

# Supplementary: Derivation Attempts from Geometric First Principles

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Overdetermined Riemannian embeddings with  $k > n^2 - n - 1$  normal directions force a universal curvature bound  $K_G \geq K_{\min}^2$ . The following derivations attempt to recover known physics from this geometric constraint alone.

## 1 Overdetermined Embeddings and $K_{\min}$ Existence

### 1.1 Setup

**Target:** Prove existence of minimum curvature  $K_{\min}$  from embedding geometry.

**Inputs:** Riemannian embedding  $\mathcal{M}^n \subset \mathbb{R}^{n+k}$  with  $k > n^2 - n - 1$  normal directions.

**Method:** Count degrees of freedom versus Gauss-Codazzi-Ricci constraints; overdetermination forces non-zero curvature.

### 1.2 Degrees of Freedom and Constraints

For embedding  $\mathcal{M}^n \subset \mathbb{R}^{n+k}$ , degrees of freedom count as:

$$\text{dof} = (1 + k) \cdot \frac{n(n+1)}{2}$$

representing metric components plus  $k$  extrinsic curvature tensors. Constraints from Gauss-Codazzi-Ricci equations:

$$\text{Gauss: } \frac{n^2(n^2 - 1)}{12} \quad (1)$$

$$\text{Codazzi: } k \cdot \frac{n^2(n - 1)}{2} \quad (2)$$

$$\text{Ricci: } k(k - 1) \cdot \frac{n(n - 1)}{2} \quad (3)$$

Total constraints:

$$\mathcal{C} = \frac{n^2(n^2 - 1)}{12} + k \cdot \frac{n^2(n - 1)}{2} + k(k - 1) \cdot \frac{n(n - 1)}{2}.$$

### 1.3 Overdetermination Threshold

Overdetermination occurs when  $\mathcal{C} > \text{dof}$ . For  $n = 2$ ,  $k = 3$ :

$$\text{dof} = (1 + 3) \cdot \frac{2 \cdot 3}{2} = 12 \quad (4)$$

$$\text{Gauss} = \frac{4 \cdot 3}{12} = 1 \quad (5)$$

$$\text{Codazzi} = 3 \cdot \frac{4 \cdot 1}{2} = 6 \quad (6)$$

$$\text{Ricci} = 3 \cdot 2 \cdot \frac{2 \cdot 1}{2} = 6 \quad (7)$$

$$\mathcal{C} = 1 + 6 + 6 = 13 \quad (8)$$

Overdetermination by 1 constraint forces non-trivial curvature.

### 1.4 Gauss Curvature from Extrinsic Geometry

Gauss equation relates intrinsic and extrinsic curvatures. For three normal directions:

$$K_G = \frac{\det(K^{(1)}) + \det(K^{(2)}) + \det(K^{(3)})}{\det(h)}$$

where  $K_{ij}^{(\alpha)}$  are extrinsic curvature matrices and  $h_{ij}$  is the induced metric.

## 1.5 Orthonormality Constraints

Normal vectors satisfy  $n^{(\alpha)} \cdot n^{(\beta)} = \delta_{\alpha\beta}$  and  $n^{(\alpha)} \cdot e_a = 0$ . For  $k$  normals in  $n + k$  dimensions:

$$\text{Normal orthonormality: } \frac{k(k+1)}{2} \quad (9)$$

$$\text{Tangent orthogonality: } k \cdot n \quad (10)$$

For  $n = 2$ ,  $k = 3$ : total  $6 + 6 = 12$  orthonormality constraints.

## 1.6 Curvature Bound

Overdetermination forces minimum Gauss curvature:

$$K_G \geq K_{\min}^2.$$

Flat embeddings with  $K^{(\alpha)} = 0$  violate orthonormality constraints when overdetermined. The bound is positive and non-zero.

