Ör: $\displaystyle\int \frac{dx}{x+\sqrt{x}} = ?$

a) $\arcsin(\sqrt{x})+c$ $\arctan(\sqrt{x})+c$ c) $\ln(\sqrt{x})+c$

d) $\ln(1+\sqrt{x})+c$ e) $2\ln(1+\sqrt{x})+c$

$x=u^2 \longrightarrow dx = 2u\,du$

$$\int \frac{2u\,du}{u^2+u} = \int \frac{2du}{u+1} = 2\ln(1+u)+c = 2\ln(1+\sqrt{x})+c$$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\left(\frac{u}{a}\right)+c$$

Ör: $\displaystyle\int \frac{x\,dx}{\sqrt{9-x^4}} = ?$

a) $\arcsin(x^2)+c$ c) $\arcsin\left(\frac{x^2}{3}\right)+c$ e) $\frac{1}{2}\arcsin\left(\frac{x^2}{3}\right)+c$

b) $\arcsin\left(\frac{x^2}{2}\right)+c$ d) $\frac{1}{6}\arcsin\left(\frac{x^2}{3}\right)+c$

tuzaklı

$x^2=u$

$2x\,dx = du$

$x\,dx = \frac{du}{2}$

$$\int \frac{\frac{du}{2}}{\sqrt{9-u^2}} = \frac{1}{2}\arcsin\left(\frac{u}{3}\right)+c$$

$$\int \frac{x\,dx}{\sqrt{x^4-9}} = \frac{1}{2}\ln\left(x^2+\sqrt{x^4-9}\right)+c \qquad \int \frac{du}{\sqrt{u^2-a^2}} = \ln\left(u+\sqrt{u^2-a^2}\right)+c$$

# RASYONEL İNTEGRALLER

* $\displaystyle\int \frac{m\,dx}{ax+b} = \frac{m}{a}\ln|ax+b|+c$

$$\frac{1}{2}\int \frac{2\,dx}{2x+1} = \frac{1}{2}\ln|2x+1|+c$$

* $\int \frac{m\,dx}{ax^2+bx+c}$ integralleri

$\Delta > 0$, $\Delta = 0$, $\Delta < 0$ dan dolayı 3 şekilde

$\Delta < 0$ ise $\int \frac{du}{a^2+(u)^2} = \frac{1}{a}\arctan\left(\frac{u}{a}\right)+c$ tipine dönüşür.

$$\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(\underbrace{x+1}_{u})^2+1} = \int \frac{du}{u^2+1} = \arctan(u)+c$$

$$= \arctan(x+1)+c$$

$\underbrace{x^2+2x+1}_{(x+1)^2}+1$

$u = x+1$

$du = dx$

$$\int \frac{dx}{x^2+2x+5} = \int \frac{dx}{(\underbrace{x+1}_{u})^2+4} = \int \frac{du}{u^2+2^2} = \frac{1}{2}\arctan\left(\frac{x+1}{2}\right)+c$$

$$\int \frac{dx}{9x^2+6x+2} = \int \frac{dx}{(\underbrace{3x+1}_{u})^2+1} = \frac{1}{3}\arctan(3x+1)+c$$

$9x^2 = (3x)^2$

$u = 3x+1$

$du = 3dx$

$\frac{du}{3} = dx$

* $\Delta = 0$ ise $\int \frac{du}{u^2} = -\frac{1}{u}+c$ şeklindedir.

$$\int \frac{6\,dx}{4x^2-12x+9} = \int \frac{6\,dx}{(2x-3)^2} = 6 \cdot \frac{1}{2}\left(-\frac{1}{2x-3}\right)+c = -\frac{3}{2x-3}+c$$

$u = 2x-3$

$du = 2dx$

$\Delta > 0$ ise BASİT KESİRLERE AYIRMA METODU kullanılır.

$$\int \frac{dx}{\underset{(x-4)(x+1)}{x^2-3x-4}} =$$

1 dereceküçüğü 0. derece sabit olur.

$$\frac{1}{(x-4)(x+1)} = \frac{\overset{\text{sabit}}{A}}{\underset{\text{1.derece}}{x^1-4}} + \frac{\overset{\text{sabit}}{B}}{\underset{\text{1.dere}}{x+1}}$$

$(x+1)$ $\quad$ $(x-4)$

$$1 = (x+1).A + (x-4).B$$

$x=-1 \Rightarrow 1=-5.B \Rightarrow B=-\frac{1}{5}$

$x=4 \Rightarrow 1=5.A \Rightarrow A=\frac{1}{5}$

int. yerine yaz

$$\int \frac{dx}{\underbrace{(x-4)(x+1)}_{\underbrace{4 \quad -1}_{5}}} = \int \frac{1/5}{x-4}dx + \int \frac{-1/5}{x+1}dx$$

$$= \frac{1}{5}\ln|x-4| - \frac{1}{5}\ln|x+1| + c$$

$$= \frac{1}{5}\ln\left|\frac{\overset{\text{küçük}}{x-4}}{x+1}\right| + c$$

$$\int \frac{dx}{\underset{\underbrace{\underbrace{(x-3)}_{3}\underbrace{(x+3)}_{-3}}_{6}}{1x^2-9}} = \frac{1}{6}\ln\left|\frac{\overset{\text{küçük}}{x-3}}{\underset{\text{büyük}}{x+3}}\right| + c \qquad \frac{1}{(x-3)(x+3)} = \frac{A}{x-3} + \frac{B}{x+3}$$

$$1 = (x+3)A + (x-3).B$$

$x=3 \rightarrow A = 1/6$

$x=-3 \rightarrow B=-1/6$

$$\int \frac{dx}{\underset{\substack{-8 \\ +1}}{x^2-7x-8}} = \frac{1}{9}\ln\left|\frac{x-8}{x+1}\right| + c$$

