Novel Noise Variance and SNR Estimation Algorithm for Wireless MIMO OFDM Systems

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***Abstract*— We present a new noise variance and SNR estimation algorithm for a 2×2 MIMO wireless OFDM system as defined in the IST-STINGRAY project. The SNR information is used to adapt parameters or reconfigure parts of the transmitter. The noise variance estimation algorithm uses only 2 OFDM training symbols from each transmitting antenna and the FFT output signals at the receiver. It does not require knowledge of the channel coefficients. Then, using the channel coefficient estimates given by a channel estimator and the estimate of the noise variance, the SNR is computed. The algorithm's performance is measured through Monte-Carlo simulations on a variety of channel models and compared to those of an MMSE algorithm using perfect channel estimates. The normalized MSE of the obtained noise variance estimate shows good results as long as the delay spread of the channel is small enough compared to the OFDM symbol period.**

I. Introduction

Diversity systems have been increasingly popular during the last years, with the advent of space-time coding techniques and multiple-input multiple-output (MIMO) systems, due to the diversity gain they can offer. Orthogonal frequency division multiplexing (OFDM), meanwhile, has proven to be useful in frequency-selective channels to achieve high data rates and has been abundantly proposed for high-data rate wireless local area networks (LAN) systems, for example. The combination of OFDM and space-time coding techniques is the current topic of many research projects around the world, for example for fixed wireless access (FWA) applications. This is the focus of the FP6 IST-STINGRAY project, in which the research presented herein takes place. FP6 is the European sixth framework programme, IST the information society technologies and STINGRAY stands for space time coding for reconfigurable wireless access systems. The reconfigurability and adaptivity features of the system will be implemented in both the transmitter and the receiver according to the channel variations and the supported services, through explicit use of feedback channel state information (CSI). The CSI is provided by the receiver to the transmitter. The CSI usually partially or entirely includes or is derived from the signal-to-noise ratio (SNR) information. The estimation of the SNR on each subcarrier as well as the overall SNR is the focus of this paper. The accuracy and reliability of the SNR estimates are important as the performance of the whole system depends on them, through the use of adaptivity and reconfigurability. Through the transmission of training symbols, we must acquire an estimate of the SNR.

Noise variance and hence SNR estimation is an old topic [1]. Various authors have been focusing on this problem, mainly for single-input single-output single carrier systems. The main areas of applications are adaptive schemes, for example [2] and [3], and turbo decoding, as in [4]. The estimation algorithms mostly focus on constant amplitude modulations, as quadrature phase shift keying (QPSK). Minimum mean square error (MMSE) and maximum likelihood (ML)-based noise variance estimators use channel estimates whereas moment-based algorithms are blind, as well as many ad-hoc algorithms, as presented in [5] and [6]. Ramesh et al. in [7] were the only authors referring to SNR estimation for diversity systems. Their algorithm is used for turbo decoding and has a long averaging interval. Our goal is rather different, as we need to acquire the SNR during the acquisition mode, using a limited amount of data. We thus have not found any trace of earlier research on our particular problem. We will also show that during acquisition, we cannot use a simple MMSE algorithm for the noise variance estimation due to the choice for the channel estimator. This is why we had to think of a new algorithm for noise variance estimation and hence SNR estimation.

In order to get an estimate of the SNR using the training symbols, we present herein a new algorithm for estimating the noise variance that does not require any knowledge about the channel coefficients. It uses only the signals at the output of the FFT at the receiver. It can be applied together with the channel estimator presented in [8], for example, to calculate the SNR. The performance of this new algorithm is evaluated through simulations.

We first present the system model by focusing on the acquisition mode, meaning the transmission of known training data. Different channel models are used to test and study the performance and limits of the algorithm. We then present the new algorithm, together with the MMSE algorithm, which will be used for comparison. Finally, we report the results from the Monte-Carlo simulations and draw some conclusions.

II. System Model

As we are focusing on noise variance and SNR estimation for the acquisition mode, based on the transmission of known

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training sequences, we show here a simplified block diagram in Figure 1. We have a 2×2 MIMO OFDM system. In the block diagram, we do not show the IFFT, the FFT and any time domain processing as we deal with frequency domain representation of the channel and we presume perfect synchronization.

