SAYISAL SİNYAL İŞLEME VİZE SINAVI ÇALIŞMA SORULARI

1.Verilen sinyalenin gerçek değerli s1(t), kampleks diğerli s2(t), çok kanallı (multichannel) s3(t), veya çok boyutlu (multidimensional) I(x,y,t) olup olmadığını belirlemyiniz. sf.7(24)

$$s_1(t) = A \sin 3\pi t$$

$$s_2(t) = Ae^{j3\pi t} = A \cos 3\pi t + jA \sin 3\pi t$$

$$\mathbf{S}_3(t) = \begin{bmatrix} s_1(t) \\ s_2(t) \\ s_3(t) \end{bmatrix}$$

$$\mathbf{I}(x, y, t) = \begin{bmatrix} I_r(x, y, t) \\ I_g(x, y, t) \\ I_b(x, y, t) \end{bmatrix}$$

3.Basit bir analog digital (A/D) konvertönin şemosını çiziniz. sf.22(39)

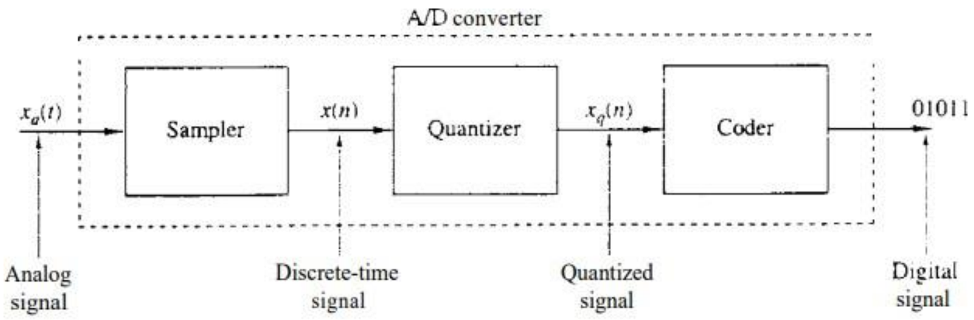

Figure 1.14 Basic parts of an analog-to-digital (AID) converter.

2.Örnek 1.4.1'e çalış, benzerini soracağım. sf.25(42)

Example 1.4.1

The implications of these frequency relations can be fully appreciated by considering the two analog sinusoidal signals

$$x_1(t) = \cos 2\pi(10)t$$
$$x_2(t) = \cos 2\pi(50)t \qquad (1.4.12)$$

which are sampled at a rate $F_s = 40$ Hz. The corresponding discrete-time signals or sequences are

$$x_1(n) = \cos 2\pi \left(\frac{10}{40}\right) n = \cos \frac{\pi}{2} n$$
$$x_2(n) = \cos 2\pi \left(\frac{50}{40}\right) n = \cos \frac{5\pi}{2} n \qquad (1.4.13)$$

However, $\cos 5\pi n/2 = \cos(2\pi n + \pi n/2) = \cos \pi n/2$. Hence $x_2(n) = x_1(n)$.

In general, the sampling of a continuous-time sinusoidal signal

$$x_a(t) = A\cos(2\pi F_0 t + \theta)$$

with a sampling rate $F_s = 1/T$ results in a discrete-time signal

$$x(n) = A\cos(2\pi f_0 n + \theta)$$

where $f_0 = F_0/F_s$ is the relative frequency of the sinusoid.

4.Örnek 1.4.2'ye çalış, benzerini soracağım. Sf.28(45)

Example 1.4.2

Consider the analog signal

$$x_a(t) = 3\cos 100\pi t$$

(a) Determine the minimum sampling rate required to avoid aliasing.

(b) Suppose that the signal is sampled at the rate $F_s = 200$ Hz. What is the discrete-time signal obtained after sampling?

(c) Suppose that the signal is sampled at the rate $F_s = 75$ Hz. What is the discrete-time signal obtained after sampling'!

(d) What is the frequency $0 < F < F_s/2$ of a sinusoid that yields samples identical to those obtained in part (c)?

Solution

(a) The frequency of the analog signal is $F = 50$ Hz. Hence the minimum sampling rate required to avoid aliasing is $F_s = 100\ Hz$.

