Medical Imaging : Group Project – Fourier Reconstruction

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Image credits: Loughborough University

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Abstract

Our project is to restructure a Shepp-Logan Head Phantom Model using a method called Direct Fourier Reconstruction. It employs the central-slice theorem to build a 2D Fourier space (in polar form) from the 1D Fourier transformed projection slices of different angles, which is then placed on a two-dimensional Cartesian coordinate. This enhances the efficiency of the inverse 2D Fourier transform. After all, a reconstructed image will be produced, and it should looks like the original Shepp-Logan Head Phantom Model. Since continuous Fourier transform is impractical in the digital world, we will use discrete Fourier transform to compute our data. And to boost up the running speed of our program, we will further use FFT to do the computation. While we transfer the 1D Fourier transformed projection slices on to a two-dimensional plane, we need to do a suitable interpolation in order to place our sample on the Cartesian grid without much shifting and errors. After all these steps, we can now do the inverse 2D Fourier transform to obtain the reconstructed images.

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I. Theory

Name of our reconstruction method: **Direct Fourier Reconstruction**

$$\hat{f}(x,y) = IF_2\{IN_2[F_{s,1}(g(s,\theta_n))]\}$$

where $g(s,\theta_n) = R[f(x,y)]$, $f(x,y)$ is the original signal,
IN_2 represents 2D Interpolation

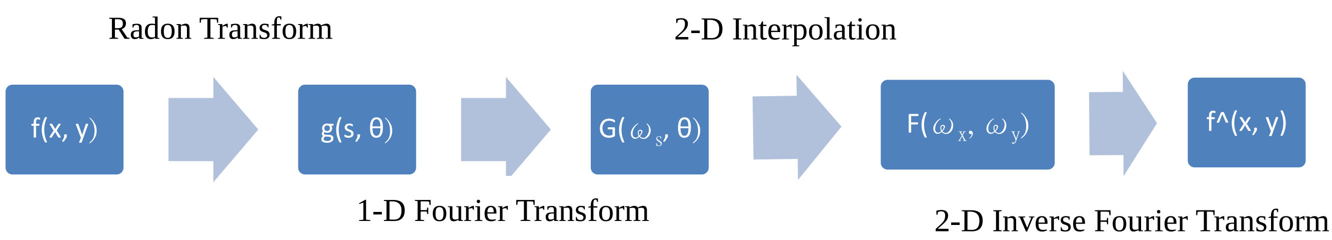

Figure 1: Flow chart of the reconstruction

As we know, the sum of the 1-D Fourier transform of a Radon function at different angles (from 0 to 179 degree) will equal to the 2-D Fourier transform of a two-dimensional function, so if we project certain amount of x-ray to a 2-D object along different angles, we will get different Radon functions at these angles. Once we have these data, we are able to generate the original 2-D image.

I.1. Radon Transform

To do the reconstruction, first we need to do a Radon transform [R] to the original signal $f(x, y)$.

$$\begin{aligned} g(s,\theta_n) &= R[f(x,y)] \\ &= \int_L f(x,y)\, dl \\ &= \iint f(x,y)\delta(x\cos\theta + y\sin\theta - s)\, dx\, dy \end{aligned}$$

This process is the same as scanning the object in different angles θ_n in CT scan.

I.2. 1-D Fourier Transform

After getting the slices, we do a 1-D Fourier transform $[F_{s,1}]$ to each slice $g(s, \theta_n)$ with respect to its spatial parameter [s].

$$G(\omega_s, \theta_n) = F_{s,1}\big(g(s, \theta_n)\big) = \int_{-\infty}^{\infty} g(s, \theta_n) e^{-i\omega s}\, ds$$

This process changes the signal from spatial domain to frequency domain.

I.3. 2-D Interpolation

Since Fast Fourier Transform (FFT) is more efficient in rectangular coordinate system, we transfer the Fourier transformed projection slices which is in polar coordinate to Cartesian coordinate system. We need to fit the slices onto the rectangular grid system, the missing data should be estimated from adjacent slices using interpolation. And to avoid extrapolation, everything outside the defined space is set to be zero. However, converting coordinate system will introduce some errors, which will show as artifact at the final reconstructed image.

I.4. 2-D Inverse Fourier Transform

After the 2-D interpolation, we can use the Fast Fourier Transform function to transform the signal from frequency domain back to spatial domain.

i.e.

$$\begin{aligned}\hat{f}(x, y) &= IF_2[F(\omega_x, \omega_y)] \\ &= \iint_{-\infty}^{\infty} F(\omega_x, \omega_y) e^{i(\omega_x x + \omega_y y)}\, d\omega_x\, d\omega_y\end{aligned}$$

This transformation is support by Central Slice Theorem.

I.5. Supporting Theorem (Central Slice Theorem (CST))

For the Fourier reconstruction, we need to make a link between the 1-D Fourier transform and the 2-D Fourier transform which allows us to switch between the space domain and the Fourier domain, and finally to get the 2-D image.

We consider the following notations:

- $f(x, y)$, the function which describes the object we want to observe
- $g(s, \theta_n)$, the Radon transform of $f(x, y)$ which represents the projection of our object along one direction and one angle θ_n
- $G(\omega_s, \theta_n)$, the 1-D Fourier transform of the projection $g(s, \theta_n)$
- $F(\omega_x, \omega_y)$, the 2-D Fourier transform of the function $f(x, y)$

The Central Slice Theorem exposes that $G(\omega_s, \theta_n)$ is equal to $F(\omega_x, \omega_y)$ in polar coordinates. i.e.

$$G(\omega_s, \theta_n) = F_{s,1}\big(g(s, \theta_n)\big) = F\big(\omega_x, \omega_y\big) = F_2[f(x, y)]$$

