik} \delta_{jl}$$

$$\overset{\substack{i=k\\j=l}}{=} B_{kl}\, \sigma(D_{kl})(1-\sigma(D_{kl})) = B_{kl} A_{kl}(1-A_{kl}) \quad \square$$