$F'(x)=f(x)$ olsun. $\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b)-F(a)$

Vize 25 soru 60' } test 5 şık
Final 40 soru 60'

$$\int_1^2 3x^2\,dx = x^3\Big|_1^2 = 2^3-1^3=7$$

$$\int_1^2 x e^x\,dx = xe^x - e^x\Big|_1^2$$

T	İ
$+\ x$	e^x
$-\ 1$	e^x
0	e^x

$$= 2e^2 - e^2 - (e-e) = e^2$$

$$\int_2^3 \frac{dx}{\underbrace{x^2+2x+1}_{(x+1)^2}} = -\frac{1}{x+1}\Big|_2^3 = -\frac{1}{4}+\frac{1}{3} = \frac{1}{12} ✓$$

$$\int_0^2 \frac{dx}{x^2+4} = \frac{1}{2}\arctan\left(\frac{x}{2}\right)\Big|_0^2 = \frac{1}{2}\cdot\frac{\pi}{4} = \frac{\pi}{8}$$

$$* \left(\int_a^b f(x)\,dx\right)' = 0$$

Leibniz Formülü

$$F(x) = \int_{u(x)}^{v(x)} f(t)\,dt \Rightarrow F'(x) = f(v(x)).v'(x) - f'(u(x)).u'(x)$$

Ör: $F(x) = \int_{x}^{x^2} \frac{t\,dt}{1+t^5}$ fonksiyonuna $x=1$ den çizilen teğetin eğimi ?

$\underbrace{\text{teğetin eğimi}}_{F'(1)=?}$

türevi

$$F'(x) = \frac{x^2}{1+x^{10}} \cdot 2x - \frac{x}{1+x^5} \cdot 1$$

türevi

$$F'(1) = \frac{2}{2} - \frac{1}{2} = \frac{1}{2}$$

* $\int_{a}^{a} f(x)\,dx = 0$

* $\int_{a}^{b} f(x)\,dx = -\int_{b}^{a} f(x)\,dx$

Ör: $\underbrace{\int_{0}^{\pi/4} \cos^2 x\,dx + \int_{0}^{\pi/4} \sin^2 x\,dx}_{\text{sınırlar aynı}} = \int_{0}^{\pi/4} 1\,dx = \frac{\pi}{4}$ ✓

Ör: $\int_{0}^{\pi/4} \cos^2 x\,dx + \int_{\pi/4}^{0} \sin^2 x\,dx = \int_{0}^{\pi/4} \cos^2 x\,dx - \int_{0}^{\pi/4} \sin^2 x\,dx$

yer değişir ise

aynı olacak

$$= \int_{0}^{\pi/4} \underbrace{(\cos^2 x - \sin^2 x)}_{\cos 2x}\,dx$$

$$= \left. \frac{\sin 2x}{2} \right|_{0}^{\pi/4}$$

$$= \frac{1}{2} \checkmark$$

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

$$\cancel{F(c)} - F(a) + F(b) - \cancel{F(c)}$$

$$\int_1^5 2x\,dx = x^2 \Big|_1^5 = 24$$

$$\int_1^3 2x\,dx + \int_3^5 2x\,dx = x^2\Big|_1^3 + x^2\Big|_3^5$$

$$= 8 + 16 = 24 \checkmark$$

Parçalı Fonksiyonların İntegralleri

$$f(x) = \begin{cases} 2x+1 & , \; x < 1 \\ \underbrace{3x^2}_{\text{alınacak fonk.}} & , \; \underbrace{x > 1}_{\text{yan şart}} \end{cases}$$

1) $$\int_{-1}^{0} \underbrace{f(x)}_{2x+1}\,dx = x^2 + x \Big|_{-1}^{0} = 0 - (1-1) = 0$$

