Anti-Türev (İlkeli)

$F'(x) = f(x)$ olsun. $\int f(x)\,dx = F(x) + c$ ifadesine belirsiz integral denir.

türev

integrant → integral değişkeni

$$\int 2x\,dx = x^2 + c$$

x'e göre

$x^2 + 1 \to 2x$

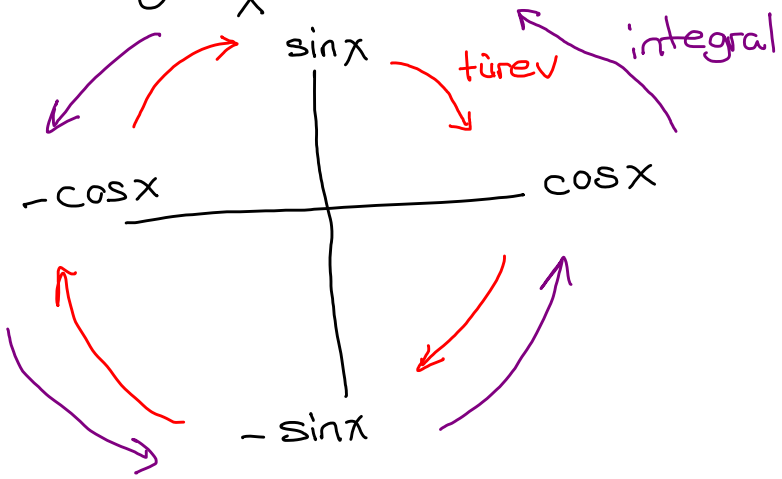
$x^2 + 5 \to 2x$

$+ c$

$$\int 2x\,da = 2x \cdot a + c$$

a'ya göre

$$\int \frac{dx}{x} = \ln x + c$$



6. $\int sec^2 x\,dx = tan\ x + c$; $\int \frac{1}{cos^2 x}\,dx = tan\ x + c$; $\int 1+\tan^2 x\,dx = \tan x + c$

7. $\int \text{cosec}^2 x dx = -cot\ x + c$; $\int \frac{1}{sin^2 x}\,dx = -cot\ x + c$; $\int 1+\cot^2 x\,dx = -\cot x + c$

8. $\int \boldsymbol{sec}\ x \cdot \tan x\,dx = \boldsymbol{sec}\ x + c$ ✓

9. $\int \text{cosec}x \cdot \boldsymbol{cot}\ x\,dx = -\text{cosec}x + c$ ✓

10. $\int \tan x\,dx = -\boldsymbol{ln\ cos}\ x + c$

11. $\int \cot x\,dx = \boldsymbol{ln\ sin}\ x + c$ →

türevi

$$\int \frac{\cos x}{\sin x}\,dx = \ln(\sin x) + c$$

kendisi

12. $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\arctan\frac{x}{a} + c$

13. $\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + c$

14. $\int \frac{dx}{\sqrt{x^2+a^2}} = \boldsymbol{ln}(x + \sqrt{x^2+a^2}) + c$

15. $\int \frac{dx}{\sqrt{x^2-a^2}} = \boldsymbol{ln}(x + \sqrt{x^2-a^2}) + c$

16. $\int \boldsymbol{cosh}\,x\,.dx = \boldsymbol{sinh}\,x + c$ →

17. $\int \boldsymbol{sinh}\,x\,.dx = \boldsymbol{cosh}\,x + c$

$$\int \frac{e^x + e^{-x}}{2}\,dx = \underbrace{\frac{e^x - e^{-x}}{2}}_{\sinh(x)} + c$$

$\int 3^{x+1}dx = \frac{3^{x+1}}{\ln 3} + c$ $\int \frac{2^{x+1}}{3^x}dx = \int 2\cdot\left(\frac{2}{3}\right)^x dx$ ($a=\frac{2}{3}$)

$= 2\cdot\frac{\left(\frac{2}{3}\right)^x}{\ln\left(\frac{2}{3}\right)} + c$

Test 30 soru 60 dk.