(b) If the signal is sampled at $F_s = 200$ Hz. the discrete-time signal is

$$x(n) = 3\cos\frac{100\pi}{200}n = 3\cos\frac{\pi}{2}n$$

(c) If the signal is sampled at $F_s = 75$ Hz. the discrete-lime signal is

$$\begin{aligned} x(n) &= 3\cos\frac{100\pi}{75}n = 3\cos\frac{4\pi}{3}n \\ &= 3\cos\left(2\pi - \frac{2\pi}{3}\right)n \\ &= 3\cos\frac{2\pi}{3}n \end{aligned}$$

(d) For the sampling rate of $F_s = 75$ Hz, we have

$$F = fF_s = 75f$$

$$F = 25\text{ Hz}$$

Clearly. the sinusoidal signal

$$\begin{aligned} y_a(t) &= 3\cos 2\pi Ft \\ &= 3\cos 50\pi t \end{aligned}$$

sampled at $F_s = 75$ samples/s yields identical samples. Hence $F = 50\ Hz$ is an alias of $F = 25$ Hz for the sampling rate $F_s = 75$ Hz.

5.Temel discrete-time sinyallerin mat. Ifadesini yazarak çiziniz. (unit sample, unite step, unit ramp, exponential sigr). Sf.45(62)

1. The *unit sample sequence* is denoted as $\delta(n)$ and is defined as

$$\delta(n) \equiv \begin{cases} 1, & \text{for } n = 0 \\ 0, & \text{for } n \neq 0 \end{cases} \tag{2.1.6}$$

In words, the unit sample sequence is a signal that is zero everywhere, except at $n = 0$ where its value is unity. This signal is sometimes referred to as a *unit impulse*. In contrast to the analog signal $\delta(t)$, which is also called a unit impulse and is defined to be zero everywhere except $t = 0$, and has unit area, the unit sample sequence is much less mathematically complicated. The graphical representation of $\delta(n)$ is shown in Fig. 2.2.

2. The *unit step signal* is denoted as $u(n)$ and is defined as

$$u(n) \equiv \begin{cases} 1, & \text{for } n \geq 0 \\ 0, & \text{for } n < 0 \end{cases} \tag{2.1.7}$$

Figure 2.3 illustrates the unit step signal.

3. The *unit ramp signal* is denoted as $u_r(n)$ and is defined as

$$u_r(n) \equiv \begin{cases} n, & \text{for } n \geq 0 \\ 0, & \text{for } n < 0 \end{cases} \tag{2.1.8}$$

This signal is illustrated in Fig. 2.4.

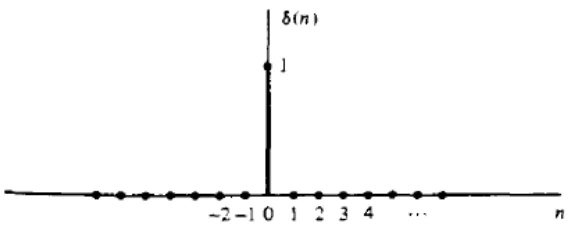

Figure 2.2 Graphical representation of the unit sample signal.

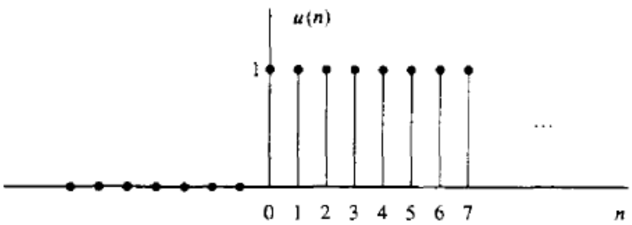

Figure 2.3 Graphical representation of the unit step signal.

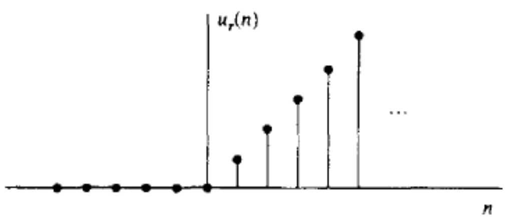

Figure 2.4 Graphical representation of the unit ramp signal.

