# İrrasyonel integraller

1) $\int \frac{dx}{\sqrt{ax^2+bx+c}}$ tipindeki integraller.

$\Delta < 0$ ise

$$\int \frac{dx}{\sqrt{x^2+1}} = \ln\left|x+\sqrt{x^2+1}\right|+c$$

$$\int \frac{dx}{\sqrt{x^2+4x+17}} = \int \frac{dx}{\sqrt{(\underbrace{x+2}_{u})^2+13}} = \int \frac{du}{\sqrt{u^2+13}}$$

$$= \ln\left|u+\sqrt{u^2+13}\right|+c = \ln\left|x+2+\sqrt{x^2+4x+17}\right|$$

$\Delta = 0$ ise irrasyonel olmaz burda işlemiyeceğiz

$$\int \frac{dx}{\sqrt{x^2+2x+1}} = \int \frac{dx}{\sqrt{(x+1)^2}} = \underbrace{\int \frac{dx}{|x+1|}}_{\text{aslı}} = \underbrace{\int \frac{dx}{x+1}}_{\text{kabul ettim}} = \ln|x+1|+c$$

Sınavda: Soru: $-2 < x < -1$ olmak üzere

$$\int \frac{dx}{\sqrt{x^2+2x+1}} = ?$$

$$\int \frac{dx}{-x-1} = -\ln(\underbrace{-x-1}_{\text{pozitif}})+c$$

~~a)~~ $\sqrt{x+1}+c$ ~~b)~~ $\ln(x+1)+c$ c) $-\ln(-x-1)+c$

~~d)~~ $-\ln(x+1)+c$ ~~e)~~ $-\sqrt{x+1}+c$

$\Delta > 0$ ise iki şekilde karşımıza gelir.

$$a>0 \Rightarrow \int \frac{dx}{\sqrt{x^2-p}} = \ln\left|x+\sqrt{x^2-p}\right|+c$$

$$\int \frac{dx}{\sqrt{x^2-4}} = \ln\left|x+\sqrt{x^2-4}\right|+c$$

$$\int \frac{dx}{\sqrt{\underbrace{x^2+16x+\overbrace{5}^{+64-59}}}} = \int \frac{dx}{\sqrt{(\underbrace{x+8}_{u})^2-59}} = \int \frac{du}{\sqrt{u^2-59}}$$

$$= \ln\left|u+\sqrt{u^2-59}\right|+c$$

$u = x+8$ yaz

$$a<0 \Longrightarrow \int \frac{dx}{\sqrt{m^2-x^2}} = \arcsin\left(\frac{x}{m}\right)+c$$

Ör: $\displaystyle\int \frac{dx}{\sqrt{2x-x^2}} = ?$ $\quad \displaystyle\int \frac{dx}{\sqrt{1-(\underbrace{x-1}_{u})^2}} = \int \frac{du}{\sqrt{1-u^2}} = \arcsin(u)+c$

$1-1$

$= \arcsin(x-1)+c$

a) $\ln(2x-x^2)+c$ b) $\sqrt{2x-x^2}+c$

c) $\ln\left|x-1+\sqrt{(x-1)^2+3}\right|+c$

d) $\arcsin(x-1)+c$ e) $\arcsin(x+1)+c$

Ör: $\displaystyle\int \frac{dx}{\sqrt{4x-x^2}} = \int \frac{dx}{\sqrt{4-4+4x-x^2}}$

$2.2x$ $\quad (x)^2$ $\quad 2^2$

$$= \int \frac{dx}{\sqrt{2^2-(\underbrace{x-2}_{u})^2}} = \arcsin\left(\frac{x-2}{2}\right)+c$$

$$* \int \frac{(mx+n)dx}{\sqrt{ax^2+bx+c}} \text{ integralleri}$$

$$\int \frac{2x\,dx}{\sqrt{x^2+1}} =$$

Uzun yol

$u^2 = x^2+1$

$\cancel{2}u\,du = \cancel{2}x\,dx$

$x\,dx = u\,du$

$$\int \frac{2\cancel{u}\,du}{\cancel{u}} = 2u+c$$

$$= 2\sqrt{x^2+1}+c$$

<u>Pratik Yol</u>: $\int \frac{f'(x)}{2\sqrt{f(x)}}dx = \sqrt{f(x)} + c$ den faydalan

$$2 \cdot \int \frac{\overbrace{2x}^{f'(x)}\,dx}{2\sqrt{\underbrace{x^2+1}_{f(x)}}} = 2 \cdot \sqrt{x^2+1} + c$$

$$\int \frac{x+5}{\sqrt{x^2-5}}dx = ?$$

$$\underbrace{\int \frac{\overbrace{2x}^{f'}dx}{2\sqrt{\underbrace{x^2-5}_{f}}}}_{\sqrt{x^2-5}} + \underbrace{5\int \frac{dx}{\sqrt{x^2-5}}}_{5\ln|x+\sqrt{x^2-5}|+c}$$

$$\sqrt{x^2-5} + 5\ln|x+\sqrt{x^2-5}|+c$$

<u>Ör</u>: $I = \int \frac{x+1\,dx}{\sqrt{\underbrace{x^2+4x+13}}} = ?$

$\downarrow$

türevi $2x+4$

1. hedef

$$I = \int \frac{2(x+1)dx}{2\sqrt{x^2+4x+13}} = \int \frac{2x\overbrace{+2}^{+4-2}dx}{2\sqrt{x^2+4x+13}}$$

$$= \int \frac{2x+4\,dx}{2\sqrt{x^2+4x+13}} + \int \frac{-\cancel{2}^{1}dx}{\cancel{2}_{1}\sqrt{\underbrace{x^2+4x+13}_{(x+2)^2+9}}}$$

$$= \sqrt{x^2+4x+13} - \ln|x+2+\sqrt{x^2+4x+13}|+c$$

$*\ \displaystyle\int \frac{dx}{\underbrace{(mx+n)}_{\frac{1}{t}}\sqrt{ax^2+bx+c}}$ integrallerinde $mx+n=\frac{1}{t}$ dönüşümü yapılabilinir.