DEĞİŞKEN DEĞİŞTİRME

ör: $\int (x+7)^{15}dx = \int u^{15}du = \frac{u^{15+1}}{16} + c = \frac{(x+7)^{16}}{16} + c$

u

$u = x+7$

$d(u) = d(x+7)$

$1\cdot du = 1\cdot dx$

$\star$ $\boxed{d(f(x)) = f'(x)dx}$

ör: $\int \underbrace{(x^2+1)^2}_{u}\,x\,dx = ?$

$u = x^2+1$

$du = d(x^2+1) = 2x\,dx$

$\int u^2\frac{du}{2} = \frac{1}{2}\cdot\frac{u^3}{3} + c$

$= \frac{(x^2+1)^3}{6} + c$

$\int x^n dx = \frac{x^{n+1}}{n+1}$

VII $\int (x^2+1)^2 dx = ?$

II $\int \underbrace{(x^2+1)^2}_{u}dx$ $x^2+1 = u$ değişken değiştir...

a) $\int u^2 du$ b) $\int u^2\frac{du}{2}$ c) $\int \frac{u^2du}{\sqrt{u}}$ d) $\int \frac{u^2du}{2\sqrt{u-1}}$ e) $\int \frac{u^2du}{2x}$ fuzuli

$\boxed{u = x^2+1} \rightarrow x = \sqrt{u-1}$

$du = 2x\,dx$

$\frac{du}{2\sqrt{u-1}} = dx$

$\int u^2\frac{du}{2\sqrt{u-1}}$ sadece u olmalı

* problem bu bize verilmede yok.

$$\int (x^2+1)^2\,dx = \int (x^4+2x^2+1)\,dx = \frac{x^5}{5}+\frac{2x^3}{3}+x+C$$

ör)

$$\int (x^2+1)^{12}\cdot x^3\,dx = \int ((\sqrt[3]{u})^2+1)^{12}\cdot u\cdot \frac{du}{3(\sqrt[3]{u})^2}$$

$u = x^3$

$du = 3x^2\,dx$

$\sqrt[3]{u} = x$

$$\int (x^2+1)^{12} x^3\,dx = \int u^{12}\cdot(\sqrt{u-1})^2\cdot\frac{du}{2\sqrt{u-1}} = \int \frac{u^{12}\cdot(u-1)\,du}{2}$$

$u = x^2+1 \rightarrow \sqrt{u-1} = x$

$du = 2x\,dx$

$\sqrt{u-1}$

$$\checkmark \int (x^2+1)^2 x^3\,dx = \int (x^4+2x^2+1)\,x^3\,dx$$

$$= \frac{x^8}{8}+\frac{2x^6}{6}+\frac{x^4}{4}+C$$

Farklı düzenlemiş:

$x\,dx$ ten

$$\int (x^2+1)^{12} x^3\,dx = \int u^{12}(u-1)\frac{du}{2}$$

$x^3 = x\cdot x^2$

$du = 2x\,dx$

$u = x^2+1 \rightarrow x^2 = u-1$

$$\frac{1}{4}\int \frac{\overbrace{4x^3dx}^{f'}}{\underbrace{1+x^4}_{f(x)}} = \frac{1}{4}\ln(1+x^4)+c$$

$$\star \int \frac{f'(x)}{f(x)}dx = \ln f(x)$$

$$\int \frac{dx}{x} = \ln x + c$$

a) $\arcsin(x)+c$ b) $\ln(1+x^4)+c$

c) $\arctan(x^2)+c$ d) $\frac{\ln(1+x^4)}{4}+c$

$\star$ $u = 1+x^4 \rightarrow du = 4x^3dx$

$$\int \frac{du}{4.u} = \frac{1}{4}\ln(u)+c$$

$$\int \frac{xdx}{\underbrace{1+x^4}_{(x^2)^2}} = ?$$

$u = f(x)$ $x^{(2)}$

$du = f'(x)dx$ x

$$\int \frac{dx}{\underset{1}{x^2}+\underset{1}{a^2}} = \frac{1}{\underset{1}{a}}\arctan\frac{x}{\underset{1}{a}}$$

a) $\frac{1}{4}\ln(1+x^4)+c$ b) $\arctan(x)+c$

c) $\frac{\arctan(1+x^4)}{4}+c$ d) $\arctan(x^4)+c$

e) $\frac{\arctan(x^2)}{2}+c$

$u = x^2$

$du = 2xdx$

$$\int \frac{\frac{du}{2}}{1+u^2} = \frac{1}{2}\arctan(u)+c$$

$$= \frac{\arctan(x^2)}{2}+c$$

$$\int \frac{2xdx}{\underbrace{9+x^4}_{(x^2)^2}} = \int \frac{du}{9+u^2} = \frac{1}{3}\arctan\left(\frac{u}{3}\right)+c$$

$u = x^2$

$du = 2xdx$

$$\int \frac{\overbrace{\cos x - \sin x}^{\text{türevi}}}{\underbrace{\cos x + \sin x}} dx$$

$$\int \frac{\text{türevi}}{\text{kendisi}} \; ? = \ln$$

a) $\sin x - \cos x + c$

b) $\cos x - \sin x + c$

d) $\ln(\sin x - \cos x) + c$

c) $\ln(\cos x + \sin x) + c$