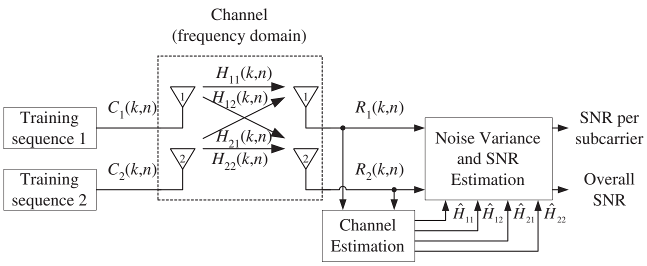

Fig. 1. Simplified block diagram of the system in the acquisition mode.

A. Transmitter

A different training sequence of two OFDM symbols is sent from each transmitting antenna simultaneously. Each OFDM symbol is composed of *N* QPSK data symbols $C_i(k,n)$ with unit magnitude, where *i* is the index of the transmitting antenna, $k = 0, 1$ is the index of the OFDM symbol, and $n = 0, ..., N-1$ is the index of the subcarrier. The length of the cyclic prefix is $3N/16$ samples and assumed to be an integer.

The training sequences' structure can be chosen freely. Anyhow, we chose to concentrate on the one presented in [8], as the channel estimator herein is simple to implement. In this case we have

$$C_1(0,n) = -C_1(1,n) = C_2(0,n) = C_2(1,n) = C(n). \quad (1)$$

B. Channel Model

The area of application of the system is FWA. In such a situation, we are facing a multipath slowly time-varying fading channel. Since the channel is slowly time-varying and the training sequences are short, we can model it by a time-invariant channel for the training sequence period. With this in mind, we investigate four different channels: (a) an AWGN channel, (b) a one-tap Rayleigh fading channel, (c) a 3-tap time-invariant fading channel with a root mean square (rms) delay spread of 2 samples and (d) a 3-tap time-invariant fading channel with a rms delay spread of 10 samples. All paths are Rayleigh fading, except the first path of channel (c), which is Ricean. The two later channel parameters are taken from [9], whose details are given in Table I. We remark that in order for the cyclic prefix to be longer than the impulse response of the channel, *N* must be at least equal to 256.

C. Receiver

Since we consider a perfectly synchronized system, a very slowly time-varying channel and a cyclic prefix longer than the channel impulse response, we can consider that there is neither intersymbol interference (ISI) nor intercarrier interference (ICI) at the output of the FFT, at the receiver side. We can thus formulate the output of the FFT in a simple manner as

$$R_j(k,n) = \sum_{i=1}^{2} H_{ij}(k,n) C_i(k,n) + n_j(k,n), \quad (2)$$

where *j* is the index of the receiving antenna and $n_j(k,n)$ is the complex AWGN of variance $2\sigma_j^2$. Both the channel estimator and the noise variance and SNR estimator know the training sequences. The channel estimator estimates the channel using the training sequences and provides one estimate per subchannel, as the channel is so slowly time varying that we consider it constant during the duration of the training sequences. The channel estimates are referred to as $\hat{H}_{ij}(n)$ where *i* and *j* are the index of the transmitting and receiving antennae, respectively, and *n* is the subcarrier index.

TABLE I

PARAMETERS FOR THE MULTIPATH CHANNELS (C) AND (D)

Channel	(c)	(d)
Delay 1st path (samples)	0	0
Rayleigh Power (dB) - 1st path	-4.52 Ricean factor = 1	-1.92
Delay 2nd path (samples)	3	12
Rayleigh Power (dB) - 2nd path	-6.51	-5.92
Delay 3rd path (samples)	7	32
Rayleigh Power (dB) - 3rd path	-11.51	-9.92

III. NOISE VARIANCE ESTIMATORS

In determining the SNR, there are two possibilities. Either we estimate the noise variance and then use the channel coefficient estimate to determine the SNR, or we determine directly the ratio of the noise variance and the signal power, which is here the square of the channel coefficient estimate. As the channel estimator provides us with the channel coefficient estimates, we focus on the noise variance estimation. In this case, there are also two possibilities in the acquisition mode. As we know the training sequences, either we use the channel estimates or we do not use them. The earlier method corresponds to the MMSE algorithm, the later one to the novel algorithm we are presenting here. We define two SNRs. The first is the SNR per subcarrier, averaged over the four subchannels, and the second is the overall SNR averaged over all subcarriers

$$\hat{\rho}_A(n) = \frac{1}{4} \sum_{j=1}^{2} \frac{\sum_{i=1}^{2} \left| \hat{H}_{ij}(n) \right|^2}{2\hat{\sigma}_j^2}, \quad (3)$$

$$\hat{\rho}_O(n) = \frac{1}{4N} \sum_{n=0}^{N-1} \sum_{j=1}^{2} \frac{\sum_{i=1}^{2} \left| \hat{H}_{ij}(n) \right|^2}{2\hat{\sigma}_j^2}. \quad (4)$$

