

Construction and implementation of various control designs for inverted pendulum

RCS Project Lab

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Introduction

In the field of control systems, the inverted pendulum stands as a classic and challenging problem, serving as a benchmark for testing the capabilities of control algorithms [1]. This project delves into the domain of inverted pendulum dynamics, focusing on the design, simulation, and implementation of control algorithms to stabilize and control the system dynamics.

The inverted pendulum, with its inherent instability, demands precise control strategies to maintain equilibrium. Our objective is to investigate and implement four known plus one new control methodologies that not only stabilize the pendulum in its upright position but also exhibit robust performance in the face of disturbances and uncertainties.

The report encompasses the theoretical foundations of control systems for inverted pendulum. Subsequently, it delves into the specific challenges posed by inverted pendulum dynamics and the various control strategies that have been proposed in the literature such as Jezierski, Mozaryn and Suski [2], book "The Inverted Pendulum in Control Theory and Robotics" [3] or the book "Advanced Control of Wheeled Inverted Pendulum Systems" [4].



(a) Rotational inverted pendulum from the company quanser [5].

(b) Linear inverted pendulum from the company quanser [6].

Figure 1: Configurations of inverted pendulum.

Within the domain of inverted pendulum systems, there are two main types of

configurations. It is rotational Fig. 1.a and linear configuration Fig. 1.b. In the case of a rotational inverted pendulum, the focus lies on managing the rotational motion of an object around a fixed pivot point. A classic example is a rigid rod attached to a pivot, and other fixed rod attached to the end of it, where the objective is to stabilize the system in an inverted position despite its inherent instability.

On the other hand, a linear inverted pendulum involves translational or linear motion along a vertical axis. This configuration is often represented by a cart on a track, with a pendulum attached to the cart. The task is to control the linear motion of the cart to maintain equilibrium with the pendulum inverted. The setup we are using for simulation and real world implementation is the linear model of inverted pendulum.

Our focus extends beyond theoretical considerations, as the project involves practical implementation on a physical inverted pendulum setup which is located at the chair of Automation and Control of RPTU. This particular setup can be seen on the picture 2. The hardware experimentation provides valuable insights into the real-world applicability of the developed algorithms, offering a bridge between theory and application.

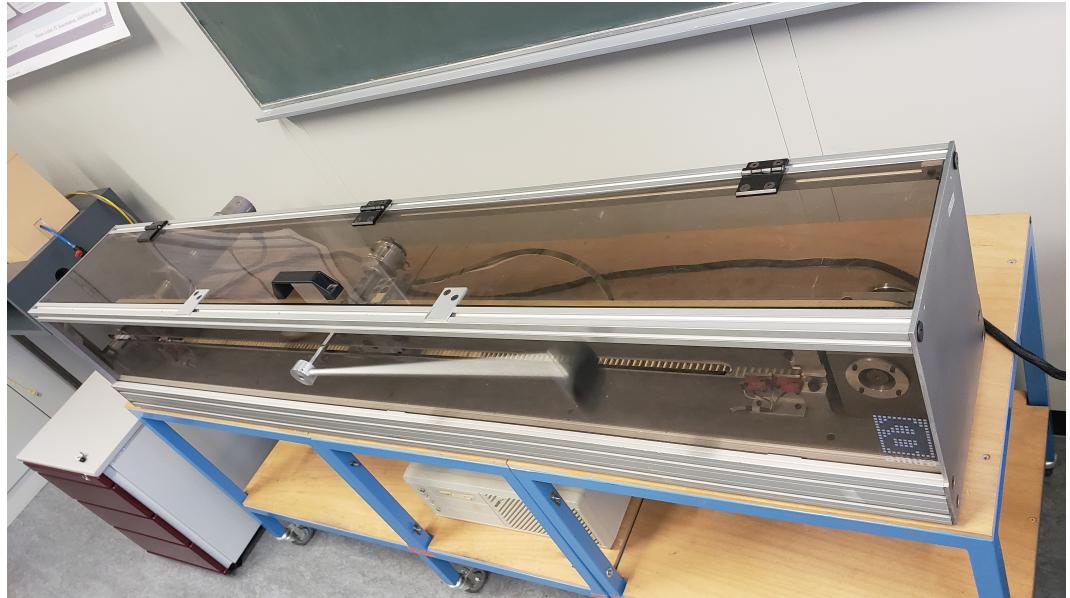


Figure 2: Inverted pendulum setup on which implementation of designed controllers was done.

This project aims to contribute to the field of control systems by presenting a comprehensive exploration of control algorithms for inverted pendulum systems. Through rigorous analysis and experimentation, this report endeavors to showcase the effectiveness and adaptability of the implemented control algorithms, opening avenues for further advancements in the control of dynamic systems.

1 Hardware

1.1 Inverted pendulum

1.2 Controller

2 Software

3 Controllers

3.1 Linear quadratic regulator (LQR)

3.1.1 Theory

The Linear Quadratic Regulator (LQR) has been presented by Rudolf E. Kalman in 1960 [7]. This optimal control algorithm is used for stabilizing linear dynamic systems by determining a control input that minimizes a quadratic cost function. LQR is an unified systematic control method for multiple-input multiple-output (MIMO) systems [7]. It is a very popular control algorithm because of its inherent robustness, where the gain and phase margin are guaranteed [8].

Central to the LQR framework is the concept of linear system dynamics. LQR is tailored for linear time-invariant systems, typically characterized by matrices A, B, C, and D, encapsulating the system's behavior [9].

At its core, LQR revolves around the minimization of a quadratic cost function over a specified time horizon. LQR reduces the amount of labor that needs to be put into the design of the controller, although the formulation of the cost function plays a crucial role in the controller performance.

The solution to the LQR problem involves online and offline calculations that can be separated to three distinct parts [7]:

1. Solving the Riccati differential equation ¹
2. Computation of the feedback matrix K^*
3. Evaluation of the feedback control law

Linear quadratic control problem can be formulated as Eq. 3.1

$$\min_{x(t), u(t), t_e} J(x(t), u(t), t_e) \text{ with } J = \frac{1}{2} x(t_e)^T S x(t_e) + \frac{1}{2} \int_0^{t_e} x(t)^T Q(t) x(t) + u(t)^T R(t) u(t) dt \quad (3.1)$$

where $x(t_e)$ is the end state vector, $Q(t)$ is the state weighting matrix, $R(t)$ is the input weighting matrix and S is the end weighting matrix. Weighting matrices Q , R and S are design parameters and with the help of them, we can change the behavior of the controller. The optimal feedback control law is give by Eq. 3.2 [7]

¹Riccati differential equation is a type of first-order ordinary differential equation that has a quadratic term in one of its variables

$$u^*(t) = K^*(t)x(t) \quad (3.2)$$

where K^* is the feedback matrix, given by Eq. 3.3 [7]

$$K^*(t) = R(t)^{-1}B(t)^TP(t) \quad (3.3)$$

3.1.2 Simulation

In the context of simulation using MATLAB for control system design, the provided code, obtained from the lab coordinator, incorporates the design of a Linear Quadratic Regulator (LQR) controller used for the control of nonlinear inverted pendulum. The primary objective is to evaluate the functionality of the code under different conditions. To assess the system's robustness and performance, disturbances have been introduced. Additionally, the LQR controller's parameters have been fine-tuned for optimal performance.

The simulation relies on MATLAB's `dlqr` command Eq. 3.4, where "dlqr" stands for "Linear-quadratic (LQ) state-feedback regulator" specifically designed for discrete-time state-space systems. This MATLAB command is instrumental in computing the state-feedback gain matrix for an LQR controller, considering both state and input weighting matrices. The resulting gain matrix is then utilized in the feedback control law to regulate the system and optimize its performance.

$$[K, P] = \text{dlqr}(A, B, Q, R) \quad (3.4)$$

Where K is the feedback matrix and P infinite horizon solution of the associated discrete-time Riccati equation, where P is in the form of Eq. 3.5

$$A^T S A - S - (A^T S B + N)(B^T S B + R)^{-1}(B^T S A + N^T) + Q = 0 \quad (3.5)$$

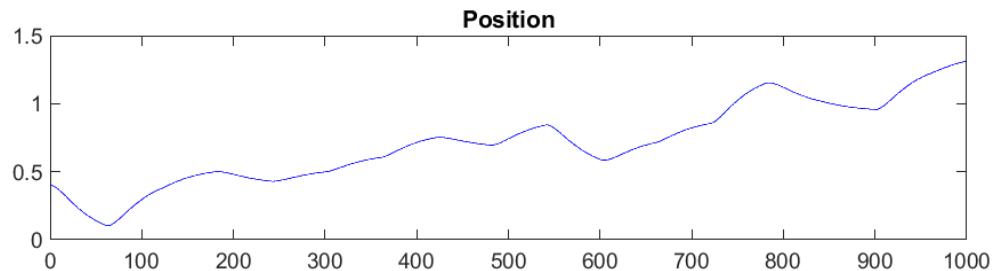


Figure 3.1: Position of the pendulum (LQR simulation).

