



# An Improved Transient Method for the Simultaneous Determination of Free-stream Reference Temperature and Convective Heat Transfer Coefficient:

## *The "Invariant h Method"*

CENGIZ CAMCI

*The Pennsylvania State University, Department of Aerospace Engineering  
Turbomachinery Heat Transfer Laboratory  
224-C Hammond Building, University Park, PA, 16802*

### **Abstract**

Heat transfer coefficient  $h$  in forced convection experiments is considered to be an indicator of the fluid dynamic status of the viscous flow between a wall and a fluid capable of transferring thermal energy via convection.  $h$  is defined as a quantity that is independent of thermal boundary conditions. By definition,  $q_w$  is linearly dependent on  $(T_{\infty} - T_w)$  by the relation  $q_w = h(T_{\infty} - T_w)$ . This paper deals with the development of a new method that can obtain heat transfer coefficient  $h$  and reference free stream temperature (adiabatic wall temp.) simultaneously by using the useful property that " $h$  does not vary in time provided that the mean flow conditions are stationary". The new "*invariant h*" method can analytically determine local free stream temperatures in internal flow cases in which heat pick-up (or heat loss) is at a non-negligible magnitude. The method eliminates the need for cumbersome and error prone free stream gas temperature measurements using conventional sensors. The general method described in this paper is applicable to many different surface temperature mapping schemes that has a capability of measuring the surface temperature at each time step during a transient experiment. The mapping techniques in this category are discrete temperature sensors, infrared imaging and temperature sensitive paints (thermographic phosphors). An extension of the "*invariant h*" method to "single shot" transient experiments can also be made for the direct determination of adiabatic wall effectiveness values in gas turbine cooling research using liquid crystal coatings.

## Nomenclature

$C_p$	Specific heat at constant pressure
FACTOR	A multiplier used to vary free stream reference temperature
FACTORx $T_{\infty}$	Suggested free stream reference temperature
$k$	Thermal conductivity for the semi-infinite wall
$h$	Convective heat transfer coefficient
	$h(t) = q_w(t)/(T_{aw} - T_w(t))$
$n$	$n$ th point in the $T_w(t)$ measurement
Nu	Local Nusselt number
Pr	Local molecular Prandtl number
$q_w$	Wall heat flux
Re	Local Reynolds Number
$\sqrt{\rho C_p k}$	Thermo-physical triple product of the semi-infinite body
$\Delta \tau$	Time step during the acquisition of $T_w(t)$
$T_{\infty}$	Reference free stream temperature
$\Delta T_{\infty}$	Deviation from the actual free stream temperature ( $T_{\infty}$ -FACTORx $T_{\infty}$ )
$T_{aw}$	Adiabatic wall temperature
$T_i$	Initial temperature of the transient experiment
$T_{\infty}$	Free stream turbulence intensity
$T_w$	Wall temperature
$t$	Time
$t_f$	Final time of the transient experiment

### Greek symbols

$\delta$	Absolute error indicator
$\beta$	Non-dimensional time
$\Gamma$	Normalized slope of $h(t)$ $\Gamma = \{[h(t)-h(t-1)]/\Delta\tau\}/h(t_f)$
$\rho$	Density

## INTRODUCTION

Accurate determination of convective heat transfer coefficient requires a precise knowledge of the free stream reference temperature  $T_{\infty}$  and the local temperature of the wall  $T_w$ . It is also a known fact that a better reference temperature for the gas stream is its corresponding “adiabatic wall temperature/recovery temperature”  $T_{aw}$ , Eckert<sup>1</sup>, Kays and Crawford<sup>2</sup>, Goldstein, Behbahani and Heppelmann<sup>3</sup>. Using adiabatic wall temperature as a reference temperature becomes more important in the compressible flow range where viscous dissipation is significant. In practice, the heat transfer coefficient  $h$  is determined by dividing the local heat flux by the difference between the adiabatic wall temperature and the wall temperature. The general method of finding  $h$  requires an accurate measurement of wall heat flux, reference temperature for the gas stream, and local wall temperature in both steady-state and transient methods. Free stream reference temperature is usually measured at the entrance of the test section once and the same

value is assigned to all downstream locations. Assigning the free stream temperature measured at the entrance to all downstream locations can easily increase the error made in the determination of  $h$ . In many steady state methods, there are surface heater strips in the form of constant heat flux surfaces or isothermal wall strip heaters. The existence of any surface heater can increase the free stream gas temperature as one moves in the downstream direction of any heat transfer model. The free stream gas has the capability of picking up non-negligible thermal energy from the wall. In the case of high speed flows, thermal energy produced via viscous dissipation mechanism may affect the local reference temperature of the free stream. In transient methods, the model is initially at a different temperature than that of the free stream fluid. Especially in internal flow cases such as serpentine cooling passages, pipes, ducts with small cross sectional area, heat pick-up (or heat loss) may be significant to alter the free stream reference temperature when compared to its inlet value. One way to solve this problem is to instrument the passage in the streamwise direction so that the free stream reference temperatures are locally measured.

This paper describes an alternative method for the simultaneous determination of local  $h$  and  $T_{\infty}$  (or  $T_{aw}$ ) in transient heat transfer experiments. The new concept introduced is capable of producing accurate local free stream reference temperatures without performing cumbersome gas stream temperature measurements using discrete sensors. The present method is extremely time efficient in transient heat transfer experiments in which heat pick-up from the model walls (or heat loss to the model surfaces) continuously modify the local gas temperature value in the streamwise direction. Since the local free stream reference temperature is the same as the adiabatic wall temperature, the new method can be considered as a direct and accurate measurement method for the local adiabatic wall temperature. By using the "*invariant h*" method discrete gas temperature measurements based on discrete sensors and associated measurement errors can be avoided in flow systems in which heat pick-up (or heat loss) by the free stream is significant.

The "*invariant h*" method presented in this paper is directly applicable to many surface temperature mapping methods including discrete thin film sensors, thermocouples, thermistor based sensors, thermographic phosphors and infrared imaging. In the current study, the acquisition of local surface temperature at every time step starting from the beginning of the transient experiment is required. In this approach, a general wall temperature to heat flux conversion scheme allows a time dependent free stream reference temperature.

The same concept can also be implemented into more time efficient single shot experiments in which a single surface thermal image at a pre-selected time is acquired to obtain local  $h$  values by using an inverse solution of one dimensional transient heat conduction equation. This specific approach requires that the free stream reference temperature is constant in time. This method is preferred frequently because of its time efficiency and simplicity. Details of the modifications required to use the new "*invariant h*" method in a single shot experiment employing thermographic liquid crystal (TLC) coatings and the applications of the method in finding adiabatic wall temperature (or effectiveness) in film cooling research is not included in this paper. An accompanying paper by Camci<sup>4</sup> presents a detailed treatment of liquid crystal and film cooling related aspects of the new method.

## A BRIEF DESCRIPTION OF THE NEW “INVARIANT h METHOD”

The convective heat transfer coefficient  $h$  assigned to a viscous layer existing between a free stream and a solid body is a strong indicator of the overall fluid dynamic status of the region. The definition of the convective heat transfer coefficient  $h=q_w/(T_{aw}-T_w)$  warrants the isolation of fluid dynamic effects from the thermal boundary conditions of the problem.  $h$  is only affected from the fluid dynamic features of the viscous flow layer. In other words, in a wide range of experimental conditions, the momentum equation and thermal energy equation are not coupled.

Any variation in the thermal driving potential ( $T_{aw}-T_w$ ) does not alter the magnitude of  $h$  provided that the local Reynolds number is kept constant at a specific location. By definition,  $q_w$  is linearly dependent on  $(T_{aw}-T_w)$  by the relation  $q_w=h(T_{aw}-T_w)$  as shown in Figure 1. When heat flux is plotted with respect to  $(T_{aw}-T_w)$ , the slope of the straight line is the local heat transfer coefficient. Therefore,  $h$  will always be the same regardless of the local heat flux  $q_w$  or the thermal driving potential value, provided that the free stream Reynolds number and turbulent flow character is kept unchanged at the specific location.

In a transient heat transfer experiment, either the free stream flow is started impulsively or the model is instantaneously injected into a steady stream. In both approaches, the flow related transients from the impulsive starting of the free stream or transients of model injection are kept to a minimum time duration when compared to the total elapsed time of the complete heat transfer experiment. The steady state flow conditions are reached rather quickly in most forced convection problems. The time scales of the viscous/turbulent diffusion of initial flow disturbances are usually much smaller than the total duration of the thermal transient created over the wall. In a typical transient experiment, time-wise change of the wall temperature is incorporated into an inverse solution of the transient heat conduction equation. It is possible to construct a “wall heat flux transient  $q_w(t)$ ” from a measured “wall temperature transient  $T_w(t)$ ” provided that the thermo-physical triple product  $\sqrt{\rho C_p k}$  of the solid body is known accurately. Many researchers in the past used this concept on heat transfer models that can be classified as semi-infinite bodies. This approach allows the user to benefit from a one-dimensional form of the transient conduction equation. However, the new “*invariant h*” method described here is equally applicable to multi-dimensional transient heat conduction models.

Once the initial flow disturbances decease, the free stream Reynolds number at a selected point on the model remains constant over the complete duration of the experiment. This is a major assumption used in the development of the new method. There are currently many facilities operating in this mode either for gas side heat transfer or internal cooling passage heat transfer research. Flow and heat transfer characteristics of the most typical of these facilities can be found in Camci, Kim, Hippensteele and Poinsatte<sup>5</sup>, Jones and Hippensteele<sup>6</sup>, Ireland and Jones<sup>7</sup>, Metzger, Bunker and Bosch<sup>8</sup>, Vedula and Metzger<sup>9</sup>, Ekkad and Han<sup>10</sup>, Drost,Hoffs and Bolcs<sup>11</sup>.

The new method uses the flow property of “*unchanging free stream Reynolds number*” over the complete duration of a transient heat transfer. Previously, the determination of the reference temperature of the free stream was mainly an experimental temperature measurement performed at a fixed point in the free stream. The number of studies documenting the variation of the reference free

stream temperature in the streamwise direction is limited. Of course, in incompressible flow the free stream total temperature and the adiabatic wall temperature are identical because of negligible viscous dissipation in the viscous layer between the wall and free stream. The present method eliminates the need to obtain reference free stream temperature  $T_{\infty}$  or  $T_{aw}$  by an independent experimental method. Since the local Re number is constant in a typical transient experiment, the local convective heat transfer coefficient  $h$  is supposed to remain constant in time. By using this "*invariant h*" property in addition to the transient conduction equation at the fluid-solid interface, one can easily obtain  $h$  and  $T_{aw}$  "simultaneously". This new approach is extremely powerful in heat transfer problems where heat pick-up (or heat loss) of the free stream gas is measurable in magnitude. An initially heated heat transfer model in an external flow experiment or an initially heated internal cooling passage (e.g. serpentine passage) may easily give rise to non-negligible free-stream temperature increase as one moves in the streamwise direction. The current response to this problem in the heat transfer community is either to ignore the gas temperature rise or to instrument the passage for gas temperature measurements at many different locations. Unfortunately, associated free stream reference temperature measurement errors significantly increase the uncertainty of the convective heat transfer coefficient  $h$ . The present study shows that the new "*invariant h*" property is extremely useful in finding the reference gas temperature or adiabatic wall temperature analytically by eliminating extra experimental efforts and errors. The new method is completely analytical and allows the researcher to obtain the local free stream reference temperature at every point where a wall temperature measurement is possible. The accurate level of the free stream reference temperature is reached by forcing the local convective heat transfer coefficient  $h$  in time to a time-wise stationary status in an iterative process. The new "*invariant h*" method is equally applicable to all discrete temperature sensor based measurements including infrared imaging and to methods in which temperature sensitive coatings (liquid crystals and thermographic paints) are used.

#### GENERIC TRANSIENT TEST CASE DEFINITION

It is essential to explain the new method using a generic transient heat transfer test case that is applicable to most forced convection heat transfer studies in the field of turbine heat transfer. An impulsively starting flow is established over a flat plate in a few miliseconds time. The heat transfer coefficient  $h$  under these conditions is assumed to be  $100 \text{ W/m}^2\text{K}$ . The free stream reference temperature at the edge of the boundary layer is constant at  $T_{\infty} = T_{aw} = 302 \text{ }^\circ\text{K}$  over the duration of the transient test that lasts about 8 seconds. It is prescribed that the model is kept at  $283 \text{ }^\circ\text{K}$  initially. Time-wise change of the wall temperature  $T_w(t)$  to be recorded by a wall sensor at the specific  $x$  location is shown in Figure 2. This temperature rise also satisfies Equation 1 for  $h=100 \text{ W/m}^2\text{K}$ . The model wall thickness satisfies the conditions required for being considered as semi-infinite. The thermo-pyphysical triple product for the plexiglass model material is about  $\sqrt{\rho C_p k} = 569 \text{ W.(sec)}^{1/2}/\text{m}^2$ . Figure 2 also shows a reconstructed wall heat flux trace  $q_w(t)=h(T_{aw}-T_w(t))=100*(302-T_w(t))$  using a one-dimensional transient heat transfer (inverse) solution for a semi-infinite body (equation 1). The heat flux  $q_w(t)$  to the wall decrease in time because of the

temperature rise at the fluid-solid interface. The corresponding heat transfer coefficient for this ideal case can be re-calculated from  $h(t) = q_w(t)/(T_{\infty} - T_w(t))$ . When  $h$  is re-calculated from  $q_w(t)$  and  $T_w(t)$  presented in Figure 2, a constant value of 100 W/m<sup>2</sup>K over the complete duration of the transient test is obviously obtained. Although the thermal boundary conditions may vary in a transient test, the resulting  $h$  is invariant (in time) because the flow features reach a steady state in an extremely short duration after the start of a transient experiment. The heat transfer coefficient  $h$  is an invariant quantity in a transient test where  $Re_x$ ,  $T_{\infty}$  and  $T_{aw}$  are constants in time.

