TRİGONOMETRİK İNTEGRALLER

1) $\int \sin^m x\,dx$, $\int \cos^m x\,dx$, $\int \sin^m x \cos^n x\,dx$ tipindeki integraller.

a) tek kuvvet varsa

$$\int \sin^3 x\,dx = \int \underbrace{\sin^2 x}_{1-\cos^2 x} \underbrace{\sin x\,dx} \quad ; \quad u=\cos x,\ du=-\sin x\,dx$$

$$= \int (1-u^2)(-du) = \frac{u^3}{3} - u + c$$

$$= \frac{\cos^3 x}{3} - \cos x + c$$

İkisi tek kuvvetli olan

Ör: $\int \cos^3 x \sin^7 x\,dx = \text{---------}$

$$\int \overset{1-\sin^2 x}{\cos^2 x} \sin^7 x \underbrace{\cos x\,dx} \qquad u=\sin x,\ du=\cos x\,dx$$

$$= \int (1-u^2)\, u^7\, du = \frac{u^8}{8} - \frac{u^{10}}{10} + c \ ; \ u=\sin x$$

kuvvetler çift ise $\quad \sin^2 x = \dfrac{1-\cos 2x}{2} \quad \cos^2 x = \dfrac{1+\cos 2x}{2}$

$$\int \cos^2 x\,dx = \int \frac{1+\cos 2x}{2}\,dx = \frac{x}{2} + \frac{\sin(2x)}{4} + c$$

ör:

$$\int \underbrace{\sin^2 x}_{\frac{1-\cos 2x}{2}} \underbrace{\cos^2 x}_{\frac{1+\cos 2x}{2}}\,dx = \int \frac{\sin^2(2x)}{4}\,dx$$

$$= \frac{1}{4}\int \frac{1-\cos 4x}{2}\,dx = \frac{x}{8} - \frac{\sin 4x}{4} \cdot \frac{1}{8} + c$$

* Ters dönüşüm Formülleri

$$\sin x \cdot \cos y = \frac{1}{2}\left[\sin(x+y) + \sin(x-y)\right]$$

$$\int \sin 5x \cdot \cos x\,dx = \frac{1}{2}\int \sin(6x) + \sin(4x)\,dx = -\frac{\cos 6x}{12} - \frac{\cos 4x}{8} + c$$

$* \int \tan x\,dx = -\ln|\cos x| + c = \ln|\sec x| + c$

$* \int \tan^2 x\,dx = \int (\tan^2 x + 1 - 1)\,dx = \tan x - x + c$

$$\int \tan^3 x\,dx = \int \tan x\,(\tan^2 x + 1 - 1)\,dx$$

$$= \int \underbrace{\tan x}_{u}\,\underbrace{(\tan^2 x + 1)\,dx}_{du} - \int \tan x\,dx$$

$$\int \tan^{\overset{n}{3}} x\,dx = \frac{\tan^2 x}{2} - \int \tan^1 x\,dx$$

$$= \frac{\tan^2 x}{2} + \ln|\cos x| + c$$

$$* \int \tan^n x\,dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\,dx$$

$$\int \tan^6 x\,dx = \frac{\tan^5 x}{5} - \underbrace{\int \tan^4 x\,dx}$$

$$= \frac{\tan^5 x}{5} - \frac{\tan^3 x}{3} + \underbrace{\int \tan^2 x\,dx}_{\tan x - x + c}$$

$*$ $\int \tan^n x \sec^m x\,dx$ tipindeki integraller

1) m çift ise $\sec^2 x\,dx$ ayrılır. $u = \tan x$ dönüşümü yapılır. $\sec^2 x = 1 + \tan^2 x$ den faydalanılır.

$$\int \tan^5 x \sec^2 x\,dx = \int u^5\,du = \frac{u^6}{6} + c$$

$u = \tan x \quad du = \sec^2 x\,dx$

$$\int \tan^5 x \sec^{\overset{\text{çift}}{4}} x\, dx = \int u^5 (1+u^2)\, du = \frac{u^6}{6} + \frac{u^8}{8} + c$$

$\tan^5 x \underbrace{\sec^2 x}_{1+\tan^2 x} \boxed{\sec^2 x\, dx}$

$u = \tan x$ yazılır.

$u = \tan x$

$du = \sec^2 x\, dx$

2) n tek ise $\int \sec x \tan x\, dx = \sec x + c$ faydanılır.