Simulation outputs

Following the MATLAB simulation, the obtained results reveal distinctive patterns Fig. 3.1, 3.2, 3.3. Notably, each spike observed across all plots corresponds to the introduced disturbance. These disturbances were deliberately added to assess the robustness of the controller under varying conditions.

Figure 3.1 shows the changing position of the pendulum carriage. Figure 3.2 shows the angle of the pendulum in radians and Fig. 3.3 shows the force i.e. torque applied, to move the pendulum. The full code can be found in appendix on the page v.

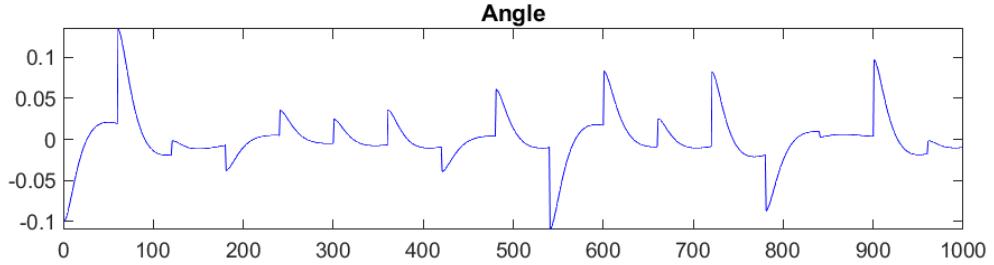


Figure 3.2: Angle of the pendulum (LQR simulation).

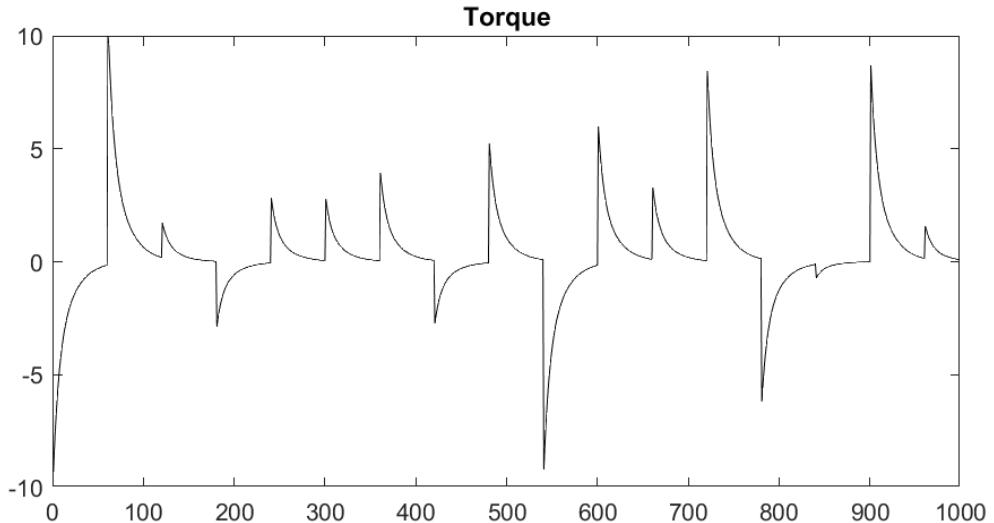


Figure 3.3: Torque of the pendulum (LQR simulation).

Examining the plots, it becomes apparent that the angle of the pendulum Fig. 3.2 achieves stabilization in the unstable upright position within a maximum of 20 samples, depending on the level of disturbance introduced. However, it's important to note that the position of the pendulum Fig. 3.1 does not attain stability; instead, it undergoes movement away from its initial 0 position (when trying to control the pendulum angle).

3.1.3 Implementation

3.2 Model predictive control (MPC)

3.2.1 Theory

Model Predictive Control (MPC) represents an optimal control methodology where the computed control actions are designed to minimize a cost function for a constrained dynamical system over a finite, receding horizon. Its primary advantage over LQR control lies in its superior performance when the process encounters limitations [10]. However, it is acknowledged that MPC require a steeper learning curve and a more intricate implementation process. Often compared to playing chess, MPC depends on a deep understanding of the plant model and subsequent predictions of its behavior, recalculating the best possible output at each step [10][11].

In the MPC framework, the controller, at each time step, receives or estimates the current state of the plant. Based on this information, it computes a sequence of control actions that minimizes the cost over the specified horizon (horizon can be sometimes infinite) [12][10]. This involves solving a constrained optimization problem, heavily relying on an internal plant model and dependent on the current system state. The controller then applies the first computed control action to the plant, disregarding the subsequent ones [12]. This iterative process repeats in each subsequent time step.

In practical applications, despite the finite horizon, MPC inherits several beneficial characteristics from traditional optimal control methodologies. It naturally accommodates multi-input multi-output (MIMO) plants, time delays, and possesses inherent robustness against model errors [11]. Furthermore, nominal stability can be assured by incorporating specific constraints. Overall, while MPC demands a more intricate knowledge of the system and involves a complex implementation, it gives big advantages in handling constraints and offering superior performance [10].

In a mathematical way we can express MPC problem as: finding the best control sequence over a future horizon of N steps Eq. 3.6

$$\min_{u_0, \dots, u_{N-1}} \sum_{k=0}^{N-1} \|y_k - r(t)\|_2^2 + \rho \|u_k - u_r(t)\|_2^2 \quad (3.6)$$

$$\begin{aligned} s.t. x_{k+1} &= f(x_k, u_k) \\ y_k &= g(x_k) \end{aligned} \quad (3.7)$$

$$u_{min} \leq u_k \leq u_{max} \quad (3.8)$$

$$y_{min} \leq y_k \leq y_{max}$$

$$x_o = x(t) \quad (3.9)$$

Where Eq. 3.7 stands as the prediction model, Eq. 3.8 as constraints and Eq. 3.9 as state feedback [11].

3.2.2 Simulation

HOW WAS THE SIMULATION DONE

Following the MATLAB simulation, the obtained results reveal distinctive patterns Fig. 3.4, 3.5, 3.6. Notably, each spike observed across all plots corresponds to the introduced disturbance. These disturbances were deliberately added to assess the robustness of the controller under varying conditions.

Figure 3.4 shows the changing position of the pendulum carriage. Figure 3.5 shows the angle of the pendulum in radians and Fig. 3.3 shows the force i.e. torque applied, to move the pendulum. The full code can be found in appendix on the page x.

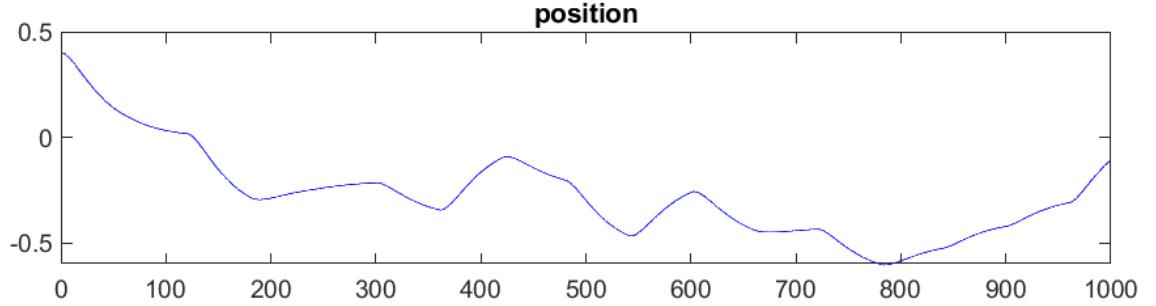


Figure 3.4: Position of the pendulum (MPC simulation).