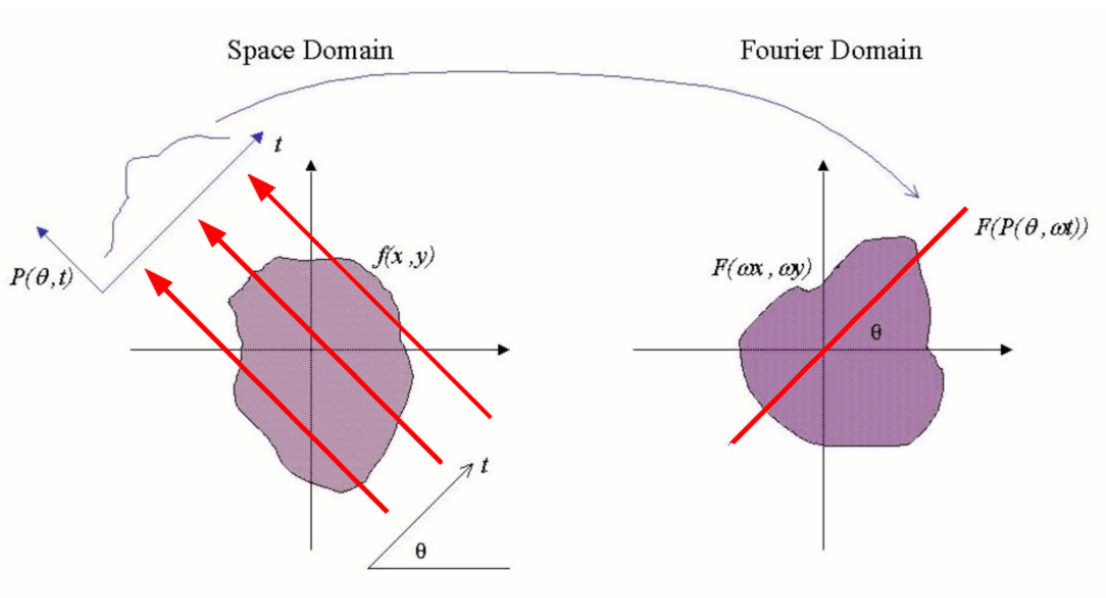

Figure 2: Radon transform

I.6. Modification to Fourier Transform

Central Slice Theorem, though powerful, cannot directly apply into programming without modifications. In the following section, we will discuss the problems and the solutions of Fourier transform.

First, the spatial data are discrete signal due to the finite size of X-ray detector. We need to modify the Fourier transform from continuous time domain to discrete time domain, which is called Discrete Time Fourier Transform (DTFT).

Performing DTFT requires infinite computer memory; therefore, it will not be used. DTFT is a discrete time sampling version of Fourier Transform whereas Discrete Fourier Transform (DFT) is a discrete frequency sampling version of DTFT. Hence DFT is a discrete time and discrete frequency sampling version of Fourier Transform.

A Fast Fourier Transform (FFT) is a faster version of the DFT that can be applied when the number of samples in the signal is a power of two. An FFT computation takes approximately $N * \log_2 (N)$ operations, whereas a DFT takes approximately N^2 operations, so the FFT is significantly faster.

II. Experiments

II.1. Basic

II.1.1. Number of sensors

By changing the phantom size, we can alter the number of sensors. As the number of sensors increase, the scanning area of each sensor will decrease. From the above figures, the drop in the number of sensors will decrease the resolution of the reconstructed images. In addition, the boundaries become harder to define and the contrast of the images becomes poor as well. Therefore, we conclude that, the more the sensors, the higher the quality of the reconstructed images.

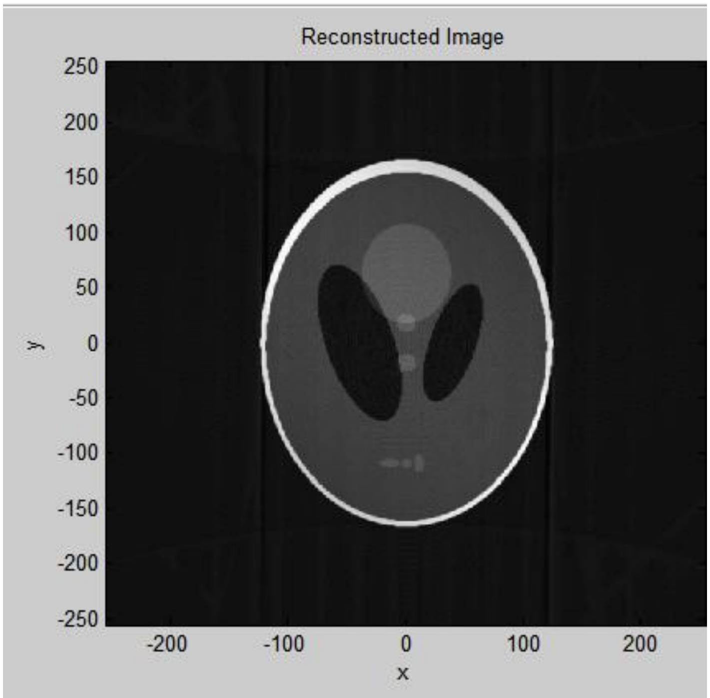

Figure 3: Number of sensors is 512 (reconstructed)

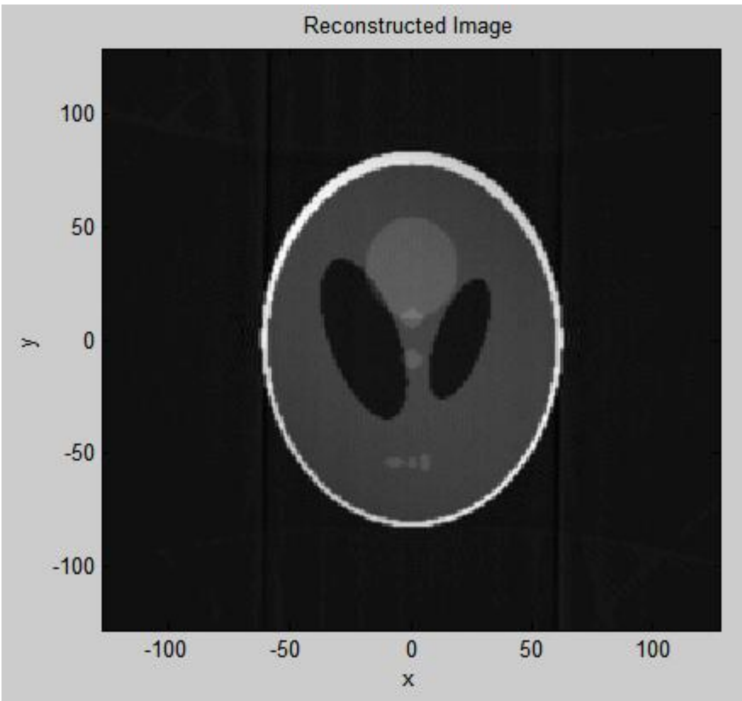

Figure 4: Number of sensors is 256 (reconstructed)

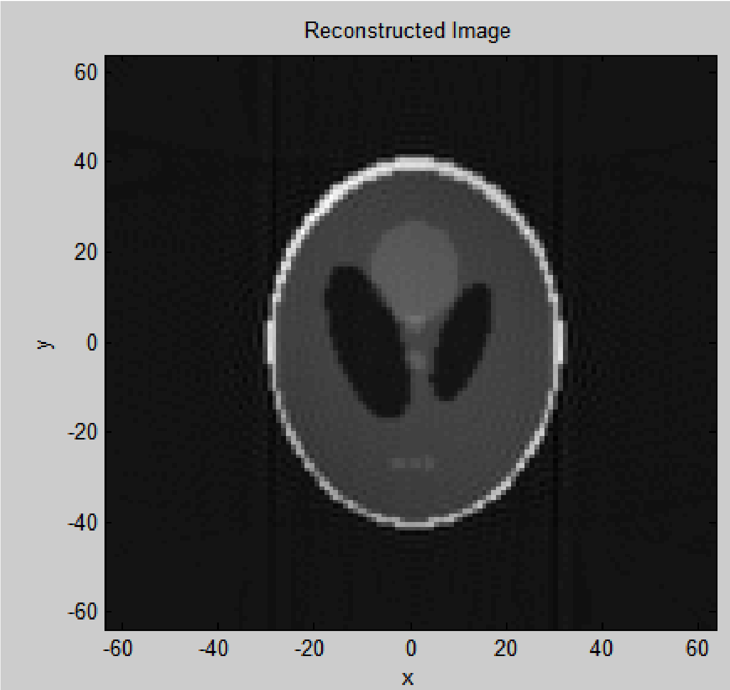

Figure 5: Number of sensors is 128 (reconstructed)