### TYPICAL UNCERTAINTY OF $h$

In the measurement of convective heat transfer coefficient, the errors made in the recording of wall temperature, reference free stream temperature and wall heat flux all contribute to  $\delta h$  at different rates. However, the most significant error contributor seems to be the reference free stream temperature as shown in Figure 3. Figure 3.a represents the best measurement quality in terms of the wall temperature error,  $\delta T_w = \pm 0.1$  °K. The figure shows the relative error on heat transfer coefficient  $\delta h/h$  in function of the reference free stream temperature error,  $\delta T_{\infty}$  (or  $\delta T_{aw}$ ). Of course better heat flux measurement efforts (with relatively lower  $\delta q_w/q_w$ ) pull down the error curve in favor of the heat transfer coefficient  $h$ . The relative error in  $h$  has a tendency to increase approximately 10 fold when the error in the measurement of the reference free stream temperature  $\delta T_{aw}$  is increased from  $\pm 0.1$  °K to  $\pm 2$  °K. Similar error increasing trend (with increasing  $\delta T_{aw}$ ) exists for the wall heat flux methods having higher error levels (e.g.  $\delta q_w/q_w = \pm 8$  %), however the relative error levels ( $\delta h/h$ ) are much higher as indicated by the dashed line in Figure 3.a.

Figure 3.b displays similar trends for a mediocre wall temperature measurement ( $\delta T_w$  5 times higher compared to the previous Figure) effort that has an uncertainty of  $\delta T_w = \pm 0.5$  °K. The same conclusions as the previous case can be drawn, however overall relative error  $\delta h/h$  follows a more elevated uncertainty character. The main result from this error analysis is that the most significant contributor to the error made in the determination of heat transfer coefficient  $h$  results from the uncertainty on "reference free stream temperature" ( $\delta T_{\infty} = \delta T_{aw}$ ). The relative contribution of various error sources in total error is plotted in the form of individual error norms in Figure 4. The percentage contribution to  $\delta h/h$  from the errors made in "reference free stream temperature" becomes significantly more important as the  $\delta T_{aw}$  increases. After a limiting value of  $\delta T_{aw} = \pm 0.6$  °K, the contribution of reference free stream temperature measurement exceeds the errors made in the measurement of wall heat flux ( $\delta q_w/q_w$ ). Wall temperature measurement errors are actually a small part of the total error on  $h$  as shown in Figure 4.

The error contribution analysis presented in Figures 3 and 4 clearly justifies the need for a better method to obtain local reference gas temperatures over a heat transfer model. Replacing time consuming and error generating discrete sensor based gas temperature measurements by "*invariant h*" method that is a robust analytical procedure is a major improvement in convective heat transfer research employing transient methods.

## DETAILS OF THE "INVARIANT h" METHOD

The detailed explanation and justification of the method uses the previously described generic test case. A hypothetical transient test having a free stream reference temperature of 302 ° K produces  $h=100 \text{ W/m}^2\text{K}$  at a selected streamwise location of a heat transfer model. The model has a uniform initial wall temperature of 283 ° K and the free stream total temperature remains constant in time after the initial transients. The wind tunnel is operated such that the Reynolds number and free stream turbulence intensity at the specific measurement location is consistent with the targeted  $h$  value of 100  $\text{W/m}^2\text{K}$ . The corresponding wall temperature rise from this test is shown in Figure 2. Finding an accurately representative  $T_w(t)$  profile that will generate  $h=100 \text{ W/m}^2\text{K}$  (constant in time) at a selected location can be achieved by using a simplified inverse solution of the transient heat conduction equation. For this case, the wall temperature transient, in the form of a non-dimensional wall temperature, can be related to heat transfer coefficient  $h$ , time and the free stream reference temperature by equation 1

$$\frac{T_i - T_{w(t)}}{T_i - T_{\infty}} = 1.0 - \exp(\beta^2) \cdot \operatorname{erfc}(\beta) \quad (1)$$

where  $\beta = h\sqrt{t} / \sqrt{\rho C_p k}$  is non-dimensional time. The wall temperature profile plotted in Figure 2 has been obtained from equation 1 by using the specified values of  $T_i$ ,  $T_{\infty}$  and  $\sqrt{\rho C_p k}$ . Figure 2 also shows the corresponding wall heat flux  $q_w(t)$  variation obtained by multiplying the specified  $h=100 \text{ W/m}^2\text{K}$  by the thermal driving potential  $[T_{\infty} - T_w(t)]$ . The slight monotonic decay of heat flux is associated with the transient rise of wall temperature during the transient experiment. Equation 1 is usually preferred in single shot experiments in which a wall temperature measurement is required at one user selected time during the transient. This method is suitable for liquid crystal based wall temperature measurements. Although extremely simple, equation 1 can not handle the cases in which there are free stream reference temperature variations in time. Equation 1 has only been used to find a baseline wall temperature profile that will be used for further development of the "*invariant h*" method. The new method will be explained and validated for the general case in which the free stream reference temperature can vary in time.

The instantaneous wall heat flux  $q_w(t)$  in the "*invariant h*" method can also be re-constructed from  $T_w(t)$  by using a convolution integral solution that can be obtained by Laplace transforming the transient heat conduction equation. For general applications in which reference free stream temperature may vary in time, the final inverse solution to 1-D transient heat conduction equation from Schultz and Jones<sup>12</sup> can be written as follows.

$$q_w(t) = \sqrt{\rho C_p k / \pi} \left\{ \frac{T_w(t)}{\sqrt{t}} + \frac{1}{2} \int_{\tau=0}^t \left[ \frac{T_w(t) - T_w(\tau)}{(t-\tau)^{1/2}} \right] d\tau \right\} \quad (2)$$

Equation 3 shows a more computationally efficient form of equation 2 that can be obtained via series summation of delayed ramp functions, as suggested by Oldfield, Jones and Schultz (1978)<sup>13</sup>.

$$q_w(t) = q_w(m\Delta\tau) = \frac{2 \cdot \sqrt{\rho C_p k}}{\sqrt{\pi \Delta\tau}} \sum_{n=0}^m (T_{n+1} + T_{n-1} - 2T_n)(m-n)^{1/2} \quad (3)$$

The heat flux trace  $q_w(t)$  re-constructed from the wall temperature transient  $T_w(t)$  using the “general method” leading to equation 2 is shown in Figure 2. From this point on, the identical methods described in equations 2 and 3 will be termed as the “general method” for wall temperature to heat flux conversion. At this point, a perturbation experiment can be performed numerically in order to see what happens to  $h$  in time when “*perturbed*” reference free-stream temperature values are substituted into

$$h(t) = q_w(t)/(T_{\infty} - T_w(t)) \quad . \quad (4)$$

Figure 5.a shows a number of  $h(t)$  curves calculated for the disturbed values of  $T_{\infty}$ . What happens to  $h(t)$  from this point on is nothing to do with neither the quality of  $T_w$  measurement nor  $q_w$  calculation using equation 2. Instead of  $T_{\infty}$ , Figure 5 uses [FACTORx $T_{\infty}$ ] in equation 4. The product [FACTORx $T_{\infty}$ ] is also termed as the “*suggested free stream temperature*” during the iterative process. The value of FACTOR has been varied from 0.970 to 1.030 with small increments of 0.001. The corresponding “*suggested free stream temperature*” values are also provided in Figure 5.a during the iterations

Of course, when FACTOR is exactly 1.000, a time invariant heat transfer coefficient  $h$  is clearly established for all times as shown in Figure 5.a. The most interesting observation is made when the distortion is at its strongest level. For FACTOR=0.95, the resulting  $h(t)$  values, have of course wrong absolute values. However the most valuable observation is that  $h(t)$  curve is not anymore invariant in time. For this case the suggested free stream temperature value is  $302 \times 0.95 = 286.9$  ° K. After the start of a transient experiment, as soon as the wall temperature reaches 286.9 ° K,  $h$  goes from high positive values to high negative values at  $t=1.6$  seconds. In this zone, the thermal driving potential ( $T_{\infty} - T_w(t)$ ) becomes very close to zero. After the wall exceeds the suggested free stream temperature negative heat transfer coefficients are obtained. For this case the wall is at a higher temperature than the suggested free stream temperature. For FACTOR=0.96, similar observations are valid, however the sign change in  $h$  is observed at a much later time at  $t=6.9$  seconds. Figure 5.b shows a close-up of the previous figure. When FACTOR is increased incrementally from 0.97 to 0.999, resulting  $h$  values have all detectable positive slopes in time. In this range there is also no sign change in  $h$  because the wall temperature at any time during the transient is always smaller than the suggested free stream temperature. The maximum wall temperature ( $T_w=290.0$  ° K) reached at the end of the transient ( $t_f=7.145$  seconds) is always smaller than the suggested free stream temperature. When FACTOR is slightly increased towards the ideal value of 1, the slope of the  $h(t)$  curve is reduced and the overall magnitude of individual  $h$  values move closer to the correct value of 100 W/m<sup>2</sup>K. The decrease on the slope of the

$h(t)$  curve when FACTOR is increased from 0.970 to 1.000 is numerically detectable. The only time  $h(t)$  has a zero slope is when the correct reference free stream temperature is used equation 2. This is an extremely useful property that repeats itself in every forced convection heat transfer experiment in the same fashion. When the perturbation parameter FACTOR goes to values greater than 1.000, the same observation is valid however, the slope of  $h(t)$  curve is negative. It is very interesting to note that in order to determine "*if the suggested FACTOR\*T<sub>∞</sub> value is proper?*" for the specific measurement, it is sufficient to monitor if a sign change in the slope of  $h(t)$  curve happens from one perturbation value to another one. In a transient experiment, after one records a  $T_w(t)$  trace and construct  $q_w(t)$ , one may suggest a set of reference free stream temperature values that are perturbed around an expected value. Monitoring the slope of  $h(t)$  curve and forcing it to approximately zero, one may find the exact actual free stream reference temperature at this specific location. The new approach described is analytical and replaces "conventional  $T_\infty$  measurement efforts" that usually introduce additional experimental errors. One may significantly improve the accuracy of heat transfer coefficient  $h$  by obtaining reference gas temperature ( $T_\infty = T_{aw}$ ) in a more precise manner compared to conventional measurement based approaches. Of course when a compressible flow problem is under investigation the free stream reference temperature to be obtained is the adiabatic wall temperature which may be different than the free stream total temperature. Figure 5.c shows the extremely sensitive nature of the overall slope of  $h(t)$  curve to variations in FACTOR. The same curves shown in Figures 5.a and 5.b are presented in a more stretched vertical scale in the actual range of  $h$ .

In summary, the "*invariant h*" method is about systematically varying an estimated "reference gas temperature" in equation 4 and numerically determining the overall slope of  $h(t)$  curve for each suggested gas temperature value (FACTOR\*T<sub>∞</sub>). The numerical process is fast and efficient. The goal is to minimize the slope of  $h(t)$  curve down to a zero value during an iterative process. The same procedure can also be performed by testing if a sign change occurs in the overall slope of  $h(t)$  curve. Since most  $h(t)$  curves are highly linear, the test can be successfully performed over a short time interval near the end of the transient test. The sign change occurs only when a correct reference free stream temperature is used in equation 5. It is also apparent from Figure 5 that the most accurate heat transfer coefficient is obtained only when  $h$  is invariant in time.

A different presentation of the same concept is given in Figure 6 by monitoring the local change in heat transfer coefficient at every time step. The local time-wise change in  $h$  was plotted with respect to time. The same data shown in Figure 5 is just plotted in a different form. However, the variation of the local slope for various FACTOR values is more visible in this form. The zero approaching feature of local slope at any time point is obvious when FACTOR is incrementally varied.