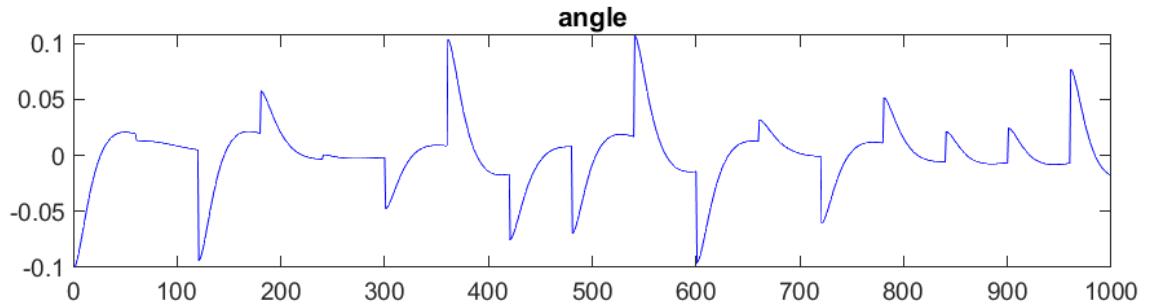


Figure 3.5: Angle of the pendulum (MPC simulation).

Examining the plots, it becomes apparent that the angle of the pendulum Fig. 3.5 achieves stabilization in the unstable upright position within a maximum of 20 samples, depending on the level of disturbance introduced. However, it's important to note that the position of the pendulum Fig. 3.4 does not attain stability; instead, it undergoes movement away from its initial 0 position (when trying to control the pendulum angle).

3.2.3 Implementation

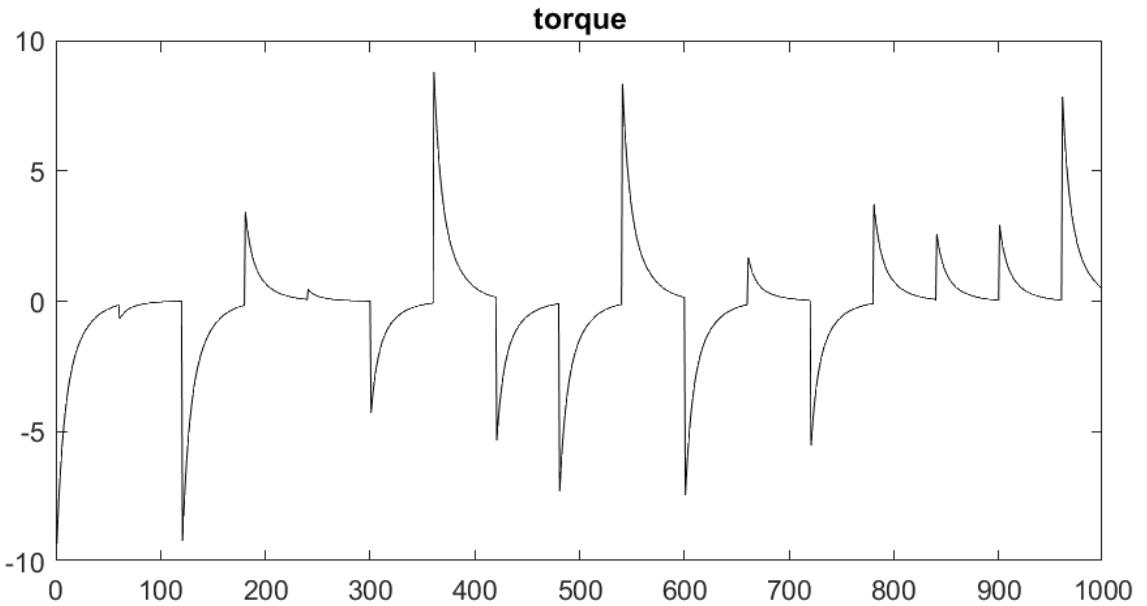


Figure 3.6: Torque of the pendulum (MPC simulation).

3.3 Sliding mode control (SMC)

3.3.1 Theory

Sliding mode control (SMC) is a nonlinear control methodology known for its impressive attributes such as precision, resilience, and straightforward tuning and application. It is applicable to both Single-Input Single-Output (SISO) and Multiple-Input Multiple-Output (MIMO) systems [13][14]. The SMC consists of a two-part controller design. In the initial phase, an N -dimensional sliding surface must be designed where, by definition, the error asymptotically approaches zero. The subsequent phase is concerned with the selection of a control law, that will make the system states be attracted to the sliding surface.

The state-feedback control law is not a continuous function of time, rather, it can transition from one continuous structure to another based on the current position in the state space. The movement of the system as it glides along the boundaries of the control structures is referred to as a sliding mode, and the geometric locus comprising these boundaries is called the sliding surface [15]. SMC exhibits robustness against model uncertainties and disturbances. However, a significant drawback is the occurrence of chattering effects around the surface, which arises as a result of the switching of the control law. This is a common phenomenon in sliding mode control and is highly undesirable in practical applications due to increased control activity and the potential excitation of unmodeled high-frequency dynamics. Nevertheless, these issues can be mitigated or even avoided through the application of appropriate strategies [13].

3.3.2 Simulation

The initial stage in simulating the sliding mode controller in MATLAB involved the creation of the sliding surface Eq. 3.10. Afterwards, the calculation of the output estimate $\hat{\tau}$ Eq. 3.12 needs to be done.

$$s = \left(\frac{d}{dt} + e \right) \tilde{\alpha} + \left(\frac{d}{dt} + \beta \right) \tilde{\rho} = \dot{\tilde{\alpha}} + e\tilde{\alpha} + \dot{\tilde{\rho}} + \beta\tilde{\rho} = -\dot{\alpha} - e\alpha - \dot{\rho} - \beta\rho = 0 \quad (3.10)$$

$$\dot{s} = -\dot{\omega} - \gamma\omega - \dot{v} - \beta v = 0 \quad (3.11)$$

$$\begin{aligned} \dot{s} &= -\frac{f_w m_p l_{sp}}{j_a(m_\omega + m_p)} v - \frac{m_p l_{sp} g}{j_a} \alpha + \frac{m_p l_{sp}}{j_a(m_w \omega + m_p)} \tau + \left(\frac{f_p}{j_a} - e \right) \omega + \\ &+ \frac{f_\omega}{m_\omega + m_p} v - \frac{1}{m_\omega + m_p} \tau - \beta v \end{aligned}$$

$$\begin{aligned} \dot{s} &= -\frac{f_\omega m_p l_{sp}}{j_a(m_\omega + m_p)} v - \frac{m_p l_{sp} g}{j_a} \alpha + \left(\frac{f_p}{j_a} - e \right) \omega + \left(\frac{f_\omega}{m_\omega + m_p} - \beta \right) v + \\ &+ \frac{m_p l_{sp} - j_a}{j_a(m_\omega + m_p)} \tau \end{aligned}$$

$$\begin{aligned} \hat{\tau} &= \frac{j_a(m_\omega + m_p)}{m_p l_{sp} - j_a} \left(\frac{f_\omega m_p l_{sp}}{j_a(m_\omega + m_p)} v + \frac{m_p l_{sp} g}{j_a} \alpha - \left(\frac{f_p}{j_a} - e \right) \omega \right. \\ &\left. - \left(\frac{f_\omega}{m_\omega + m_p} - \beta \right) v \right) \end{aligned} \quad (3.12)$$

$$\begin{aligned} \hat{\tau} &= \frac{0,072(3,2 + 0,329)}{0,329 * 0,44 - 0,072} \left(\frac{6,2 * 0,329 * 0,44}{0,072 * (3,2 + 0,329)} v + \frac{0,329 * 0,44 * 9,81}{0,072} \alpha - \right. \\ &\left. - (0,125 - e) \omega - \left(\frac{6,2}{3,2 + 0,329} - \beta \right) v \right) \end{aligned}$$

$$\begin{aligned} \hat{\tau} &= 12,335239v + 68,877358\alpha - 3,492138(0,125 - e)\omega - \\ &- 3,492138(1,7568 - \beta)v - (10 * sign(s)) \end{aligned} \quad (3.13)$$

where in Eq. 3.11 ω stands for the pendulum angle, $\dot{\omega}$ is the angular velocity of the pendulum, v is position of the base of the pendulum and \dot{v} is the linear velocity of the base of the pendulum. Variables γ and β in Eq. 3.11 are controller design parameters and they need to be tuned for the best performance. Variables $f_\omega, f_p, m_p, m_\omega, l_{sp}, j_a, g$ were obtained from the state space realization of the simulated plant model.