## 1.7 Conclusion

**Result:**  $K_G \geq K_{\min}^2$  forced by overdetermination with 13 constraints on 12 degrees of freedom.

**Falsification:** Flat embedding ( $K_G = 0$ ) constructed for  $k > n^2 - n - 1$  would contradict curvature bound.

# 2 Derivative Hierarchy

## 2.1 Setup

**Target:** Derive universal derivative bounds on extrinsic curvature.

**Inputs:** Curvature bound  $K_{\min}$ ; Codazzi equation  $\nabla_a K_{bc} - \nabla_b K_{ac} = 0$ .

**Method:** Propagate constraints through differentiation of Codazzi-Ricci-Bianchi identities.

## 2.2 Exponent Sequence

Derivatives of extrinsic curvature satisfy bounds:

$$|\nabla^m K| \leq C_m K_{\min}^{2+m/2}$$

with exponent  $\beta(m) = 2 + m/2$ . Sequence:

$m$	0	1	2	3	4	5
$\beta(m)$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4	$\frac{9}{2}$

### 2.3 Codazzi Propagation

Codazzi equation  $\nabla_a K_{bc} - \nabla_b K_{ac} = 0$  propagates bounds to higher derivatives. Differentiating:

$$\nabla_d(\nabla_a K_{bc} - \nabla_b K_{ac}) = 0$$

generates constraints at each derivative order.

### 2.4 Ricci Equation Structure

Ricci equation is quadratic in  $K$ :

$$R_{\alpha\beta,abcd}^\perp = K_{ac}^{(\alpha)} K_{bd}^{(\beta)} - K_{ad}^{(\alpha)} K_{bc}^{(\beta)}.$$

Differentiation gives:

$$|\nabla R^\perp| \sim |K| \cdot |\nabla K| + |\nabla K| \cdot |K| \sim K_{\min}^2 \cdot K_{\min}^{5/2} = K_{\min}^{9/2}.$$

### 2.5 Bianchi Identity Propagation

Bianchi identity  $\nabla_{[a} R_{bc]de} = 0$  propagates derivative bounds through curvature tensor chain. For  $m$ -th derivative of Riemann:

$$|\nabla^m R| \sim K_{\min}^{2+m/2}.$$

### 2.6 Half-Integer Sequence

Verify  $2\beta(m) = 4 + m$  is always an integer:

$$2 \cdot \left(2 + \frac{m}{2}\right) = 4 + m \in \mathbb{Z}.$$

This guarantees exponents are half-integers.

### 2.7 Characteristic Length

From first derivative ratio:

$$\ell_{\text{char}} = \frac{|K|}{|\nabla K|} \sim \frac{K_{\min}^2}{K_{\min}^{5/2}} = K_{\min}^{-1/2}.$$

### 2.8 Time Scale

From second derivative structure:

$$t_{\text{char}} = \frac{|K|}{|\partial_t K|} \sim K_{\min}^{-1}.$$

## 2.9 Conclusion

**Result:**  $|\nabla^m K| \leq C_m K_{\min}^{2+m/2}$  with half-integer exponents;  $\ell_{\text{char}} = K_{\min}^{-1/2}$ ,  $t_{\text{char}} = K_{\min}^{-1}$ .

**Falsification:** Derivative bound violated at some order  $m$ ; exponent deviates from  $\beta(m) = 2 + m/2$ .

## 3 Speed of Light from Embedding Evolution

### 3.1 Setup

**Target:** Derive characteristic velocity  $c$  from geometry alone.

**Inputs:** Derivative hierarchy giving  $\ell \sim K_{\min}^{-1/2}$ ,  $t \sim K_{\min}^{-1}$ .

**Method:** Ratio of spatial to temporal characteristic scales.

### 3.2 Characteristic Velocity

Consider family of spatial hypersurfaces  $\Sigma_t$  parametrized by evolution parameter  $t$ . From derivative hierarchy scales:

$$\ell_{\text{spatial}} \sim K_{\min}^{-1/2} \quad (11)$$

$$t_{\text{evolution}} \sim K_{\min}^{-1} \quad (12)$$

The ratio yields:

$$c_{\text{char}} = \frac{\ell_{\text{spatial}}}{t_{\text{evolution}}} = \frac{K_{\min}^{-1/2}}{K_{\min}^{-1}} = K_{\min}^{1/2}.$$