6.Örnek 2.2.1'e çalış, benzerini soracağım. Sf.57(74)

Example 2.2.1

Determine the response of the following sytems to the input signal

$$x(n) = \begin{cases} |n|, & -3 \le n \le 3 \\ 0, & \text{otherwise} \end{cases}$$

(a) $y(n) = x(n)$

(b) $y(n) = x(n-1)$

(c) $y(n) = x(n+1)$

(d) $y(n) = \frac{1}{3}[x(n+1) + x(n) + x(n-1)]$

(e) $y(n) = max\{x(n+1), x(n), x(n-1)\}$

(f) $y(n) = \sum_{k=-\infty}^{n} x(k) = x(n) + x(n-1) + x(n-2) + \cdots$ (2.2.3)

Solution First, we determine explicitly the sample values of the input signal

$$x(n) = \{\ldots, 0, 3, 2, 1, \underset{\uparrow}{0}, 1, 2, 3, 0, \ldots\}$$

Next, we determine the output of each system using its input–output relationship.

(a) In this case the output is exactly the same as the input signal. Such a system is known as the *identity* system.

(b) This system simply delays the input by one sample. Thus its output is given by

$$x(n) = \{\ldots, 0, 3, 2, 1, \underset{\uparrow}{0}, 1, 2, 3, 0, \ldots\}$$

(c) In this case the system "advances" the input one sample into the future. For example, the value of the output at time $n = 0$ is $y(0) = x(1)$. The response of this system to the given input is

$$x(n) = \{\ldots, 0, 3, 2, 1, 0, \underset{\uparrow}{1}, 2, 3, 0, \ldots\}$$

(d) The output of this system at any time is the mean value of the present, the immediate past, and the immediate future samples. For example, the output at time $n = 0$ is

$$y(0) = \frac{1}{3}[x(-1) + x(0) + x(1)] = \frac{1}{3}[1 + 0 + 1] = \frac{2}{3}$$

Repeating this computation for every value of n, we obtain the output signal

$$y(n) = \{\ldots, 0, 1, \tfrac{5}{3}, 2, 1, \tfrac{2}{3}, 1, 2, \tfrac{5}{3}, 1, 0, \ldots\}$$

(e) This system selects as its output at time n the maximum value of the three input samples $x(n-1)$, $x(n)$, and $x(n+1)$. Thus the response of this system to the input signal $x(n)$ is

$$y(n) = \{0, 3, 3, 3, 2, \underset{\uparrow}{1}, 2, 3, 3, 3, 0, \ldots\}$$

(f) This system is basically an *accumulator* that computes the running sum of all the past input values up to present time. The response of this system to the given input is

$$y(n) = \{\ldots, 0, 3, 5, 6, \underset{\uparrow}{6}, 7, 9, 12, 0, \ldots\}$$

7.A constant multiplier …. Blok diagremlarını çiziniz. Sf.60(77)

A constant multiplier. This operation is depicted by Fig. 2.14, and simply represents applying a scale factor on the input $x(n)$. Note that this operation is also memoryless.

Figure 2.14 Graphical representation of a constant multiplier.

8.Y(n)=nx(n) time invariant/variant olup olmadığını belirle. Benzeri Örnek2.2.4 Sf.64(81)