Figure 7 shows the heat transfer coefficient obtained exactly at the end of the transient run ( $t_f=7.145$  seconds) with respect to its associated suggested free stream temperature. Of course every individual  $h$  point on this curve comes from a suggested free stream temperature value defined as (FACTOR\*T<sub>∞</sub>). Since there is only one correct free stream temperature, only one point on this curve is physically meaningful. When FACTOR is 1.0 we know that the resulting heat transfer coefficient is correct at  $h=100$  W/m<sup>2</sup>K. Although Figure 7 shows the functional

relationship between the suggested free stream temperature and the resulting heat transfer coefficient at the end of the transient run, it is not sufficient to direct us to the correct free stream temperature during the iterative process for actual experiments. The idea of “*invariant h*” explained in previous paragraphs can be expressed mathematically as the slope of the heat transfer coefficient variation in time. Figure 5.c clearly shows that when  $h$  is invariant in time, the local slope is zero and the resulting  $h$  has the most accurate value which is the only physically admissible heat transfer coefficient. Figure 8 presents the relationship between “resulting” heat transfer coefficient and its time-wise slope at the end of the transient run for each suggested free stream temperature value. Monitoring the heat transfer coefficient against its slope is certainly a very effective method of isolating the correct free stream temperature and its associated heat transfer coefficient. When FACTOR varies from a low (0.970) to a high value (1.030) by an increment of 0.001, whenever a slope value of zero is reached, the correct  $h$  value and its associated free stream temperature is reached. The use of FACTOR in this generic case was only permitted for testing the character of the suggested method. In laboratory applications, instead of using  $(\text{FACTOR} * T_{\infty})$  one may start testing the slope of heat transfer variation by using a suggested starting value for the reference temperature of the free stream. The choice of the starting value is straight forward because the researcher have always the knowledge of an approximate free stream temperature. For example, in a transient test to be performed in a pedestal type cooling passage the inlet value of the free stream gas is more than sufficient to start the iterative process. Further examination of the relationship between  $h$  and its slope shows that the (almost) second order curve as shown in Figure 8 can be linearized by normalizing the slope using the  $h$  value at the end of the transient run when  $t=t_f$ . For this case, a new quantity  $\Gamma$  showing the status of the slope of  $h$  is defined as follows.

$$\Gamma = \{\text{SLOPE}/h(t_f)\} * 100 \quad (5)$$

Figure 9 contains data from the last 5 points of the generic transient experiment summarized in Figure 2. Isolating the correct  $h$  and its associated “reference free stream temperature” is a matter of finding the proper level of suggested free stream temperature that is corresponding to  $\Gamma=0$  in a simple iterative process. The trial is the suggested free stream temperature, and the error is the finite level of  $\Gamma$ . By forcing error  $\Gamma$  to zero, the heat transfer coefficient and reference temperature of the free stream can be obtained simultaneously and accurately. All five points at the end of the transient run provided the same functional dependency as shown in Figure 9. Using just one of the five points in Figure 9 is sufficient to obtain the correct  $h$  and its associated “reference free stream temperature”.

#### SUGGESTED PROCEDURE AND APPLICATION OF THE METHOD

A flow chart of the suggested numerical procedure summarizing the “invariant  $h$  method” that can be built into a computer procedure is given in Figure 10. The

procedure can be applied at every point on a heat transfer model exposed to stationary flow conditions. A wall temperature transient is recorded by known methods of non-intrusive measurement techniques such as flush mounted thermocouples, thin film resistance sensors, thermistors, infrared imaging and thermographic phosphors emitting laser induced fluorescence. The general method described here requires the capturing of the complete transient in wall temperature with uniform time step  $\Delta\tau$ . The next step is to convert the measured  $T_w(t)$  to  $q_w(t)$  by using either Equation 2 or 3. It should be noted that these two equations allow the user to work with reference free stream temperatures that may actually vary during the transient experiment. The only condition to be satisfied here is the maintenance of constant free stream Reynolds number at the measurement location. Since most transient heat transfer facilities are designed with stationary free stream flow conditions in mind, this is a realistic and straight forward condition to satisfy. Usually, the first short period containing the flow disturbances originating from the impulsive start-up of the free stream or the injection of the model into the free stream is much shorter when compared to the complete duration of the experiment. After constructing the wall heat flux transient a starting suggested value for the free stream temperature is needed. A rough guess on this value is sufficient to start the trial and error procedure. In most facilities, the free stream temperature measured at the inlet of the test section is a good enough starting value. For example in a transient test in which the model is initially heated, as one moves along the streamwise direction, the free stream temperature at the edge of the boundary layer at different locations may vary because of different heat pick up rates from the heated model surface. When the model is cooled by the relatively colder free stream air, thermal energy is absorbed by the free stream. For this case, starting from the test section inlet value of free stream temperature and increasing the suggested value at each step by about 0.001 times the value of the initial guess may allow the user apply the procedure described in Figure 9. The incremental change needs to be sufficiently small so that the sign change of the normalized slope  $\Gamma$  can be detected with sufficient resolution. The process of incrementally varying “suggested” free stream temperature continues until a zero slope (or a sign change) is detected. At this point the resulting  $h$  and the suggested free stream temperature are recorded as the final measurement results. The iterative process required after the recording of the wall temperature transient is completely analytical and straight-forward to implement numerically in any computer program.

Although the new concept is explained using 1-D transient heat transfer models, the method can also be implemented in multi-dimensional methods that can be used in the future, because of the significant progress made in the transient temperature mapping techniques for complex curved surfaces. The “*invariant h*” method as described in this study requires surface temperature mapping techniques that are capable of recording local temperatures in every time step of a transient experiment. Surface thermocouples, thin film sensors, infrared imaging systems and thermographic phosphor based temperature mapping methods are a few of these techniques. The implementation of the “*invariant h*” concept for single shot heat transfer experiments using liquid crystal based temperature mapping is currently under progress. In this method, a special form of the transient heat conduction equation requires the measurement of wall temperature only at one pre-selected time point. Another important application area of the new concept is the determination of

adiabatic film cooling effectiveness for film cooling studies. Implementation of the "invariant  $h$ " method in liquid crystal thermography based research without and with film cooling is described in a separate publication

## EXPERIMENTAL CONFIRMATION OF THE NEW "INVARIANT $h$ METHOD"

The specific method described in this paper is based on the fact that a wrong determination of the free stream reference temperature can cause a significant error in the calculation of heat transfer coefficient  $h$  although the wall heat flux and wall temperature is measured properly. There are two excellent examples of this observation in the literature by Goldstein and Behbahani<sup>15</sup> and Kim<sup>16</sup>. They used a heated impinging jet impinging to a flat plate initially kept at ambient temperature. Although the round jet is heated in this case, the surrounding still air is at ambient temperature.

A schematic of our current transient heat transfer experiments performed for this arrangement at  $Re=30,000$  based on the jet diameter (nozzle exit) and velocity is shown in Figure 11. It is obvious that the entrainment of the cold surrounding air to the heated jet tends to cool the fluid in the wall jet layer as the jet fluid moves toward the impingement plate. The heating potential of the wall jet is also reduced when one moves away from the stagnation point in radially outward direction. and finally may even disappear far away from the stagnation region by the cooling influence of the entrained fluid. The past studies clearly showed that the heating potential for this problem should be considered as the temperature difference ( $T_{rec}-T_w$ ) instead of ( $T_{jmax}-T_w$ ) especially when the jet temperature at the nozzle exit  $T_{jmax}$  is different from the surrounding fluid temperature in still air.

The correct reference temperature  $T_{ref}$  for the free stream is the recovery (adiabatic) temperature of the wall at a specific radial position on the impingement plate. The measurement of the adiabatic temperature of the wall at infinitely long time after the start of a transient experiment is sufficient to find  $T_{rec}=T_{aw}$ . It is important to wait for a long duration (at least an hour for the present experiment) to reach adiabatic conditions on the impingement plate during the acquisition of  $T_{aw}$ . It should be noted that during a transient experiment although the wall temperature increases in time, the reference temperature of the free stream (the edge of the wall jet fluid) is in steady state at a level of  $T_{ref}=T_{aw}$ . Figure 12 shows the measured distribution of  $(T_{rec}-T_i)/(T_{jmax}-T_i)$  in function of  $r/D$  where  $T_i$  is still air temperature.  $T_i$  also shows the initial temperature of the transient heat transfer experiment.

What happens to Nusselt number (or  $h=Nu \cdot k/D$ ) when one uses the wrong free stream reference temperature is shown in Figure 13 for  $H/D=6$ . The distribution shown as  $Nu'$  is obtained from a transient heat transfer experiment using nozzle exit temperature  $T_{jmax}$  as free stream reference temperature. The same value of  $T_{ref}=T_{jmax}$  was used for all radial positions on the impingement plate. The distribution denoted as  $Nu$  is for the case in which a measured local adiabatic wall temperature is used for each radial position as reference free stream temperature. The difference between measured local  $Nu$  and  $Nu'$  can be very large as shown in Figure 13. At  $r/D=2.4$ , the measured (correct) heat transfer coefficient  $h=Nu \cdot k/D$  can be as high as 300 % of corresponding  $h'=Nu' \cdot k/D$  obtained with  $T_{jmax}$ . The difference between  $Nu$  and  $Nu'$  is monotonically reduced when one moves radially inward toward the stagnation point.

Of course entrainment of cold surrounding air into the heated jet fluid is much more significant at outer radius locations than the stagnation point. At the stagnation point  $\text{Nu}$  is about 20 % higher than  $\text{Nu}'$ . The experiments clearly show the amount of energy lost into relatively cold still air from the heated wall jet needs to be taken into account. When a wrong free stream reference temperature is used in a transient experiment significant errors may be introduced into heat transfer coefficient distributions as shown in Figure 13.

The transient heat transfer data shown in Figures 13 and 14 also show a second interesting feature. The impinging jet experiment is executed by using a nozzle exit flow that is in steady state after an extremely short time duration (a few miliseconds). The total duration of the transient experiment is about 90 seconds. The heat transfer coefficient  $h$  is supposed to be "invariant" in time during the whole duration of the transient experiment. Although the wall temperature has a strong transient, the hydrodynamic conditions in the viscous flow zone need to be invariant in time. When the reference temperature is taken as  $T_{j,\max}$ , not only the measured heat transfer coefficient levels are wrong, they also have strong time dependency as shown in Figure 14. It is very interesting to note that when measured local adiabatic wall temperature is used as  $T_{\text{ref}}$ , time dependency of  $h=\text{Nu}.k/D$  is eliminated as shown in Figure 14. The observations made up to this point form an experimental confirmation of the "*invariant h*" method presented in this paper. By using the fact that  $h$  remains invariant during a transient experiment one can simultaneously obtain the local reference temperature belonging to free stream fluid by forcing the timewise slope of  $h$  to zero.

## CONCLUSIONS

This paper deals with the explanation of a new method for the simultaneous and accurate determination of the convective heat transfer coefficient  $h$  and the corresponding reference free stream temperature in a typical forced convection heat transfer experiment. The new technique named "*invariant h*" method is applicable into transient heat transfer experiments in which free stream hydrodynamic conditions are in a steady state.

The new "*invariant h*" method can analytically determine local free stream reference temperatures in internal flow cases in which heat pick-up (or heat loss) is at a non-negligible magnitude. The method eliminates the need for cumbersome and error prone free stream gas temperature measurements using conventional sensors. The general method described in this paper is applicable to many different surface temperature mapping schemes that has a capability of mapping the surface at each time step during a transient experiment. The mapping techniques in this category are discrete thin film sensors, thermocouples, termistor based sensors, infrared imaging and temperature sensitive paints (thermographic phosphors). An extension of the "*invariant h*" method can also be made for the direct determination of adiabatic wall effectiveness values in gas turbine cooling research using liquid crystal coatings. The new method is explained by using a generic heat transfer case. However an experimental confirmation of the technique is presented using a heated impinging jet configuration. The new method is extremely useful in internal and external flow configurations in which the free-stream reference temperature is continuously

modified by a heated wall or a heated free stream. Internal serpentine cooling passages, internal disk cavity systems, complex impinging jet arrays in internal passages are only a few of the turbomachinery related configurations in which a time efficient and accurate determination of free stream reference temperature is essential during the measurement of the heat transfer coefficient.

The same concept can also be implemented into more time efficient single shot experiments in which a single surface thermal image at a pre-selected time is acquired to obtain local  $h$  values by using an inverse solution of one dimensional transient heat conduction equation. This specific approach requires that the free stream reference temperature is constant in time. This method is preferred frequently because of its time efficiency and simplicity. Details of the modifications required to use the new "*invariant h*" method in a single shot experiment employing thermographic liquid crystal (TLC) coatings and the applications of the method in finding adiabatic wall temperature (or effectiveness) in film cooling research is not included in this paper. An accompanying paper by Camci<sup>4</sup> presents a detailed treatment of liquid crystal and film cooling related aspects of the new method.