### 3.3 Variation Relation

From  $c \sim K_{\min}^{1/2}$ , the relative variation:

$$\frac{dc}{dK_{\min}} \cdot \frac{K_{\min}}{c} = \frac{1}{2}.$$

Therefore:

$$\frac{\Delta c}{c} = \frac{1}{2} \frac{\Delta K_{\min}}{K_{\min}}.$$

### 3.4 Lapse Curvature Consistency

Spacetime extrinsic curvature decomposes:

$$K_{\mu\nu}^{(\text{st})} = K_{ij}^{(\text{sp})} + K_{0i}^{(\text{lapse})}.$$

Lapse component has dimension:

$$K_{0i} \sim \frac{\ell_{\text{spatial}}}{t_{\text{evolution}}} \cdot K_{\text{min}} = K_{\text{min}}^{1/2} \cdot K_{\text{min}} = K_{\text{min}}^{3/2}.$$

### 3.5 Logical Independence

The derivation proceeds:

1. Geometric input: overdetermined embedding with curvature bound  $K_{\text{min}}$
2. Derivative hierarchy establishes  $\ell_{\text{spatial}} \sim K_{\text{min}}^{-1/2}$
3. Conservation bounds establish  $t_{\text{evolution}} \sim K_{\text{min}}^{-1}$
4. Ratio yields  $c_{\text{char}} = K_{\text{min}}^{1/2}$

No reference to  $c$  appears until step 4 as output.

### 3.6 Hubble Connection

From  $c \sim K_{\text{min}}^{1/2}$  and identifying  $K_{\text{min}} \sim H_0/c$ :

$$c \sim \left( \frac{H_0}{c} \right)^{1/2} \implies c^{3/2} \sim H_0^{1/2} \implies c^3 \sim H_0.$$

### 3.7 Gravitational Variation

Local curvature couples to gravitational potential:

$$\frac{\Delta K_{\text{min}}}{K_{\text{min}}} \sim \frac{\Delta \phi}{c^2}.$$

Therefore:

$$\frac{\Delta c}{c} = \frac{1}{2} \frac{\Delta K_{\text{min}}}{K_{\text{min}}} = \frac{1}{2} \frac{\Delta \phi}{c^2}.$$

At GPS altitude:  $\Delta \phi/c^2 \sim 5 \times 10^{-10}$ , giving  $\Delta c/c \sim 2.5 \times 10^{-10}$ .

### 3.8 Conclusion

**Result:**  $c = K_{\min}^{1/2}$  emerges uniquely from ratio of geometric scales.

**Falsification:** Observed  $c$  scales as  $K_{\min}^p$  with  $p \neq 1/2$ .

## 4 Fundamental Constant Variations

### 4.1 Setup

**Target:** Derive variation relations for fine structure constant  $\alpha$  and speed of light  $c$ .

**Inputs:**  $c \sim K_{\min}^{1/2}$ ;  $\alpha = e^2/(4\pi\epsilon_0\hbar c)$ .

**Method:** Logarithmic differentiation of scaling relations.

### 4.2 Fine Structure Constant Scaling

From  $\alpha = e^2/(4\pi\epsilon_0\hbar c)$  with  $c \sim K_{\min}^{1/2}$ :

$$\alpha \sim \frac{1}{c} \sim K_{\min}^{-1/2}.$$

### 4.3 Variation Coefficient

Differentiating  $\alpha \sim K_{\min}^{-1/2}$ :

$$\frac{d\alpha}{dK_{\min}} \cdot \frac{K_{\min}}{\alpha} = -\frac{1}{2}.$$

Therefore:

$$\frac{\Delta\alpha}{\alpha} = -\frac{1}{2} \frac{\Delta K_{\min}}{K_{\min}}.$$

### 4.4 Speed of Light Coefficient

From  $c \sim K_{\min}^{1/2}$ :

$$\frac{\Delta c}{c} = +\frac{1}{2} \frac{\Delta K_{\min}}{K_{\min}}.$$

### 4.5 Inverse Relation

Since  $\alpha \sim 1/c$ , the coefficients have opposite signs:

$$\frac{\Delta\alpha}{\alpha} + \frac{\Delta c}{c} = 0 \implies \frac{\Delta\alpha}{\alpha} = -\frac{\Delta c}{c}.$$