$u = \sec x$ dönüşümü yapılır.

$$\int \tan^{\overset{\text{tek}}{5}} x . \sec^4 x\, dx = \int \underbrace{\tan^4 x}_{(\sec^2 x - 1)^2} . \sec^3 x . \underbrace{\tan x \sec x\, dx}_{du}$$

$u = \sec x$

$du = \sec x . \tan x\, dx$

$$= \int (u^2 - 1)^2\, u^3\, du$$

$u^2 - 1 = t$, $2u\, du = dt$, $u\, du = \frac{dt}{2}$

$$= \int t^2 (t+1) \frac{dt}{2} = \frac{t^4}{8} + \frac{t^3}{6} + c$$

$$\int \tan^4 x . \sec^{\overset{\text{çift}}{4}} x\, dx = ? \qquad \int u^4 . (1+u^2)\, du$$

$u = \tan x$

$du = \sec^2 x\, dx$

$$= \frac{u^5}{5} + \frac{u^7}{7} + c$$

$u = \tan x$ yaz.

$$\int \cot^{\overset{\text{tek}}{3}} x \operatorname{cosec}^3 x\, dx = \int \cot^2 x . \operatorname{cosec}^2 x . \cot x \operatorname{cosec} x\, dx$$

$u = \operatorname{cosec} x \to du = -\operatorname{cosec} x . \cot x\, dx$

1) $1 + \operatorname{cosec}^2 x = \cot^2 x$

$$= \int (1+u^2)\, u^2\, (-du)$$

2) $(\operatorname{cosec} x)' = -\operatorname{cosec} x . \cot x$

$$* \int \cot x \, dx = \ln|\sin x| + c$$

$$* \int \cot^2 x \, dx = \int (\cot^2 x + 1 - 1) dx = -\cot x - x + c$$

$$* \int \cot^n x \, dx = -\frac{\cot^{n-1} x}{n-1} - \int \cot^{n-2} x \, dx$$

$$\int \frac{\sin^6 x \, dx}{\cos^8 x} = ?$$

$$\int \frac{\sin^6 x}{\cos^6 x} \cdot \frac{1}{\cos^2 x} dx = \int \tan^6 x \sec^2 x \, dx = \frac{\tan^7 x}{7} + c \checkmark$$

$$\int \frac{\overset{\text{türevi}}{\overline{\cos x \, dx}}}{\underbrace{1 + \sin x}_{\text{kendisi}}} = \ln|1 + \sin x| + c$$

$$\underline{\underline{\text{ör:}}} \int \frac{\overset{\cos x + \sin x + \cos x - \sin x}{2 \cos x \, dx}}{\sin x + \cos x} = ? \qquad \int \left(1 + \frac{\text{türevi}}{\text{kendisi}}\right) dx = x + \ln|\sin x + \cos x| + c$$

$$\int \frac{A f(x) + B f'(x)}{f(x)} dx = Ax + B \ln|f(x)| + c$$

$$f(x) = \sin x + \cos x$$

$$2\cos x = A f(x) + B f'(x)$$

$$= A(\sin x + \cos x) + B(\cos x - \sin x)$$

$$2\cos x = \underbrace{(A - B)}_{0} \sin x + \underbrace{(A + B)}_{2} \cos x$$

$$A = B \longrightarrow A = B = 1$$

$$\int \frac{12\sin x - 5\cos x}{2\sin x - 3\cos x}\,dx = ? \qquad \int \frac{3f(x) + 2f'(x)}{f(x)}\,dx$$

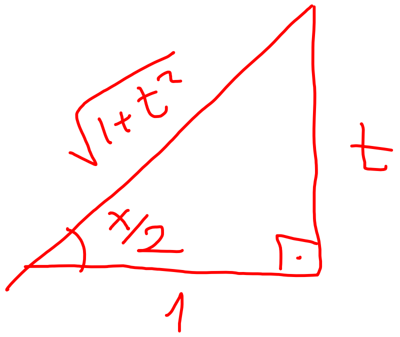
$$f(x) = 2\sin x - 3\cos x \qquad = x + 2\ln|2\sin x - 3x| + c$$

