1. Given that

$$y = 8x^3 - 4\sqrt{x} + \frac{3x^2 + 2}{x}, \quad x > 0$$

find $\frac{dy}{dx}$.

(Total 6 marks)

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2. Given that $y = x^4 + x^{\frac{1}{3}} + 3$, find $\frac{dy}{dx}$.

(Total 3 marks)

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3. The curve C has equation

$$y = \frac{(x+3)(x-8)}{x}, \quad x > 0$$

(a) Find $\frac{dy}{dx}$ in its simplest form.

(4)

(b) Find an equation of the tangent to C at the point where $x = 2$

(4)

(Total 8 marks)

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4. Given that $y = 2x^3 + \frac{3}{x^2}$, $x \neq 0$, find

(a) $\frac{dy}{dx}$

(3)

(b) $\int y \, dx$, simplifying each term.

(3)

(Total 6 marks)

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1. $\frac{3x^2+2}{x} = 3x + 2x^{-1}$ M1 A1

$(y' =) 24x^2, -2x^{-\frac{1}{2}}, +3 - 2x^{-2}$ M1 A1 A1 A1

$\left[24x^2 - 2x^{-\frac{1}{2}} + 3 - 2x^{-2}\right]$

Note

1^{st} M1 for attempting to divide(one term correct)

1^{st} A1 for both terms correct on the same line, accept $3x^1$ for $3x$ or $\frac{2}{x}$ for $2x^{-1}$

These first two marks may be implied by a correct differentiation at the end.

2^{nd} M1 for an attempt to differentiate $x^n \to x^{n-1}$ for at least one term of their expression

"Differentiating" $\frac{3x^2+2}{x}$ and getting $\frac{6x}{1}$ is M0

2^{nd} A1 for $24x^2$ only

3^{rd} A1 for $-2x^{-\frac{1}{2}}$ allow $\frac{-2}{\sqrt{x}}$. Must be simplified to this, not e.g. $\frac{-4}{2}x^{-\frac{1}{2}}$

4^{th} A1 for $3 - 2x^{-2}$ allow $\frac{-2}{x^2}$. Both terms needed. Condone $3 + (-2)\,x^{-2}$

If "$+c$" is included then they lose this final mark

They do not need one line with all terms correct for full marks. Award marks when first seen in this question and apply ISW.

Condone a mixed line of some differentiation and some division e.g. $24x^2 - 4x^{\frac{1}{2}} + 3x + 2x^{-1}$ can score 1^{st} M1A1 and 2^{nd} M1A1

Quotient / Product Rule

$\dfrac{x(6x)-(3x^2+2)\times 1}{x^2}$ or $6x(x^{-1}) + (3x^2 + 2)(-x^{-2})$

$\dfrac{3x^2-2}{x^2}$ or $3 - \dfrac{2}{x^2}$ (o.e.)

1st M1 for an attempt: $\frac{P-Q}{x^2}$ or $R + (-S)$ with one of P, Q or R, S correct.

1st A1 for a correct expression

4th A1 same rules as above

[6]

2. $x^4 \to kx^3$ or $x^{1/3} \to kx^{-2/3}$ or $3 \to 0$ (k a non-zero constant) M1

$\left(\dfrac{dy}{dx}=\right)4x^3$.........., with '3' differentiated to zero (or 'vanishing') A1

$\left(\dfrac{dy}{dx}=\right)$ .... $+\dfrac{1}{3}x^{-2/3}$ or equivalent, e.g. $\dfrac{1}{3\sqrt[3]{x^2}}$ or $\dfrac{1}{3\left(\sqrt[3]{x}\right)^2}$ A1

<u>**Note**</u>

1st A1 requires $4x^3$, <u>and</u> 3 differentiated to zero.

Having '$+C$' loses the 1st A mark.

Terms not added, but otherwise correct, e.g. $4x^3$, $\dfrac{1}{3}x^{-2/3}$ loses the 2nd A mark.

[3]

3. (a) $y = \dfrac{x^2 - 5x - 24}{x} = x - 5 - 24x^{-1}$

(or equiv., e.g. $x + 3 - 8 - \dfrac{24}{x}$) M1 A1

$\dfrac{dy}{dx} = 1 + 24x^{-2}$ or $\dfrac{dy}{dx} = 1 + \dfrac{24}{x^2}$ M1 A1 4

Note

1^{st} M: Mult. out to get $x^2 + bx + c$, $b \neq 0$, $c \neq 0$ and dividing by x (not x^2).
Obtaining one correct term, e.g. x…….. is sufficient evidence of a division attempt.

2^{nd} M: Dependent on the 1^{st} M:

Evidence of $x^n \to kx^{n-1}$ for one x term (i.e. not just the constant term) is sufficient). Note that mark is not given if, for example, the numerator and denominator are differentiated separately.

A mistake in the 'middle term', e.g. $x + 5 - 24x^{-1}$, does not invalidate the 2^{nd} A mark, so M1 A0 M1 A1 is possible.

(b) $x = 2$: $y = -15$ Allow if seen in part (a). B1

$\left(\dfrac{dy}{dx} =\right) 1 + \dfrac{24}{4} = 7$ Follow-through from candidate's non-constant $\dfrac{dy}{dx}$. B1ft

This must be simplified to a "single value".

$y + 15 = 7(x - 2)$ (or equiv., e.g. $y = 7x - 29$)
Allow $\dfrac{y+15}{x-2} = 7$ M1 A1 4

Note

B1ft: For evaluation, using $x = 2$, of their $\dfrac{dy}{dx}$, even if unlabelled or called y.

M: For the equation, in any form, of a straight line through (2, '–15') with candidate's $\dfrac{dy}{dx}$ value as gradient.

Alternative is to use (2, '–15') in $y = mx + c$ to find a value for c, in which case $y = 7x + c$ leading to $c = -29$ is sufficient for the A1).

(See general principles for straight line equations at the end of the scheme).

Final A: 'Unsimplified' forms are acceptable, but…

$y - (-15) = 7(x - 2)$ is A0 (unresolved 'minus minus').

[8]

4. (a) $\frac{dy}{dx} = 6x^2 - 6x^{-3}$ M1 A1 A1 3

Note

M1 for an attempt to differentiate $x^n \to x^{n-1}$

1st A1 for $6x^2$

2nd A1 for $-6x^{-3}$ or $-\frac{6}{x^3}$ Condone $+ -6x^{-3}$ here. Inclusion of $+c$ scores A0 here.

(b) $\frac{2x^4}{4} + \frac{3x^{-1}}{-1}(+C)$ M1 A1

$\frac{x^4}{2} - 3x^{-1} + C$ A1 3

Note

M1 for some attempt to integrate an x term of the given y. $x^n \to x^{n+1}$

1st A1 for **both** x terms correct but unsimplified– as printed or better. Ignore $+c$ here

2nd A1 for both x terms correct and simplified and $+c$. Accept $-\frac{3}{x}$ but NOT $+ -3x^{-1}$

Condone the +c appearing on the first (unsimplified) line but missing on the final (simplified) line

Apply ISW if a correct answer is seen

If part (b) is attempted first and this is clearly labelled then apply the scheme and allow the marks. Otherwise assume the first solution is for part (a).

[6]