Finally, the sliding mode controller was implemented as written in code 3.1

```
1 s = - qd(2,k) - alpha * q(2,k) - qd(1,k) - beta * q(1,k);
2 tau(1,k) = 12.335239*qd(1,k)+68.877358*q(2,k)-3.492138*(0.125-alpha)*qd
(2,k)-3.492138*(1.7568-beta)*qd(1,k)-(var_k * sign(s));
```

Code 3.1: Sliding mode control implementation in MATLAB.

Following the MATLAB simulation, the obtained results reveal distinctive patterns Fig. 3.7, 3.8, 3.9. Notably, each spike observed across all plots corresponds to the introduced disturbance. These disturbances were deliberately added to assess the robustness of the controller under varying conditions.

Figure 3.7 shows the changing position of the pendulum carriage. Figure 3.8 shows the angle of the pendulum in radians and Fig. 3.9 shows the force i.e. torque applied, to move the pendulum. The full code can be found in appendix on the page xvi.

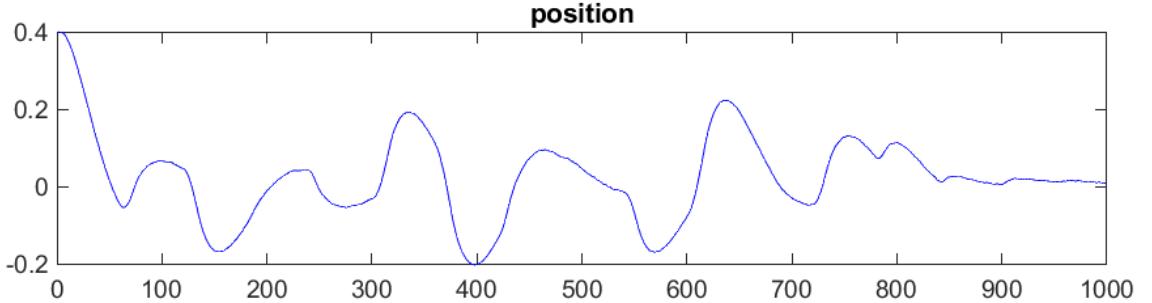


Figure 3.7: Position of the pendulum (SMC simulation).

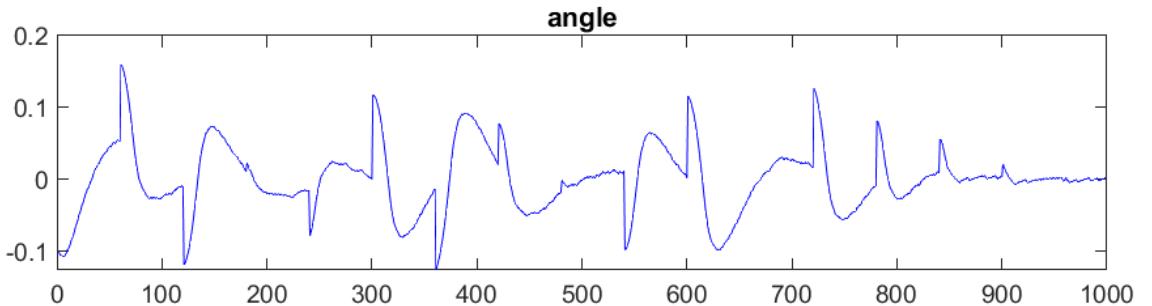


Figure 3.8: Angle of the pendulum (SMC simulation).

Examining the plots, it becomes apparent that the angle of the pendulum Fig. 3.8 achieves stabilization in the unstable upright position within a maximum of 40 samples, depending on the level of disturbance introduced. The position of the pendulum Fig. 3.7 attain stability within a maximum of 40 samples. We can clearly see the rapid switching of the control law on the Fig. 3.9.

3.3.3 Implementation

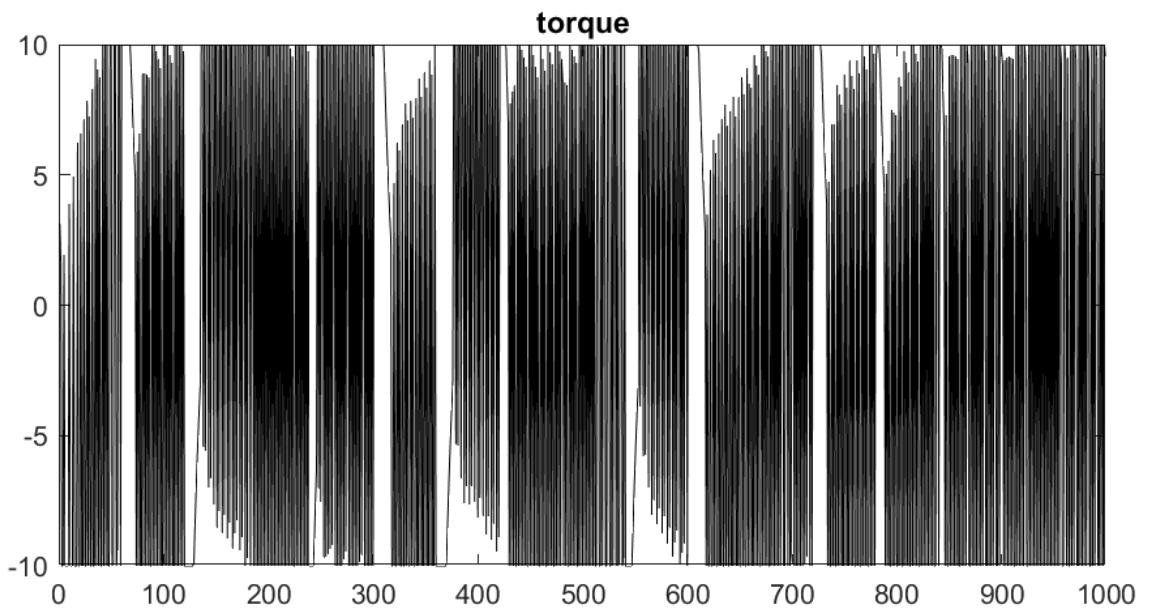


Figure 3.9: Torque of the pendulum (SMC simulation).

3.4 Fuzzy controller

3.4.1 Theory

3.4.2 Simulation

3.4.3 Implementation

3.5 Impedance control

3.5.1 Theory

3.5.2 Simulation

3.5.3 Implementation

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Source code of linear quadratic regulator (Simulation)

```

1 clear all;
2 clc
3
4 %% Parameters and Initialization
5 Ts = 0.02; % Sampling Time
6 time = 20; % Total simulation time
7
8 % Preallocate arrays for system variables
9 q = zeros(2, time/Ts); % Position and angle
10 qd = zeros(2, time/Ts); % Linear and angular velocity
11 qdd = zeros(2, time/Ts); % Linear and angular acceleration
12 q(:, 1) = [0.4; -0.1]; % Initial values for position and angle
13 tau = zeros(1, time/Ts); % Voltage
14 q_r(1) = 0; % Reference value of position
15 q_r(2) = 0; % Reference value of angle
16
17 qd_r = zeros(2, 1); % Reference value of linear and angular velocity
18
19 %% State-space
20 m_p = 0.329; m_w = 3.2; l_sp = 0.44; f_w = 6.2;
21 f_p = 0.009; gra = 9.81; j_a = 0.072; Ts = 0.02;
22
23 % Continuous-time state-space matrices
24 A_c = [0 1 0 0
25 0 -f_w/(m_w+m_p) 0 0
26 0 0 0 1
27 0 (f_w*m_p*l_sp)/(j_a*(m_w+m_p)) (m_p*l_sp*gra)/j_a -f_p/j_a];
28 B_c = [0 ; 1/(m_w+m_p) ; 0 ; -m_p*l_sp/((m_w+m_p)*j_a)];
29 C_c = [1 0 0 0
30 0 1 0 0
31 0 0 1 0
32 0 0 0 1];
33 D_c = [0;0;0;0];
34
35 % Convert to discrete-time
36 sys_cont = ss(A_c, B_c, C_c, D_c);
37 sys_d = c2d(sys_cont, Ts);
38
39 A = sys_d.A; B = sys_d.B; C = sys_d.C; D = sys_d.D;
40
41 %% Pendulum parameters
42 KF=2.6; M0=3.2; M1=0.329; M=M0+M1; ls=0.44; inert=0.072; N_val=0.1446;
43 N01_sq=0.23315; Fr=6.2; C=0.009; gra=9.81;
44
45 a32 = -N_val^2/N01_sq*gra; a33 = -inert*Fr/N01_sq; a34 = N_val*C/N01_sq;
46 a35 = inert*N_val/N01_sq; a42 = M*N_val*gra/N01_sq; a43 = N_val*Fr/N01_sq;
47 a44 = -M*C/N01_sq;
48 a45 = -N_val^2/N01_sq; b3=inert/N01_sq; b4=-N_val/N01_sq;
49 b3_hat = inert/N01_sq+0.1; b4_hat = -N_val/N01_sq+0.1;
50
51 %% Animation parameters
52 xmin = -1;
53 xmax = +1;
54
55 figure;
56 h = [];
57 h(1) = subplot(4,2,1);
58 h(2) = subplot(4,2,3);
59 h(3) = subplot(4,2,5);
60 h(4) = subplot(4,2,7);
61 h(5) = subplot(2,2,2);
62 h(6) = subplot(2,2,4);
63 Disturbance=1;
64