### Acknowledgments

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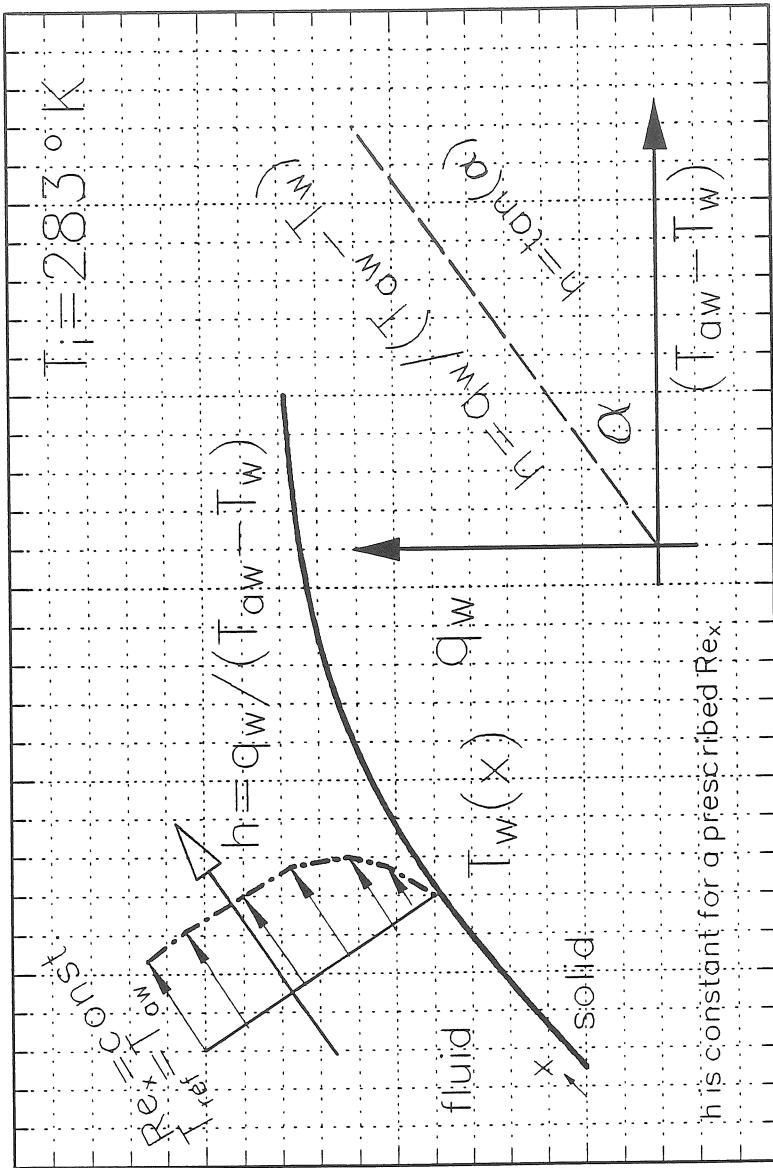
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Figure 1  
Generic test case definition and nomenclature

FIGONE



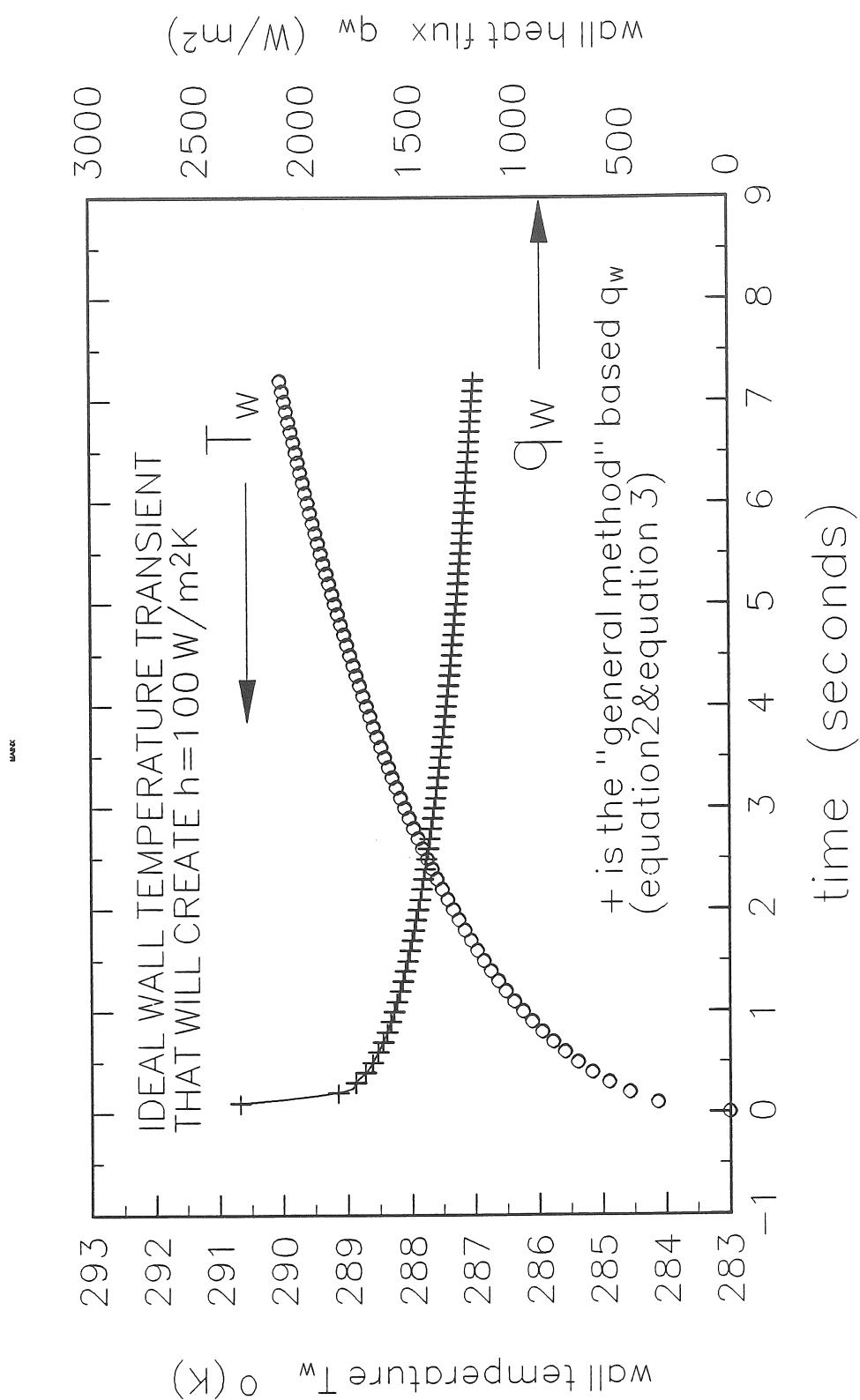


Figure 2  
Wall temperature transient recorded at a specific point on heat transfer surface  
and its corresponding converted wall heat flux trace evaluated from the "general method" (Equation 2)  
 $T_{\infty} = T_{ref} = T_w = 302 \text{ }^\circ\text{K}$     $T_i = 283 \text{ }^\circ\text{K}$

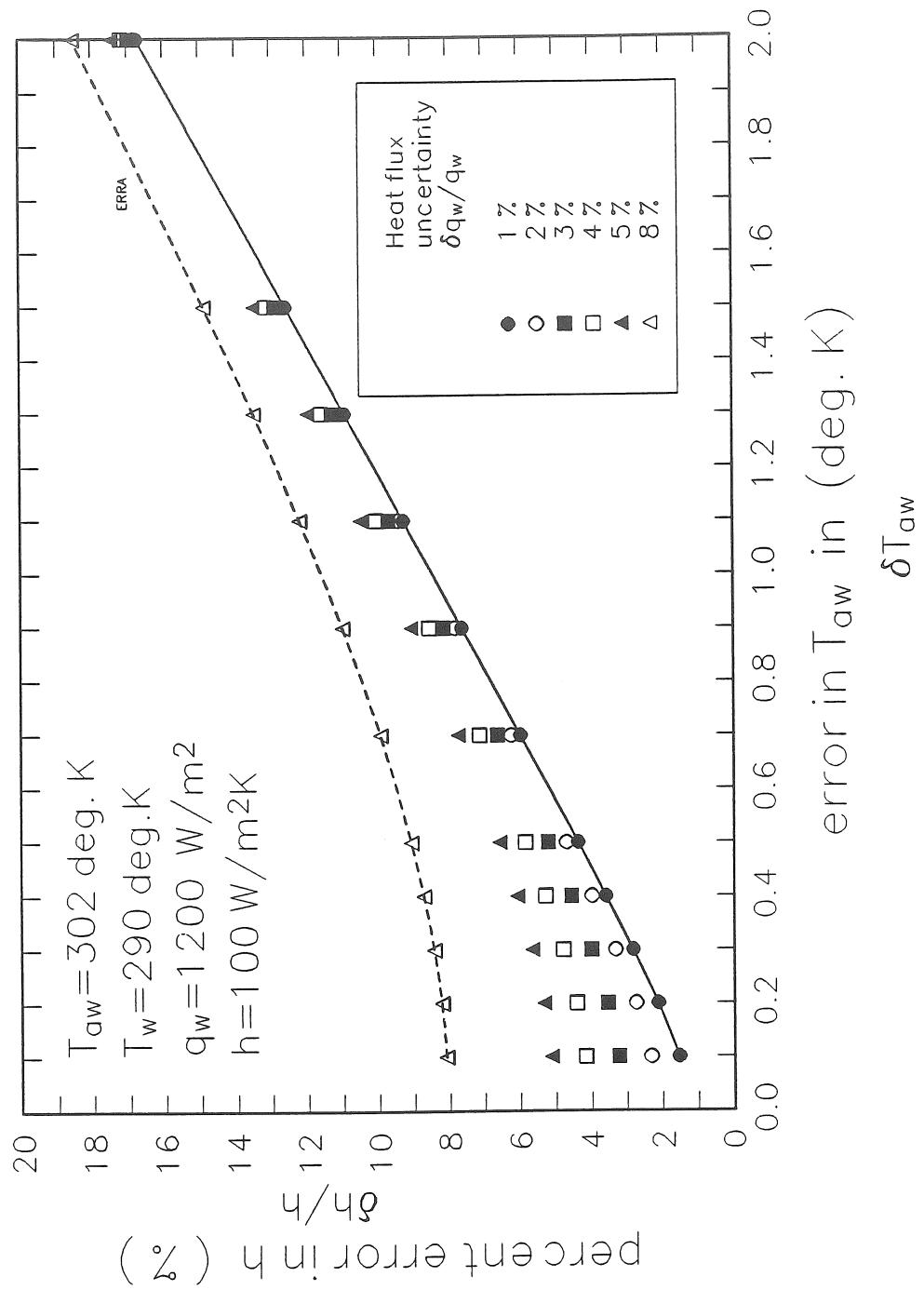


Figure 3.a  
 Uncertainty of heat transfer coefficient  $\delta h/h$  for  $\delta T_w = \pm 0.1 \text{ }^\circ \text{K}$   
 excellent wall temperature measurement capability

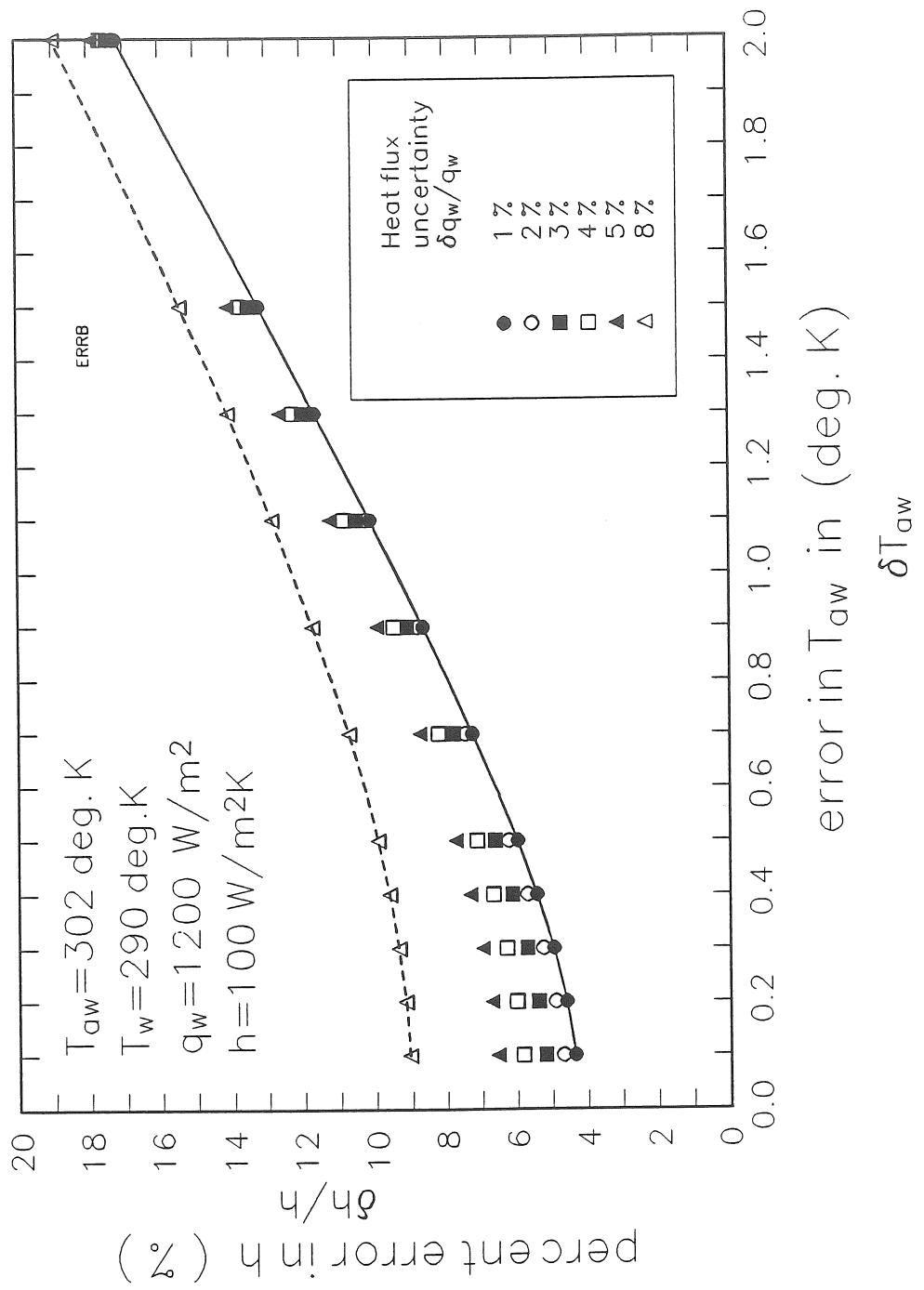


Figure 3.b  
 Uncertainty of heat transfer coefficient  $\delta h/h$  for  $\delta T_w = \pm 0.5^\circ \text{K}$   
 average wall temperature measurement capability