A. MMSE Noise Variance Estimator

The MMSE noise variance estimator is defined as [10]

$$2\hat{\sigma}^2_{j,MMSE} = \frac{1}{2}\sum_{k=0}^{1}\left|R_j(k,n) - \sum_{i=1}^{2}\hat{H}_{ij}(n)C_i(k,n)\right|^2 . \quad (5)$$

B. Novel Noise Variance Estimator

As stated in the introduction, we use the training sequences following (1) together with the channel estimation method presented in [8]. This is a simple channel estimation method that eases the stress on the implementation. The channel coefficient estimates are given by

$$\begin{aligned}\hat{H}_{1j}(n) &= \frac{1}{2\left|C(n)\right|^2}C^*(n)\left(R_j(0,n) - R_j(1,n)\right),\\ \hat{H}_{2j}(n) &= \frac{1}{2\left|C(n)\right|^2}C^*(n)\left(R_j(0,n) + R_j(1,n)\right).\end{aligned} \quad (6)$$

With these channel coefficient estimates, the MMSE algorithm cannot be applied, as (5) leads to zero. We thus need to calculate the noise variance without using the channel estimates given by the channel estimator. For each receiving antenna, we have two received signals and 3 unknown variables, as we consider the channel constant during the transmission of the training symbols. By adding and subtracting the two received signals at the output of the FFT at each receiver, we get two new signals

$$\begin{aligned}R_{j,1}(n) &= R_j(0,n) - R_j(1,n)\\ &= 2H_{1j}(n)C(n) + n_j(0,n) - n_j(0,n)\\ &= 2H_{1j}(n)C(n) + n_{j,1}(n),\\ R_{j,2}(n) &= R_j(0,n) + R_j(1,n)\\ &= 2H_{2j}(n)C(n) + n_j(0,n) + n_j(0,n)\\ &= 2H_{2j}(n)C(n) + n_{j,2}(n).\end{aligned} \quad (7)$$

As we still have too many unknown parameters for the number of equations at hand, we need to make another assumption and restrict our system to be able to estimate the noise variance. We assume that the channel is slowly varying in the frequency domain, which allows us to consider that the channel degradation is the same on adjacent subcarriers. This is basically valid for channels with a delay spread small compared to the OFDM symbol period. For two adjacent subcarriers indexed n and $n-1$, using (7), we can formulate the new equation

$$\begin{aligned}&\left|C(n-1)R_{j,p}(n) - C(n)R_{j,p}(n-1)\right|^2 =\\ &\quad \left|n_{j,k}(n)\right|^2 + \left|n_{j,k}(n-1)\right|^2\\ &\quad -2\Re\left[C(n)C^*(n-1)n^*_{j,p}(n)n_{j,p}(n-1)\right],\end{aligned} \quad (8)$$

with $p = 1, 2$ and $\Re$ is the real part extraction operator. The statistical average of the right hand side of equation (8) is equal to $8\sigma_j^2$. Having ergodic processes, we replace statistical averaging by time averaging, and we obtain the following algorithm

$$\begin{aligned}2\hat{\sigma}^2_{j,NA} = \; &\frac{1}{8(N-1)}\\ &\cdot\sum_{n=1}^{N-1}\Big[\left|C(n-1)\left(R_j(0,n) + R_j(1,n)\right)\right.\\ &\left.-C(n)\left(R_j(0,n-1) + R_j(1,n-1)\right)\right|^2\\ &+\left|C(n-1)\left(R_j(0,n) - R_j(1,n)\right)\right.\\ &\left.-C(n)\left(R_j(0,n-1) - R_j(1,n-1)\right)\right|^2\Big],\end{aligned} \quad (9)$$

where we consider that all N subcarriers are modulated with non-zero QPSK symbols with unit magnitude.