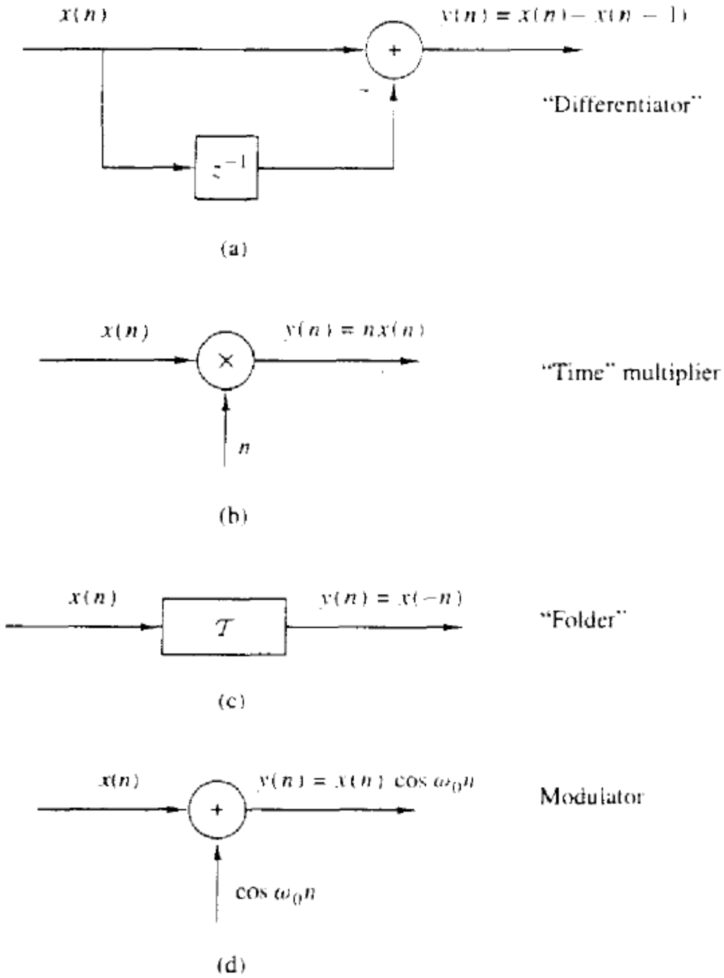

Figure 2.19 Examples of a time-invariant (a) and some time-variant systems (b)–(d).

Example 2.2.4

Determine if the systems shown in Fig. 2.19 are time invariant or time variant.

Solution

(a) This system is described by the input-output equations

$$y(n) = \mathcal{T}[x(n)] = x(n) - x(n-1) \tag{2.2.15}$$

Now if the input is delayed by k units in time and applied to the system, it is clear from the block diagram that the output will be

$$y(n,k) = x(n-k) - x(n-k-1) \tag{2.2.16}$$

On the other hand, from (2.2.14) we note that if we delay $y(n)$ by k units in time, we obtain

$$y(n-k) = x(n-k) - x(n-k-1) \tag{2.2.17}$$

Since the right-hand sides of (2.2.16) and (2.2.17) are identical, it follows that $y(n,k) = y(n-k)$. Therefore, the system is time invariant.

(b) The input-output equation for this system is

$$y(n) = \mathcal{T}[x(n)] = nx(n) \tag{2.2.18}$$

The response of this system to $x(n-k)$ is

$$y(n,k) = nx(n-k) \tag{2.2.19}$$

Now if we delay $y(n)$ in *(2.2.18)* by k units in time, we obtain

$$\begin{aligned} y(n-k) &= (n-k)x(n-k) \\ &= nx(n-k) - kx(n-k) \end{aligned} \tag{2.2.20}$$

This system is time variant, since $y(n,k) \neq y(n-k)$.

(c) This system is described by the input-output relation

$$y(n) = \mathcal{T}[x(n)] = x(-n) \tag{2.2.21}$$

The response of this system to $x(n-k)$ is

$$y(n,k) = \mathcal{T}[x(n-k)] = x(-n-k) \tag{2.2.22}$$

Now, if **we** delay the output $y(n)$, as given by (2.2.21), by k units in **time,** the **result** will be

$$y(n-k) = x(-n+k) \tag{2.2.23}$$

Since $y(n,k) \neq y(n-k)$, the system is time variant.

(d) The input-output equation for this system is

$$y(n) = x(n)\cos\omega_0 n \tag{2.2.24}$$

The response of this system to $x(n-k)$ is

$$y(n,k) = x(n-k)\cos\omega_0 n \tag{2.2.25}$$

If the expression in *(2.2.24)* is delayed by k units and the result is compared to (2.2.25), it is evident that the system is time variant.

9.Y (n) = Ax(n)+B linear-/non-linear olup olmadığını belirle. Benzeri örnek2.2.5 Sf.67(84)

Example 2.2.5

Determine if the systems described by the following input–output equations are linear or nonlinear.