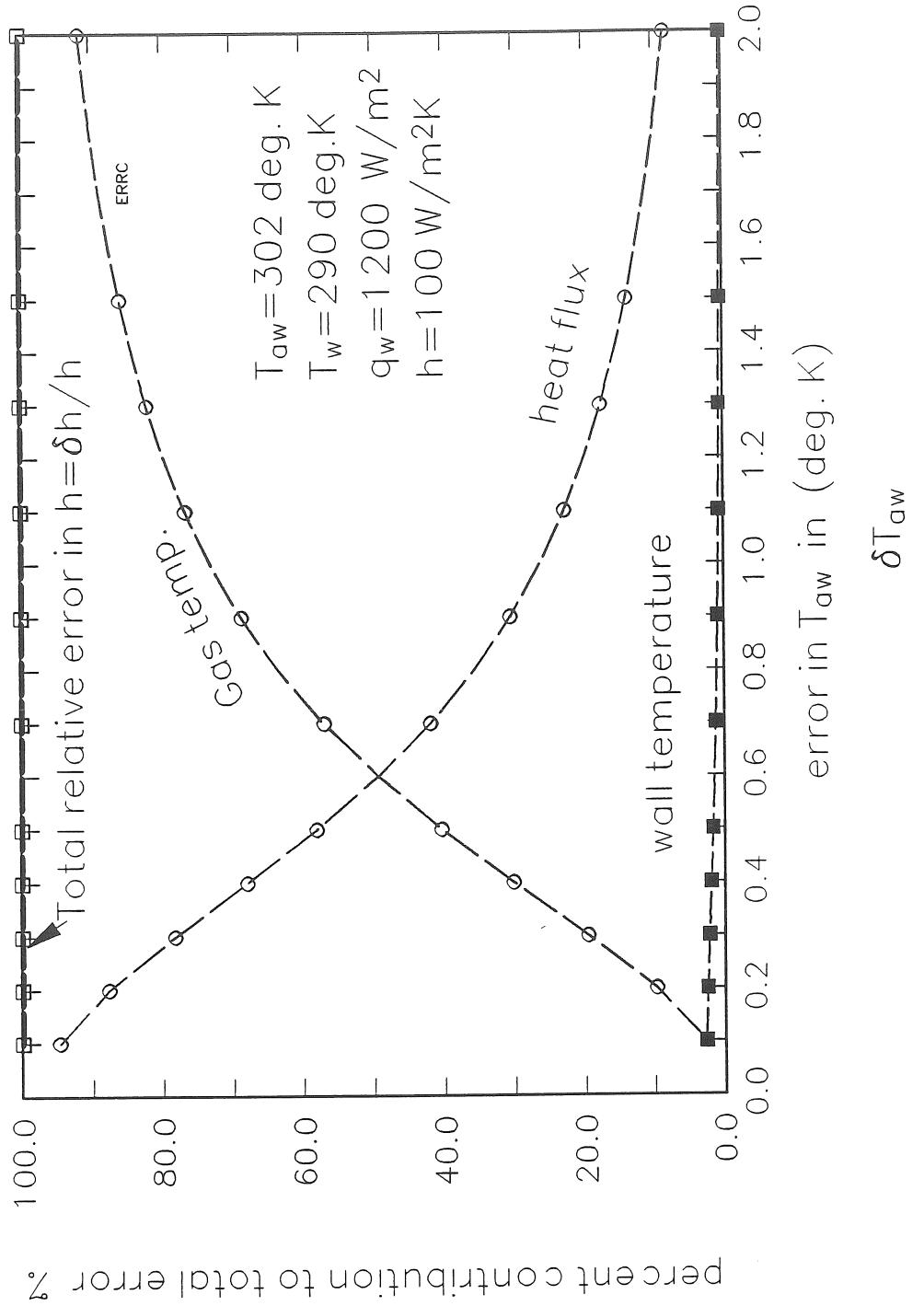


Figure 4  
 Contribution of individual errors to  $\delta h/h$   
 (percentage of error norms)

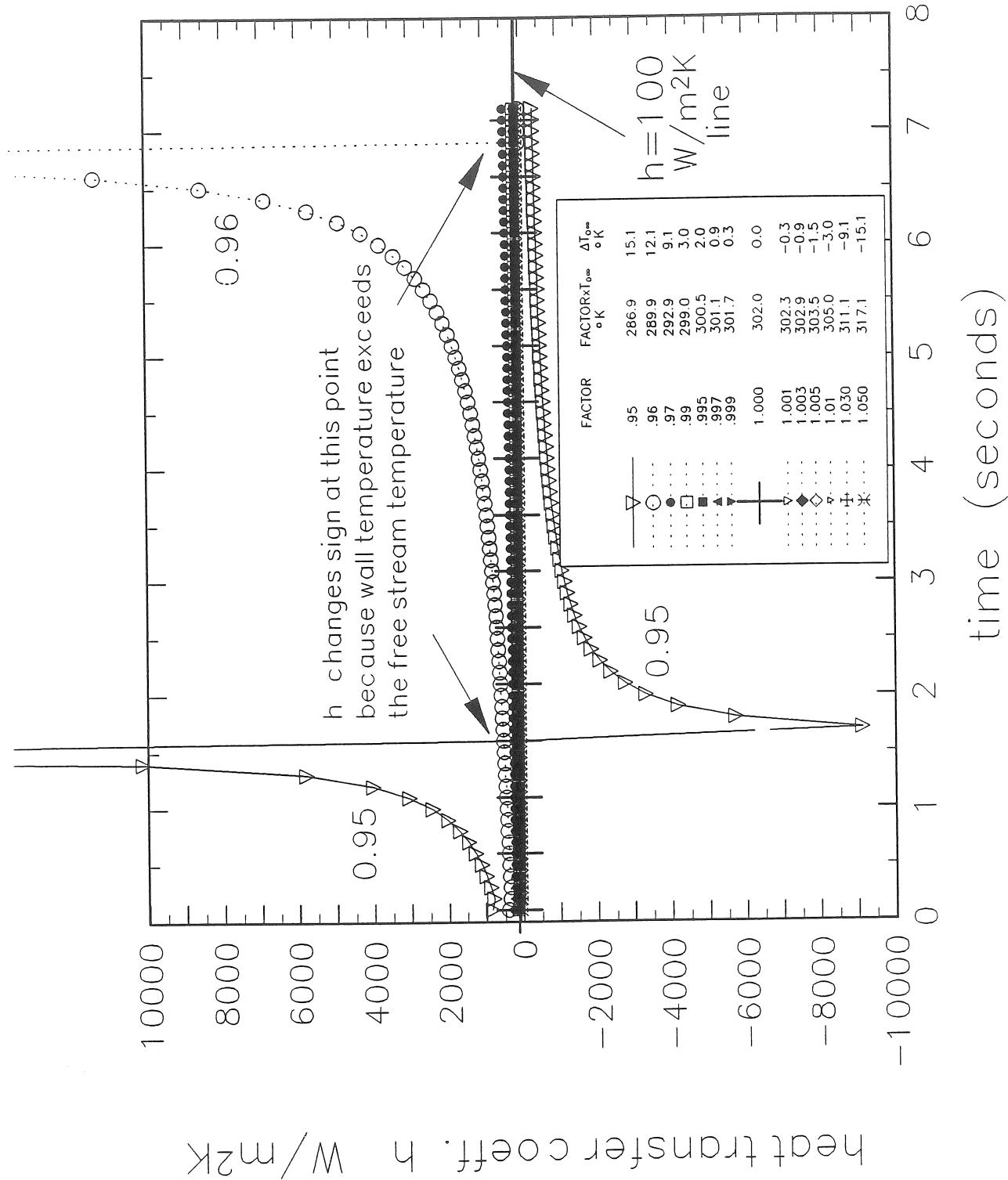


Figure 5.a  
Influence of suggested (iteratively altered) free stream temperature [FACTOR\*T<sub>∞</sub>] on heat transfer coefficient [compressed h range]

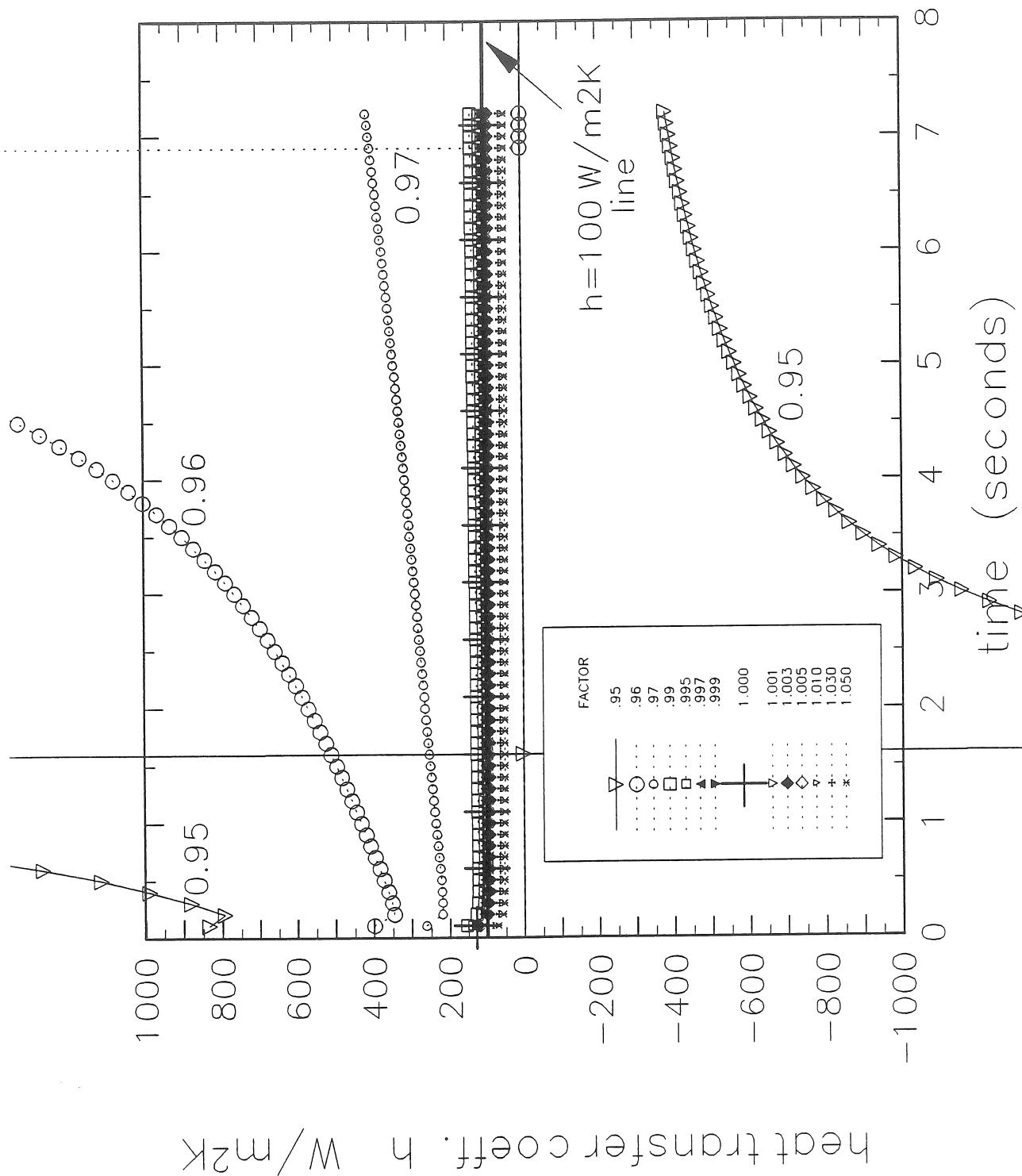


Figure 5.b  
Influence of suggested (iteratively altered) free stream temperature [FACTOR \*  $T_{\infty}$ ] on heat transfer coefficient, [enlarged  $h$  range]

