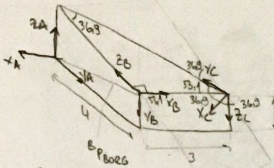
ROBOT TEORİSİ

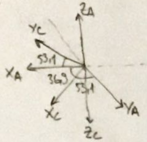
Vize Çıkmış Sorular − 2020

(1) A, B, C koordinat sistemleri yanda verilen şekildeki gibi konumlandırılmışlardır. Buna göre;

1. ${}^{C}_{A}R$ ve ${}^{C}_{B}R$ dönme matrislerini bulunuz.
2. A koordinat sisteminde tanımlanmış ${}^{A}P = [2\ 3\ 0]^T$ noktasının B koordinat sistemine göre konumunu bulunuz.

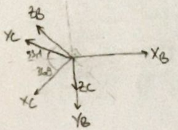


1.



$$
{}^{C}_{A}R = \begin{bmatrix} \cos 36{,}9 & \cos 53{,}1 & \cos 90 \\ \cos 53{,}1 & \cos 143 & \cos 90 \\ \cos 90 & \cos 90 & \cos 180 \end{bmatrix}
$$

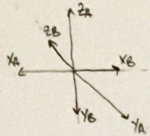
$$
= \begin{bmatrix} 0{,}8 & 0{,}6 & 0 \\ 0{,}6 & -0.8 & 0 \\ 0 & 0 & -1 \end{bmatrix}
$$



$$
{}^{C}_{B}R = \begin{bmatrix} \cos 143 & \cos 90 & \cos 127 \\ \cos 127 & \cos 90 & \cos 36 \\ \cos 90 & \cos 0 & \cos 90 \end{bmatrix}
$$

$$
= \begin{bmatrix} -0.8 & 0 & -0.6 \\ -0.6 & 0 & 0{,}8 \\ 0 & 1 & 0 \end{bmatrix}
$$

2. ${}^{B}P = {}^{B}_{A}T \cdot {}^{A}P$



$$
{}^{B}_{A}T = \begin{bmatrix} \cos 180 & \cos 90 & \cos 90 & 0 \\ \cos 90 & \cos 90 & \cos 180 & 2 \\ \cos 90 & \cos 180 & \cos 90 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{B}P = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 \\ 0 & -1 & 0 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ 1 \\ 1 \end{bmatrix}
$$

(2) Bir robotun uç işlevcisine sabit (roll, pitch, yaw) bir koordinat sistemi yerleştiriliyor. Robotun uç işlevcisi bu sabit koordinat sisteminin eksenleri olan z boyunca γ, daha sonra x boyunca β ve son olarak Y boyunca α kadar döndürülerek aşağıdaki dönme matrisi elde ediliyor. Bu matristen faydalanarak α, β, γ açılarını bulunuz.

$$
R_{XYZ}(\beta, \alpha, \gamma) = \begin{bmatrix} 0{,}9980 & -0.0143 & 0.0617 \\ 0{,}0151 & 0.9997 & -0.0186 \\ -0.0414 & 0.0192 & 0.9990 \end{bmatrix}
$$

$$
R_{XYZ}(\beta, \alpha, \gamma) = R_y(\alpha) \cdot R_x(\beta) \cdot R_z(\gamma)
$$

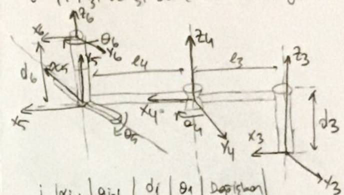
$$
= \begin{bmatrix} c\alpha & 0 & s\alpha \\ 0 & 1 & 0 \\ -s\alpha & 0 & c\alpha \end{bmatrix} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & c\beta & -s\beta \\ 0 & s\beta & c\beta \end{bmatrix} \begin{bmatrix} c\gamma & -s\gamma & 0 \\ s\gamma & c\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}}
$$

$$
= \begin{bmatrix} c\alpha & 0 & s\alpha \\ 0 & 1 & 0 \\ -s\alpha & 0 & c\alpha \end{bmatrix} \begin{bmatrix} c\gamma & -s\gamma & 0 \\ c\beta s\gamma & c\beta c\gamma & -s\beta \\ s\beta s\gamma & s\beta c\gamma & c\beta \end{bmatrix}
$$

$$
= \begin{bmatrix} c\alpha c\gamma + s\alpha s\beta s\gamma & -c\alpha s\gamma + s\alpha s\beta c\gamma & s\alpha c\beta \\ c\beta s\gamma & c\beta c\gamma & -s\beta \\ -s\alpha c\gamma + c\alpha s\beta s\gamma & s\alpha s\gamma + c\alpha s\beta c\gamma & c\alpha c\beta \end{bmatrix}
$$

(3) Yanda verilen RRPRRR robotun;

1. Her eklemine birer koordinat sistemi yerleştiriniz.
2. Yerleştirdiğiniz koordinat sisteminden yararlanarak DH parametreleri tablosunu oluşturunuz.
3. ${}^{0}_{1}T$, ${}^{2}_{3}T$ ve ${}^{4}_{5}T$ dönüşüm matrislerini yazınız.



i	α_{i-1}	a_{i-1}	d_i	θ_i	Değişken
4	0	l_3	d_3	θ_4	Q_4
5	90	l_4	0	θ_5	Q_5
6	−90	0	d_6	θ_6	Q_6

$$
{}^{4}_{5}T = \begin{bmatrix} c\theta_5 & -s\theta_5 & 0 & a_4 \\ \cancel{s\theta_5 c(90)} & \cancel{c\theta_5 c(90)} & -s(90) & \cancel{-s(90).0} \\ s\theta_5 c(90) & c\theta_5 s(90) & c(90) & \cancel{d(90).0} \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{4}_{5}T = \begin{bmatrix} c\theta_5 & s\theta_5 & 0 & l_4 \\ 0 & 0 & -1 & 0 \\ s\theta_5 & c\theta_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Final Çıkmış Sorular