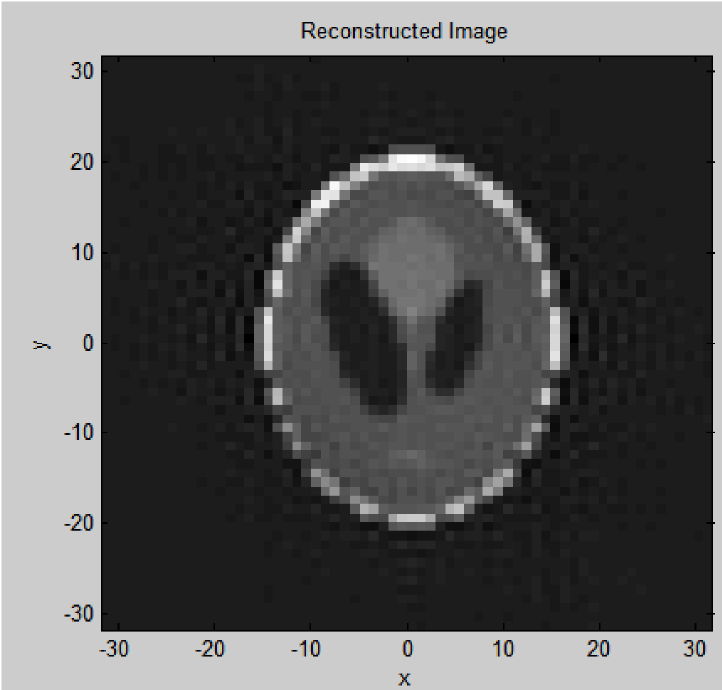

Figure 6: Number of sensors is 64 (reconstructed)

II.1.2. Number of projection slices

As the number of projection slices decreases, the reconstructed images become blurry and have many artifacts. The artifacts arise from the missing data due to lack of number of slice. The sharpness can be improved by capturing more slices; this will also significantly reduce artifacts.