(a) $y(n) = nx(n)$ **(b)** $y(n) = x(n^2)$ **(c)** $y(n) = x^2(n)$

(d) $y(n) = Ax(n) + B$ **(e)** $y(n) = e^{x(n)}$

Solution

(a) For two input sequences $x_1(n)$ and $x_2(n)$, the corresponding outputs are

$$\begin{aligned} y_1(n) &= nx_1(n) \\ y_2(n) &= nx_2(n) \end{aligned} \tag{2.2.31}$$

A linear combination of the two input sequences results in the output

$$\begin{aligned} y_3(n) &= \mathcal{T}[a_1x_1(n) + a_2x_2(n)] = n[a_1x_1(n) + a_2x_2(n)] \\ &= a_1nx_1(n) + a_2nx_2(n) \end{aligned} \tag{2.2.32}$$

On the other hand, a linear combination of the two outputs in (2.2.31) results in the output

$$a_1y_1(n) + a_2y_2(n) = a_1nx_1(n) + a_2nx_2(n) \tag{2.2.33}$$

Since the right-hand sides of (2.2.32) and (2.2.33) are identical, the system is linear.

(b) As in part (a), we find the response of the system to two separate input signals $x_1(n)$ and $x_2(n)$. The result is

$$\begin{aligned} y_1(n) &= x_1(n^2) \\ y_2(n) &= x_2(n^2) \end{aligned} \tag{2.2.34}$$

The output of the system to a linear combination of $x_1(n)$ and $x_2(n)$ is

$$y_3(n) = \mathcal{T}[a_1x_1(n) + a_2x_2(n)] = a_1x_1(n^2) + a_2x_2(n^2) \tag{2.2.35}$$

Finally, a linear combination of the two outputs in (2.2.36) yields

$$a_1y_1(n) + a_2y_2(n) = a_1x_1(n^2) + a_2x_2(n^2) \tag{2.2.36}$$

By comparing (2.2.35) with (2.2.36), we conclude that the system is linear.

(c) The output of the system is the square of the input. (Electronic devices that have such an input–output characteristic and are called square-law devices.) From our previous discussion it is clear that such a system is memoryless. We now illustrate that this system is nonlinear.

The responses of the system to two separate input signals are

$$\begin{aligned} y_1(n) &= x_1^2(n) \\ y_2(n) &= x_2^2(n) \end{aligned} \tag{2.2.37}$$

The response of the system to a linear combination of these two input signals is

$$\begin{aligned} y_3(n) &= \mathcal{T}[a_1x_1(n) + a_2x_2(n)] \\ &= [a_1x_1(n) + a_2x_2(n)]^2 \\ &= a_1^2x_1^2(n) + 2a_1a_2x_1(n)x_2(n) + a_2^2x_2^2(n) \end{aligned} \tag{2.2.38}$$

On the other hand, if the system is linear, it would produce a linear combination of the two outputs in (2.2.37), namely,

$$a_1y_1(n) + a_2y_2(n) = a_1x_1^2(n) + a_2x_2^2(n) \tag{2.2.39}$$

Since the actual output of the system, as given by (2.2.38), is not equal to (2.2.39), the system is nonlinear.

(d) Assuming that the system is excited by $x_1(n)$ and $x_2(n)$ separately, we obtain the corresponding outputs

$$\begin{aligned} y_1(n) &= Ax_1(n) + B \\ y_2(n) &= Ax_2(n) + B \end{aligned} \tag{2.2.40}$$

A linear combination of $x_1(n)$ and $x_2(n)$ produces the output

$$\begin{aligned} y_3(n) &= \mathcal{T}[a_1x_1(n) + a_2x_2(n)] \\ &= A[a_1x_1(n) + a_2x_2(n)] + B \\ &= Aa_1x_1(n) + a_2Ax_2(n) + B \end{aligned} \tag{2.2.41}$$

On the other hand, if the system were linear, its output to the linear combination of $x_1(n)$ and $x_2(n)$ would be a linear combination of $y_1(n)$ and $y_2(n)$, that is,

$$a_1y_1(n) + a_2y_2(n) = a_1Ax_1(n) + a_1B + a_2Ax_2(n) + a_2B \tag{2.2.42}$$

Clearly, (2.2.41) and (2.2.42) are different and hence the system fails to satisfy the linearity test.