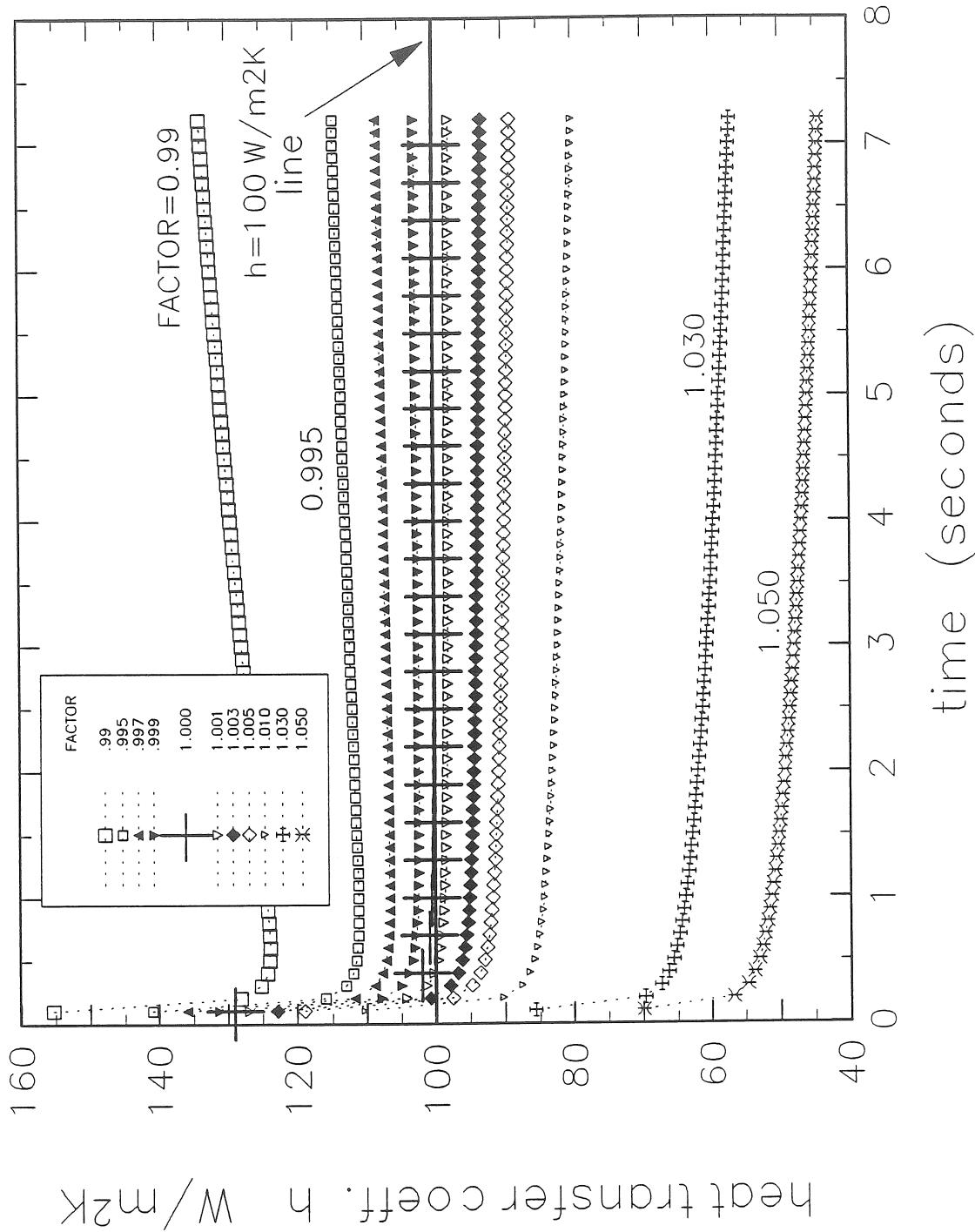
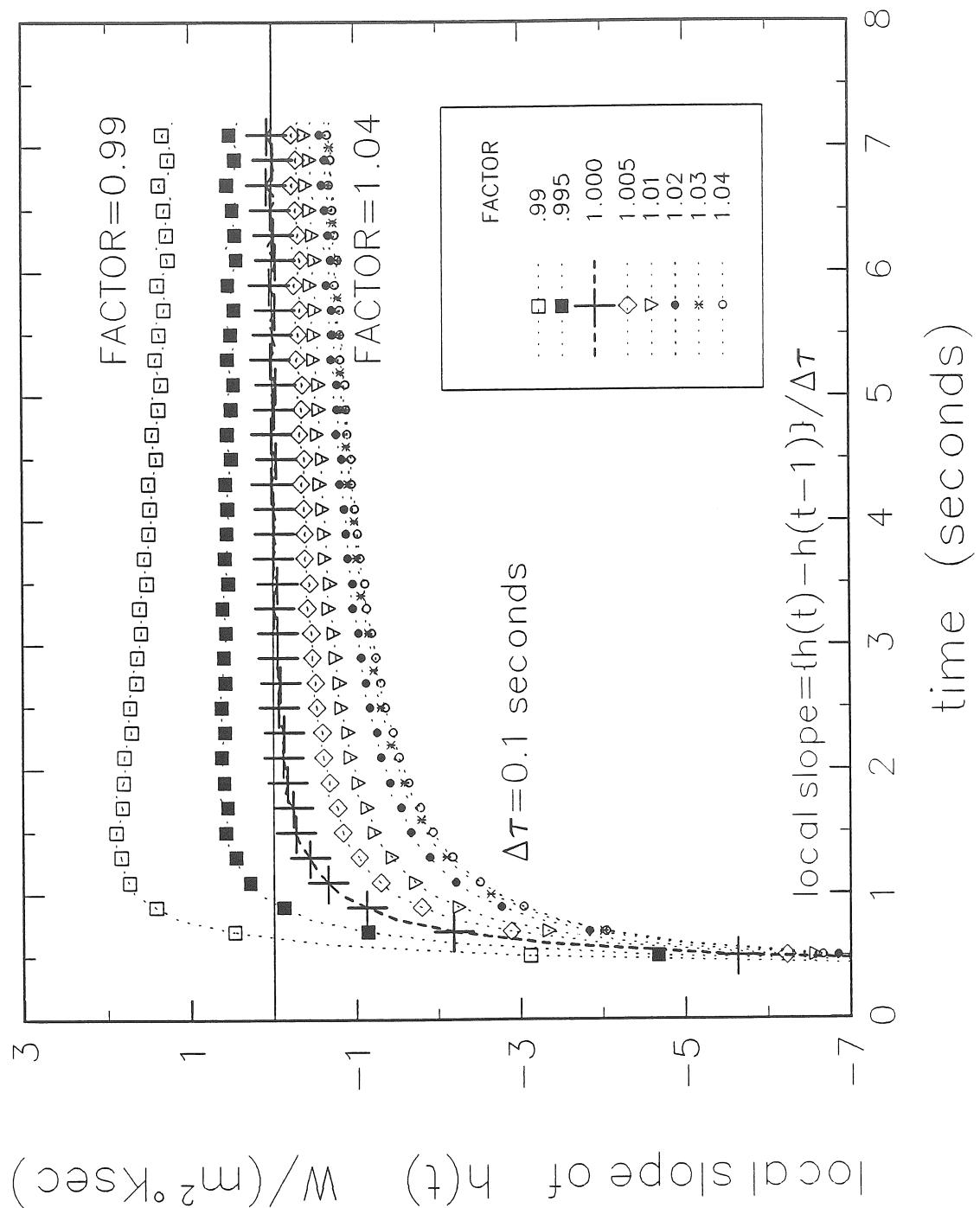


Figure 5.c

Influence of suggested (iteratively altered) free stream temperature [FACTOR \*  $T_{\infty}$ ] on heat transfer coefficient [actual  $h$  range]

Figure 6  
Local slope of heat transfer coefficient  $h$



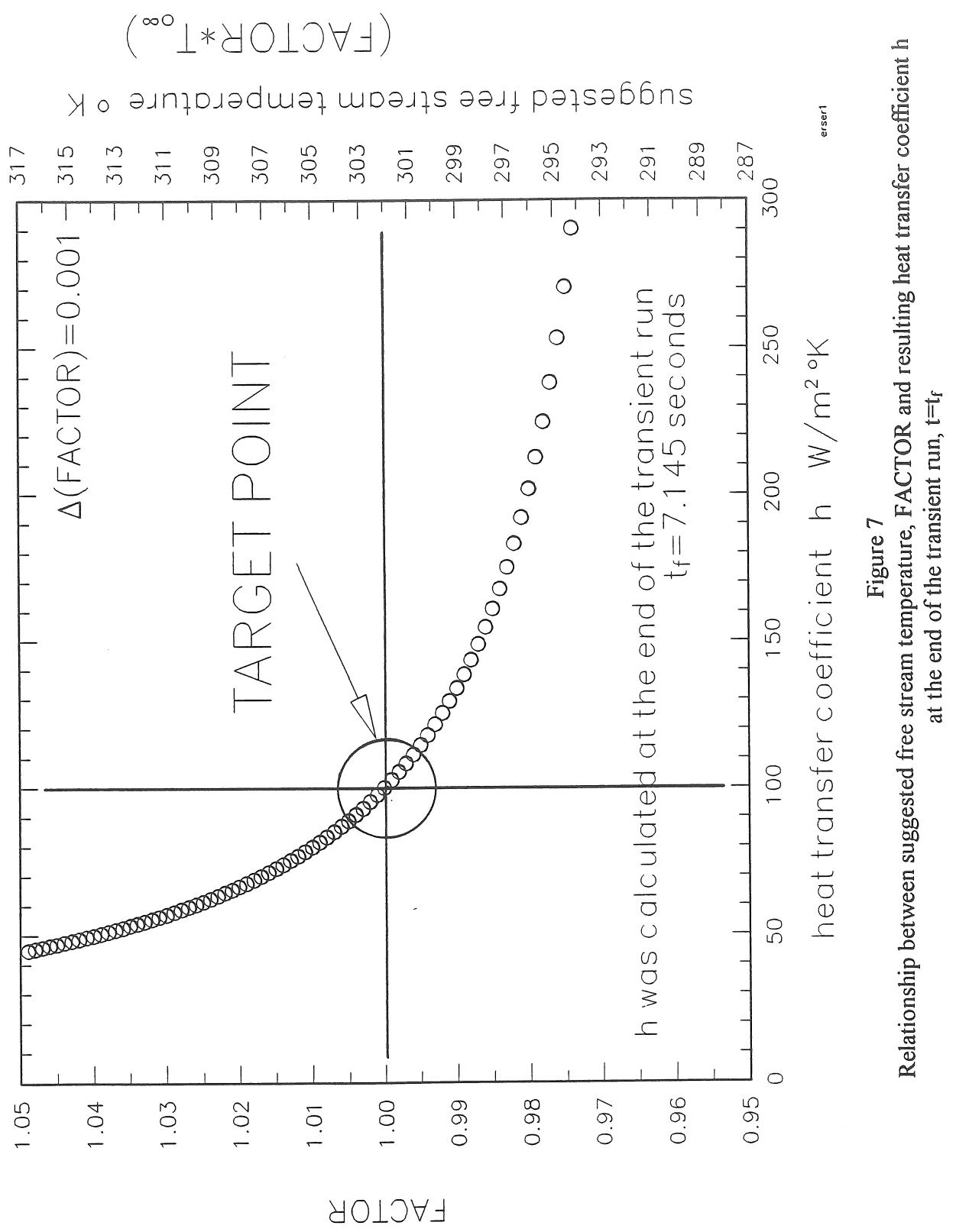
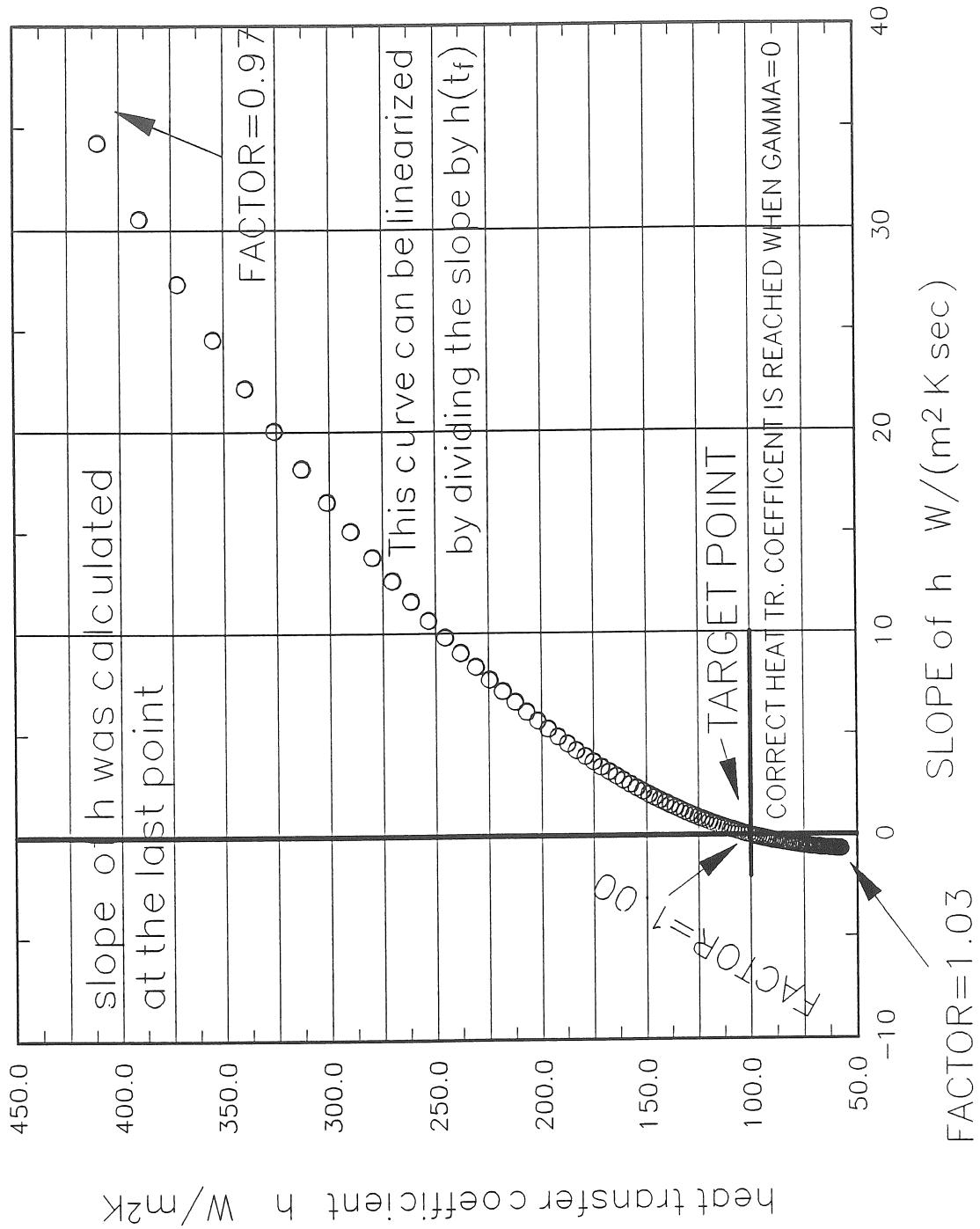
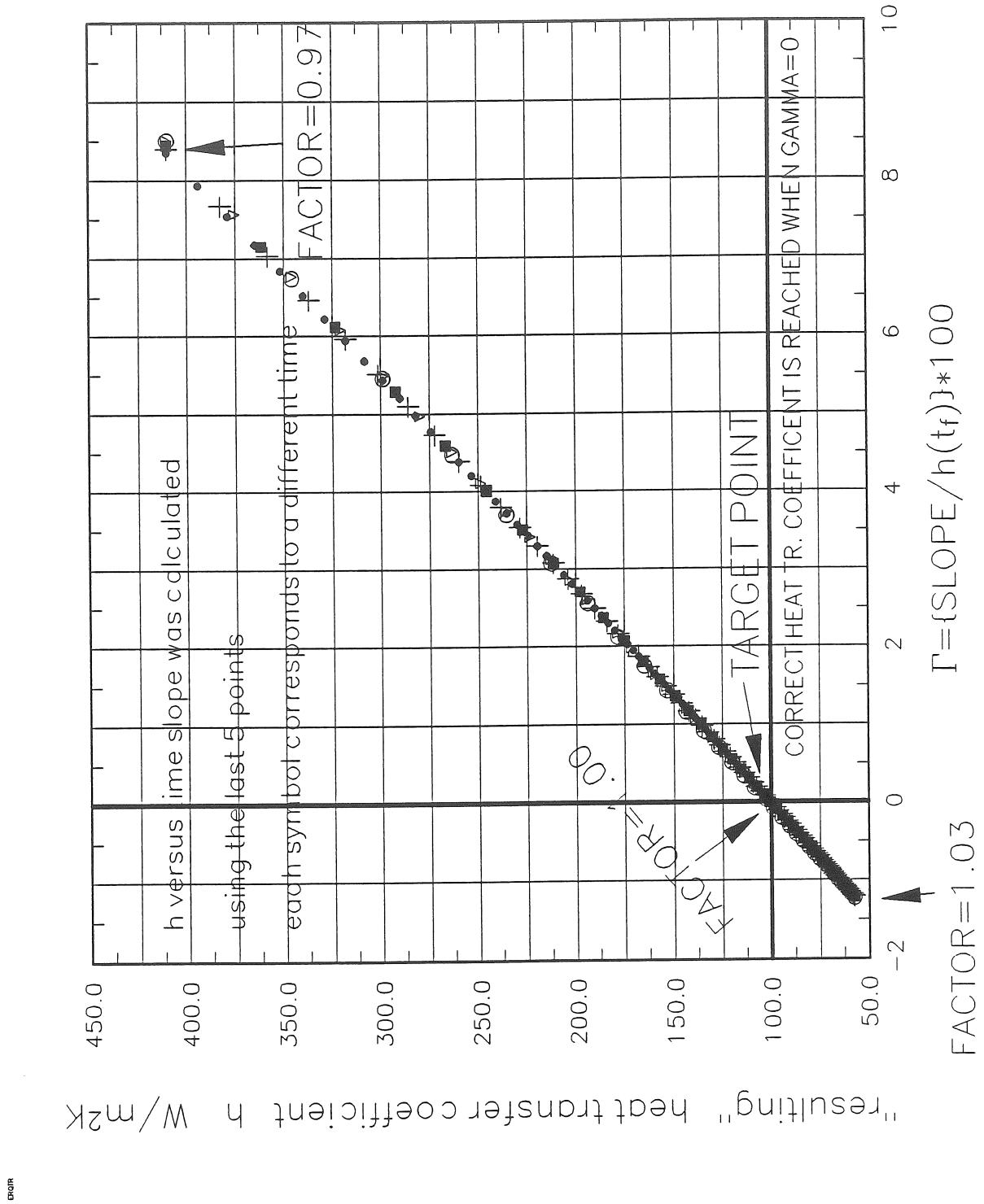


