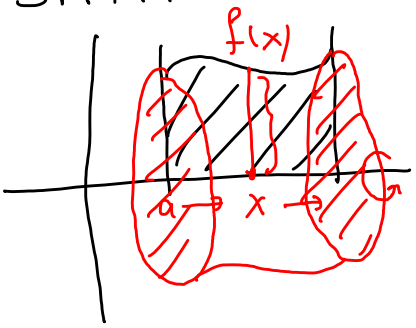
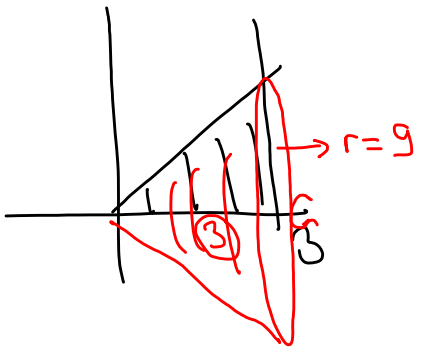
Dönel Yüzey Hacmi

$y=f(x)$ eğrisi $x=a, x=b$ doğruları ve x ekseni ile sınırlı bölgenin x ekseni etrafında döndürülmesi ile meydana gelen dönel cismin hacmi



$$\pi \int_a^b (f(x))^2 \, dx$$

Ör: $y=3x$ doğrusu, $x=3$ doğrusu ve x ekseni ile sınırlı bölgenin x ekseni e.d.m.g.d.c.h=?

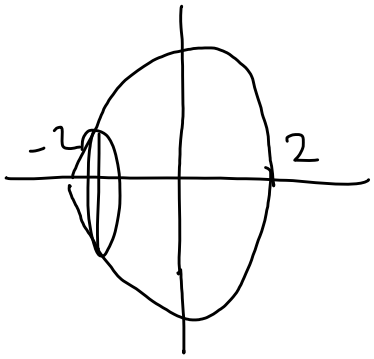


$$\pi \int_0^3 (3x)^2 \, dx = 9\pi \left.\frac{x^3}{3}\right|_0^3 = 81\pi$$

Koni Hacmi

$$\frac{\pi}{3} r^2 h = \frac{\pi}{3} 81.3 = 81\pi$$

Ör: $y=4-x^2$ eğrisinin x ekseni e.d.m.g.d.c.h=?

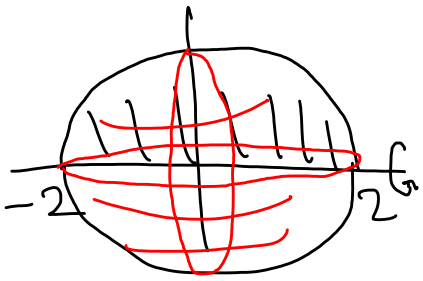


$$2.\pi \int_0^2 (4-x^2)^2 \, dx$$

$$2\pi \int_0^2 (16-8x^2+x^4) \, dx$$

$$2\pi \left(16x - 8\frac{x^3}{3} + \frac{x^5}{5}\Big|_0^2\right)$$

$y=\sqrt{4-x^2}$ eğrisi ve x ekseni ile sınırlı bölgenin x ekseni e.d.m.g.d.c.h=?

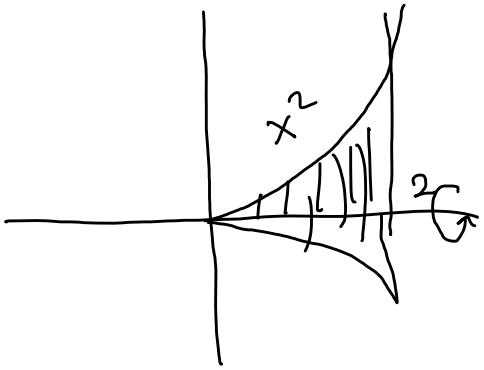


Küre $\frac{4}{3}\pi r^3 = \frac{4\pi}{3}8 = \frac{32\pi}{3}$

$$\pi\int_{-2}^{2}(4-x^2)\,dx = \pi\left(4x-\frac{x^3}{3}\Big|_{-2}^{2}\right)$$

$$=\frac{32\pi}{3}$$

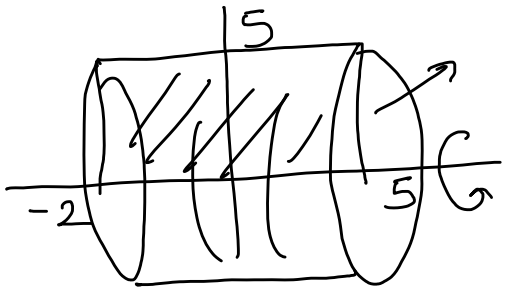
Ör: $y=x^2$ parabolü, $x=2$ doğrusu ve x ekseni ile sınırlı bölgenin xekseni e.d.m.g.d.c.h=?



$$\pi\int_{0}^{2}(x^2)^2\,dx=\frac{32\pi}{5}$$

Ör: $y=5$ doğrusu, $x=-2$, $x=5$ doğruları ve x ekseni ile sınırlı bölgenin x ekseni e.d.m.g.d.c. nedir?

