

28.05.2020 Final Sınavı

①

$f(x) = \frac{3}{2}$  fonksiyonunun grafiği ile  $2 \leq x \leq 6$  aralığı arasındaki alan için 2 eşit genişlikte bölünmüş ve üst toplam kullanılarak bir kestirimde bulunun.

$$\Delta x = \frac{6-2}{2} = 2$$

$$2 \leq x \leq 4 \rightarrow f(4)$$

$$4 \leq x \leq 6 \rightarrow f(6)$$

$$= \frac{3}{2} \cdot 2 + \frac{3}{2} \cdot 2$$

$$= 1 + \frac{3}{2}$$

$$= \frac{5}{2}$$

$$= \frac{5}{2}$$

②

$f(x) = x^2$  fonksiyonunun grafiği ile  $2 \leq x \leq 6$  aralığı arasındaki alan için 2 eşit genişlikte bölünmüş ve alt toplam kullanılarak bir kestirimde bulunun.

$$\Delta x = \frac{6-2}{2} = 2$$

$$2 \leq x \leq 4 \rightarrow f\left(\frac{2+4}{2}\right) = f(3)$$

$$4 \leq x \leq 6 \rightarrow f\left(\frac{4+6}{2}\right) = f(5)$$

$$= 5^2 \cdot 2 + 3^2 \cdot 2$$

$$= 90 + 18$$

$$= 108$$

③

Belirli integral

$\int_{-2}^2 \sqrt{4-x^2} dx = a$ , ( $a > 0$ ),  $a$  eşitliğinde  $a$  kaçtır?

$$\Rightarrow 2 \cdot \frac{1}{2} \pi \cdot 2 = 2\pi$$

$$\int_{-2}^2 \sqrt{4-x^2} dx = 2\pi$$

$$2(4-x^2)^{1/2} \Big|_{-2}^2 = 0$$

$$2(4-x^2)^{3/2} \Big|_{-2}^2 = 0$$

$$2(4-x^2)^{3/2} - 2(4-x^2)^{3/2} = 0$$

$$4-x^2 \geq 0$$

$$4-x^2 \geq 0$$

$$x^2 \leq 4$$

④ Belirli integral - İntegrasyon Tekniği

$$\frac{d}{dx} \int_0^{\sin x} \frac{1}{\sqrt{1-u^2}} du = ?$$

$$= (\sin x)' \cdot \frac{1}{\sqrt{1-\sin^2 x}} = 1$$

$$= \cos x \cdot \frac{1}{\sqrt{1-\sin^2 x}} = 1$$

⑤ Belirli integral - İntegrasyon Tekniği

$$\frac{d}{dx} \int_0^x e^{x^2} dx = ?$$

$$= (x^2)' \cdot e^{x^2} = 2x \cdot e^{x^2}$$

$$= \frac{x^2}{2} \cdot e^{x^2}$$

$$= \frac{e^{x^2}}{2x}$$

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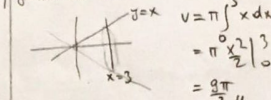
$$= \frac{e^{x^2}}{2x}$$

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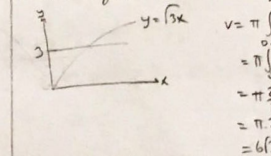
⑦ Belirli integralin Uygulamaları - Hacim

$y = x$ ,  $x = 0$ ,  $x = 3$  ile sınırlı bölgenin  $x$  eksenine etrafında döndürülmesi ile üretilen hacmi  $\frac{9\pi}{2}$  olarak bulunun.



⑧ Belirli integralin Uygulamaları - Hacim

Aşağıdaki bölgeyi  $y$ -ekseni etrafında döndürerek üretilen hacmi  $\frac{9\pi}{2}$  olarak bulunun.



⑨ Belirli integralin Uygulamaları - Dönüşüm Tekniği

$$\int_0^1 (x^2-5)^4 dx = ?$$

$$x^2-5 = u$$

$$2x = du$$

$$x = \frac{du}{2}$$

$$= \int_{-5}^{-4} u^4 \cdot \frac{1}{2} du$$

$$= \frac{1}{2} \cdot \frac{u^5}{5} \Big|_{-5}^{-4}$$

$$= \frac{1}{10} (4^5 - (-5)^5)$$

$$= \frac{1}{10} (1024 + 3125)$$

$$= \frac{4149}{10}$$

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⑩ Belirli integral Uygulamaları - Epi Ürünü

$x = \frac{3}{2}$  ve  $y = \frac{1}{2}$  eksenleri ile  $y = 2$  eksenine kadar uzanarak bölünür.

$$\left(\frac{3}{2} - \frac{1}{2}\right)^2 = \frac{4}{4} - \frac{2}{4} = \frac{2}{4} = \frac{1}{2}$$

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⑪ Belirli integral Uygulamaları - Epi Ürünü

$x = \frac{1}{2}$  ve  $y = \frac{1}{2}$  eksenleri ile  $y = 2$  eksenine kadar uzanarak bölünür.