Ör: $\int \frac{dx}{x^3+x} = ?$

$x^2+1=u$

$$\frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

$(x^2+1)$     $x$

$$\int \frac{dx}{x^3+x} = \int \frac{dx}{x} + \frac{1}{2}\int \frac{-2x\,dx}{x^2+1}$$

$$= \ln|x| - \frac{1}{2}\ln|x^2+1| + c$$

$$1 = Ax^2 + A + Bx^2 + Cx$$

$$1 = \underbrace{(A+B)}_{0}x^2 + \underbrace{C}_{0}x + A$$

$A=1$

$C=0$

$B=-1$

$$\int \frac{dx}{x^3-4x} = ?$$

$$\frac{1}{x(x-2)(x+2)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+2}$$

$$1 = (x-2)(x+2)A + x(x+2)B + x(x-2)C$$

$x=2 \rightarrow 1 = 8B \rightarrow B = 1/8$

$x=-2 \rightarrow 1 = 8C \rightarrow C = 1/8$

$x=0 \rightarrow 1 = -4A \rightarrow A = -1/4$

$$\int \frac{-1/4}{x}dx + \int \frac{1/8}{x-2}dx + \int \frac{1/8\,dx}{x+2}$$

$$= -\frac{1}{4}\ln x + \frac{1}{8}\ln|x^2-4| + c$$

$*$ $\int \frac{mx+n\;dx}{ax^2+bx+c}$ integralleri

$\Delta=0$ payda

$$\int \frac{x+5\,dx}{\underbrace{x^2+2x+1}_{(x+1)^2}} = \int \frac{(x+1)+4}{(x+1)^2}dx = \int \frac{dx}{x+1} + \int \frac{4}{(x+1)^2}du$$

$$= \ln|x+1| + 4\cdot\left(-\frac{1}{x+1}\right) + c$$

2.yol $\displaystyle\int \frac{x+5\,dx}{(x+1)^2} = \int \frac{(u+4)\,du}{u^2} = \int \frac{du}{u} + 4\int \frac{du}{u^2} = \ln u - \frac{4}{u} + c$

$x+1=u$

$dx=du$

$u=x+1$ ✓

$\Delta>0$ ise direk basit kesir

$$\int \frac{3x+7\,dx}{x^2-4} = \frac{3}{2}\int \frac{2x\,dx}{x^2-4} + 7\int \frac{dx}{x^2-4}$$

$$= \frac{3}{2}\ln|x^2-4| + \frac{7}{4}\underbrace{\ln\left|\frac{x-2}{x+2}\right|}_{\ln|x-2|-\ln|x+2|} + c$$

$$\int \frac{3x+7}{x^2-4}dx$$

$$\frac{3x+7}{(x-2)(x+2)} = \frac{A}{x-2} + \frac{B}{x+2}$$

$$3x+7=(x+2)A+(x-2)B$$

$$x=2 \to 13=4A \Rightarrow A=13/4$$

$$x=-2 \to 1=-4B \Rightarrow B=-1/4$$

$$\int \frac{3x+7}{x^2-4}dx = \frac{13}{4}\ln|x-2| - \frac{1}{4}\ln|x+2| + c$$

$\Delta<0$ ise

$$\int \frac{x+5}{x^2+2x+5}dx = ?$$

1.yol $\displaystyle\int \frac{x+5\,dx}{(x+1)^2+4} = \int \frac{u+4}{u^2+4}du$

$x+1=u$

$dx=du$

$$= \frac{1}{2}\int \frac{2u\,du}{u^2+4} + 4\int \frac{du}{u^2+4}$$

$$= \frac{1}{2}\ln|u^2+4| + 4\cdot\frac{1}{2}\arctan\left(\frac{u}{2}\right) + c$$

$$* \int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\left(\frac{u}{a}\right) + c$$

$$\frac{1}{2}\int\frac{2(x+5)\,dx}{x^2+2x+5}=\frac{1}{2}\int\frac{\overset{2x+2+8}{2x+10}\,dx}{x^2+2x+5}$$

türevi $2x+2$

$$=\frac{1}{2}\left[\underbrace{\int\frac{2x+2\,dx}{x^2+2x+5}}+8\int\frac{dx}{(x+1)^2+4}\right]$$

$$=\frac{1}{2}\ln|x^2+2x+5|+\frac{4}{2}\arctan\left(\frac{x+1}{2}\right)+c$$

Not: der [Pay] $\geq$ der [Payda] önce polinom bölmesi yap sonra düzenle

$$\int\frac{x^3\,dx}{x^2+1}=\int\left(x^1+\frac{-x}{x^2+1}\right)dx$$

$$\begin{array}{r|l} x^3 & x^2+1 \\ \cline{2-2} -/\; x^3+x & x \\ \hline -x & \end{array}$$

$$=\frac{x^2}{2}-\frac{1}{2}\ln|x^2+1|+c$$

$$\int\frac{x^4\,dx}{x^2+1}=?$$

$$\int\left(x^2-1+\frac{1}{x^2+1}\right)dx=\frac{x^3}{3}-x+\arctan(x)+c$$

$$\begin{array}{r|l} x^4 & x^2+1 \\ \cline{2-2} -/\; x^4+x^2 & x^2-1 \\ \hline -x^2 & \\ -/\; -x^2-1 & \\ \hline +1 & \end{array}$$