KUTUPSAL KOORDİNATLAR

A(x, y) kartezyen koordinatlarda verilmiş olsun.

Kutupsal koordinatlardaki karşılığı (r, θ) olmak üzere

$$r = \sqrt{x^2 + y^2} \quad \text{ve} \quad \theta = \arctan\left(\frac{y}{x}\right)$$

yani

$$\tan\theta = \frac{y}{x} \quad \text{tir.}$$

Kartezyen (x, y)	Kutupsal (r, θ)

$(1, 1)$ $\longrightarrow$

$$r = \sqrt{1^2 + 1^2} = \sqrt{2}$$
$$\tan\theta = \frac{1}{1} \Rightarrow \frac{\pi}{4}$$

$\left(\sqrt{2}, \frac{\pi}{4}\right)$
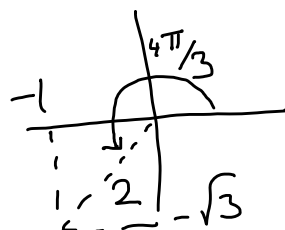
2.yol:

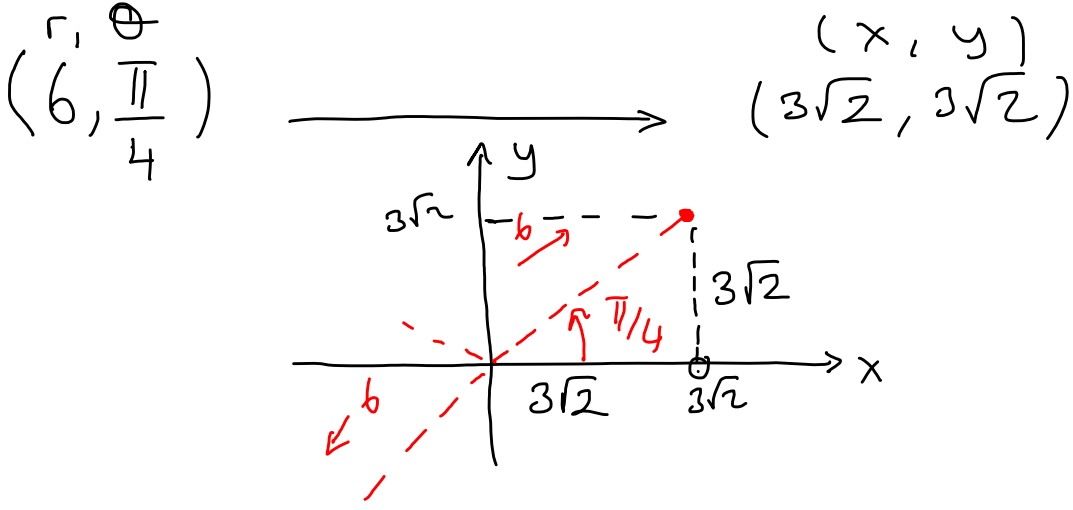
$(-1, -\sqrt{3})$ $\longrightarrow$

4. bölge

$$r = \sqrt{1 + 3} = 2$$

$$\tan\theta = \frac{-\sqrt{3}}{-1} = \sqrt{3} \Rightarrow \theta = \frac{\pi}{3}$$

$\left(2, \frac{4\pi}{3}\right)$

$$\pi + \frac{\pi}{3} = \frac{4\pi}{3}$$

$$\left(6, \frac{\pi}{4}\right) \quad \overset{r, \theta}{} \longrightarrow \quad \overset{(x, y)}{(3\sqrt{2}, 3\sqrt{2})}$$

(r, θ) verilirse $\quad x = r\cos\theta \, , \, y = r\sin\theta$ olarak

alınır.

