CNCM Math Bowl 3rd Place Match

CNCM Administration

Problems

Within each of the (infinitely many) circular cross sections of a sphere parallel to the $x - y$ plane centered at the origin with radius 6, a square is inscribed. Find the volume inside the sphere outside the 3-dimensional shape created by the cross sections.

What unit fraction is closest to the value of $\frac{e^{12}}{\pi^{13}}$? A unit fraction is a fraction with one as its numerator and an integer as its denominator.

Compute

$$\lim_{n\to 1} \sum_{i=1}^{6} \sum_{j=1}^{i} \frac{n^j - 1}{\log n}$$

Answer: $\boxed{56}$

Find d in the equation $x^4 - 3x^3 - \frac{9}{4}x^2 + dx = 0$ where the roots of the equation form an arithmetic sequence with common difference $\frac{3}{2}$ and mean of $\frac{3}{4}$

Find all angles $0 < \theta \leq 2\pi$ that satisfy $\tan\theta \sec\theta = -\frac{2}{3}$.
$\frac{7\pi}{6}, \frac{11\pi}{6}$

Find all integer solutions to

$$\sin x = \frac{x^2}{125\pi^2}$$

0

Find the greatest integer y for which $5^{15} > 243^y$

Compute $\frac{4.3*5.4}{5}$ in decimal form.

Three cards are drawn from a deck without replacement. Then, the three cards are returned to the deck, the deck is shuffled, and a fourth card is drawn. Given that the three cards originally drawn were all of different suits, what is the probability that the fourth card is a heart and the three original cards were all not hearts?

Find the sum of all x that satisfy the equation $3 + f(x) = f(g(x))$ where f(x) $= \dfrac{x+4}{x-2}$ and $g(x) = x - 2$

How many ordered pairs (a, b, c) of positive integers less than 5 are there such that $a!b! = c!$?

There exists a quarter-circle with radius two. The circumcircle of the right triangle that shares the entirety of two sides with the quarter-circle is not contained entirely within the quarter-circle. Find the area of this circumcircle that lies outside the original quarter-circle.

Suppose $f(g(x)) = 5x$. $f(x) = x^2 + 2x + 1$. Find all possible functions $g(x)$.

Suppose $f(x) = g(x)$ has 4 solutions x, with f and g being continuous functions. Both sides are multiplied by $x^3 - 5x^2 + 7x - 3$. What is the maximum number of solutions this modified equation can have?

If $a+b = 12$, $b+c = 13$, $ac = 20$, and a, b, c are positive integers, what is the value of $\dfrac{b^a}{a^c}+abc+ab-c$?