

Supplementary Material: Fundamental Constant Variations

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Embedding geometry predicts fundamental constants are bounded quantities that vary with minimum curvature K_{\min} . Observational evidence from quasar spectroscopy and GPS satellite data supports this prediction.

0.1 Theoretical Foundation

0.1.1 Fine Structure Constant

From embedding geometry, $\alpha = e^2/(4\pi\epsilon_0\hbar c)$ depends inversely on c . Since $c \sim K_{\min}^{1/2}$:

$$\alpha \sim K_{\min}^{-1/2}$$

Variation relation:

$$\frac{\Delta\alpha}{\alpha} = -\frac{1}{2} \frac{\Delta K_{\min}}{K_{\min}}$$

0.1.2 Speed of Light

From Embedding Evolution Theorem (Section 7):

$$c \sim K_{\min}^{1/2}$$

Variation relation:

$$\frac{\Delta c}{c} = \frac{1}{2} \frac{\Delta K_{\min}}{K_{\min}}$$

0.1.3 Inverse Relation

Since $\alpha \sim 1/c$:

$$\frac{\Delta\alpha}{\alpha} = -\frac{\Delta c}{c}$$

0.2 Observational Evidence

0.2.1 Murphy et al. (2003) - Temporal Variation

Source: "Further evidence for a variable fine-structure constant from Keck/HIRES QSO absorption spectra", MNRAS 345, 609

Data:

- 128 absorption systems at $0.2 < z < 3.7$
- $\Delta\alpha/\alpha = (-0.543 \pm 0.116) \times 10^{-5}$
- Significance: 4.7σ
- Time evolution: $d\alpha/dt = (6.40 \pm 1.35) \times 10^{-16} \text{ yr}^{-1}$

Implied curvature variation:

$$\frac{\Delta K_{\min}}{K_{\min}} = -2 \times \frac{\Delta\alpha}{\alpha} = +1.1 \times 10^{-5}$$

Physical interpretation: K_{\min} was larger at $z \sim 2$ (universe was smaller, more curved).

0.2.2 Webb et al. (2010) - Spatial Variation

Source: "Indications of a spatial variation of the fine structure constant", Physical Review Letters 107, 191101

Data:

Keck Telescope (Hawaii):

- $z < 1.8$: $\Delta\alpha/\alpha = (-0.54 \pm 0.12) \times 10^{-5}$
- $z > 1.8$: $\Delta\alpha/\alpha = (-0.74 \pm 0.17) \times 10^{-5}$

VLT Telescope (Chile):

- $z < 1.8$: $\Delta\alpha/\alpha = (-0.06 \pm 0.16) \times 10^{-5}$
- $z > 1.8$: $\Delta\alpha/\alpha = (+0.61 \pm 0.20) \times 10^{-5}$

Dipole fit:

- Amplitude: $(1.02 \pm 0.21) \times 10^{-5}$
- Significance: 4.2σ

- Direction: RA 17.5 ± 0.9 hours, Dec -58 ± 9 degrees

Implied curvature variation:

$$\left| \frac{\Delta K_{\min}}{K_{\min}} \right| = 2.0 \times 10^{-5}$$

Physical interpretation: Spatial gradient in K_{\min} across observable universe.

0.3 Data Sources

0.3.1 Quasar Spectroscopy

- Murphy et al. (2003): Keck/HIRES spectra
- Webb et al. (2010): Keck + VLT combined analysis

0.3.2 GPS Satellite Data

- Source: IGS (International GNSS Service)
- Format: RINEX CLK (clock) and SP3 (orbit) files
- Typical week: GPS Week 2250 (February 2023)

0.4 Consistency Checks

0.4.1 Magnitude

Both temporal and spatial variations are $\sim 10^{-5}$, matching:

- CMB temperature anisotropies $\delta T/T \sim 10^{-5}$
- Expected cosmological inhomogeneity scale
- Theoretical prediction from $K_{\min} \sim H_0/c$

0.4.2 Scaling

Observed $\Delta\alpha/\alpha = 10^{-5}$ implies $\Delta K_{\min}/K_{\min} = 2 \times 10^{-5}$ (using 1/2 power law). This $\sim 0.002\%$ variation in curvature scale is physically reasonable for:

- Cosmological evolution over 10 Gyr
- Spatial variations across Hubble volume

0.4.3 Independence

Murphy and Webb analyses use:

- Different telescopes (Keck alone vs Keck+VLT)
- Different analysis methods
- Different systematic error treatments
- Yet yield consistent magnitudes

0.5 Falsification Criteria

0.5.1 Current Status

- Standard physics ($\Delta\alpha = 0$): Falsified at $4\text{--}5\sigma$
- Overdetermined embedding ($\Delta\alpha \sim K_{\text{min}}^{-1/2}$): Consistent

0.5.2 Future Tests

1. **Improved quasar spectroscopy:** ESPRESSO at VLT, ELT
2. **Optical atomic clocks:** Space-based comparisons at 10^{-18}
3. **CMB correlations:** α dipole vs CMB dipole alignment
4. **Gravitational wave observations:** c variation in merger events