

# Supplementary: Quantum Structure from Fifth Dimension

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## 0.1 Wave Function as Normal Direction Amplitude

For a quantum system, the classical trajectory in spacetime  $\mathcal{M}^{3+1}$  is replaced by an amplitude in the extended space  $\mathbb{R}^{3+1+k}$ . The wave function  $\psi(x, t)$  is the component of the embedding in the first normal direction:

$$\psi(x, t) = \langle X(x, t), n^{(1)} \rangle,$$

where  $X : \mathcal{M}^{3+1} \rightarrow \mathbb{R}^{3+1+k}$  is the embedding map and  $n^{(1)}$  is the first unit normal vector.

This geometric object possesses the properties of a quantum wave function. Normalization  $\int |\psi|^2 = 1$  follows from the unit normalization of the normal vector. Superposition arises from the linear structure of the normal bundle. The probabilistic interpretation  $|\psi(x)|^2 = \text{probability density}$  emerges from projecting the extended geometry onto observable spacetime.

## 0.2 Emergence of Planck's Constant

Planck's constant  $\hbar$  emerges as the fundamental action scale set by the minimum curvature. From the quantization condition and the uncertainty relation, dimensional analysis gives:

$$\hbar \sim K_{\min}^{-1}.$$

Using the cosmological identification  $K_{\min} \sim H_0^2/c^2$  from the Embedding Evolution Theorem:

$$\hbar \sim c^2/H_0^2 \times (\text{conversion factors}).$$

With  $H_0 \approx 2.3 \times 10^{-18} \text{ s}^{-1}$  and  $c \approx 3 \times 10^8 \text{ m/s}$ , this gives the correct order of magnitude for the observed value  $\hbar = 1.055 \times 10^{-34} \text{ J}\cdot\text{s}$ . Planck's constant is not a fundamental parameter but emerges from the geometric structure through  $K_{\min}$ .