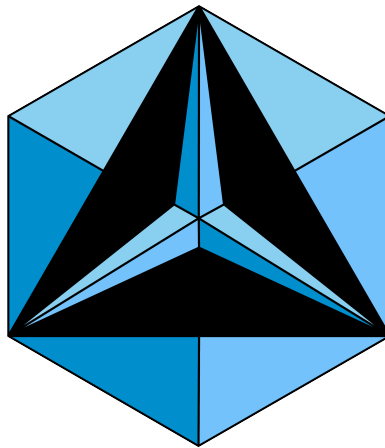


# CNCM Online Round 3

CNCM Administration

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### 3 Problems

**Problem 1.** Harry, who is incredibly intellectual, needs to eat carrots  $C_1, C_2, C_3$  and solve *Daily Challenge* problems  $D_1, D_2, D_3$ . However, he insists that carrot  $C_i$  must be eaten only after solving *Daily Challenge* problem  $D_i$ . In how many satisfactory orders can he complete all six actions?

**Problem 2.** Consider rectangle  $ABCD$  with  $AB = 6$  and  $BC = 8$ . Pick points  $E, F, G, H$  such that the angles  $\angle AEB, \angle BFC, \angle CGD, \angle AHD$  are all right. What is the largest possible area of quadrilateral  $EFGH$ ?

**Problem 3.** Let  $a_1 = 1$  and  $a_{n+1} = a_n \cdot p_n$  for  $n \geq 1$  where  $p_n$  is the  $n$ th prime number, starting with  $p_1 = 2$ . Let  $\tau(x)$  be equal to the number of divisors of  $x$ . Find the remainder when

$$\sum_{n=1}^{2020} \sum_{d|a_n} \tau(d)$$

is divided by 91 for positive integers  $d$ . Recall that  $d|a_n$  denotes that  $d$  divides  $a_n$ .

**Problem 4.** Hari is obsessed with cubics. He comes up with a cubic with leading coefficient 1, rational coefficients and real roots  $0 < a < b < c < 1$ . He knows the following three facts:  $P(0) = -\frac{1}{8}$ , the roots form a geometric progression in the order  $a, b, c$ , and

$$\sum_{n=1}^{\infty} (a^n + b^n + c^n) = \frac{9}{2}$$

The value  $a + b + c$  can be expressed as  $\frac{m}{n}$ , where  $m, n$  are relatively prime positive integers. Find  $m + n$ .

**Problem 5.** How many positive integers  $N$  less than  $10^{1000}$  are such that  $N$  has  $x$  digits when written in base ten and  $\frac{1}{N}$  has  $x$  digits after the decimal point when written in base ten? For example, 20 has two digits and  $\frac{1}{20} = 0.05$  has two digits after the decimal point, so 20 is a valid  $N$ .

**Problem 6.** Triangle  $ABC$  has side lengths  $AB = 13, BC = 14$ , and  $CA = 15$ . Let  $\Gamma$  denote the circumcircle of  $\triangle ABC$ . Let  $H$  be the orthocenter of  $\triangle ABC$ . Let  $AH$  intersect  $\Gamma$  at a point  $D$  other than  $A$ . Let  $BH$  intersect  $AC$  at  $F$  and  $\Gamma$  at  $G$ . Suppose  $DG$  intersects  $AC$  at  $X$ . Compute the greatest integer less than or equal to the area of quadrilateral  $HDXF$ .

**Problem 7.** A subset of the positive integers  $S$  is said to be a *configuration* if  $200 \notin S$  and for all nonnegative integers  $x$ ,  $x \in S$  if and only if both  $2x \in S$  and  $\lfloor \frac{x}{2} \rfloor \in S$ . Let the number of subsets of  $\{1, 2, 3, \dots, 130\}$  that are equal to the intersection of  $\{1, 2, 3, \dots, 130\}$  with some configuration  $S$  equal  $k$ . Compute the remainder when  $k$  is divided by 1810.