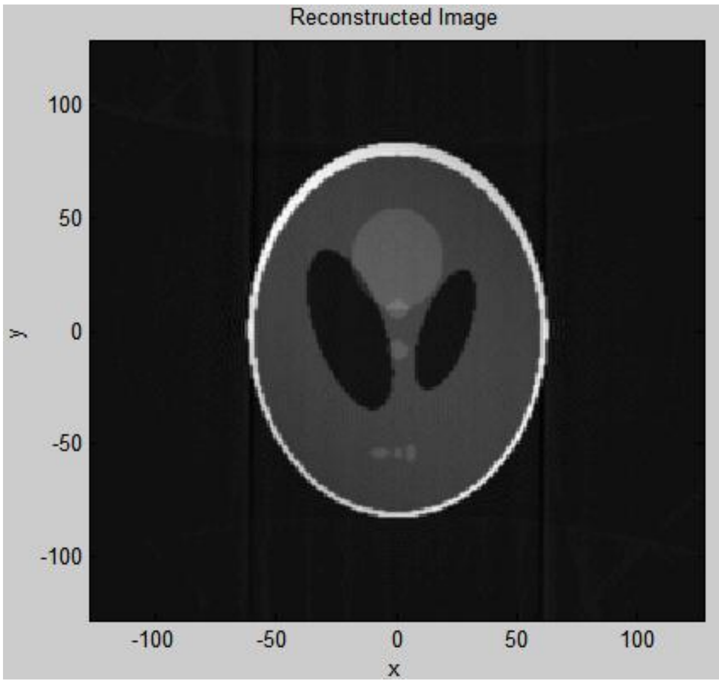

Figure 7: 180 slices covering 180^o, 1^o per slice

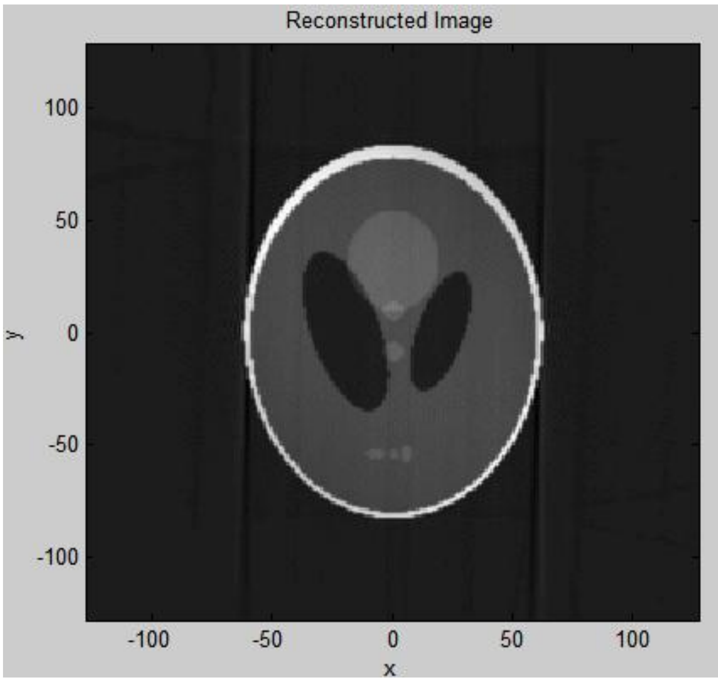

Figure 8: 90 slices covering 180^o, 2^o per slice

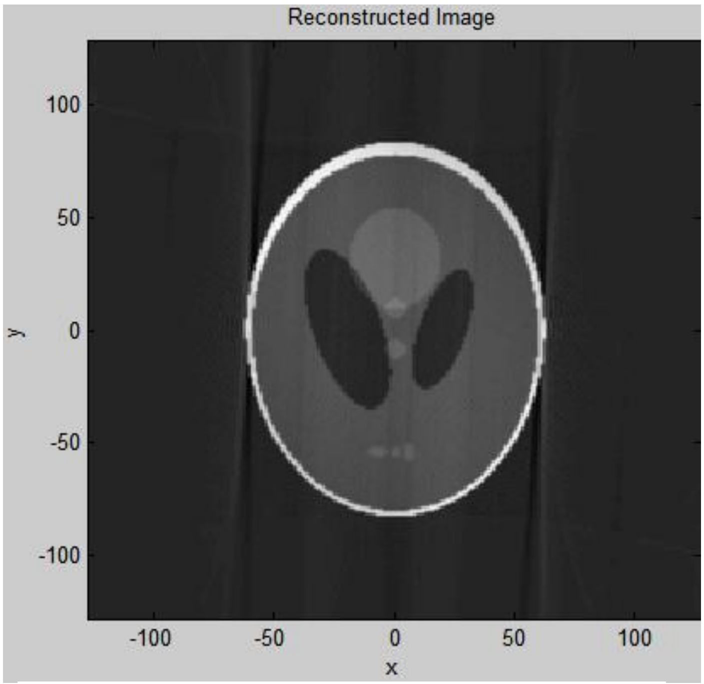

Figure 9: 45 slices covering 180^o, 4^o per slice

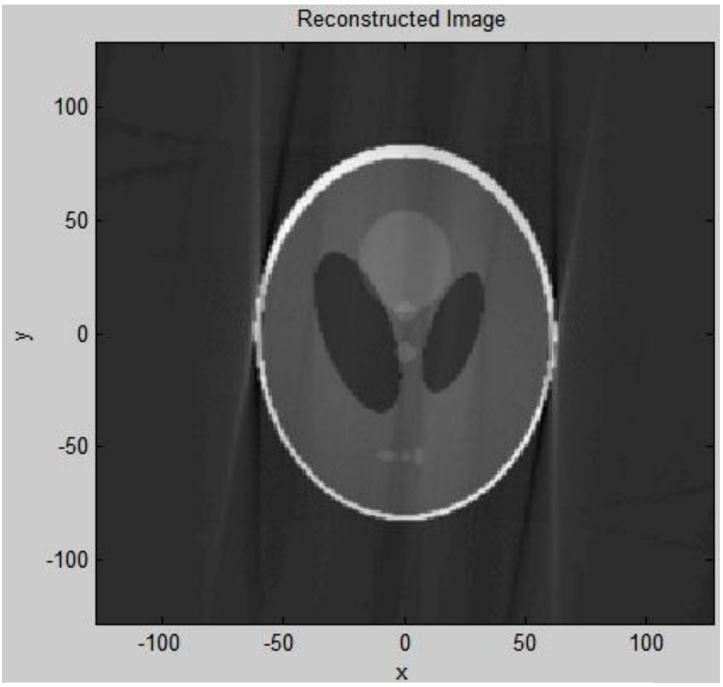

Figure 10: 20 slices covering 180^o, 9^o per slice

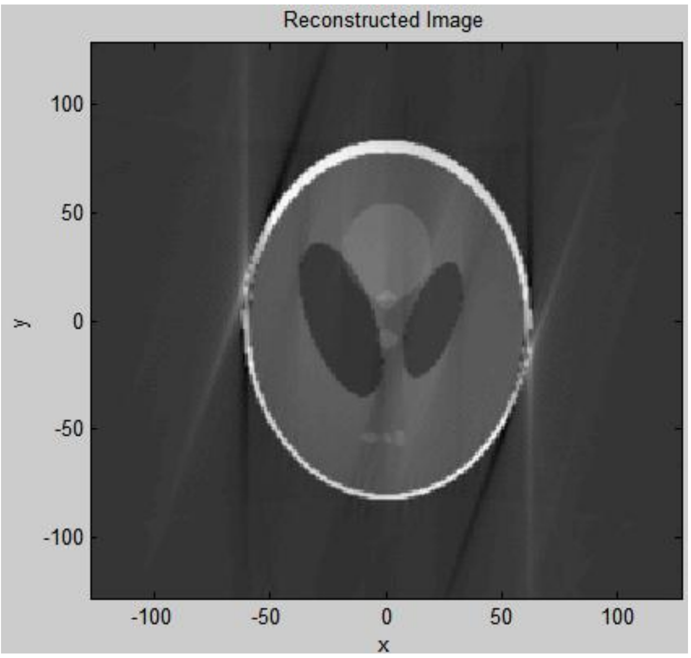

Figure 11: 10 slices covering 180°, 18° per slice

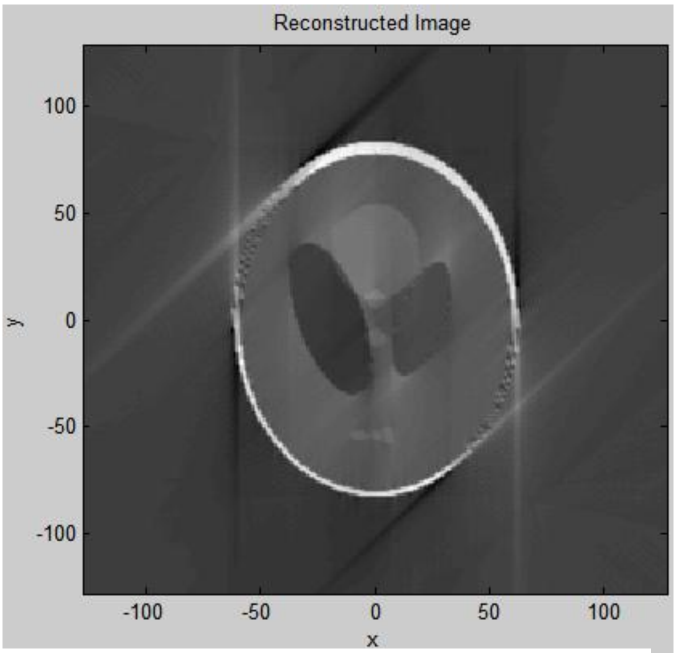

Figure 12: 4 slices covering 180°, 45° per slice

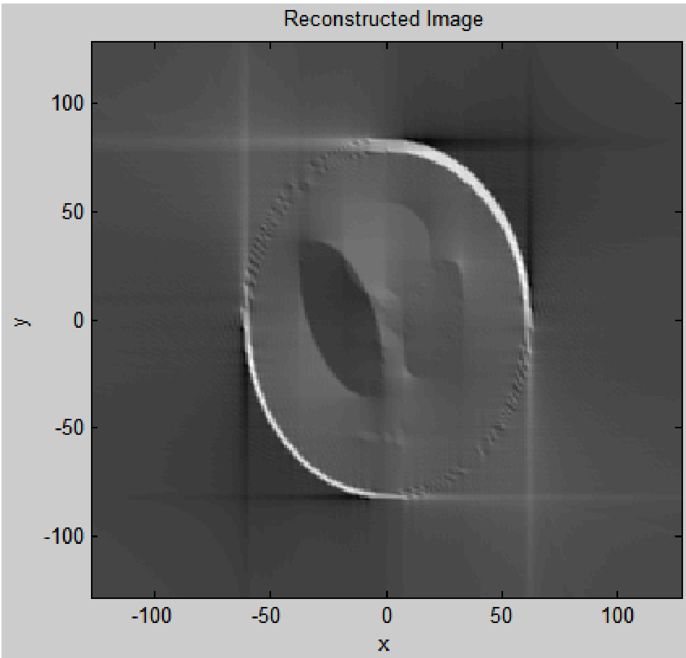

Figure 13: 2 slices covering 180°, 90° per slice

II.1.3. Scan angle (<180, >180)