Figure 7  
Relationship between suggested free stream temperature, FACTOR and resulting heat transfer coefficient  $h$  at the end of the transient run,  $t=t_f$



**Figure 8**  
 Relationship between “resulting”  $h$  and its timewise slope  
 at the end of the transient run  $t=t_f$



**Figure 9**  
 Linearized curve for  $h$  versus time-wise slope variation

## invariant h method

A Transient Heat Transfer Experiment

Start an impulsively starting free stream flow  $Re = \text{constant}$  in time

Measure and record wall temperature  $T_w(t)$   
history at a selected sampling rate

Convert  $T_w(t)$  to  $q_w(t)$  using an inverse solution of  
transient heat conduction equation  
see equations 2&3 for the conversion

**GUESS A STARTING VALUE FOR THE  
FREE STREAM REFERENCE TEMPERATURE**

Calculate an estimate of  $h(t)$  from equation 2  
using the GUESSED value of the free stream temperature  
for each time point using measured  $T_w(t)$

Numerically determine the AVERAGE SLOPE of the  
individual  $h(t)$  curve corresponding to GUESSED  
free stream reference temperature

Slightly increase or decrease  
the guessed free stream temperature  
suggested incremental value is  
about 0.001 of the guessed  
free stream temperature

NO

IF(SLOPE=0)

?

YES

THE SUGGESTED LEVEL OF FREE STREAM GAS TEMPERATURE IS CORRECT  
simultaneous determination of  $h$  and free stream gas temperature is complete

Figure 10

A flow chart of the suggested procedure for the “invariant h” method

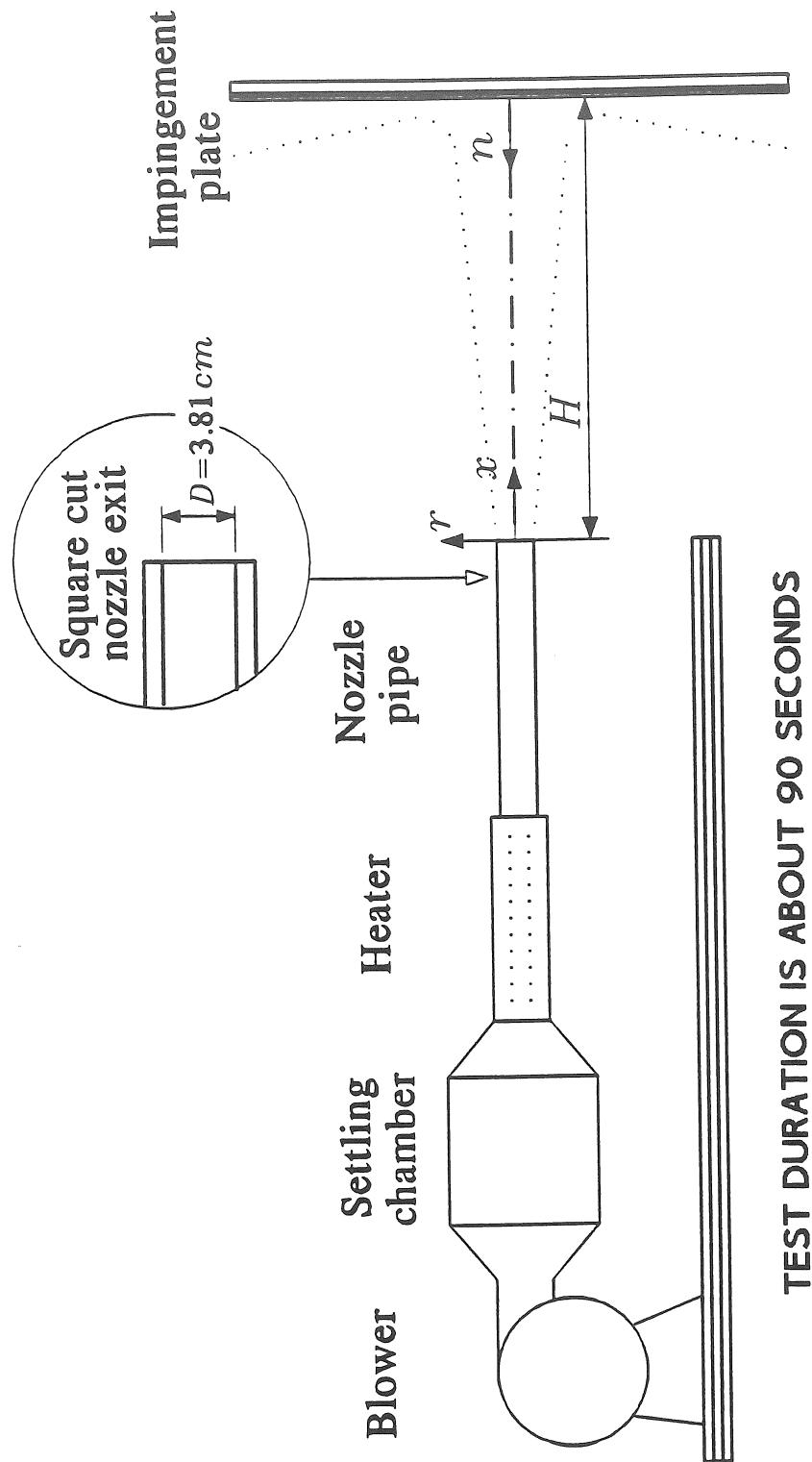
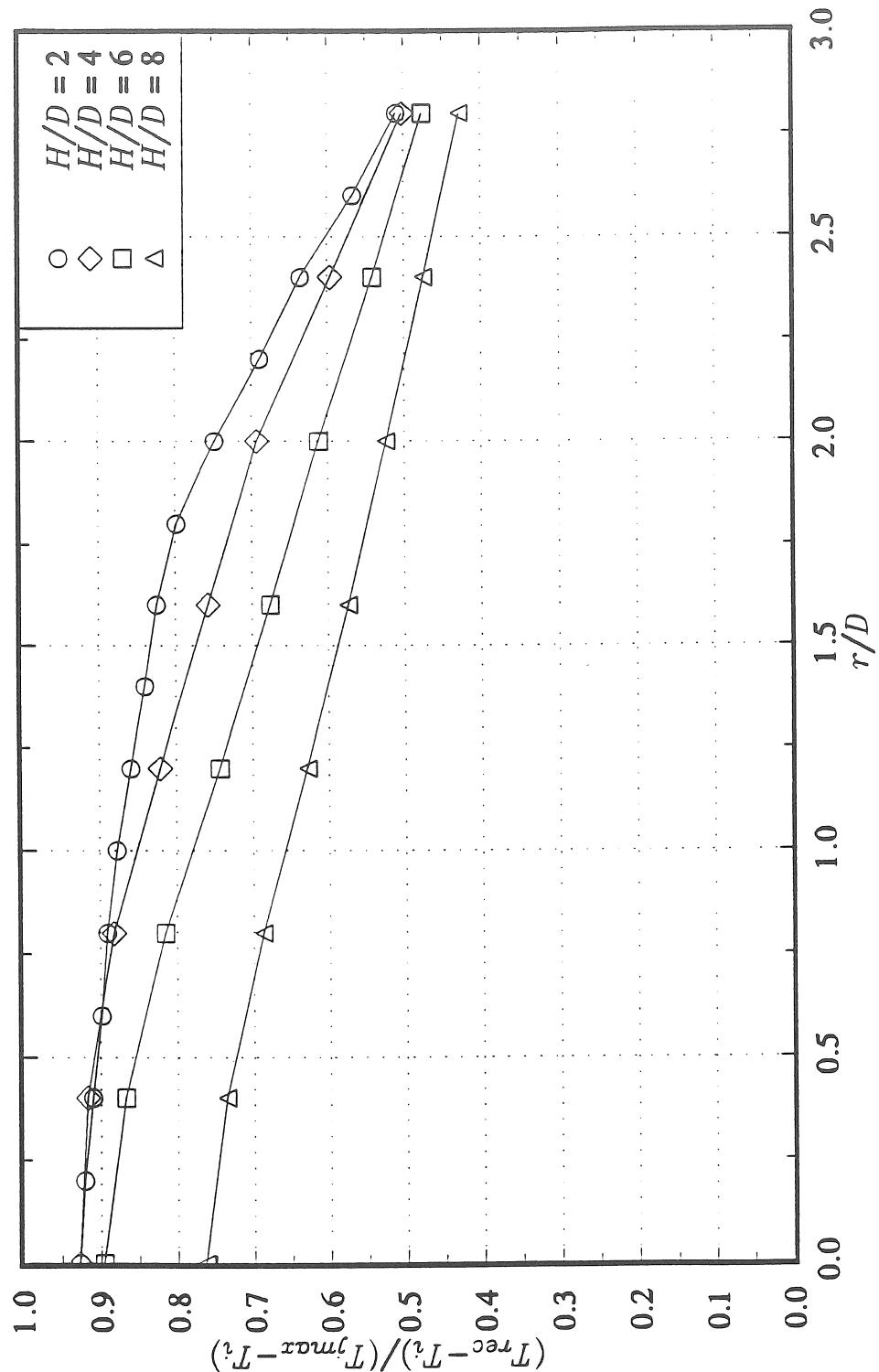