$$12\sin x - 5\cos x = A(2\sin x - 3\cos x) + B(2\cos x + 3\sin x)$$

$$3 \quad 2 = (2A + 3B)\sin x + (2B - 3A)\cos x$$

$$12 = 2A + 3B$$
$$-5 = -3A + 2B$$

$*$ $\tan\frac{x}{2} = t$ dönüşümü



$$\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2} = 2 \cdot \frac{t}{\sqrt{1+t^2}} \cdot \frac{1}{\sqrt{1+t^2}} = \frac{2t}{1+t^2}$$

$$\boxed{\sin x = \frac{2t}{1+t^2}}$$

$$\cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2} = \frac{1}{1+t^2} - \frac{t^2}{1+t^2} = \frac{1-t^2}{1+t^2}$$

$$\boxed{\cos x = \frac{1-t^2}{1+t^2}}$$

$$\tan\frac{x}{2} = t \longrightarrow (1 + \underbrace{\tan^2\frac{x}{2}}_{t^2}) \cdot \frac{1}{2}\,dx = dt$$

$$\boxed{dx = \frac{2dt}{1+t^2}}$$

Ör:
$$\int \frac{dx}{\sin x} = \int \frac{\frac{2dt}{\cancel{1+t^2}}}{\frac{2t}{\cancel{1+t^2}}} = \int \frac{dt}{t} = \ln|t| + c$$
$$= \ln\left|\tan\frac{x}{2}\right| + c$$

$$\int \frac{dx}{1+\sin x} = ?$$

$\tan\frac{x}{2} = t$

$$\int \frac{\frac{2dt}{1+t^2}}{1+\frac{2t}{1+t^2}} = \int \frac{2dt}{(t+1)^2} = -\frac{2}{t+1} + c$$

$\tan\frac{x}{2}$

KISMİ İNTEGRASYON METODU

$$\int d(u.v) = \int v.du + \int u\,dv$$

$$u.v = \int v.du + \int u.dv \quad \star$$

$$\boxed{\int u.dv = u.v - \int v\,du}$$

U'nun öncelik sırası

L → logaritma

A → terstrigonometrik

P → polinom

T → trigonometrik

Ü → üstel

ör: $\int \underbrace{x}_{u}^{P} \underbrace{e^x dx}_{dv}^{Ü} = \quad u.v - \int v.du$

$= x.e^x - \int e^x dx = xe^x - e^x + c$

$u = x$

$du = dx$

$\int dv = \int e^x dx \Rightarrow v = e^x$

pratik:

	Türev	İntegral
+	x	e^x
−	1	e^x
	0	e^x

$$\int xe^x dx = xe^x - e^x + c$$

$$\int x^2 \sin 2x \, dx = ?$$

$x^2 \downarrow P$, $\sin 2x \downarrow T$

$u = x^2$

	T	İ
+	x^2	$\sin 2x$
−	$2x$	$-\frac{\cos 2x}{2}$
+	2	$-\frac{\sin 2x}{4}$
	0	$+\frac{\cos 2x}{8}$

$$u \cdot v - \int v \cdot du$$

$$\int x^5 \ln x \, dx = \ln x \cdot \frac{x^6}{6} - \int \frac{x^6}{6} \frac{dx}{x} = \frac{x^6 \ln x}{6} - \frac{x^6}{36} + c$$

$x^5 \downarrow P$, $\ln x \downarrow L$

$u = \ln x \longrightarrow du = \frac{dx}{x}$

$\int dv = \int x^5 dx \longrightarrow v = \frac{x^6}{6}$

$$\int \arcsin x \, dx = x \arcsin x - \int \frac{x \, dx}{\sqrt{1-x^2}}$$

$\arcsin x \downarrow u$, $dx \downarrow dv$

$$= x \arcsin x + \frac{\sqrt{1-x^2}}{2} + c$$

$v = x$

$u = \arcsin x$

$du = \frac{dx}{\sqrt{1-x^2}}$

$$I = \int \underbrace{\sin x}_{u} \cdot \underbrace{e^x\,dx}_{dv} = \sin x \cdot e^x - \int \underbrace{\cos x}_{u}\, \underbrace{e^x\,dx}_{dv}$$

$$du = \cos x\,dx \qquad = \sin x\, e^x - \cos x\, e^x + \underbrace{\int -\sin x\, e^x\,dx}_{-I}$$

$$I = \frac{e^x(\sin x - \cos x)}{2} + c$$