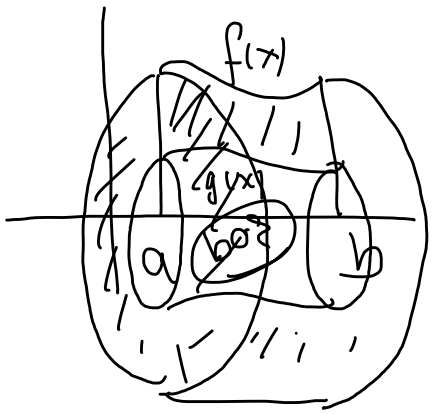
hacmi?



Silindir Hacmi

$\pi r^2 h = \pi 5^2 . 7$

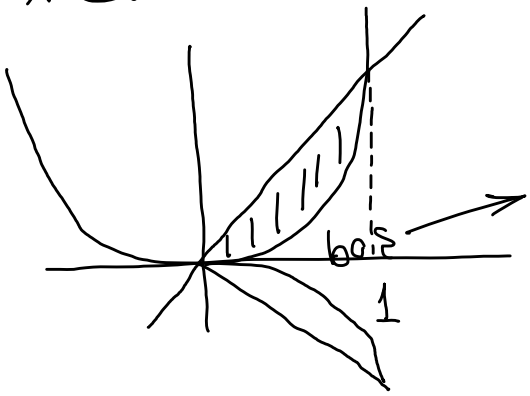
$= 175\pi$



Dış − İç

$$\pi \int_a^b (f(x))^2 dx - \pi \int_a^b (g(x))^2 dx$$

$y = x$ eğrisi, $y = x^2$ parabolü ile sınırlı bölgenin x ekseni e.d.m.g.d.c. h = ? Örnek!

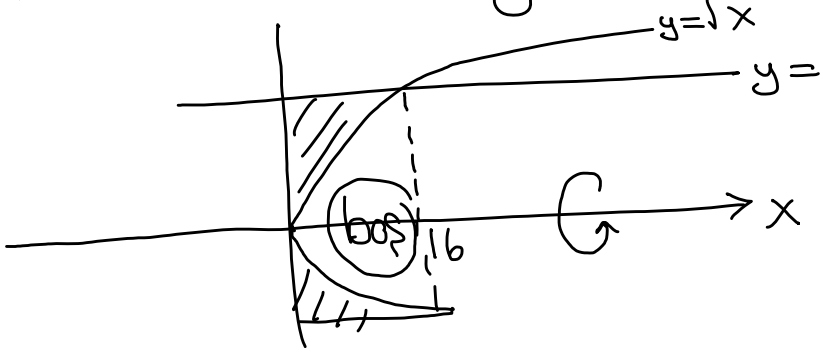


dış − iç

(koni) $\pi \int_0^1 (x^2)^2 dx$

$\frac{\pi}{3} - \frac{\pi}{5} = \frac{2\pi}{15}$

Ör: $y = 4$ doğrusu, $y = \sqrt{x}$ eğrisi ve y ekseni ile sınırlı bölgenin x ekseri e.d.m.g.d.c. h=?



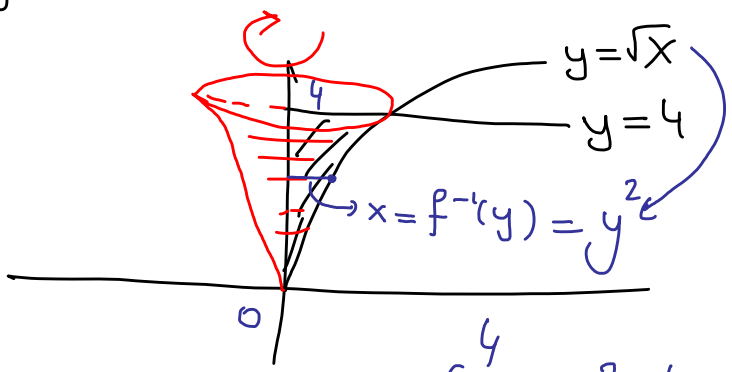
$\sqrt{x} = 4$

$x = 16$

Dış — iç

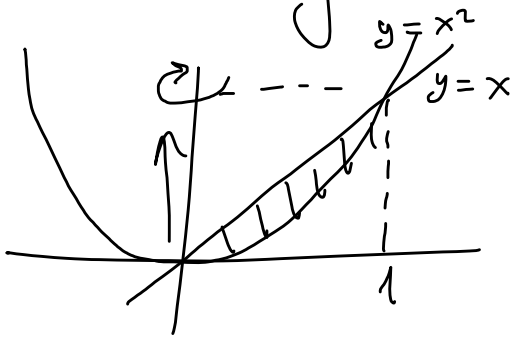
$$\pi \int_0^{16} (4)^2 dx - \pi \int_0^{16} (\sqrt{x})^2 dx$$

y ekseni etrafında döndürülseydi ?



$$\pi \int_0^4 (y^2)^2 dy = \pi \frac{y^5}{5}\Big|_0^4$$

Örnek 1 y ekseni etrafında



Dış — İç (kesik)

$$\pi \int_0^1 \underbrace{(\sqrt{y})^2}_{f^{-1}(y)} dy - \frac{\pi}{3} 1^2 \cdot 1$$

$$\frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$$