The algorithm can also be applied to arbitrary training sequences, for which

$$\begin{aligned}R_{j,1}(n) &= C^*_2(0,n)R_j(0,n) - C^*_2(1,n)R_j(1,n)\\ &= H_{1j}(n)\cdot[C^*_2(0,n)C_1(0,n)\\ &\quad -C^*_2(1,n)C_1(1,n)]\\ &\quad +C^*_2(0,n)n_j(0,n) - C^*_2(1,n)n_j(1,n),\\ R_{j,2}(n) &= C^*_1(0,n)R_j(0,n) - C^*_1(1,n)R_j(1,n)\\ &= H_{2j}(n)\cdot[C^*_1(0,n)C_2(0,n)\\ &\quad -C^*_1(1,n)C_2(1,n)]\\ &\quad +C^*_1(0,n)n_j(0,n) - C^*_1(1,n)n_j(1,n).\end{aligned} \quad (10)$$

We do not go further into details in this paper, but the final equation for the algorithm is easily calculated from (10), following the example given in (8).

IV. Simulations Results

In order to check the performance of the new noise variance estimator, we test it under different channel models, using Monte-Carlo simulation method. All simulations were performed under Co-Centric System Studio software. The SNR estimator is using the noise variance from the new algorithm and the channel coefficients derived from Jeon's channel estimator [8]. The results are compared to the MMSE noise variance with perfect knowledge of the channel and the SNR estimate using the output of the MMSE noise variance estimator and the perfect channel coefficients.

In the simulation, there are $N = 256$ subcarriers, all modulated, and the cyclic prefix is 48 samples long. There are 2 OFDM training symbols sent on each transmitting antenna and the symbols $C(n)$ are arbitrary QPSK symbols with unit magnitude. The evaluation of the performance is done in terms of normalized MSE (NMSE) of the estimated values following

$$NMSE = \frac{1}{M}\sum_{m=0}^{M-1}\left(\frac{\hat{\upsilon} - \upsilon}{\upsilon}\right)^2, \quad (11)$$

where M is the number of data over which the NMSE is measured, $\hat{\upsilon}$ is the estimated value of the noise variance or the SNR and υ is its true value. Those values are of course instantaneous values, as the channel changes from one acquisition period to the other one. The NMSEs obtained can thus be compared with the results presented in [5] and [6].

A. AWGN Channel

In the AWGN channel (a), the NMSE, measured over $M = 1000$ data, of the noise variance for the new algorithm is independent of the true noise variance and equals $3 \cdot 10^{-3}$, whereas the MMSE algorithm with perfect channel knowledge reaches a NMSE of $1.93 \cdot 10^{-3}$. The new algorithm perform as well as the MMSE estimator using channel coefficients obtained from the method presented in [8] and averaged over 32 subcarriers. As for the SNR, the NMSE depends on the true SNR according to the performance of the channel estimator as shown in Figure 2. In [5] and [6], for a SNR of 10 dB, the lowest NMSEs obtained for binary phase shift keying (BPSK) signals in a single carrier system are about $3 \cdot 10^{-2}$ for 64 symbols, which corresponds to our results. Thus the new algorithm performs well in an AWGN channel, at least as well as algorithms designed for single carrier systems.

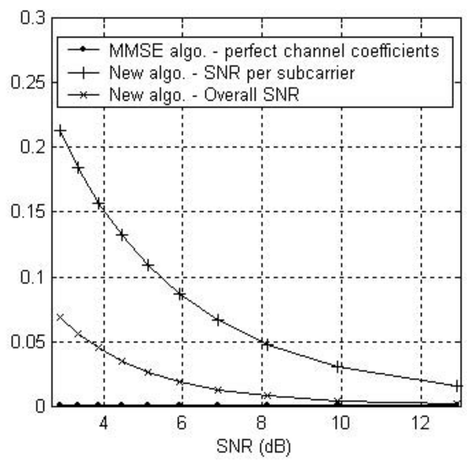

Fig. 2. NMSE of the SNR estimates in AWGN channel.

B. One-Path Rayleigh Fading Channel

In the channel model (b), whether or not the subchannels have the same power for the Rayleigh path, the noise variance NMSEs, measured over $M = 2000$ data, do not change compared to the AWGN channel case. Indeed, the channel model is time-invariant and frequency-nonselective. As for the SNR estimates, the NMSEs vary depending on the type of channel we are facing. When the power of the Rayleigh paths are the same on all the subchannels, the SNR NMSE is larger than in the case when they differ. This is due to the channel estimates, as we already noticed that the noise variance estimates are not dependent on the symmetry in the power of the Rayleigh paths on each subchannel.