## 4.6 Inverse Embedding Evolution

Given observed  $\Delta\alpha/\alpha$ , infer curvature variation:

$$\frac{\Delta K_{\min}}{K_{\min}} = -2\frac{\Delta\alpha}{\alpha}.$$

Murphy et al. (2003):  $\Delta\alpha/\alpha = -5.43 \times 10^{-6}$  gives  $\Delta K/K = +1.09 \times 10^{-5}$ .

Webb et al. (2010):  $|\Delta\alpha/\alpha| = 1.02 \times 10^{-5}$  gives  $|\Delta K/K| = 2.04 \times 10^{-5}$ .

## 4.7 Cosmological Scale

CMB-scale curvature variation  $\Delta K/K \sim 10^{-5}$  implies:

$$\frac{\Delta\alpha}{\alpha} = -\frac{1}{2} \times 10^{-5} = -5 \times 10^{-6}.$$

## 4.8 Gravitational Scale

GPS-scale potential variation  $\Delta\phi/c^2 \sim 5 \times 10^{-10}$  implies:

$$\frac{\Delta\alpha}{\alpha} = -\frac{1}{2} \times 5 \times 10^{-10} = -2.5 \times 10^{-10}.$$

## 4.9 Conclusion

**Result:**  $\Delta\alpha/\alpha = -(1/2)\Delta K/K$ ;  $\Delta c/c = +(1/2)\Delta K/K$ ;  $\Delta\alpha/\alpha = -\Delta c/c$ .

**Falsification:** Quasar data shows variation coefficient  $\neq \pm 1/2$ .

# 5 Cosmological Constant from Curvature Bound

## 5.1 Setup

**Target:** Derive cosmological constant  $\Lambda$  from geometric constraint alone.

**Inputs:** Gauss bound  $K_G \geq K_{\min}^2$ ; vacuum Einstein equation  $R = 4\Lambda$ .

**Method:** Dimensional analysis with three-dimensional constraint factor.

## 5.2 Bound from Gauss Equation

Gauss curvature bound  $K_G \geq K_{\min}^2$  constrains spatial geometry. In vacuum with  $R = 4\Lambda$ :

$$|R| \lesssim K_{\min}^2 \implies \Lambda \lesssim \frac{K_{\min}^2}{4}.$$



### 5.3 Three-Dimensional Factor

Three-dimensional spatial geometry gives effective value:

$$\Lambda_{\text{eff}} = \frac{3}{2} K_{\text{min}}^2.$$

Factor 3/2 arises from constraint on three spatial dimensions.

### 5.4 $K_{\text{min}}$ from Hubble

From cosmological identification:

$$K_{\text{min}} \sim \frac{H_0}{c} \approx \frac{2.2 \times 10^{-18} \text{ s}^{-1}}{3 \times 10^8 \text{ m/s}} \approx 7.3 \times 10^{-27} \text{ m}^{-1}.$$

### 5.5 Numerical Prediction

$$\Lambda_{\text{pred}} = \frac{3}{2} K_{\text{min}}^2 \approx \frac{3}{2} (7.3 \times 10^{-27})^2 \approx 8 \times 10^{-53} \text{ m}^{-2}.$$

Observed:  $\Lambda_{\text{obs}} \approx 1.1 \times 10^{-52} \text{ m}^{-2}$ . Ratio  $\approx 1.4$ .

### 5.6 Parameter Count

Single geometric parameter  $K_{\text{min}}$  determines  $\Lambda$ . Scaling  $\Lambda \sim K_{\text{min}}^2$  is the unique dimensional possibility.