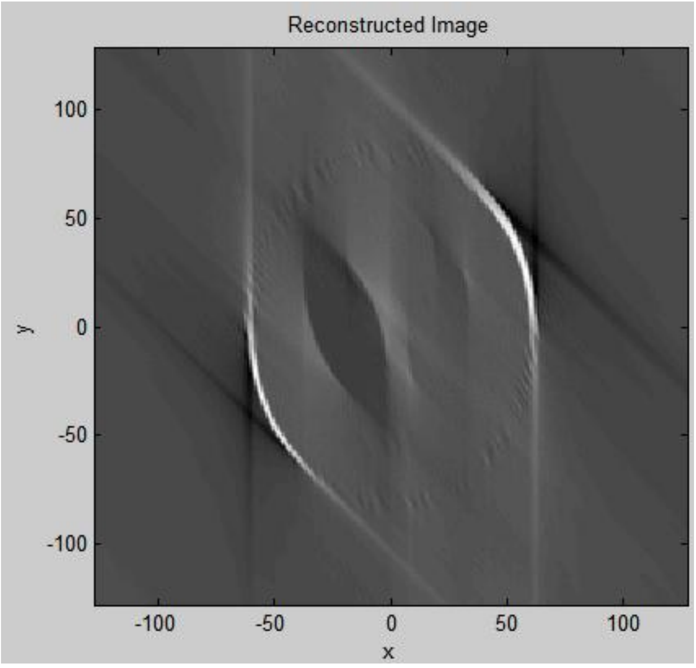

Figure 14: Scan from 0^o to 45^o, 1^o per slice

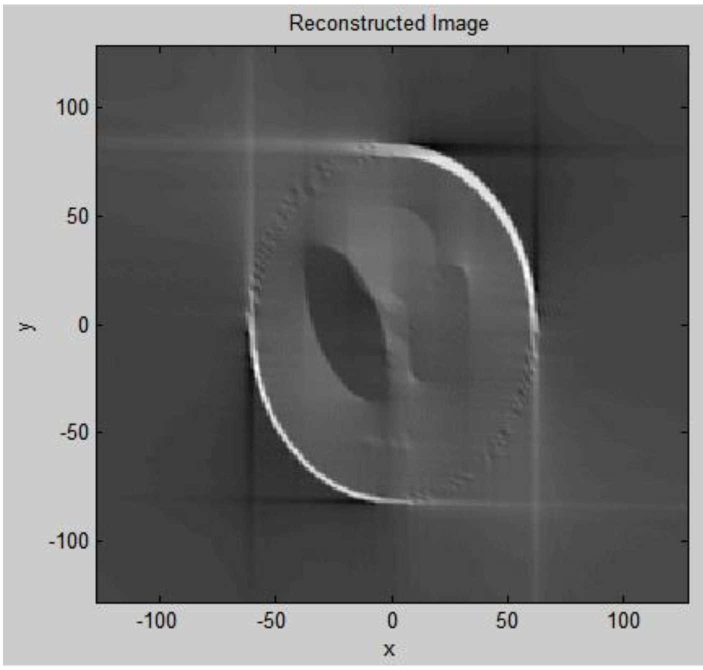

Figure 15: Scan from 0^o to 90^o, 1^o per slice

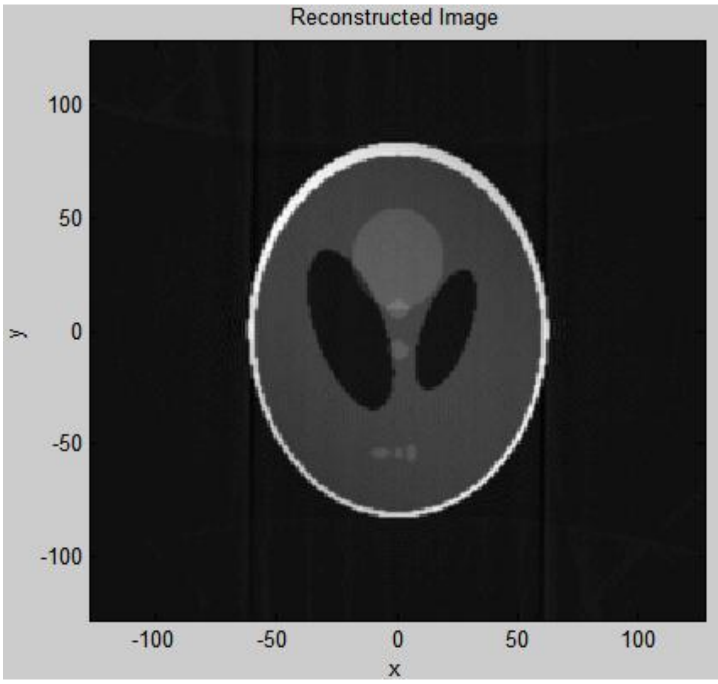

Figure 16: Scan from 0^o to 180^o, 1^o per slice

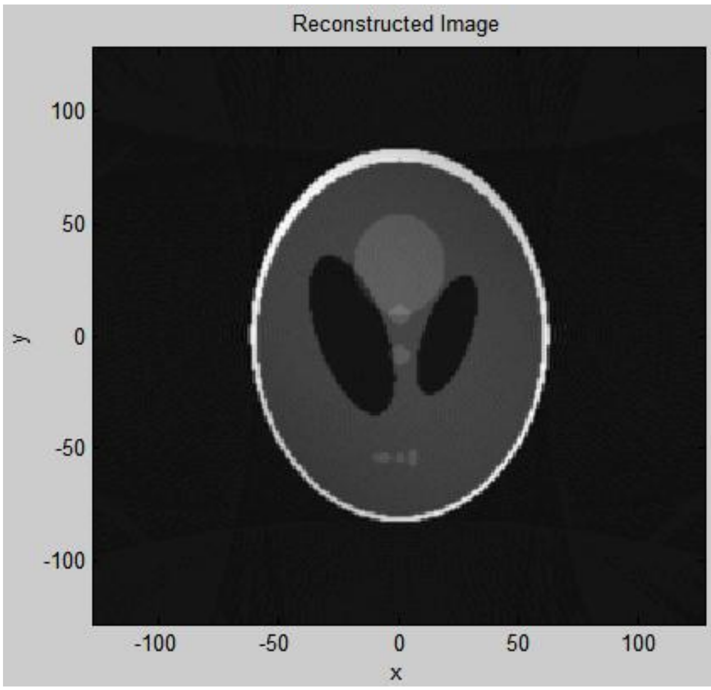

Figure 17: Scan from 0^o to 270^o, 1^o per slice

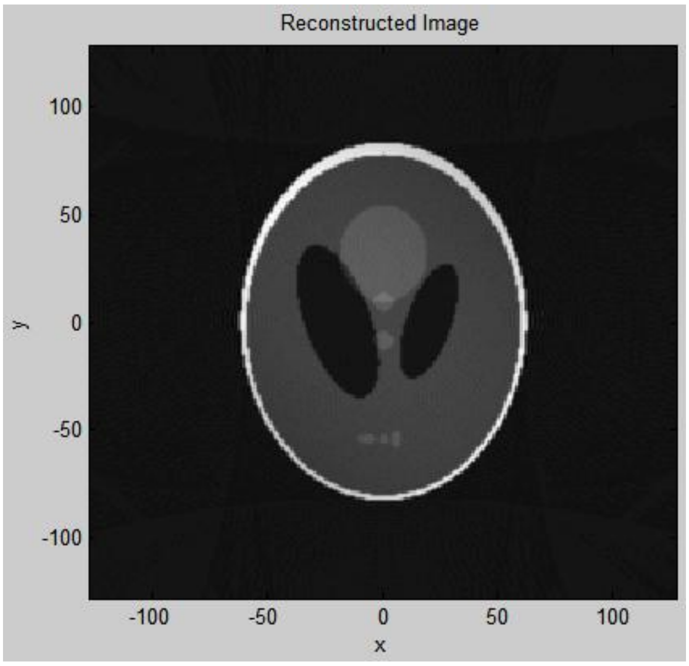

Figure 18: Scan from 0^o to 360^o, 1^o per slice

The image resolution increases as the scanning angle increases from 45^o to 180^o. From 180^o to 270^o, there will be fewer artifacts and from 270^o onwards, there is not observable change.