⇒ İleri Kinematik

08.06.2021 - 1

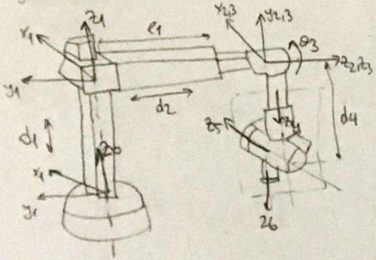
Aşağıda eklem yapısı PPRRRR olan altı eklemli bir robot verilmiştir.

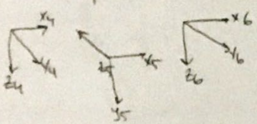
a) Robotun eklemlerine koordinat sistemleri yerleştiriniz.

b) DH parametrelerini tablo halinde yazınız.

c) ${}^0_1T$, ${}^1_2T$, ${}^2_3T$ dönüşüm matrislerini yazınız.

d) ${}^0_3T$ dönüşüm matrisini bulunuz.





i	α_{i-1}	a_{i-i}	d_i	θ_1	değişken
1	0	0	d_1	0	d_1
2	90	0	l_1+d_2	0	d_2
3	0	0	0	θ_3	θ_3
4	90	0	d_4	$90+\theta_4$	θ_4
5	90	0	0	θ_5	θ_5
6	-90	0	0	θ_6	θ_6

⇒ Ters Kinematik

08.06.2021 - 2

Dönüşüm matrisleri aşağıdaki gibi verilen RPP robotun verilen bir uç işlevci dönüşüm matrisi için eklem değerlerini veren denklemleri bulunuz.

$$
{}^0_1T=\begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\quad
{}^1_2T=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\quad
{}^2_3T=\begin{bmatrix} 1 & 0 & 0 & l_1 \\ 0 & 0 & -1 & -d_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$ {}^0_3T = {}^0_1T\,{}^1_2T\,{}^2_3T $$

$$ \left[{}^0_1T\right]^{-1} {}^0_3T = {}^1_2T\,{}^2_3T $$

$$
\begin{bmatrix} c\theta_1 & s\theta_1 & 0 & 0 \\ -s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} r_{11} & r_{12} & r_{13} & P_x \\ r_{21} & r_{22} & r_{23} & P_y \\ r_{31} & r_{32} & r_{33} & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = {}^1_2T\,{}^2_3T
$$

$$
\begin{bmatrix} \cdot & \cdot & \cdot & c\theta_1 P_x + s\theta_1 P_y \\ \cdot & \cdot & \cdot & -s\theta_1 P_x + c\theta_1 P_y \\ \cdot & \cdot & \cdot & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}
=
\begin{bmatrix} \cdot & \cdot & \cdot & l_1 \\ \cdot & \cdot & \cdot & -d_3 \\ \cdot & \cdot & \cdot & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$ c\theta_1 P_x + s\theta_1 P_y = l_1 $$

$$ -s\theta_1 P_x + c\theta_1 P_y = -d_3 $$

$$ P_z = d_2 $$

$$ \theta_1 = \mathrm{Atan2}(P_y, P_x) \mp \mathrm{Atan2}\left(\sqrt{P_y^2+P_x^2-l_1^2},\ l_1\right) $$

→ Jakobiyen

08.06.2021-3

Uç işlevci dönüşüm matrisi aşağıdaki gibi verilen üç eklemli bir RPP robotun uç işlevcisinin doğrusal hızları için Jakobiyen matrisini bulunuz.

$$
{}^0_3T=\begin{array}{c} \begin{bmatrix} c\theta_1 & 0 & s\theta_1 & l_1 c\theta_1 + d_3 s\theta_1 \\ s\theta_1 & 0 & -c\theta_1 & l_1 s\theta_1 - d_3 c\theta_1 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}
\begin{array}{l} P_x, \theta_1 \\ P_y, d_2 \\ P_z, d_3 \\ \end{array}
$$

$$
{}^3J_v=\begin{bmatrix} -l_1 s\theta_1 + d_3 c\theta_1 & 0 & s\theta_1 \\ l_1 c\theta_1 + d_3 s\theta_1 & 0 & -c\theta_1 \\ 0 & 1 & 0 \end{bmatrix}
$$

21.01.2021-2

DH parametreleri tablosu aşağıda verilen robotun uç işlevcisinin doğrusal hızlarını veren Jakobiyen matrisini bulun.

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	h_1	θ_1
2	0	l_1	0	θ_2
3	180	l_2	d_3	0

$$
{}^0_1T=\begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & h_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^1_2T=\begin{bmatrix} c\theta_2 & -\sin\theta_2 & 0 & l_1 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^2_3T=\begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$ {}^0_3T = \underbrace{{}^0_1T\,{}^1_2T}\,{}^2_3T $$

$$
=\begin{bmatrix} c\theta_1 c\theta_2 - s\theta_1 s\theta_2 & -c\theta_1 s\theta_2 - s\theta_1 c\theta_2 & 0 & c\theta_1 l_1 \\ s\theta_1 c\theta_2 + c\theta_1 s\theta_2 & -s\theta_1 s\theta_2 + c\theta_1 c\theta_2 & 0 & s\theta_1 l_1 \\ 0 & 0 & 1 & h_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot {}^2_3T
$$