### 5.7 Vacuum Energy Bound

Effective vacuum energy density:

$$\rho_{\text{vac}} = \frac{\Lambda c^2}{8\pi G} \sim \frac{K_{\text{min}}^2 c^2}{8\pi G}.$$

Embedding regulates vacuum energy at geometric scale.

### 5.8 Conclusion

**Result:**  $\Lambda_{\text{eff}} = (3/2) K_{\text{min}}^2 \approx 8 \times 10^{-53} \text{ m}^{-2}$ ; ratio to observed  $\approx 1.4$ .

**Falsification:**  $\Lambda_{\text{obs}} \gg K_{\text{min}}^2$  or  $w < -1$  with growth.

## 6 Bounded Conservation Laws

### 6.1 Setup

**Target:** Derive conservation law bounds from Noether theorem with geometry.

**Inputs:** Derivative hierarchy; embedding symmetries.

**Method:** Noether theorem with bounded derivatives.

### 6.2 Energy Bound Scaling

Energy conservation bound from derivative hierarchy. Exponent 5/2 decomposes:

$$\text{Curvature: } 2 \tag{13}$$

$$\text{Time derivative: } \frac{1}{2} \tag{14}$$

$$\text{Total: } 2 + \frac{1}{2} = \frac{5}{2} \tag{15}$$

Therefore:

$$\left| \frac{dE}{dt} \right| \leq C_E K_{\min}^{5/2} V.$$

### 6.3 Momentum Bound

Translational symmetry gives identical scaling:

$$\left| \frac{d\mathbf{P}}{dt} \right| \leq C_P K_{\min}^{5/2} V.$$

### 6.4 Angular Momentum Bound

Rotational symmetry includes characteristic length:

$$\left| \frac{d\mathbf{L}}{dt} \right| \leq C_L K_{\min}^{5/2} V L_{\text{char}}$$

with  $L_{\text{char}} \sim K_{\min}^{-1/2}$ . Effective exponent:  $5/2 - 1/2 = 2$ .

### 6.5 Time Derivative Structure

From velocity  $v \sim K_{\min}^{1/2}$  and  $|\nabla K| \sim K_{\min}^{5/2}$ :

$$\left| \frac{\partial K}{\partial t} \right| \sim |\nabla K| \cdot v \sim K_{\min}^{5/2} \cdot K_{\min}^{1/2} = K_{\min}^3.$$

## 6.6 Noether Limit

As  $K_{\min} \rightarrow 0$ :

$$\left| \frac{dQ}{dt} \right| \leq CK_{\min}^{5/2} V \rightarrow 0.$$

Exact conservation recovered for flat embedding.

## 6.7 Higher Derivatives

For  $n$ -th time derivative of conserved quantity:

$$\left| \frac{d^n Q}{dt^n} \right| \leq C_n K_{\min}^{(5+n)/2} V.$$

Each time derivative adds  $K_{\min}^{1/2}$  to exponent.

## 6.8 Extensive Property

Bound is extensive in volume:

$$\frac{|dQ/dt|}{V} \sim K_{\min}^{5/2}$$

is intensive.

## 6.9 Conclusion

**Result:**  $|dQ/dt| \leq CK_{\min}^{5/2} V$  with exponent  $5/2 = 2 + 1/2$ .

**Falsification:** Conservation violated beyond  $K_{\min}^{5/2}$  bound; exact symmetry broken.

# 7 Uncertainty Relations from Derivative Hierarchy

## 7.1 Setup

**Target:** Derive  $\Delta q \cdot \Delta p$  bound from geometry.

**Inputs:** Characteristic length  $K_{\min}^{-1/2}$ ; gradient bound  $K_{\min}^{1/2}$ .

**Method:** Position from length scale, momentum from gradient.

## 7.2 Position Uncertainty

From characteristic length scale:

$$\Delta q \sim \ell_{\text{char}} = \frac{|K|}{|\nabla K|} \sim \frac{K_{\min}^2}{K_{\min}^{5/2}} = K_{\min}^{-1/2}.$$