II.1.4. Interpolation method

While comparing different methods of interpolations, it is clear that using nearest point method will give lots of artifacts than the others. Although none of the above four methods can give a completely artifact-free reconstructed image, linear, spline and cubic methods are not too bad, the quality of the reconstructed images are quite high in fact.

With regarding the computation time, using nearest point and linear methods give relatively shorter period to compute the images than using spline and cubic methods. Hence, among the four methods mentioned, using linear method will produce a high quality images as well as using less computation time.

However, while considering the contrast of the image, the linear one gives the less contrast, whereas, the cubic method ranks the second last. Unlikely, the spline method and nearest point methods provide with the greatest contrast.

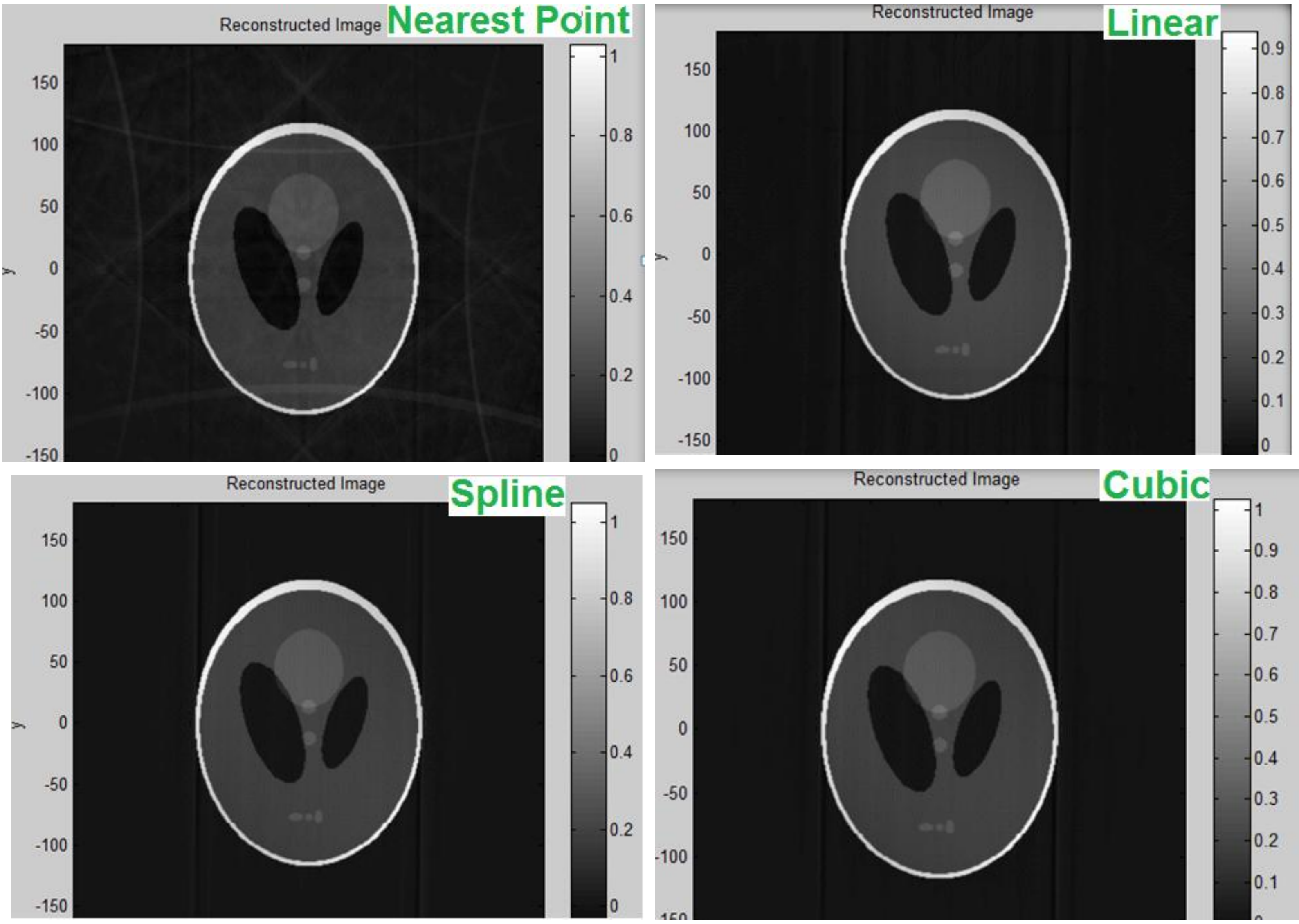

Figure 19: Different interpolation methods

II.2. Advanced

II.2.1. Noise

There must be noise in real-life CT scanning such as background noise or interference from other machines. In order to simulate the actual performance of CT scanning, we need to manually input some noise. In our program, we add signal-to-noise ratio (SNR) after the Radon transform. In this part, we will focus on how the SNR and image quality are related to the number of sensors, the angle of each slice and the method of interpolation.

There is no observable change in the image when we adjust the number of slices from 180 to 720, the method of interpolation (linear, cubic, spline) or the ration of oversampling.

Relation between noise and number of sensors

With higher SNR, the image quality increase as the number of sensors increase. However with lower SNR, increase the number sensors can produce a more detailed image but the contrast decrease.