$$\int \frac{dx}{x\sqrt{x^2+4}}$$

1. yol: irrasyonel mantığı ile çöz.

$$x=\frac{1}{t} \longrightarrow dx=-\frac{dt}{t^2}$$

$$\int \frac{-\frac{dt}{t^2}}{\frac{1}{t}\sqrt{\frac{1}{t^2}+4}} = \int \frac{-dt}{\sqrt{1+\underbrace{4t^2}_{(2t)^2}}} = -\frac{1}{2}\ln\left|2t+\sqrt{1+4t^2}\right|+c$$

$u=2t$

$du=2dt$

$t=\frac{1}{x}$ yaz.

Ör: $\displaystyle\int \frac{dx}{\underbrace{(x-1)}_{\frac{1}{t}}.\sqrt{x^2+2x+3}}$ irrasyonel integrali için uygun dönüşüm yapılırsa hangi int. elde edilir?

a) $\displaystyle\int \frac{dt}{\sqrt{1+4t^2}}$ b) $\displaystyle\int \frac{dt}{t\sqrt{t^2+1}}$ c) $\displaystyle\int \frac{dt}{\sqrt{7t^2+4t+1}}$

d) $\displaystyle\int \frac{-dt}{\sqrt{7t^2+4t+1}}$

e) Hiç biri

$$x-1=\frac{1}{t} \longrightarrow dx=\frac{-dt}{t^2}$$

$$x=\frac{1}{t}+1=\frac{1+t}{t}$$

$$\int \frac{-\frac{dt}{\cancel{t^2}\ t}}{\frac{1}{\cancel{t}}\sqrt{\left(\frac{1+t}{t}\right)^2+\left(\frac{1+t}{t}\right).2+\frac{3}{1}}} = \int \frac{-\frac{dt}{\cancel{t}}}{\frac{\sqrt{(t+1)^2+(1+t).2.t+3t^2}}{\cancel{t}}}$$

$t \qquad t^2$

$$= \int \frac{-dt}{\sqrt{7t^2+4t+1}}$$

$$* \int \frac{P_n(x)\,dx}{\sqrt{ax^2+bx+c}}$$ , $P_n(x)$ n.derece polinom temsil ediyor

$$\left(\int \frac{P_n(x)\,dx}{\sqrt{ax^2+bx+c}}\right)' = \left(Q_{n-1}(x).\sqrt{ax^2+bx+c} + \lambda \int \frac{dx}{\sqrt{ax^2+bx+c}}\right)'$$

oluşturulan $Q_{n-1}(x)$ polinomunun kat sayıları ve $\lambda$ değeri bulunur.

$P_2(x)$ $\quad Q_{2-1}(x) = Q_1(x)$ $\quad$ bir yol

$$\int \frac{x^2\,dx}{\sqrt{x^2+1}} = (Ax+B).\sqrt{x^2+1} + \lambda \int \frac{dx}{\sqrt{x^2+1}}$$

Her iki tarafın türevini alıyoruz.

$$\frac{x^2}{\sqrt{x^2+1}} = \frac{A.\sqrt{x^2+1}}{1} + \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+1}}.(Ax+B) + \frac{\lambda}{\sqrt{x^2+1}}$$

$(\sqrt{x^2+1})$

$$x^2 = A(x^2+1) + x(Ax+B) + \lambda$$

$$x^2 = \underbrace{(A+A)}_{2A}x^2 + \underbrace{B}_{0}x + \underbrace{\lambda + A}_{0}$$

$A = 1/2$

$$\lambda = -\frac{1}{2}$$

$$\int \frac{x^2\,dx}{\sqrt{x^2+1}} = \frac{1}{2}x.\sqrt{x^2+1} - \frac{1}{2}\underbrace{\int \frac{dx}{\sqrt{x^2+1}}}_{\ln|x+\sqrt{x^2+1}|+c}$$

Ör: $\int \frac{(x^2-4)\,dx}{\sqrt{x^2+1}}$ integrali irrasyonel int yöntemi ile çözülürse

1) $\lambda = ?$    2) $Q_n(x)$ için $Q_n(1) = ?$

$$\int \frac{\overbrace{(x^2-4)}^{2.\ derece}\,dx}{\sqrt{x^2+1}} = \overbrace{(Ax+B)}^{1.\ derece}\sqrt{x^2+1} + \lambda\int\frac{dx}{\sqrt{x^2+1}}$$

Türev Al

$$\frac{x^2-4}{\sqrt{x^2+1}} = \frac{A\sqrt{x^2+1}}{\frac{1}{\sqrt{x^2+1}}} + \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+1}}(Ax+B) + \frac{\lambda}{\sqrt{x^2+1}}$$

$$\underbrace{1x^2 - 4 = 2A}_{1/2 = A}x^2 + \underset{0}{Bx} + \underbrace{\lambda + A}_{-4}$$

$\lambda = -4 - \frac{1}{2} = -\frac{9}{2}$

$Q_n(x) = \frac{x}{2}$

$Q_n(1) = \frac{1}{2}$