```

```

65 %% Controller Parameters
66 Q = diag([1, 1, 1e7, 1e2]); % State penalization
67 R = 1e7; % Input penalization
68 Np = 10; % Prediction horizon
69 [K, P] = dlqr(A, B, Q, R);
70
71 for k = 1:time/Ts-1
72
73 % Control Algorithm
74 tau(1,k) = -K*[q(1,k); qd(1,k); q(2,k); qd(2,k)]; % LQR
75
76 % Voltage Limitation
77 if abs(tau(:,k)) > 10
78 tau(:,k) = sign(tau(:,k))*10;
79 end
80
81 % Inverted Pendulum Math. Model
82 beta_x2 = (1 + N_val^2/N01_sq*(sin(q(2,k)))^2)^(-1);
83 qdd(:,k+1) = [beta_x2*(a32*sin(q(2,k))*cos(q(2,k))+a33*qd(1,k)+...
84 a34*cos(q(2,k))*(qd(2,k))+a35*sin(q(2,k))*qd(2,k)^2+b3*tau(:,k));
85 beta_x2*(a42*sin(q(2,k))+a43*cos(q(2,k))*qd(1,k)+...
86 a44*(qd(2,k))+a45*cos(q(2,k))*sin(q(2,k))*(qd(2,k))^2+b4*cos(q(2,k))*tau(:,...
87 k))] ;
88 qd(:,k+1) = qd(:,k) + qdd(:,k+1)*Ts;
89 q(:,k+1) = q(:,k) + qd(:,k+1)*Ts;
90 q(2,k+1) = mod(q(2,k+1)+pi,2*pi)-pi;
91
92 % Disturbance
93 if mod(k,60)==0 && Disturbance==1
94 x = rand();
95
96 if x > 0.5 && k ~= 200
97 q(2,k+1) = q(2,k+1) + rand() * 0.12;
98 else
99 q(2,k+1) = q(2,k+1) - rand() * 0.12;
100 end
101
102 if k == 200
103 q(2,k+1) = q(2,k+1) - 0.1;
104 end
105 end
106
107 % Plot Animation
108 plot(0, 'Parent', h(6));
109 hold on;
110 p1 = -q(1,k);
111 p2 = -q(1,k) + 1s * exp(1i*(q(2,k)+pi/2));
112 line(real([p1,p2]), imag([p1,p2]));
113 plot(real(p2), imag(p2), '.', 'markersize', 40);
114 hold off;
115
116 % Center plot w.r.t. object
117 if q(1) > xmax
118 xmin = xmin + 0.1;
119 xmax = xmax + 0.1;
120 elseif q(1) < xmin
121 xmin = xmin - 0.1;
122 xmax = xmax - 0.1;
123 end
124
125 % Update animation
126 grid on;
127 axis([h(1)], [0 time/Ts-1 -1 1]); title('position')
128 axis([h(2)], [0 time/Ts-1 -0.5 0.5]);
129 axis([h(3)], [0 time/Ts-1 -1 1]);
130 axis([h(4)], [0 time/Ts-1 -5 5]);
131 axis([h(5)], [0 time/Ts-1 -10 10]);
132 axis([h(6)], [xmin-.2 xmax+.2 -.5 .5]);
133

```

```

134 % Update subplots
135 drawnow;
136
137 plot(q(1,1:k), 'b', 'Parent', h(1)); %Position
138 plot(q(2,1:k), 'b', 'Parent', h(2)); % Angle
139 plot(qd(1,1:k), 'b', 'Parent', h(3)); % Velocity
140 plot(qdd(1,1:k), 'b', 'Parent', h(4)); % Angular velocity
141 plot(tau(1:k), 'k', 'Parent', h(5)); % Torque
142
143 title(h(1), 'Position');
144 title(h(2), 'Angle');
145 title(h(3), 'Velocity');
146 title(h(4), 'Angular Velocity');
147 title(h(5), 'Torque');
148 end

```

Code 3.2: Source code of linear quadratic regulator (Simulation).

Source code of linear quadratic regulator (Real implementation)

```

1 % Pause for 3 seconds before starting the code execution
2 pause(3);
3
4 % Clear workspace, close all figures, and clear command window
5 clear all;
6 close all;
7 clc;
8
9 % Add necessary paths for libraries
10 addpath(genpath('CLSS Praxis'), genpath('hudaqlib'))
11
12 % Hudaq device initialization
13 dev = HudaqDevice('MF634');
14
15 % Total experiment samples
16 s = 2000;
17 % Sampling period
18 Ts = 0.02;
19
20 % States during the experiment
21 x = zeros(s,4);
22 % Sample states
23 z2 = zeros(4,1);
24 % Initial values
25 x(1,:) = [(AIRead(dev,1)/0.15/100) 0.01*round(-13.1*AIRead(dev,3)) (-AIRead
    (dev,2)/0.96*pi/180) 0];
26
27 % Force array to store calculated force values
28 Force = zeros(s,1);
29
30 % Pendulum parameters
31 m_p = 0.329; m_w = 3.2; l_sp = 0.44; f_w = 6.2;
32 f_p = 0.009; gra = 9.81; j_a = 0.072; Ts = 0.02;
33
34 % Continuous-time state-space matrices
35 A_c = [ 0      1      0      0
          0      -f_w/(m_w+m_p) 0      0
          0      0      0      1
          0      (f_w*m_p*l_sp)/(j_a*(m_w+m_p)) (m_p*l_sp*gra)/j_a -f_p/j_a ];
36 B_c = [ 0      ; 1/(m_w+m_p) ; 0      ; -m_p*l_sp/((m_w+m_p)*j_a) ];
37 C_c = [ 1      0      0      0
          0      1      0      0
          0      0      0      1];
38 D_c = [ 0;0;0;0];
39
40 % Convert to discrete-time
41 sys_cont = ss(A_c,B_c,C_c,D_c);
42 sys_d = c2d(sys_cont,Ts);
43
44 A = sys_d.A; B = sys_d.B; C = sys_d.C; D = sys_d.D;
45
46 % LQR controller parameters
47 Q = diag([100, .1, 100, .1]); % State penalization
48 R = 1e-2; % Input penalization
49 Np = 60; % Prediction horizon
50 [K, P] = dlqr(A, B, Q, R);
51
52 % Loop through each sample
53 for i = 1:s-1
54
55 % LQR CONTROL ALGORITHM
56 Force(i) = ctranspose(-K*x(i,:));
57
58 % Save the data
59
60
61
62
63
64

```

```

65 z2(1) = AIRead(dev,1)/0.15/100;      % Position of cart (meter)
66 z2(2) = 0.01*round(-13.1*AIRead(dev,3));    % Speed of cart (m/s)
67 z2(3) = -AIRead(dev,2)/0.96*pi/180;   % Angle of pendulum (radian)
68
69 % Voltage limitation
70 if abs(Force(i)) > 10
71 Force(i) = sign(Force(i))*10;
72 end
73
74 % Apply calculated voltage
75 tic
76 while toc < Ts
77 DOWriteBit(dev, 1, 2, 1);    % Activation pendulum
78 DOWriteBit(dev, 1, 2, 0);    % Channel 1 consists of DO0..DO7
79 DOWriteBit(dev, 1, 2, 1);    % DO2 Requires continuous pulse
80 AOWrite(dev, 2, Force(i));  % Apply calculated voltage
81 end
82
83 % Angular speed calculation (derivative)
84 z2_winkel = -AIRead(dev,2)/0.96*pi/180;
85 z2(4) = (z2_winkel - z2(3))/Ts;    % Angular speed of pendulum
86
87 % Update states
88 x(i+1,:) = ctranspose(z2);
89
90 % Check if the pendulum is out of range
91 if abs(z2(1)) > 0.3 || abs(z2(3)*180/pi) > 10
92 disp('Please bring me back !');
93 pause(3);    % Wait for 3 seconds
94 end

```

Code 3.3: Source code of linear quadratic regulator (Real implementation).