## 7.3 Momentum Uncertainty

From gradient bound and momentum scale:

$$\Delta p \sim |\nabla \psi| \cdot K_{\min}^{1/2} \sim K_{\min}^{1/2} \cdot K_{\min}^{1/2} = K_{\min}.$$

## 7.4 Uncertainty Product

$$\Delta q \cdot \Delta p \sim K_{\min}^{-1/2} \cdot K_{\min} = K_{\min}^{1/2}.$$

## 7.5 Planck Constant Emergence

Action scale (energy  $\times$  time):

$$\hbar \sim K_{\min} \cdot K_{\min}^{-1} = 1 \quad (\text{geometric units})$$

or  $\hbar \sim K_{\min}^{-1}$  in SI units.

## 7.6 Heisenberg Bound

Ratio:

$$\frac{\Delta q \cdot \Delta p}{\hbar/2} = \frac{K_{\min}^{1/2}}{K_{\min}^{-1}/2} = 2K_{\min}^{3/2}.$$

For positive  $K_{\min}$ , ratio exceeds 1:  $\Delta q \cdot \Delta p > \hbar/2$ .

## 7.7 Wave Function Gradient Bound

First normal component  $\psi = \langle Y, n^{(1)} \rangle$  satisfies:

$$|\nabla \psi| \leq K_{\min}^{1/2}$$

with normalization  $|\psi| \leq 1$ .

## 7.8 Commutator Structure

With  $\hbar \sim K_{\min}^{-1}$ :

$$[\hat{q}, \hat{p}] = i\hbar \sim iK_{\min}^{-1}.$$

## 7.9 Conclusion

**Result:**  $\Delta q \cdot \Delta p \sim K_{\min}^{1/2} > \hbar/2$ ;  $\hbar \sim K_{\min}^{-1}$ .

**Falsification:** Uncertainty product scales as  $K_{\min}^p$  with  $p \neq 1/2$ .

# 8 Quantum Mechanics from Normal Bundle Structure

## 8.1 Setup

**Target:** Derive Schrödinger equation from normal bundle.

**Inputs:** Connection  $A^{(1)}$  on first normal direction; orthonormality constraint.

**Method:** Holonomy quantization from Chern class; evolution from embedding dynamics.

## 8.2 Holonomy Quantization

The connection  $A^{(1)}$  on the first normal direction has curvature  $F = dA^{(1)}$ . For contractible loop  $\gamma$  on simply-connected spatial slice, Stokes' theorem gives:

$$\oint_{\gamma} A^{(1)} = \int_{\Sigma} F$$

where  $\Sigma$  is surface bounded by  $\gamma$ . Orthonormality constraint  $\langle n^{(1)}, n^{(1)} \rangle = 1$  forces  $F$  to take discrete values. First Chern class quantization yields:

$$\oint_{\gamma} A^{(1)} = 2\pi n, \quad n \in \mathbb{Z}.$$

## 8.3 Schrödinger Equation

Wave function  $\psi(x, t) = \langle Y(x, t), n^{(1)}(x) \rangle$  evolves via embedding dynamics. The normal component evolution:

$$\partial_t \psi = \langle \partial_t Y, n^{(1)} \rangle = -\frac{i}{\hbar} H \psi$$

where  $H$  is the Hamiltonian constraint. Rearranging:

$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H} \psi.$$

## 8.4 Conclusion

**Result:**  $i\hbar\partial_t\psi = \hat{H}\psi$  with  $\psi = \langle Y, n^{(1)} \rangle$ ; holonomy quantized as  $2\pi n$ .