The reason that this system fails to satisfy the linearity test is not that the system is nonlinear (in fact, the system is described by a linear equation) but the presence of the constant B. Consequently, the output depends on both the input excitation and on the parameter $B \neq 0$. Hence, for $B \neq 0$, the system is not relaxed. If we set $B = 0$, the system is now relaxed and the linearity test is satisfied.

(e) Note that the system described by the input–output equation

$$y(n) = e^{x(n)} \tag{2.2.43}$$

is relaxed. If $x(n) = 0$, we find that $y(n) = 1$. This is an indication that the system is nonlinear. This, in fact, is the conclusion reached when the linearity test, is applied.

10.Y(n)= X(n)-X(n-1) casual/non-casual olup olmediğini berlile. Benzeri Örnek2.2.6 Sf.69(86)

Example 2.2.6

Determine if the systems described by the following input–output equations are causal or noncausal.

(a) $y(n) = x(n) - x(n-1)$ **(b)** $y(n) = \sum_{k=-\infty}^{n} x(k)$ **(c)** $y(n) = ax(n)$

(d) $y(n) = x(n) + 3x(n+4)$ **(e)** $y(n) = x(n^2)$ **(f)** $y(n) = x(2n)$

(g) $y(n) = x(-n)$

Solution The systems described in parts (a), (b), and (c) are clearly causal, since the output depends only on the present and past inputs. On the other hand, the systems in parts (d), (e), and (f) are clearly noncausal, since the output depends on future values of the input. The system in (g) is also noncausal, as we note by selecting, for example, $n = -1$, which yields $y(-1) = x(1)$. Thus the output at $n = -1$ depends on the input at $n = 1$, which is two units of time into the future.

11.LTI (linear time-Invariant) sistemlerinin berleştirince (associative) özelliğini şekil çizerek açıklayın. Sf83(101)

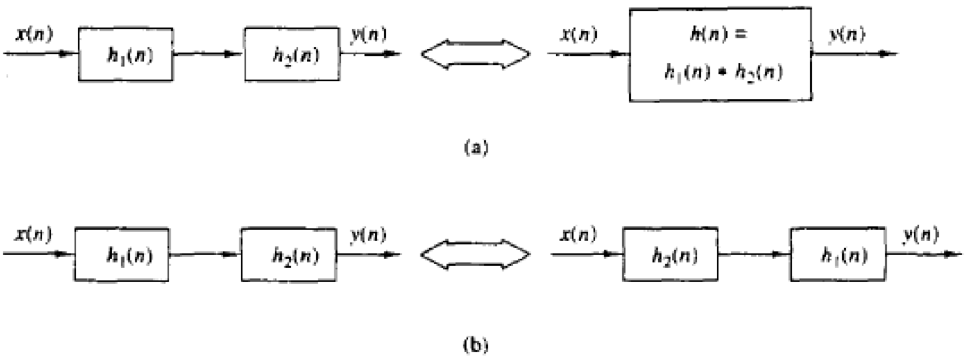

Figure 2.26 Implications of the associative (a) and the associative and commutative (b) properties of convolution.

12.Örnek 3.1.1'e (d) ve (f) çalış, benzerini soracağım. **Sf152(169)**

Example 3.1.1

Determine the z-transforms of the following *finite-duration* signals.

(a) $x_1(n) = \{1, 2, 5, 7, 0, 1\}$

(b) $x_2(n) = \{1, 2, \underset{\uparrow}{5}, 7, 0, 1\}$

(c) $x_3(n) = \{0, 0, 1, 2, 5, 7, 0, 1\}$

(d) $x_4(n) = \{2, 4, \underset{\uparrow}{5}, 7, 0, 1\}$

(e) $x_5(n) = \delta(n)$

(f) $x_6(n) = \delta(n - k), k > 0$

(g) $x_7(n) = \delta(n + k), k > 0$

Solution From definition (3.1.1), we have

(a) $X_1(z) = 1 + 2z^{-1} + 5z^{-2} + 7z^{-3} + z^{-5}$, ROC: entire z-plane except $z = 0$