First simulation: SNR = 30 dB, angle of each slice = 1° and linear interpolation

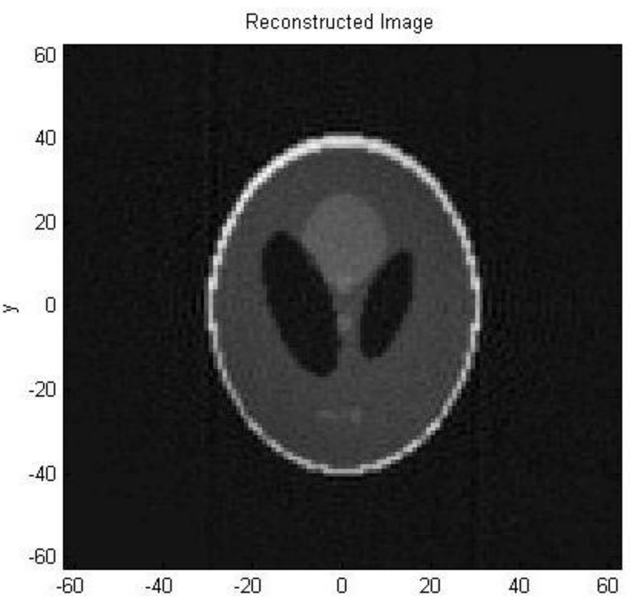

Figure 20: Number of sensors = 129

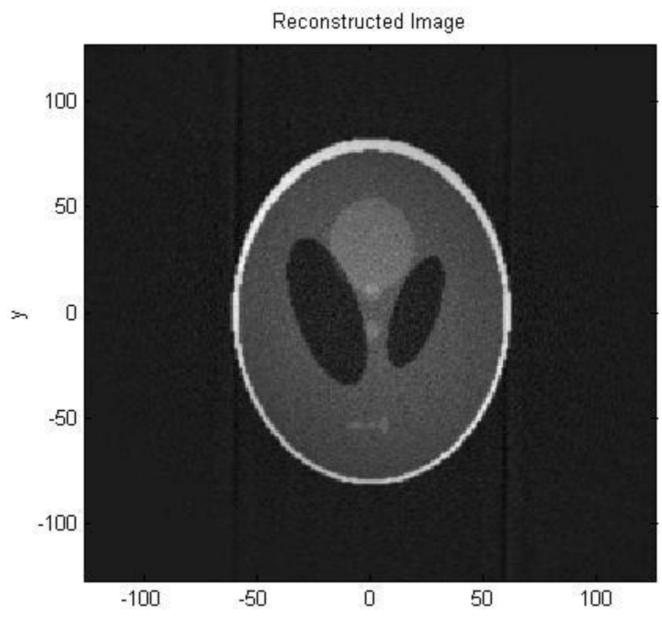

Figure 21: Number of sensors = 255

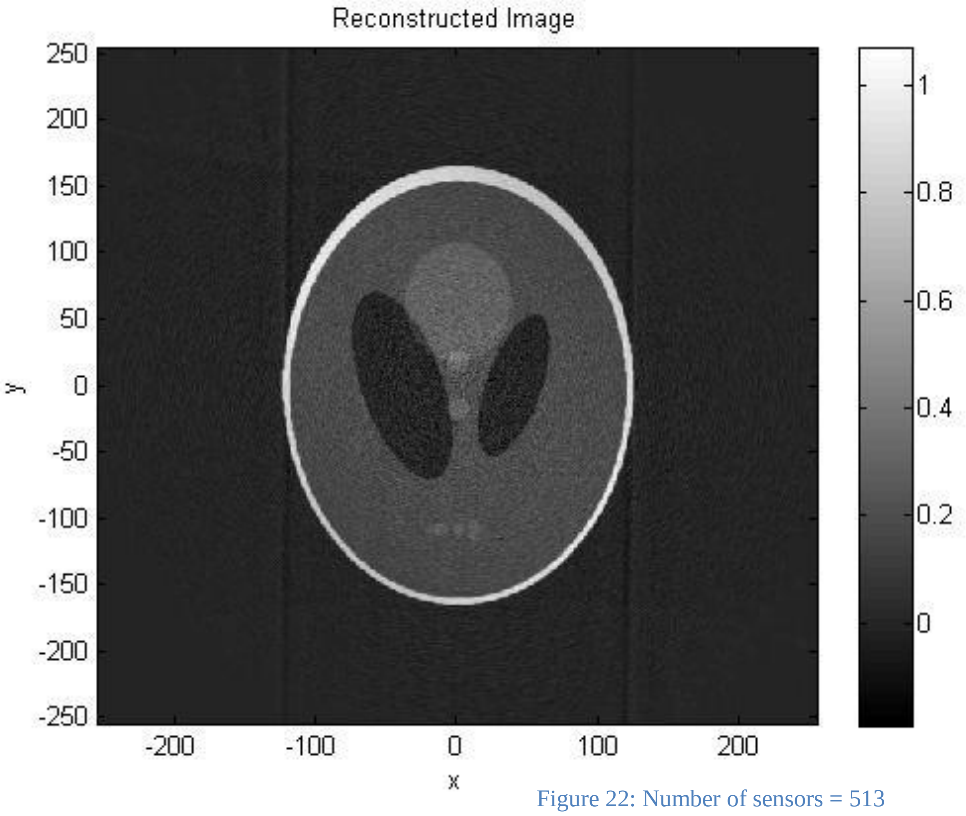

Figure 22: Number of sensors = 513

Second simulation: SNR = 15 dB, angle of each slice = 1^{o} and linear interpolation