$$\left(4, \frac{\pi}{6}\right) \longrightarrow \begin{array}{l} x = 4\cos\dfrac{\pi}{6} = 2\sqrt{3} \\[2mm] y = 4\sin\dfrac{\pi}{6} = 2 \end{array} \quad (2\sqrt{3}, 2)$$
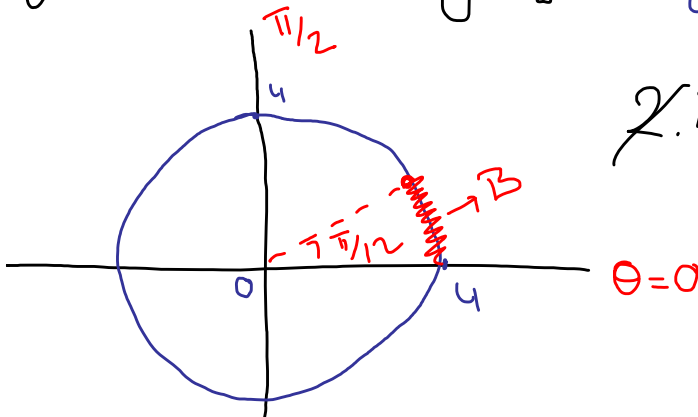
$\underline{\text{Ör:}}$ $y = \dfrac{x}{\sqrt{3}}$ eğrisinin kutupsal koordinattaki

karşılığı? $\qquad r\sin\theta = \dfrac{r\cos\theta}{\sqrt{3}} \implies \tan\theta = \dfrac{1}{\sqrt{3}} \implies \theta = \dfrac{\pi}{6}$

a) $r = \dfrac{\pi}{3}$ b) $\theta = \dfrac{\pi}{3}$ c) $\theta = \dfrac{\pi}{6}$ d) $r = \cos\theta$ e) Hiçbiri

$\underline{\text{Ör:}}$ $B = \left\{ (r, \theta) : r = 4, \ 0 \le \theta \le \dfrac{\pi}{12} \right\}$ ise

→ x ekseni ile yaptığı açı

B eğrisinin uzunluğu? → orijin uzaklığı

$$2 \cdot 4 \cdot \pi \cdot \dfrac{\frac{\pi}{12}}{2\pi} = \frac{\pi}{3}$$

$$B = \left\{ (r, \theta) : \underline{0 \leq r \leq 4}, \ 0 \leq \theta \leq \frac{\pi}{3} \right\}$$

ise

1) B bölgesinin alanı
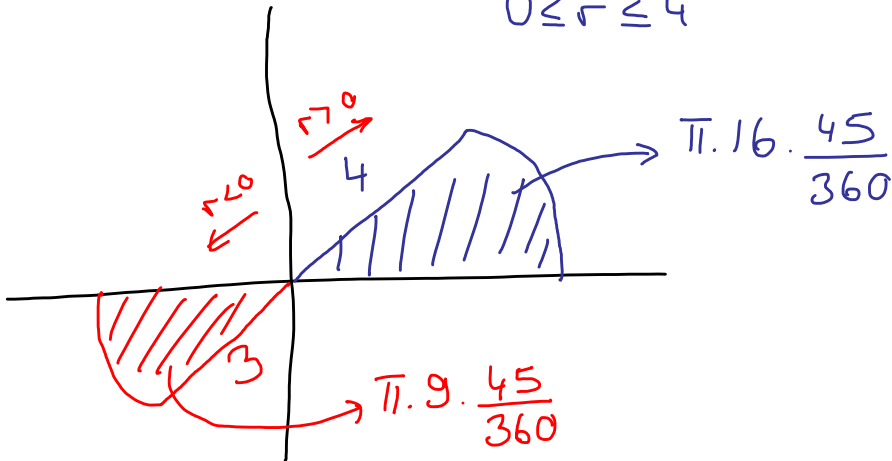
$$\pi \cdot 4^2 \cdot \frac{60}{360} = \frac{8\pi}{3}$$
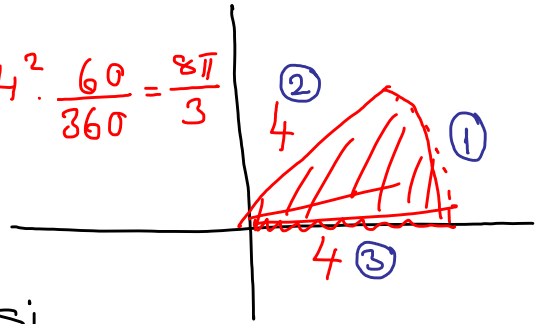
2) B bölgesinin çevresi

$$① + ② + ③ = \frac{\pi}{3} + 4 + 4$$

$$B = \left\{ (r, \theta) : \bigcirc -3 \leq r \leq 4, \ 0 \leq \theta \leq \frac{\pi}{4} \right\}$$