Source code of model predictive controller (Simulation)

```

1 % Clear workspace, command window, and import CasADi library
2 clear all;
3 clc
4
5 %% Parameters and Initialization
6 Ts = 0.02; % Sampling Time
7 time = 20; % Total simulation time
8
9 % Preallocate arrays for system variables
10 q = zeros(2, time/Ts); % Position and angle
11 qd = zeros(2, time/Ts); % Linear and angular velocity
12 qdd = zeros(2, time/Ts); % Linear and angular acceleration
13 q(:, 1) = [0.4; -0.1]; % Initial values for position and angle
14 tau = zeros(1, time/Ts); % Voltage
15
16 q_r(1) = 0; % Reference value of position
17 q_r(2) = 0; % Reference value of angle
18 Q = diag([1, 1, 1e7, 1e2]); % State penalisation
19 R = 1e7; % Input penalisation
20 Np = 10; % Prediction horizon
21
22 qd_r = zeros(2, 1); % Reference value of linear and angular velocity
23
24 %% State-space
25 m_p = 0.329; m_w = 3.2; l_sp = 0.44; f_w = 6.2;
26 f_p = 0.009; gra = 9.81; j_a = 0.072; Ts = 0.02;
27
28 % Continuous-time state-space matrices
29 A_c = [0 1 0 0
30 0 -f_w/(m_w+m_p) 0 0
31 0 0 0 1
32 0 (f_w*m_p*l_sp)/(j_a*(m_w+m_p)) (m_p*l_sp*gra)/j_a -f_p/j_a];
33 B_c = [0 ; 1/(m_w+m_p) ; 0 ; -m_p*l_sp/((m_w+m_p)*j_a)];
34 C_c = [1 0 0 0
35 0 1 0 0
36 0 0 1 0
37 0 0 0 1];
38 D_c = [0;0;0;0];
39
40 % Convert to discrete-time
41 sys_cont = ss(A_c,B_c,C_c,D_c);
42 sys_d = c2d(sys_cont,Ts);
43
44 A = sys_d.A; B = sys_d.B; C = sys_d.C; D = sys_d.D;
45 [K, P] = dlqr(A, B, Q, R); % Linear quadratic regulator for
        calculating the P matrix
46
47 %% Pendulum parameters
48 KF=2.6; M0=3.2; M1=0.329; M=M0+M1; ls=0.44; inert=0.072; N_val=0.1446;
49 N01_sq=0.23315; Fr=6.2; C=0.009; gra=9.81;
50
51 a32 = -N_val^2/N01_sq*gra ; a33 = -inert*Fr/N01_sq; a34 = N_val*C/N01_sq;
52 a35 = inert*N_val/N01_sq; a42 = M*N_val*gra/N01_sq; a43 = N_val*Fr/N01_sq;
        a44 = -M*C/N01_sq;
53 a45 = -N_val^2/N01_sq; b3=inert/N01_sq; b4=-N_val/N01_sq;
54 b3_hat = inert/N01_sq+0.1; b4_hat = -N_val/N01_sq+0.1;
55
56 %% Animation parameters
57 xmin = -1;
58 xmax = +1;
59
60 figure;
61 h = [];
62 h(1) = subplot(4,2,1);
63 h(2) = subplot(4,2,3);

```

```

64 h(3) = subplot(4,2,5);
65 h(4) = subplot(4,2,7);
66 h(5) = subplot(2,2,2);
67 h(6) = subplot(2,2,4);
68
69 Disturbance = 1;
70
71 % Hessian , F matrices of cost function
72 [H, F] = Controller_MPC_CostFunction(A, B, Np, Q, R, P);
73 s = zeros(1, Np);
74 s(1, 1) = 1;
75
76 for k = 1:time/Ts-1
77
78 % CONTROL ALGORITHM - Explicit MPC
79 tau(1, k) = -s*(H(-1))*F*[q(1, k); qd(1, k); q(2, k); qd(2, k)];
80
81 % Voltage limitation
82 if abs(tau(:, k)) > 10
83 tau(:, k) = sign(tau(:, k)) * 10;
84 end
85
86 % Inverted pendulum math model
87 beta_x2 = (1 + N_val^2/N01_sq*(sin(q(2, k)))^2)^(-1);
88 qdd(:, k+1) = [beta_x2*(a32*sin(q(2, k))*cos(q(2, k))+a33*qd(1, k)+...
89 a34*cos(q(2, k))*(qd(2, k))+a35*sin(q(2, k))*qd(2, k)^2+b3*tau(:, k));
90 beta_x2*(a42*sin(q(2, k))+a43*cos(q(2, k))*qd(1, k)+...
91 a44*(qd(2, k))+a45*cos(q(2, k))*sin(q(2, k))*(qd(2, k))^2+b4*cos(q(2, k))*...
tau(:, k))];
92
93 qd(:, k+1) = qd(:, k) + qdd(:, k+1)*Ts;
94 q(:, k+1) = q(:, k) + qd(:, k+1)*Ts;
95 q(2, k+1) = mod(q(2, k+1) + pi, 2*pi) - pi;
96
97 % Disturbance
98 if mod(k, 60) == 0 && Disturbance == 1
99 x = rand();
100 if x > 0.5 && k ~= 200
101 q(2, k+1) = q(2, k+1) + rand() *0.12;
102 else
103 q(2, k+1) = q(2, k+1) - rand() *0.12;
104 end
105 if k == 200
106 q(2, k+1) = q(2, k+1) - 0.1;
107 end
108 end
109
110 % Plotting animation
111 plot(0, 'Parent', h(6));
112 hold on;
113 p1 = -q(1, k);
114 p2 = -q(1, k) + 1s*exp(1i*(q(2, k)+pi/2));
115 line(real([p1, p2]), imag([p1, p2]));
116 plot(real(p2), imag(p2), '.', 'markersize', 40);
117 hold off;
118
119 % Center plot w.r.t. object
120 if q(1) > xmax
121 xmin = xmin + 0.1;
122 xmax = xmax + 0.1;
123 elseif q(1) < xmin
124 xmin = xmin - 0.1;
125 xmax = xmax - 0.1;
126 end
127
128 grid on;
129 axis([h(1)], [0 time/Ts-1 -1 1]); title('position')
130 axis([h(2)], [0 time/Ts-1 -0.5 0.5]);
131 axis([h(3)], [0 time/Ts-1 -1 1]);
132 axis([h(4)], [0 time/Ts-1 -5 5]);

```

```

133 axis([h(5)], [0 time/Ts-1 -10 10]);
134 axis([h(6)], [xmin-.2 xmax+.2 -.5 .5]);
135
136 % Update animation
137 drawnow;
138
139 % Plotting system variables
140 plot(q(1, 1:k), 'b', 'Parent', h(1)); % Position
141 plot(q(2, 1:k), 'b', 'Parent', h(2)); % Angle
142 plot(qd(1, 1:k), 'b', 'Parent', h(3)); % Velocity
143 plot(qdd(1, 1:k), 'b', 'Parent', h(4)); % Angular velocity
144 plot(tau(1:k), 'k', 'Parent', h(5)); % Torque
145
146 title(h(1), 'position');
147 title(h(2), 'angle');
148 title(h(3), 'velocity');
149 title(h(4), 'angular velocity');
150 title(h(5), 'torque');
151 end

```

Code 3.4: Source code of model predictive controller (Simulation).