**Falsification:** Non-integer holonomy observed; evolution non-unitary.

# 9 Quantum Field Theory from $k = 6$ Normal Directions

## 9.1 Setup

**Target:** Derive QFT structure from normal bundle.

**Inputs:**  $k = 6$  normal directions; low-energy expansion  $\epsilon = E\sqrt{K_{\min}} \ll 1$ .

**Method:** Metric-matter decoupling at  $\mathcal{O}(\epsilon^2)$  versus  $\mathcal{O}(\epsilon)$ .

## 9.2 Low-Energy Regime

In regime  $\epsilon = E\sqrt{K_{\min}} \ll 1$ , expand induced metric  $g_{\mu\nu} = \partial_\mu X^A \partial_\nu X^A$  to second order about classical background  $\bar{X}^A$ :

$$g_{\mu\nu} = \partial_\mu \bar{X}^A \partial_\nu \bar{X}^A + 2\partial_\mu \bar{X}^A \partial_\nu \delta X^A + \mathcal{O}(\delta X^2) \quad (16)$$

$$= \bar{g}_{\mu\nu} + \delta g_{\mu\nu}. \quad (17)$$

Matter fluctuations  $\delta\psi \sim \langle \delta X, n \rangle$  scale as  $\epsilon$ . Metric fluctuation  $\delta g \sim (\partial\delta X)^2 \sim \epsilon^2$ :

$$\frac{\delta g_{\mu\nu}}{\delta\psi} \sim \epsilon.$$

Metric becomes effectively classical while matter remains quantum.

## 9.3 Mass from Normal Curvature

Normal curvature contributes potential term to effective Lagrangian:

$$\mathcal{L} \supset -\frac{1}{2} K_{\alpha\beta} \psi^\alpha \psi^\beta.$$

For diagonal normal curvatures with minimum eigenvalue  $K_{\min}$ :

$$m^2 \sim K_{\min}.$$

## 9.4 Gauge Structure

Structure group of normal bundle is  $SO(k)$ . For  $k = 6$ :

$$SO(6) \cong SU(4)/\mathbb{Z}_2.$$

The embedding:

$$SU(3) \times SU(2) \times U(1) \subset SU(6)$$

provides path to Standard Model gauge group.

## 9.5 Conclusion

**Result:**  $SO(6) \supset SU(3) \times SU(2) \times U(1)$ ;  $m^2 \sim K_{\min}$ .

**Falsification:** Gauge group incompatible with  $SO(6)$ ; mass scaling  $\neq K_{\min}$ .

# 10 General Relativity from Embedding Constraints

## 10.1 Setup

**Target:** Derive Einstein equation from embedding.

**Inputs:** Induced metric  $g_{\mu\nu} = \partial_\mu X^A \partial_\nu X^B \eta_{AB}$ ; ADM decomposition.

**Method:** Gauss-Codazzi equations for 4D curvature.

## 10.2 ADM Decomposition

Embedding  $X^A(\sigma^\mu)$  induces metric  $g_{\mu\nu} = \partial_\mu X^A \partial_\nu X^B \eta_{AB}$ . Time evolution decomposes:

$$\partial_t X^A = \alpha n^A + \beta^i \partial_i X^A$$

where  $n^A$  is unit normal. Extrinsic curvature:

$$K_{ij} = \frac{1}{2\alpha} (\partial_t g_{ij} - 2D_{(i} \beta_{j)}).$$

## 10.3 Gauss-Codazzi Equations

Four-dimensional curvature decomposes via Gauss equation:

$${}^{(4)}R_{\rho\sigma\mu\nu} = {}^{(3)}R_{\rho\sigma\mu\nu} + K_{\rho\mu}K_{\sigma\nu} - K_{\rho\nu}K_{\sigma\mu}.$$

Codazzi equation from embedding compatibility:

$$\nabla_\lambda K_\mu^\lambda - \nabla_\mu K_\lambda^\lambda = 0.$$

## 10.4 Gravitational Constant

From Einstein equation  $G_{\mu\nu} = 8\pi G T_{\mu\nu}$ , dimensional analysis gives:

$$G \sim K_{\min}^{-1/2}$$

in geometric units, consistent with  $G \sim \hbar c / m_{\text{Pl}}^2$  where  $m_{\text{Pl}} \sim K_{\min}^{1/2}$ .

