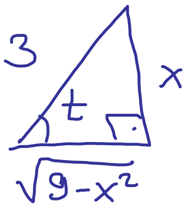
Bazı Özel Dönüşümler

* $\sqrt{a^2-x^2}$ dışında kökü ifade bulundurmayan integrallerde $a\sin t = x$ uygulanabilir.

* $$\int \frac{dx}{\sqrt{9-x^2}} = \arcsin\left(\frac{x}{3}\right)+c$$

$$\boxed{\int \frac{f'(x)\,dx}{2\sqrt{f(x)}} = \sqrt{f(x)}+c}$$

* $$-\int \frac{-2x\,dx}{2\sqrt{9-x^2}} = ? \quad -\sqrt{9-x^2}+c \checkmark$$



2.yol: $x = 3\sin t \rightarrow dx = 3\cos t\,dt$

$$\int \frac{3\sin t\,\cancel{3\cos t}\,dt}{\underbrace{\sqrt{9-9\sin^2 t}}_{\cancel{3}\cancel{\cos t}}} = -3\cos t + c$$

$$= -3\cdot\frac{\sqrt{9-x^2}}{3}+c = -\sqrt{9-x^2}$$

3.yol: $9-x^2 = u^2$

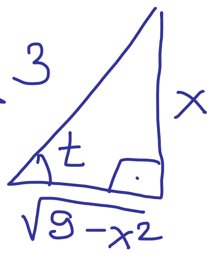
$-2x\,dx = 2u\,du$

$x\,dx = -u\,du$

$$\int \frac{-\cancel{u}\,du}{\cancel{u}} = -u+c$$

$$= -\sqrt{9-x^2}+c$$

Ör: $$\int \frac{dx}{x\sqrt{9-x^2}} = ?$$



$\boxed{x = 3\sin t} \Rightarrow dx = 3\cos t\,dt$

$$\int \frac{\cancel{3\cos t}\,dt}{3\sin t\,\underbrace{\sqrt{9-9\sin^2 t}}_{\cancel{3\cos t}}} = \frac{1}{3}\int \frac{dt}{\sin t}$$

$$= \frac{1}{3}\ln\left|\tan\frac{t}{2}\right|+c$$

$t \rightarrow \arcsin\left(\frac{x}{3}\right)$

Ör: $$\int \frac{dx}{x^2\sqrt{9-x^2}} = ? \quad \int \frac{\cancel{3}\cancel{\cos t}\,dt}{9\sin^2 t\ \cancel{3}\cancel{\cos t}} = -\frac{\cot(t)}{9}+c = -\frac{\sqrt{9-x^2}}{9x}+c$$

a) $\dfrac{x}{\sqrt{9-x^2}}+c$ b) $\dfrac{\sqrt{9-x^2}}{x}+c$ c) $\dfrac{9x}{\sqrt{9-x^2}}$ d) $\dfrac{x}{9\sqrt{9-x^2}}$

* $\sqrt{x^2-a^2}$ dışında köklü ifade bulundurmayan integrallerde $x=a\sec t$ dönüşümü yapılır.

$$\int \frac{dx}{\sqrt{x^2-4}} = \ln|x+\sqrt{x^2-4}|+c$$

* $$\int \frac{2x\,dx}{2\sqrt{x^2-4}} = \sqrt{x^2-4}+c$$

2. tercih
$x^2-4=u^2$

3. tercih
$x=2\sec t$

Ör:
$$\int \frac{dx}{x\sqrt{x^2-4}} = ?$$

$$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\left(\frac{x}{a}\right)+c$$

$\boxed{x=2\sec t} \longrightarrow t=\operatorname{arcsec}\left(\frac{x}{2}\right)$

$dx=2\sec t\tan t\,dt$

$$\int \frac{\cancel{2}\cancel{\sec t}\,\cancel{\tan t}\,dt}{\cancel{2}\cancel{\sec t}\underbrace{\sqrt{4\sec^2 t-4}}_{2\cancel{\tan t}}} = \frac{t}{2}+c = \frac{1}{2}\operatorname{arcsec}\left(\frac{x}{2}\right)+c$$

$$\int \frac{dx}{x\sqrt{x^2-4}}$$

Ayben; $u=x^2-4$ dönüşümü ile bakalım.
$du=2x\,dx$

$$=\int \frac{x.dx}{x^2\sqrt{x^2-4}} = \int \frac{\frac{du}{2}}{(u+4).\sqrt{u}} = \int \frac{\frac{\cancel{2}t\,dt}{\cancel{2}}}{(t^2+4).\cancel{t}}$$

$u=t^2$
$du=2t\,dt$

$$=\frac{1}{2}\arctan\left(\frac{t}{2}\right)+c$$

$$=\frac{1}{2}\arctan\left(\frac{\sqrt{x^2-4}}{2}\right)+c$$

$$\boxed{\sec^2 x=\tan^2 x+1}$$

$* \sqrt{x^2+a^2}$ için $x=a\tan t$ dönüşümü uygulanabilinir.

