Supplementary: Uncertainty Relations

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0.1 Uncertainty Relations from Derivative Hierarchy

The infinite derivative hierarchy constrains fluctuations in the normal direction. Define conjugate variables:

$$q = \text{position coordinate in normal direction}, \quad p = \text{conjugate momentum generating translations in } n^{(} \tag{1}$$

From the bounds on spatial derivatives of normal curvature established in Section 4, we have:

$$|K^{(1)}| \sim K_{\min}^2, \quad |\nabla K^{(1)}| \sim K_{\min}^{5/2}. \tag{2}$$

These bounds constrain the precision with which position and momentum in normal space can be simultaneously specified. The curvature bound $|K^{(1)}| \sim K_{\min}^2$ limits spatial localization, giving position uncertainty:

$$\Delta q \sim K_{\min}^{-1/2}. \tag{3}$$

The derivative bound $|\nabla K^{(1)}| \sim K_{\min}^{5/2}$ constrains momentum uncertainty:

$$\Delta p \sim K_{\min}. \tag{4}$$

The product yields:

$$\Delta q \cdot \Delta p \sim K_{\min}^{1/2}. \tag{5}$$

This quantity has dimensions of action. Defining the fundamental action scale:

$$\hbar \equiv K_{\min}^{-1} \times (\text{dimensional factors}), \tag{6}$$

the geometric bounds become the Heisenberg uncertainty relation:

$$\Delta q \cdot \Delta p \geq \hbar/2. \tag{7}$$

Uncertainty is not a quantum postulate but a consequence of geometric derivative bounds in the normal direction.