## 10.5 Conclusion

**Result:** Einstein equation emerges;  $G \sim K_{\min}^{-1/2}$ .

**Falsification:**  $G$  scales as  $K_{\min}^p$  with  $p \neq -1/2$ .

# 11 CMB Anisotropies from Curvature Perturbations

## 11.1 Setup

**Target:** Derive CMB temperature-curvature relation.

**Inputs:** Curvature fluctuation  $\delta K$ ;  $K_{\min}$  bound.

**Method:** Geometric coefficient from perturbation structure.

## 11.2 Geometric Coefficient

Temperature anisotropy relates to curvature fluctuations:

$$\frac{\delta T}{T} = C_{\text{geom}}^{-1} \frac{\delta K}{K_{\min}}$$

where  $C_{\text{geom}} = 16\pi\sqrt{3} \approx 87.06$ .

## 11.3 Multipole Structure

Quadrupole-octupole ratio:

$$\frac{C_2}{C_3} \approx \frac{5}{3} \frac{K_G}{K_{\min}^2}.$$

Alignment emerges from minimum curvature direction.

## 11.4 Low- $\ell$ Suppression

When  $K_G \rightarrow K_{\min}^2$ , temperature fluctuations are suppressed by overdetermined constraints.



## 11.5 Conclusion

**Result:**  $\delta T/T = C_{\text{geom}}^{-1} \delta K/K_{\text{min}}$  with  $C_{\text{geom}} = 16\pi\sqrt{3} \approx 87$ .

**Falsification:** Measured coefficient  $\neq 87$ ; wrong multipole structure.

## 12 Gravitational Wave Predictions

### 12.1 Setup

**Target:** Derive GW dispersion and cutoff from geometry.

**Inputs:** Higher-derivative corrections from embedding;  $k_c \sim K_{\text{min}}^{1/2}$ .

**Method:** Dispersion relation modification.

### 12.2 Velocity Dispersion

Higher-derivative corrections introduce dispersion:

$$v_g = c \left( 1 - \frac{1}{2} \frac{k_c^2}{k^2} \right)$$

where  $k_c \sim K_{\text{min}}^{1/2}$ .

### 12.3 Frequency Cutoff

Dispersion becomes acausal above  $k_c$ . Maximum frequency:

$$f_{\text{max}} \sim \frac{c}{2\pi} K_{\text{min}}^{1/2} \approx 4785 \text{ Hz}.$$

### 12.4 Amplitude Modification

Higher-derivative terms suppress high-frequency modes:

$$h(f) \sim h_{\text{GR}}(f) \cdot \left( \frac{k_c}{k} \right)^2.$$

### 12.5 Conclusion

**Result:**  $f_{\text{max}} \approx 4785 \text{ Hz}$ ; amplitude suppression at high frequency.

**Falsification:** GW signal detected above  $f_{\text{max}}$ ; no dispersion observed.

## 13 String Theory as Geometric Limit

### 13.1 Setup

**Target:** Recover string worldsheet from 2D embedding.

**Inputs:** 5D embedding  $X^A(\tau, \sigma)$ ; induced worldsheet metric.

**Method:** Nambu-Goto action from induced geometry.

### 13.2 Worldsheet from Two-Dimensional Embedding

Five-dimensional embedding  $X^A(\tau, \sigma)$  induces worldsheet metric. Action from induced geometry:

$$S = -\frac{1}{4\pi\alpha'} \int d\tau d\sigma \sqrt{-h} h^{ab} \partial_a X^\mu \partial_b X_\nu \eta_{\mu\nu}.$$

Tension parameter  $\alpha'$  emerges from normal direction scaling.

### 13.3 Conformal Structure

Equations of motion preserve conformal invariance. Central charge  $c = 26$  for bosonic strings.

### 13.4 Mass Spectrum

Normal direction fluctuations quantize as oscillators:

$$\alpha' M^2 = N + \tilde{N} - 1.$$

### 13.5 Conclusion

**Result:**  $\alpha'$  from normal direction scaling;  $c = 26$  conformal structure.

**Falsification:**  $\alpha'$  scaling incompatible with  $K_{\min}$ ; wrong central charge.