$$0 \leq r \leq 4$$

$$\pi \cdot 16 \cdot \frac{45}{360}$$

$$\pi \cdot 9 \cdot \frac{45}{360}$$

$r \quad \theta \qquad\qquad (x, y)$

$$\left(\bigcirc 4, \frac{\pi}{4} \right) \longrightarrow$$

zıt yön

$(-2\sqrt{2}, -2\sqrt{2})$

$$x = -4 \cdot \cos\theta = -2\sqrt{2}$$

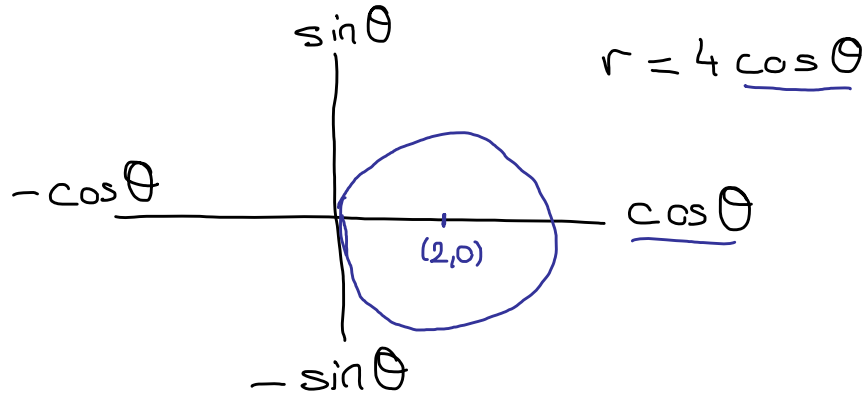$$y = -4 \cdot \sin\frac{\pi}{4} = -2\sqrt{2}$$

$$\theta = \pi/4$$
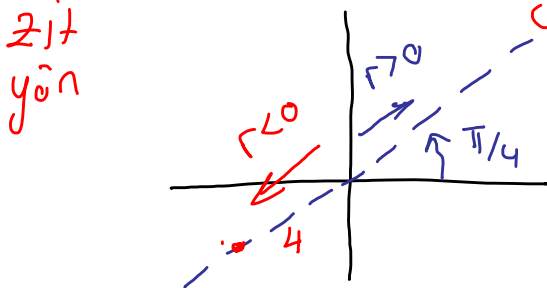
1/ $r = 4\cos\theta$ eğrisinin uzunluğu?

$$r^2 = 4\underbrace{r\cos\theta}$$

$$x^2 + y^2 = 4\underbrace{X}$$

$$x^2 - 4x + 4 \quad + y^2 = 0 + 4$$

$$\underbrace{(x-2)^2} + y^2 = \boxed{2}^2$$

Merkezi $(2,0)$ y.ç. $= 2$

olan çember

Çevresi $2 \cdot \pi \cdot 2 = 4\pi$

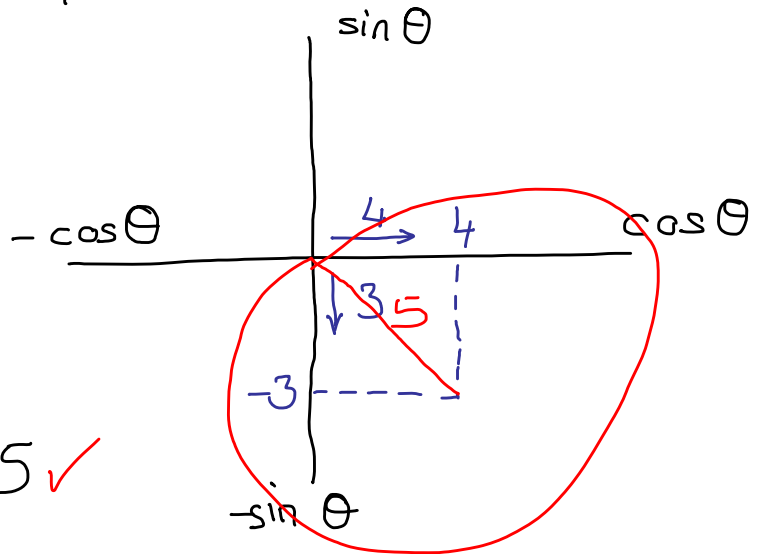| $x = r\cos\theta$ |
| $y = r\sin\theta$ |
| $r = \sqrt{x^2 + y^2}$ |
| $\tan\theta = \dfrac{y}{x}$ |

