Supplementary: CMB Temperature Anisotropies

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0.1 CMB Temperature Anisotropies

Curvature fluctuations around the minimum:

$$K(\mathbf{x}) = K_{\min} + \delta K(\mathbf{x}).$$

These relate to temperature anisotropies through:

$$\delta K = C_{\text{geom}} \times K_{\min} \times \frac{\delta T}{T},$$

where the geometric coefficient:

$$C_{\text{geom}} = 16\pi\sqrt{3} \approx 87.06.$$

0.2 Origin of the Geometric Coefficient

$C_{\text{geom}} = 8\pi \times 2 \times \sqrt{3}$ decomposes as:

- 8π: from Einstein-Hilbert action, appearing in $G_{\mu\nu} = 8\pi G T_{\mu\nu}$
- **2**: from Gauss equation relating 2D to 3D curvature, $R_3 = 2K_G$
- $\sqrt{3}$: from three independent normal directions in 5D embedding

CMB coefficients inherit the same geometric structure as Einstein's field equations.

0.3 Physical Basis

Energy density scales as $\rho \propto T^4$ (Stefan-Boltzmann), giving $\delta\rho/\rho \approx 4(\delta T/T)$. The Friedmann equation $R = 16\pi\rho/3$ yields $\delta R/R \approx 4(\delta T/T)$. Since $K \sim \sqrt{R}$:

$$\frac{\delta K}{K_{\min}} \approx \frac{\delta T}{T},$$

with embedding geometry providing $C_{\text{geom}} = 16\pi\sqrt{3}$.

0.4 CMB Configuration Space Interpretation

CMB configuration space is defined by:

$$\Phi_{\text{CMB}} = (T_{\text{photon}}(\mathbf{x}), \tau(\mathbf{x}), \rho_e(\mathbf{x}), \rho_H(\mathbf{x})).$$

Recombination becomes a geometric constraint:

$$\rho_e(\mathbf{x}) = \rho_H(\mathbf{x}).$$

Last scattering surface is defined by optical depth:

$$\tau(\mathbf{x}) = \int \sigma_T \rho_e(\chi)\, d\chi = 1.$$

Temperature anisotropies:

$$\frac{\Delta T}{T}(\mathbf{x}) = \frac{\delta T_{\text{photon}}(\mathbf{x})}{T_{\text{photon}}(\mathbf{x})}\Big|_{\tau(\mathbf{x})=1}.$$

CMB observables become geometric properties of spatial configurations, consistent with time elimination.