Source code of model predictive controller (Real implementation)

```

1 % Pause for 3 seconds to allow initialization
2 pause(3);
3
4 % Clear workspace, close all figures, and clear command window
5 clear all;
6 close all;
7 clc;
8
9 % Add necessary paths for CLSS Praxis and hudaqlib
10 addpath(genpath('CLSS Praxis'), genpath('hudaqlib'))
11
12 % Create a HudaqDevice object for MF634
13 dev = HudaqDevice('MF634');
14
15 % Number of samples and sampling period
16 s = 2000; % Total experiment samples
17 Ts = 0.02; % Sampling period
18
19 % Initialize arrays for storing system states and control input
20 x = zeros(s, 4); % States during the experiment
21 z2 = zeros(4, 1); % Sample states
22 x(1, :) = [(AIRead(dev, 1)/0.15/100) 0.01*round(-13.1*AIRead(dev,
3)) (-AIRead(dev, 2)/0.96*pi/180) 0]; % Initial values
23 Force = zeros(s, 1); % Control input (force applied)
24
25 %% State-space
26 m_p = 0.329; m_w = 3.2; l_sp = 0.44; f_w = 6.2;
27 f_p = 0.009; gra = 9.81; j_a = 0.072; Ts = 0.02;
28
29 % Define continuous-time state-space matrices
30 A_c = [ 0      1      0      0
31          0 -f_w/(m_w+m_p)      0      0
32          0      0      0      1
33          0 (f_w*m_p*l_sp)/(j_a*(m_w+m_p)) (m_p*l_sp*gra)/j_a -f_p/j_a ];
34 B_c = [ 0 ; 1/(m_w+m_p) ; 0 ; -m_p*l_sp/((m_w+m_p)*j_a) ];
35 C_c = [ 1 0 0 0
36 0 1 0 0
37 0 0 1 0
38 0 0 0 1];
39 D_c = [ 0; 0; 0; 0];
40
41 % Convert to discrete-time
42 sys_cont = ss(A_c, B_c, C_c, D_c);
43 sys_d = c2d(sys_cont, Ts);
44
45 A = sys_d.A; B = sys_d.B; C = sys_d.C; D = sys_d.D;
46
47 % Hessian, F matrices of cost function
48 [H, F] = Controller_MPC_CostFunction(A, B, Np, Q, R, P);
49 a = zeros(1, Np);
50 a(1, 1) = 1;
51
52 for i = 1:s-1
53
54 % MPC CONTROL ALGORITHM
55 Force(i) = ctranspose(-a*(H^(-1))*F*x(i, :));
56
57 % Save the data
58 z2(1) = AIRead(dev, 1)/0.15/100; % Position of the cart (
59 % meter)
60 z2(2) = 0.01*round(-13.1*AIRead(dev, 3)); % Speed of the cart (m/s)
61 z2(3) = -AIRead(dev, 2)/0.96*pi/180; % Angle of the pendulum (
% radian)

```

```

62      % Voltage limitation
63      if abs(Force(i)) > 10
64          Force(i) = sign(Force(i))*10;
65      end
66
67      % Apply calculated voltage
68      tic
69      while toc < Ts
70          DOWriteBit(dev, 1, 2, 1); % Freischaltung Pendel
71          DOWriteBit(dev, 1, 2, 0); % Channel 1 consists of DO0..DO7
72          DOWriteBit(dev, 1, 2, 1); % DO2 Requires continuous pulse
73          AOWrite(dev, 2, Force(i)); % Apply calculated voltage
74      end
75
76      % Angular speed calculation (derivative)
77      z2_winkel = -AIRead(dev, 2)/0.96*pi/180;
78      z2(4) = (z2_winkel - z2(3))/Ts; % Angular speed of the pendulum
79
80      % Update system states
81      x(i+1, :) = ctranspose(z2);
82
83      % Check if the pendulum is out of range
84      if abs(z2(1)) > 0.3 || abs(z2(3)*180/pi) > 10      % Der Pendel ist
85          ausser Bereich
86          disp('Please bring me back !');
87          pause(3); % Wait 3 seconds
end

```

Code 3.5: Source code of model predictive controller (Real implementation).

Source code of cost function callculation for MPC

```

1  function [H, F] = CostFunctionMPC(A, B, np, Q, R, P)
2
3  [nx, nu] = size(B); % Get system dimensions
4
5  phi = zeros(np*nx, nx); % Preallocate matrix Phi
6  for i = 1:np % Create matrix M using powers of matrix A
7  phi(i*nx-nx+1:i*nx, :) = A^i;
8  end
9
10 % Get prediction matrix N
11 gamma = zeros(nx*np, nu*np); % Preallocate matrix Gamma
12 NN = zeros(nx*np, nu); % Preallocate auxiliary matrix NN with
13 % zeros
14 for i = 1:np % Calculate auxiliary matrix NN
15 NN(i*nx-nx+1:i*nx, :) = A^(i - 1) * B;
16 end
17 for i = 1:np % By shifting NN matrix create matrix Gamma
18 gamma(i*nx-nx+1:end, i*nu-nu+1:i*nu) = NN(1:np*nx-(i - 1)*nx, :);
19 end
20 % Get cost function matrices H,F
21 psi = kron(eye(np), R);
22 omega = zeros(np*nx, np*nx); % Preallocate matrix Omega
23 for i = 1:np-1 % Create matrix Omega
24 omega(i*nx-nx+1:i*nx, i*nx-nx+1:i*nx) = Q;
25 end
26 omega(np*nx-nx+1:np*nx, np*nx-nx+1:np*nx) = P; % Add last diagonal element
27 H=2*(psi+gamma'*omega*gamma);
28 F=2*gamma'*omega*phi;
29
30 end

```

Code 3.6: Source code of cost function callculation for MPC.

Source code of sliding mode controller (Simulation)

```

1 clear all;
2 clc;
3
4 %% Parameters and Initialization
5 Ts = 0.02; % Sampling Time
6 time = 20; % Total simulation time
7 alpha = 6;
8 beta = 1.8; % Variable that changes the speed of switching or duty cycle
9 var_k = 10;
10
11 % Initial values
12 q = zeros(2, time / Ts); % Position and angle
13 qd = zeros(2, time / Ts); % Linear and angular velocity
14 qdd = zeros(2, time / Ts); % Linear and angular acceleration
15 q(:, 1) = [0.4; -0.1]; % Initial values for position and angle
16 tau = zeros(1, time / Ts); % Voltage
17
18 q_r(1) = 0; % Reference value of position
19 q_r(2) = 0; % Reference value of angle
20
21 qd_r = zeros(2, 1); % Reference value of linear and angular velocity
22
23 %% State-space
24 % Define system matrices for continuous-time and discrete-time systems
25 m_p = 0.329; m_w = 3.2; l_sp = 0.44; f_w = 6.2;
26 f_p = 0.009; gra = 9.81; j_a = 0.072; Ts = 0.02;
27
28 A_c = [0 1 0 0;
29 0 -f_w / (m_w + m_p) 0 0;
30 0 0 1;
31 0 (f_w * m_p * l_sp) / (j_a * (m_w + m_p)) (m_p * l_sp * gra) / j_a -f_p /
32 j_a];
33 B_c = [0; 1 / (m_w + m_p); 0; -m_p * l_sp / ((m_w + m_p) * j_a)];
34 C_c = [1 0 0 0;
35 0 1 0 0;
36 0 0 1 0];
37 D_c = [0; 0; 0; 0];
38
39 % Convert to discrete-time system
40 sys_cont = ss(A_c, B_c, C_c, D_c);
41 sys_d = c2d(sys_cont, Ts);
42
43 A = sys_d.A; B = sys_d.B; C = sys_d.C; D = sys_d.D;
44
45 %% Pendulum parameters
46 KF = 2.6; M0 = 3.2; M1 = 0.329; M = M0 + M1; ls = 0.44; inert = 0.072;
47 N_val = 0.1446;
48 N01_sq = 0.23315; Fr = 6.2; C = 0.009; gra = 9.81;
49
50 % Define coefficients for inverted pendulum math model
51 a32 = -N_val^2 / N01_sq * gra; a33 = -inert * Fr / N01_sq; a34 = N_val * C
52 / N01_sq;
53 a35 = inert * N_val / N01_sq; a42 = M * N_val * gra / N01_sq; a43 = N_val *
54 Fr / N01_sq;
55 a44 = -M * C / N01_sq; a45 = -N_val^2 / N01_sq; b3 = inert / N01_sq; b4 =
56 -N_val / N01_sq;
57 b3_hat = inert / N01_sq + 0.1; b4_hat = -N_val / N01_sq + 0.1;
58
59 %% Animation parameters
60 xmin = -1;
61 xmax = +1;
62
63 % Create subplots for animation
64 figure;