C. Time-Invariant Multipath Fading Channel

The channel models (c) and (d) are multipath fading channels, thus they are frequency-selective. One of the main assumption leading to the new algorithm being that the channel degradation on adjacent subcarriers will be equal, we can see how the severity of the frequency selectivity affects the new noise variance estimation algorithm. As we can see from Figure 3, the noise variance NMSEs, measured over $M = 4000$ data, depends on the true instantaneous noise variance. The smaller the noise variance to be estimated, the larger the error on the estimation. As expected, the more severe the frequency selectivity the larger the error, since channel (c) is less frequency selective than channel (d).

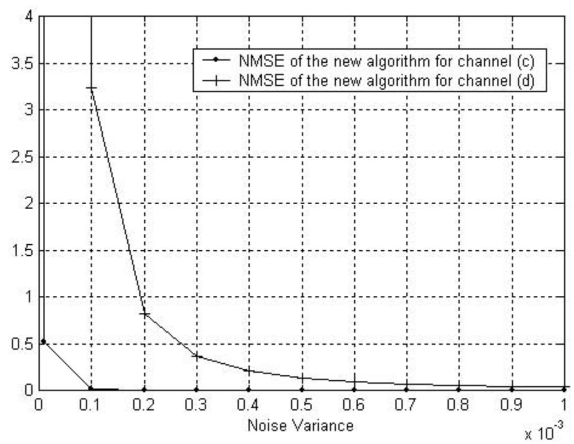

Fig. 3. NMSE of the noise on channel (c) and (d).

The dependence of the noise variance estimation error of the new algorithm on the true noise variance can be further seen from Figure 4, where the SNR NMSEs are plotted for different true average channel SNRs. As we can see, when the frequency selectivity is too severe, the SNR estimates become unreliable and the new algorithm can obviously not be used. For high SNRs, the noise variance estimation becomes the main impact on the SNR estimation error. For low SNRs, most of the SNR estimation error comes from the error in the channel coefficients estimation. For low SNRs, we can also see that the NMSE in channel (d) is better than the one in channel (a), meaning that the errors in the noise variance estimator and the channel coefficient estimator compensate each others. This clearly shows how unreliable the algorithm is in such an environment.

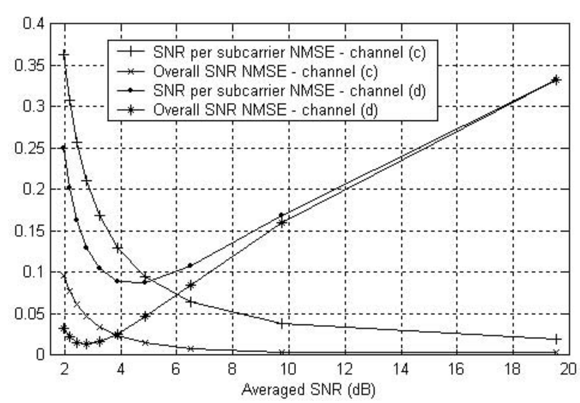

Fig. 4. NMSE of the SNR on channel (c) and (d).

V. CONCLUSION

In this paper, we have defined and simulated a new noise variance and SNR estimation algorithm for 2×2 MIMO OFDM system in the acquisition mode when transmitting known training data. The new algorithm is a frequency domain algorithm and does not use any channel coefficient estimates in estimating the noise variance. The main requirements leading to the use of the algorithm are to have a quasi time-invariant channel whose delay spread is small compared to the OFDM symbol period. The algorithm was tested on several channels, ranging from AWGN channel to multipath highly frequency selective fading channel. The channel estimator used for calculating the SNR was the one described in [8]. The performance of the new algorithm was compared to an MMSE algorithm using perfect channel estimates.

The NMSE of the new algorithm's noise variance estimates does not depend on the SNR for frequency-nonselective channels. The SNR NMSE increases as the averaged SNR decreases, due to the errors on the channel estimates. In a highly frequency selective multipath channel the performance of the new algorithm in terms of SNR NMSE degrades as the SNR increases since the noise variance NMSE increases as the true noise variance decreases. As expected, the new algorithm is sensitive to frequency selectivity and is unreliable when used in such an environment. It is also limited to constant envelope modulations. Some improvements may be reached by further averaging in a decision directed mode. A mathematical analysis of the performance of the new algorithm would also help understanding the real effect of frequency selectivity as well as its limits.

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