$$\int \frac{6dx}{\sqrt{9+x^2}} = 6\ln\left|x+\sqrt{x^2+9}\right|+c$$

$$\int \frac{\overset{2.2}{4}x\,dx}{2\sqrt{9+x^2}} = 2\sqrt{9+x^2}+c$$

Ör: $\displaystyle\int \frac{dx}{x\sqrt{9+x^2}} = ?$

$\boxed{x=3\tan t}$

$dx=3\sec^2 t\,dt$

$t=\arctan\left(\frac{x}{3}\right)$

$$\int \frac{\cancel{3}\overset{\sec t}{\cancel{\sec^2}}t\,dt}{3\tan t\underbrace{\sqrt{9+9\tan^2 t}}_{\cancel{3}.\cancel{\sec}t}} = \frac{1}{3}\int \frac{\frac{1}{\cancel{\cos t}}\,dt}{\frac{\sin t}{\cancel{\cos t}}} = \frac{1}{3}\int \frac{dt}{\sin t}$$

$$= \frac{1}{3}\ln\left|\tan\left(\frac{t}{2}\right)\right|$$

$$\int \frac{\sqrt[2]{3x+1}+\sqrt[3]{3x+1}}{\sqrt[4]{3x+1}}\,dx = \int \frac{u^6+u^4}{\underset{1}{\cancel{u^3}}} \cdot 4\,\underset{u^8}{\cancel{u^{11}}}\,du$$

$3x+1=u$ okek(2,3,4)=12

$3dx=12u^{11}dt$

$$= \frac{4u^{15}}{15}+\frac{4u^{13}}{13}+c \;;\; u=(3x+1)^{1/12}$$

a) $\displaystyle\int \frac{dx}{3\sin x+4\cos x} = ?$

$\tan\frac{x}{2}=t \qquad dx=\frac{2dt}{1+t^2}$

$\sin x=\frac{2t}{1+t^2} \qquad \cos x=\frac{1-t^2}{1+t^2}$

$$\int \frac{\frac{2dt}{1+t^2}}{\frac{3.2t}{1+t^2}+\frac{4(1-t^2)}{1+t^2}} = \int \frac{2\,dt}{-4t^2+6t+4} = \int \frac{-dt}{2t^2-3t-2}$$

$$\frac{-1}{2t^2-3t-2}=\frac{A}{2t+1}+\frac{B}{t-2}$$

$2t \quad 1$

$t \quad -2$

$$-1=(t-2)A+B(2t+1)$$

$t=2 \Rightarrow -1=5B \Rightarrow B=-1/5$

$\underbrace{t=0}_{\text{keyfi}} \quad -1=-2A+B.1$

$+2/5=A$

$$\int \frac{-2/5}{2t+1}dt+\int\frac{-1/5\,dt}{t-2}$$

$$=+\frac{2}{5}\cdot\frac{1}{2}\ln|2t+1|-\frac{1}{5}\ln|t-2|$$

$t=\tan\frac{x}{2}$ yazılır.

1.yol $\quad u=\sin 2x$

$du=2\cos 2x\,dx$

a) $\int \underbrace{\sin^3(2x)}_{\text{bu da parçalanabilinir di.}} \cos^3(2x)\,dx$ (tek, tek) $=\int \sin^3(2x)\cos^2(2x)\,\cos 2x\,dx$

$$=\int u^3(1-u^2)\left(\frac{du}{2}\right)$$

$$=\frac{u^4}{2}-\frac{u^6}{2}+c$$

$\int x^5\cos(x^3)\,dx$ integralini hesaplayınız. ($x^3.x^2$)

$x^3=u \rightarrow 3x^2dx=du$

Türev	İnt.
$+\ u$	$\cos u$
$-\ 1$	$\sin u$
0	$-\cos u$

$$\int u.\cos u.\frac{du}{3}=\frac{1}{3}(u\sin u+\cos u)+c$$

$$=\frac{1}{3}(x^3\sin x^3+\cos x^3)+c$$

b) $\int \frac{\sqrt{x+4}}{x}dx \quad \boxed{x+4=u^2}$

$dx=2u\,du$

$$=\int\frac{u.2u\,du}{u^2-4}=\int\left(2+\frac{8}{u^2-4}\right)du=2u+8.\frac{1}{4}\ln\left|\frac{u-2}{u+2}\right|+c$$

$\sqrt{x+4}=u$ yazılır.

$$\int \frac{\sin^2 x}{\cos^3 x}\,dx = $$

1.yol: $\int \underbrace{\tan^2 x}_{(\sec^2 x - 1)} \cdot \sec x \, dx$

2.yol: $\int \frac{1-\cos^2 x}{\cos^3 x}\,dx = \underbrace{\int \frac{dx}{\cos^3 x}}_{I} - \int \frac{dx}{\cos x}$

$\tan\frac{x}{2} = t$ den

u: $\sec x$ dv: $\sec^2 x\,dx$

$$I = \int \frac{1}{\cos x} \cdot \frac{dx}{\cos^2 x}$$

$$\int \sec x\,dx = \int \frac{\overbrace{\sec x(\sec x + \tan x)}^{\text{türevi}}}{\underbrace{\sec x + \tan x}_{\text{kendisi}}}\,dx = \ln|\sec x + \tan x| + c$$