(b) $X_2(z) = z^2 + 2z + 5 + 7z^{-1} + z^{-3}$, ROC: entire z-plane except $z = 0$ and $z = \infty$

(c) $X_3(z) = z^{-2} + 2z^{-3} + 5z^{-4} + 7z^{-5} + z^{-7}$, ROC: entire z-plane except $z = 0$

(d) $X_4(z) = 2z^2 + 4z + 5 + 7z^{-1} + z^{-3}$, ROC: entire z-plane except $z = 0$ and $z = \infty$

(e) $X_5(z) = 1$[i.e., $\delta(n) \overset{z}{\longleftrightarrow} 1$], ROC: entire z-plane

(f) $X_6(z) = z^{-k}$[i.e., $\delta(n - k) \overset{z}{\longleftrightarrow} z^{-k}$], $k > 0$, ROC: entire z-plane except $z = 0$

(g) $X_7(z) = z^k$[*i.e.*, $\delta(n + k) \overset{z}{\longleftrightarrow} z^k$], $k > 0$, ROC: entire z-plane except $z = \infty$

13. Örnek 3.1.2'ye çalış, benzerini soracağım. Sf153(170)

Example 3.1.2

Determine the z-transform of the signal

$$x(n) = (\tfrac{1}{2})^n u(n)$$

Solution The signal $x(n)$ consists of an infinite number of nonzero values

$$x(n) = \{1, (\tfrac{1}{2}), (\tfrac{1}{2})^2, (\tfrac{1}{2})^3, \ldots, (\tfrac{1}{2})^n, \ldots\}$$

The z-transform of $x(n)$ is the infinite power series

$$\begin{aligned} X(z) &= 1 + \tfrac{1}{2}z^{-1} + (\tfrac{1}{2})^2 z^{-2} + (\tfrac{1}{2})^n z^{-n} + \cdots \\ &= \sum_{n=0}^{\infty} (\tfrac{1}{2})^n z^{-n} = \sum_{n=0}^{\infty} (\tfrac{1}{2}z^{-1})^n \end{aligned}$$

This is an infinite geometric series. We recall that

$$1 + A + A^2 + A^3 + \cdots = \frac{1}{1-A} \qquad \text{if } |A| < 1$$

Consequently, for $|\frac{1}{2}z^{-1}| < 1$, or equivalently, for $|z| > \frac{1}{2}$, $X(z)$ converges to

$$X(z) = \frac{1}{1 - \frac{1}{2}z^{-1}} \qquad \text{ROC: } |z| > \tfrac{1}{2}$$

We see that in this case, the z-transform provides a compact alternative representation of the signal $x(n)$.

Let us express the complex variable z in polar form as

$$z = re^{j\theta} \tag{3.1.4}$$

where $r = |z|$ and $\theta = \measuredangle z$. Then $X(z)$ can be expressed as

$$X(z)|_{z=re^{j\theta}} = \sum_{n=-\infty}^{\infty} x(n) r^{-n} e^{-j\theta n}$$

In the ROC of $X(z)$, $|X(z)| < \infty$. But

$$\begin{aligned} |X(z)| &= \left| \sum_{n=-\infty}^{\infty} x(n) r^{-n} e^{-j\theta n} \right| \\ &\le \sum_{n=-\infty}^{\infty} |x(n) r^{-n} e^{-j\theta n}| = \sum_{n=-\infty}^{\infty} |x(n) r^{-n}| \end{aligned} \tag{3.1.5}$$

Hence $|X(z)|$ is finite if the sequence $x(n)r^{-n}$ is absolutely summable.

The problem of finding the ROC for $X(z)$ is equivalent to determining the range of values of r for which the sequence $x(n)r^{-n}$ is absolutely summable. To elaborate, let us express (3.1.5) as

$$\begin{aligned} |X(z)| &\le \sum_{n=-\infty}^{-1} |x(n) r^{-n}| + \sum_{n=0}^{\infty} \left| \frac{x(n)}{r^n} \right| \\ &\le \sum_{n=1}^{\infty} |x(-n) r^n| + \sum_{n=0}^{\infty} \left| \frac{x(n)}{r^n} \right| \end{aligned} \tag{3.1.6}$$