Figure 11  
Experimental confirmation of the “invariant  $h$ ” method,  
Heated impinging jet details

Figure 12  
 Experimental confirmation of the “invariant h” method,  
 Distribution of measured recovery temperature ( $T_{aw}=T_{rec}$ ) on the impingement plate  
 $Re=30,000$ , jet maximum temperature at nozzle exit  $T_{jmax}=333$  °K,  $T_i=296$  °K



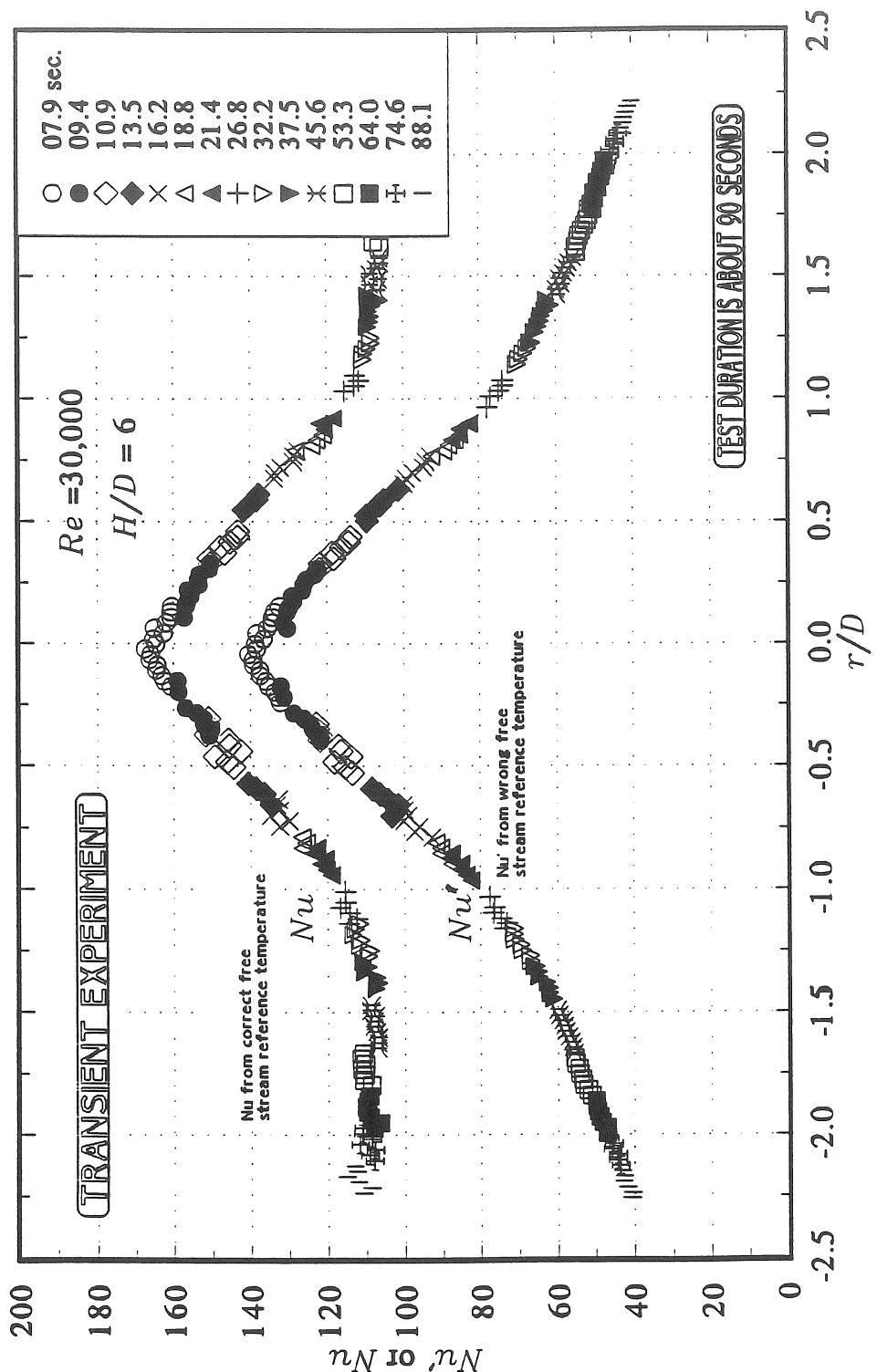


Figure 13  
 Experimental confirmation of the “invariant h” method,  
 $Nu$  number distribution obtained from the correct free stream reference temperature and  
 $Nu'$  distribution resulting from  $T_{j\max}$  at the exit of the nozzle.  
 $Re=30,000$ , jet maximum temperature at nozzle exit  $T_{j\max} = 333^{\circ}\text{K}$ ,  $T_i=296^{\circ}\text{K}$

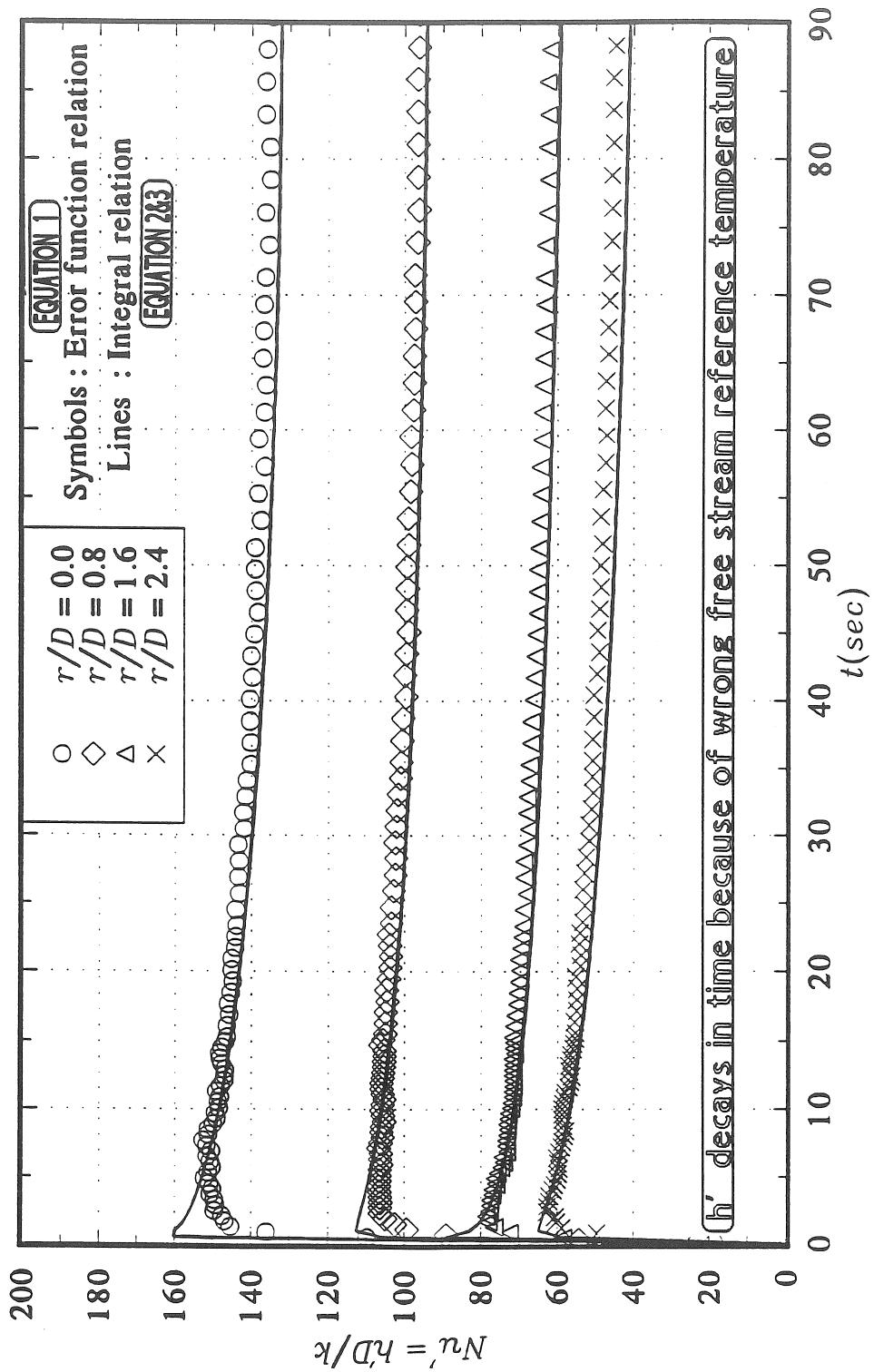


Figure 14  
Experimental confirmation of the “invariant  $h'$ ” method,  
 $Nu' = h'D/k$  decays in time when the reference free stream temperature is wrong

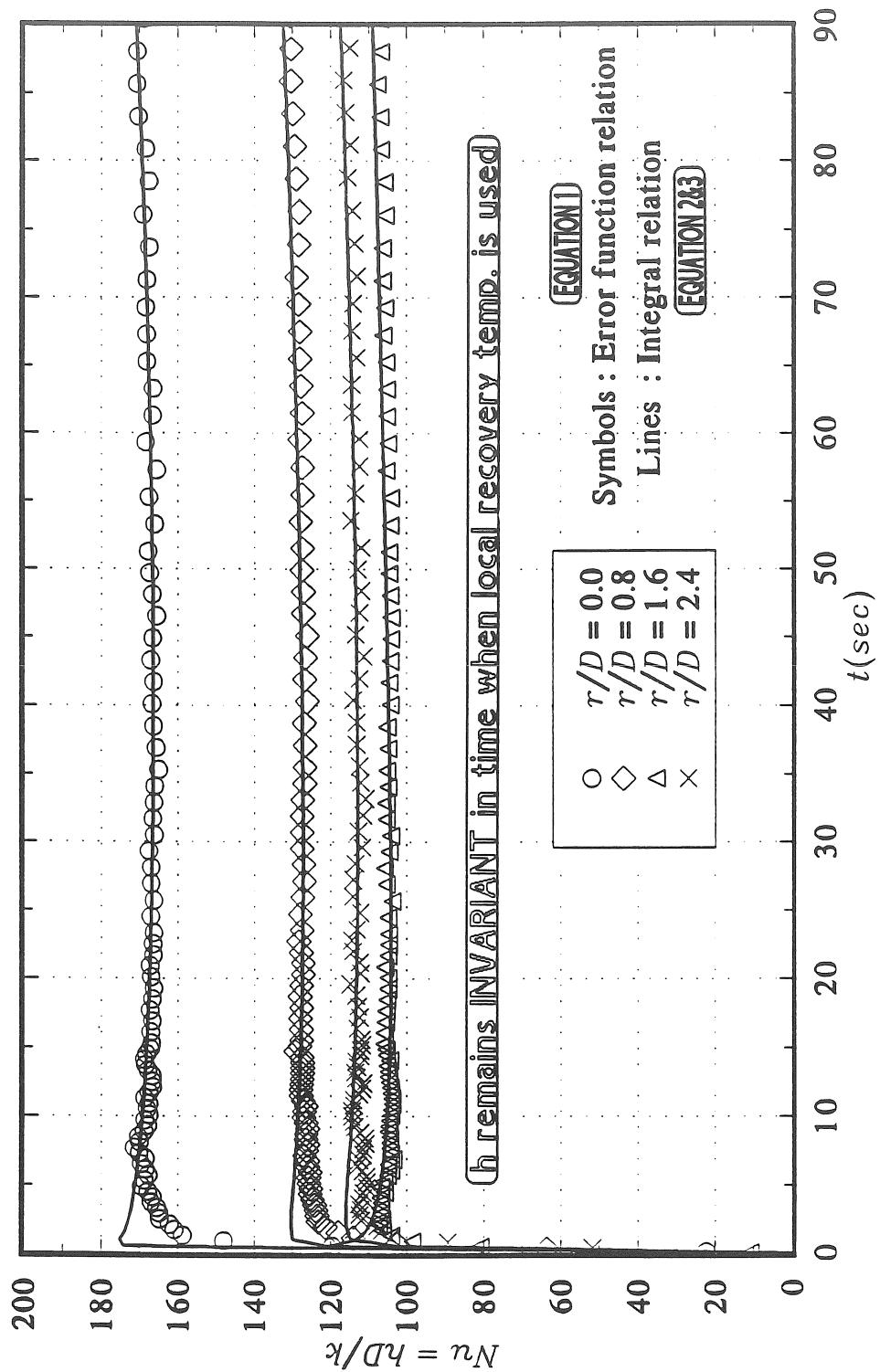


Figure 15  
Experimental confirmation of the “invariant  $h$ ” method,  
 $Nu = hD/k$  stays invariant when local recovery temperature is used as a free stream reference temperature