$-1 < x < 0$

2) $$\int_1^3 \underbrace{f(x)}_{3x^2}\,dx = x^3 \Big|_1^3 = 26$$

$1 < x < 3$

3) $$\int_{-1}^{3} f(x)\,dx = \int_{-1}^{1} 2x+1\,dx + \int_1^3 3x^2\,dx$$

$$= x^2 + x\Big|_{-1}^{1} + x^3\Big|_1^3 = 2 + 26 = 28$$

Signum Fonksiyonunun İntegrali

$$\int_3^8 \operatorname{sgn}(\underbrace{x-5}) \, dx = \underbrace{\int_3^5 \underbrace{\operatorname{sgn}(x-5)}_{-1} dx}_{-2} + \underbrace{\int_5^8 \underbrace{\operatorname{sgn}(x-5)}_{1} dx}_{3} = 1$$

içini 0 yapan x'ler kritik nokta

$$\int_3^8 x^{\operatorname{sgn}(x-5)} dx = \int_3^5 x^{-1} dx + \int_5^8 x^{1} dx$$

$$\ln x \Big|_3^5 + \frac{x^2}{2}\Big|_5^8$$

$\Delta < 0$ ve $a = 1 > 0$

$$\int_1^5 \frac{x^{\operatorname{sgn}(x^2-x+1)} dx}{\operatorname{sgn}(\underbrace{-1+x-x^2})} = ?$$

$\Delta < 0$ ve $a = -1 < 0$

$$\int_1^5 \frac{x^1}{-1} dx = -\frac{x^2}{2}\Big|_1^5 = -12$$

Tam Değer Fonksiyonunun int.

$a \in \mathbb{Z}$

$\forall \; a \leq x < a+1$

$\Rightarrow \; [\![x]\!] = a$

$$\int_3^4 \underbrace{[\![x]\!]}_{3} dx = \int_3^4 3 \, dx$$

$3 \leq x < 4$

$$= 3x \Big|_3^4 = 3 \checkmark$$

$$[\![1{,}2{,}8]\!] = 2$$

$-1 \quad 0 \quad 1 \quad 2 \cdot 2{,}8$ den büyük olmayan en büyük tamsayı

$$\int_2^3 \underbrace{[\![x]\!]}_{2} dx = 2(3-2) = 2$$

$$\int_2^4 [\![x]\!] dx = \int_2^3 2\,dx + \int_3^4 3\,dx = 5 ✓$$

Ör: $\int_0^n [\![x]\!] dx = 45$ ise $n \in \mathbb{N} = ?$

$$\int_0^1 0\,dx + \int_1^2 1\,dx + \dots + \int_{n-1}^n (n-1)\,dx = 45$$

$$\frac{(n-1).n}{2} = 45 \qquad 9 . 10$$

a) 8 b) 9 c) 10 d) 11 e) Hiçbiri

$$\int_1^7 [\![x]\!] dx = 1+2+\dots+6 = \frac{6.7}{2} = 21$$

$$\int_1^3 [\![x]\!]^{sgn(x-2)} dx = \int_1^2 1^{-1} dx + \int_2^3 2^1 dx = 1+2 = 3 ✓$$

Mutlak Değer Fonksiyonunun integrali

$$\int_1^5 |2x+1|\, dx = \int_1^5 2x+1\, dx = x^2+x \Big|_1^5 = 28$$

$1 < x < 5$ her x için $+$

$x = -\frac{1}{2}$ kritik nokta değil

$$\int_1^5 |\underbrace{2x-4}_{=0}|\,dx = \int_1^2 -2x+4\,dx + \int_2^5 (2x-4)\,dx$$

$$x = 2$$

x	$1 \longrightarrow 2$	$2 \longrightarrow 5$
	$-2x+4$	$2x-4$

$$\int_{-1}^{8} |\underbrace{x^2-4x}_{=0}|\,dx = \int_{-1}^{0} x^2-4x\,dx + \int_0^4 -x^2+4x\,dx + \int_4^8 x^2-4x\,dx$$

$$x = 0 \qquad x = 4$$

$$* \quad \int_a^b d(f(x)) = f(x)\Big|_a^b = f(b) - f(a)$$

$$\int_0^{\pi/4} d(\sin x) = \sin x \Big|_0^{\pi/4} = \frac{1}{\sqrt{2}}$$

$$\int_0^{\pi/4} \frac{d(\sin x)}{\cos x} = \int_0^{\pi/4} \frac{\cancel{\cos x}\,dx}{\cancel{\cos x}} = \frac{\pi}{4}$$