$r = 4\cos\theta$

$r = 8\cos\theta \ominus 6\sin\theta$ çemberinin merkezi ve yarı çapı?
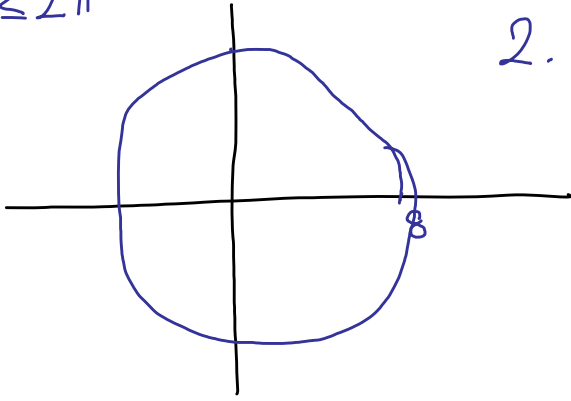
a) $(8, -6)$ y.ç $= 3$

b) $(4, -6)$ y.ç $= 6$

c) $(4, -3)$ ȳ.ç $= 5$ ✓

d) Hiçbiri

$r = 8$ eğrisinin uzunluğu ?

$0 \leq \theta \leq 2\pi$

$2 \cdot 8 \cdot \pi = 16\pi$

Kardiyoidler

1)
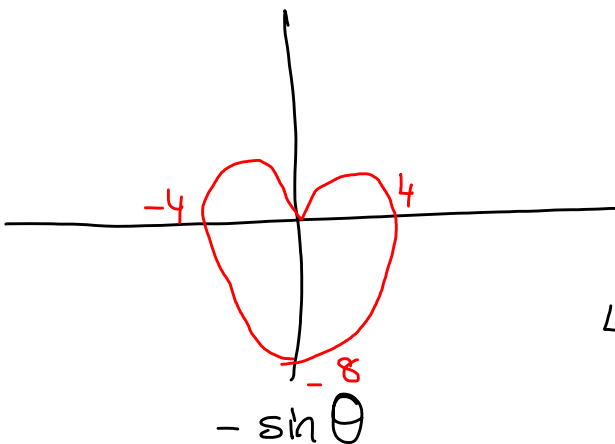
$\theta = \pi/2$

$\sin\theta$

$r = 2(1 + \cos\theta)$

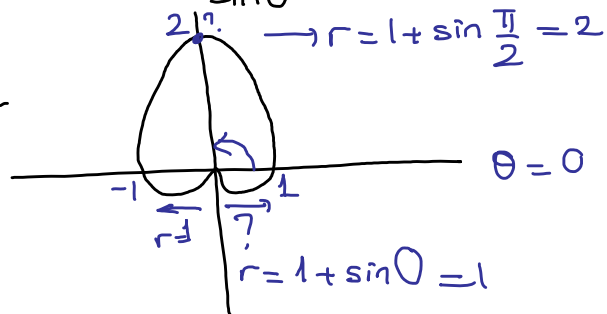$-\cos\theta$

$\cos\theta, \quad \theta = 0$

$-\sin\theta$

2)

$r = 4(1 - \sin\theta)$

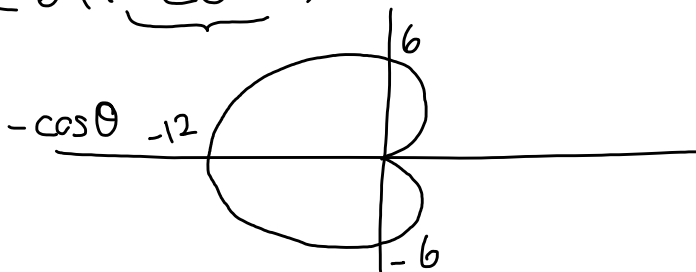$-\sin\theta$

4) $r = 1 + \sin\theta$

$\sin\theta$

$\rightarrow r = 1 + \sin\frac{\pi}{2} = 2$

$\theta = 0$

$r = 1 + \sin 0 = 1$

3) $r = 6(1 - \cos\theta)$

$-\cos\theta$

Çemberler ve Kardiyoidler

Genel eğriler

$$y = \frac{x}{\sqrt{3}} \implies \theta = \frac{\pi}{3} \quad \text{gibi}$$