```

```

61 h = [];
62 h(1) = subplot(4, 2, 1);
63 h(2) = subplot(4, 2, 3);
64 h(3) = subplot(4, 2, 5);
65 h(4) = subplot(4, 2, 7);
66 h(5) = subplot(2, 2, 2);
67 h(6) = subplot(2, 2, 4);
68
69 Disturbance = 1;
70
71 %% Control
72 for k = 1:time / Ts - 1
73 % Control algorithm
74 s = -qd(2, k) - alpha * q(2, k) - qd(1, k) - beta * q(1, k);
75 tau(1, k) = 12.335239 * qd(1, k) + 68.877358 * q(2, k) - 3.492138 * (0.125
    - alpha) * qd(2, k)...
76 - 3.492138 * (1.7568 - beta) * qd(1, k) - (var_k * sign(s));
77
78 % Voltage limitation
79 if abs(tau(:, k)) > 10
80 tau(:, k) = sign(tau(:, k)) * 10;
81 end
82
83 % Inverted pendulum math model
84 beta_x2 = (1 + N_val^2 / N01_sq * (sin(q(2, k)))^2)^(-1);
85 qdd(:, k + 1) = [beta_x2 * (a32 * sin(q(2, k)) * cos(q(2, k)) + a33 * qd(1,
    k) + ...
86 a34 * cos(q(2, k)) * (qd(2, k)) + a35 * sin(q(2, k)) * qd(2, k)^2 + b3 *
    tau(:, k));
87 beta_x2 * (a42 * sin(q(2, k)) + a43 * cos(q(2, k)) * qd(1, k) + ...
88 a44 * (qd(2, k)) + a45 * cos(q(2, k)) * sin(q(2, k)) * (qd(2, k))^2 + b4 *
    cos(q(2, k)) * tau(:, k))];
89
90 % Update position and velocity
91 qd(:, k + 1) = qd(:, k) + qdd(:, k + 1) * Ts;
92 q(:, k + 1) = q(:, k) + qd(:, k + 1) * Ts;
93 q(2, k + 1) = mod(q(2, k + 1) + pi, 2 * pi) - pi;
94
95 % Disturbance
96 if mod(k, 60) == 0 && Disturbance == 1
97 x = rand();
98
99 if x > 0.5 && k ~= 200
100 q(2, k + 1) = q(2, k + 1) + rand() * 0.12;
101 else
102 q(2, k + 1) = q(2, k + 1) - rand() * 0.12;
103 end
104
105 if k == 200
106 q(2, k + 1) = q(2, k + 1) - 0.1;
107 end
108 end
109
110 % Animation
111 plot(0, 'Parent', h(6));
112 hold on;
113 p1 = -q(1, k);
114 p2 = -q(1, k) + 1s * exp(1i * (q(2, k) + pi / 2));
115 line(real([p1, p2]), imag([p1, p2]));
116 plot(real(p2), imag(p2), '.', 'markersize', 40);
117 hold off;
118
119 % Center plot w.r.t. object
120 if q(1) > xmax
121 xmin = xmin + 0.1;
122 xmax = xmax + 0.1;
123 elseif q(1) < xmin
124 xmin = xmin - 0.1;
125 xmax = xmax - 0.1;
126 end

```

```

127 grid on;
128 axis([h(1)], [0 time / Ts - 1 -1 1]);
129 title('position');
130 axis([h(2)], [0 time / Ts - 1 -0.5 0.5]);
131 title('angle');
132 axis([h(3)], [0 time / Ts - 1 -1 1]);
133 title('velocity');
134 axis([h(4)], [0 time / Ts - 1 -5 5]);
135 title('angular velocity');
136 axis([h(5)], [0 time / Ts - 1 -10 10]);
137 title('torque');
138
139 % Update animation
140 drawnow;
141
142 % Plot results
143 plot(q(1, 1:k), 'b', 'Parent', h(1)); % Position
144 plot(q(2, 1:k), 'b', 'Parent', h(2)); % Angle
145 plot(qd(1, 1:k), 'b', 'Parent', h(3)); % Velocity
146 plot(qdd(1, 1:k), 'b', 'Parent', h(4)); % Angular velocity
147 plot(tau(1:k), 'k', 'Parent', h(5)); % Torque
148 title(h(1), 'position');
149 title(h(2), 'angle');
150 title(h(3), 'velocity');
151 title(h(4), 'angular velocity');
152 title(h(5), 'torque');
153 end

```

Code 3.7: Source code of sliding mode controller (Simulation).

Source code of sliding mode controller (Real implementation)

```

1 pause(3);
2 clear all;
3 close all;
4 clc;
5
6 addpath(genpath('CLSS Praxis'),genpath('hudaqlib'));
7 dev = HudaqDevice('MF634');
8 mn = 25;
9 mkdir Messung_25;
10
11 s = 2000; %total experiment sample
12 Ts = 0.02; % sampling period
13
14 x = zeros(s,4); %states during experiment
15 z2 = zeros(4,1);% sample states
16 x(1,:) = [(AIRead(dev,1)/0.15/100) 0.01*round(-13.1*AIRead(dev,3)) (-AIRead
    (dev,2)/0.96*pi/180) 0]; % initial values
17 Force = zeros(s,1);
18
19 %% State-space
20 m_p = 0.329; m_w = 3.2; l_sp = 0.44; f_w = 6.2;
21 f_p = 0.009; gra = 9.81; j_a = 0.072; Ts = 0.02;
22
23 A_c = [ 0 1 0 0
24 0 -f_w/(m_w+m_p) 0 0
25 0 0 0 1
26 0 (f_w*m_p*l_sp)/(j_a*(m_w+m_p)) (m_p*l_sp*gra)/j_a -f_p/j_a];
27 B_c = [0 ; 1/(m_w+m_p) ; 0 ; -m_p*l_sp/((m_w+m_p)*j_a)];
28 C_c = [ 1 0 0 0
29 0 1 0 0
30 0 0 1 0
31 0 0 0 1];
32 D_c = [ 0 ; 0 ; 0 ; 0 ];
33
34 sys_cont = ss(A_c,B_c,C_c,D_c);
35 sys_d = c2d(sys_cont,Ts);
36
37 A = sys_d.A;B = sys_d.B;C = sys_d.C;D = sys_d.D;
38
39 %% Implementation
40 alpha = 6;
41 beta = 1.8; % variable that changes the speed of switching resp the duty
    cycle
42 var_k = 10;
43
44 for i = 1:s-1
45
46 % CONTROL ALGORITHM
47
48 s1 = - x(i,4) - alpha * x(i,3) - x(i,2) - beta * x(i,1);
49 Force(i) = 12.335239*x(i,2)+68.877358*x(i,3)-3.492138*(0.125-alpha)*x(i,4)
    -3.492138*(1.7568-beta)*x(i,2)-(var_k * sign(s1));
50
51 % save the data
52 z2(1) = AIRead(dev,1)/0.15/100; % position of cart (meter)
53 z2(2) = 0.01*round(-13.1*AIRead(dev,3)); % speed of cart (m/s)
54 z2(3) = -AIRead(dev,2)/0.96*pi/180; % angle of pendulum (radian)
55
56 % voltage limitation
57 if abs(Force(i))>10
58 Force(i) = sign(Force(i))*10;
59 end
60
61 % apply calculated voltage
62 tic

```

```

63 while toc<Ts
64 DOWriteBit(dev,1,2,1)           % Freischaltung Pendel
65 DOWriteBit(dev,1,2,0)           % channel 1 besteht aus DO0..DO7
66 DOWriteBit(dev,1,2,1)           % DO2 requires continuous impuls
67 AOWrite(dev, 2, Force(i));     % apply calculated voltage
68 end
69
70 % angular speed calculation (derivative)
71 z2_winkel = -AIRead(dev,2)/0.96*pi/180;
72 z2(4) = (z2_winkel-z2(3))/Ts; % angular speed of pendulum
73
74 x(i+1,:)= z2';
75
76 if abs(z2(1)) > 0.3 || abs(z2(3)*180/pi) > 10      % Der Pendel ist ausser
    Bereich
77 disp('Please bring me back !');
78 pause(3);          % wait 3 second
79 end
80
81 end

```

Code 3.8: Source code of sliding mode controller (Real implementation).