$$\left(\frac{1}{2} - \frac{1}{2}\right)^2 = \frac{0}{4} - \frac{0}{4} = \frac{0}{4} = 0$$

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24) Belirli İntegral Uygulamaları - Alan

$y = \ln x$  ile  $x$ -ekseni arasındaki alan  $x=1$  den  $x=2$ 'ye kadar bulunur.

$$\begin{aligned} &= \int_1^2 \ln x \, dx \\ &= \ln x \cdot x - x \Big|_1^2 \\ &= \ln 2 \cdot 2 - 2 - (\ln 1 \cdot 1 - 1) \\ &= \ln 2 \cdot 2 - 2 + 1 \\ &= 2 \ln 2 - 1 \end{aligned} \quad \begin{aligned} &\ln x = u \quad dx = du \\ &\frac{1}{x} dx = du \quad x = u \\ &= \ln x \cdot x - \int x \cdot \frac{1}{x} dx \\ &= \ln x \cdot x - x \end{aligned}$$

24) Belirli İntegral Uygulamaları - Alan

$y = (4 + 8x^2) = 3$ ,  $x=0$ ,  $y=0$  ve  $x=3$  ile sınırlı bölgenin alanını bulunuz.

$$\begin{aligned} y &= \frac{3}{4 + 8x^2} \\ &= \int_0^3 \frac{3}{4 + 8x^2} dx \\ &= \int_0^3 \frac{3}{2^2 + (2x)^2} dx \\ &= 3 \arctan \frac{2x}{2} \Big|_0^3 \\ &= 3 \arctan \frac{3}{1} \end{aligned}$$

24) Belirli İntegral Uygulamaları - Alan

$y = \frac{x-14}{x^2-7x-8}$ ,  $y=0$ ,  $x=1$  ve  $x=3$  ile sınırlı bölge.

Alan bulunur.

$$\begin{aligned} &= \int_1^3 \frac{x-14}{x^2-7x-8} dx \\ &\rightarrow \frac{x-14}{(x-8)(x+1)} = \frac{A}{x-8} + \frac{B}{x+1} \\ &Ax + A + Bx - 8B = x - 14 \\ &-A + B = 1 \rightarrow A = 1 - B \\ &\frac{A - 8B = -14}{-A + B = 1} \\ &\frac{-8B = -15}{B = \frac{15}{8}} \end{aligned}$$

$$\begin{aligned} &= \int_1^3 \left( \frac{1}{8} \frac{1}{x-8} + \frac{15}{8} \frac{1}{x+1} \right) dx \\ &= -\frac{1}{8} \ln|x-8| + \frac{15}{8} \ln|x+1| \Big|_1^3 \\ &= -\frac{1}{8} \ln 7 + \frac{15}{8} \ln 4 - \left( -\frac{1}{8} \ln 7 + \frac{15}{8} \ln 2 \right) \\ &= -\frac{1}{8} \ln 7 + \frac{15}{8} \ln 4 + \frac{1}{8} \ln 7 - \frac{15}{8} \ln 2 \\ &= \frac{1}{8} (\ln 7 - \ln 7) + \frac{15}{8} (\ln 4 - \ln 2) \\ &= \frac{1}{8} \ln 2 + \frac{15}{8} \ln 2 \end{aligned}$$

24) İntegral Altına Yıkılma - Kısmi Kesirlerin İntegrasyonu

$$\begin{aligned} &\frac{5x+38}{x^2+9x+12} \text{ kısmi kesirler ayrıştırılır.} \\ &\frac{5x+38}{(x+6)(x+2)} = \frac{A}{x+6} + \frac{B}{x+2} \\ &5x+38 = A(x+2) + B(x+6) \\ &-2A+B=5 \\ &\frac{2A+6B=38}{4B=28} \\ &B=7 \\ &A=-2 \end{aligned}$$

25) Geliştirilmiş İntegraller

$\int_{-\infty}^{\infty} 20e^x dx$  genelleştirilmiş integralin hesaplanması.

$$\begin{aligned} &\lim_{t \rightarrow -\infty} \int_t^1 20e^x dx \\ &\lim_{t \rightarrow -\infty} 20 \cdot e^x \Big|_t^1 \\ &\lim_{t \rightarrow -\infty} 20 \cdot e^1 - 20 \cdot e^{-t} \\ &= \frac{20}{e} + 20 \cdot \frac{1}{e^0} \\ &= \frac{20}{e} + 20 \end{aligned}$$

25)

$y = \frac{21}{\sqrt{36-48x}}$  fonksiyonunun  $x=0$ 'den  $x=\frac{3}{2}$ 'ye orantıları hesaplanır.

$$\begin{aligned} &= \frac{1}{\frac{3}{2}} \int_0^{\frac{3}{2}} \frac{21}{\sqrt{6^2-(4x)^2}} dx \\ &= \frac{2}{3} \cdot 21 \arcsin \frac{4x}{6} \Big|_0^{\frac{3}{2}} \\ &= 49 \arcsin \frac{4 \cdot \frac{3}{2}}{6} - 49 \arcsin \frac{0}{6} \\ &= 49 \frac{\pi}{6} \end{aligned}$$