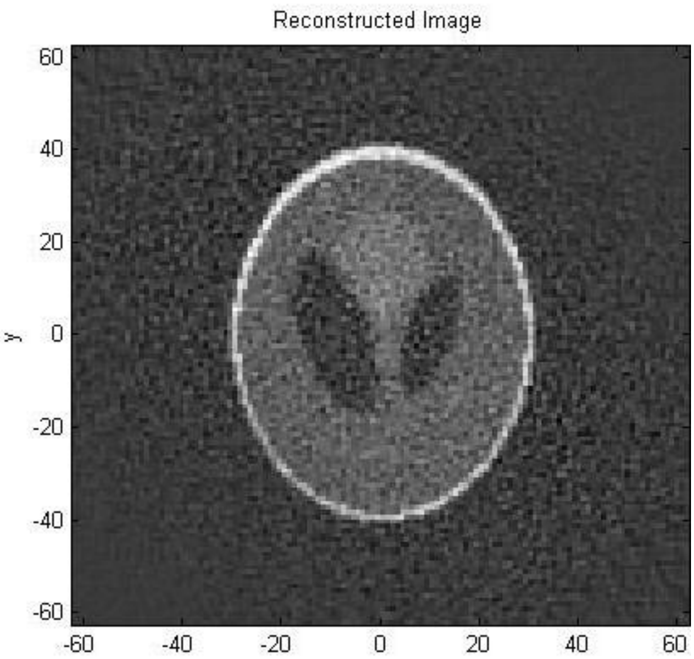

Figure 23: Number of sensors = 129

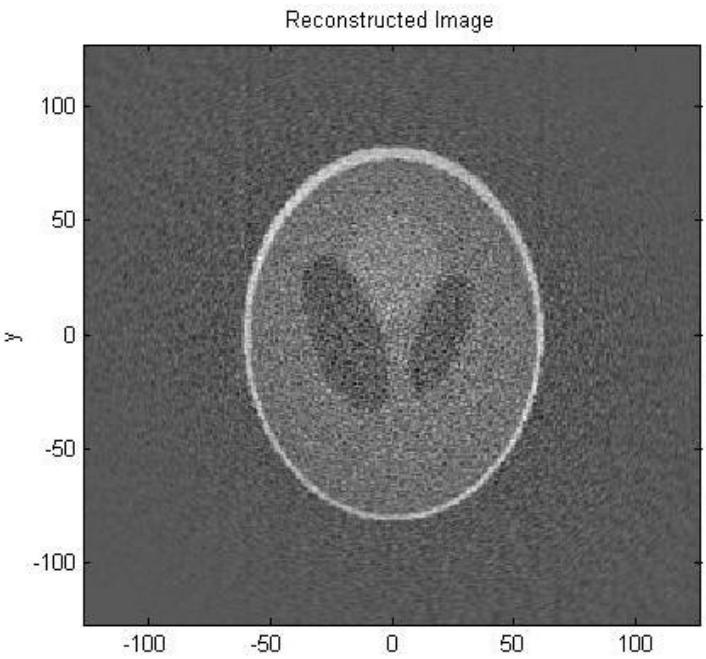

Figure 24: Number of sensors = 255

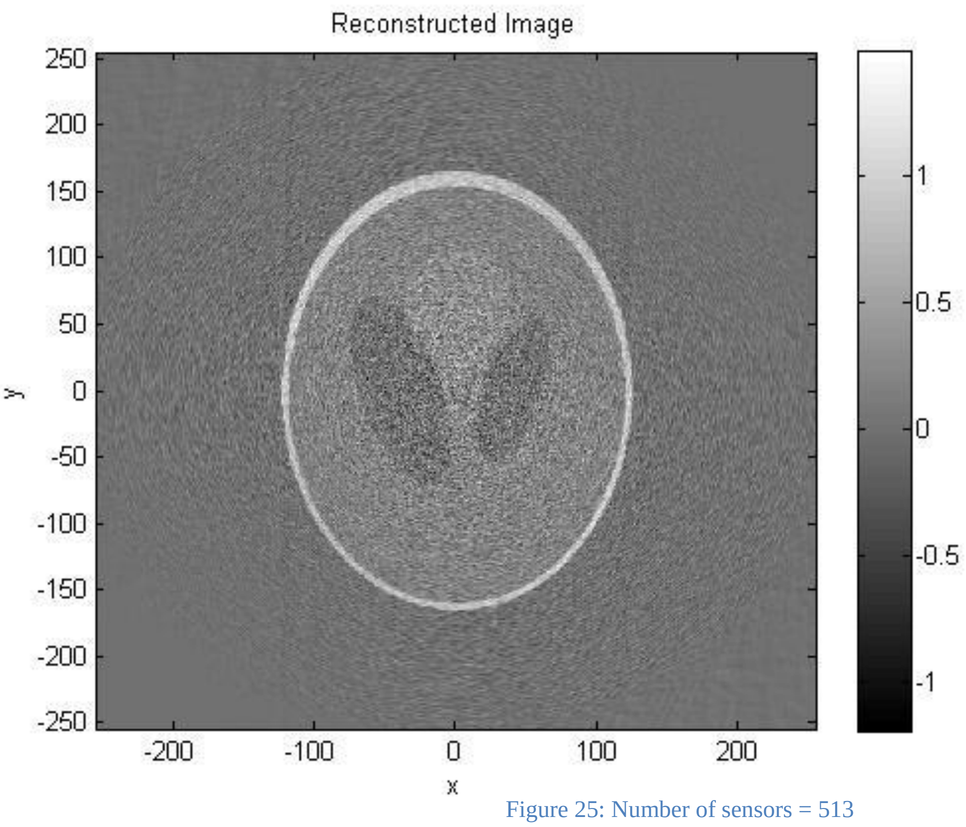

Figure 25: Number of sensors = 513

II.2.2. Sensor Damage

In the sinogram, each s value in the vertical axis corresponds to an x-ray sensor. Every damaged sensor contributes to a black straight horizontal line on the sinogram.

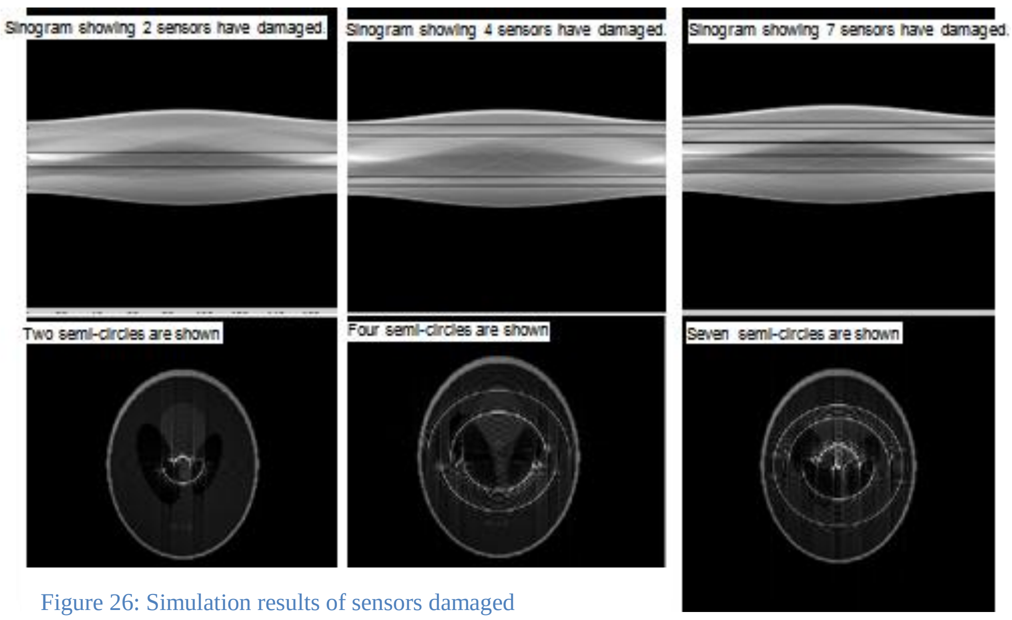

Figure 26: Simulation results of sensors damaged

Obviously, the damaged sensors corrupt the reconstructed image significantly. Each black line in the sinogram corresponds to a semi-circle artifact in the reconstructed image. The semi-circle appears in the upper half when the black line in the sinogram has a positive value; while the semi-circle appears in the lower half when the black line in the sinogram has a negative value.

To deal with this damaged sensors problem without replacing those sensors, we tried to scan the object by 360^{o} instead of 180^{o}, however this make the image quality even worse.

Conclusion

Direct Fourier reconstruction could be simulated in the processes below using Matlab. Firstly, we do Radon Transform with the Phantom and then secondly with a 1-D Fourier Transform. Thirdly, convert the data from polar coordinate system to Cartesian coordinate system. Fourthly, we apply inverse 2-D Fourier Transform to the interpolated data to give the reconstructed image.

Apart from these, we have additionally tested on how the reconstructed images' quality change when we altered number of sensors, number of projection slices, degree of covering angles, and signal to noise ratio. The consequence of having damaged sensors is also investigated.

Direct Fourier Reconstruction uses efficient algorithm to give a good quality image, with necessary details in the Phantom can be conserved in the reconstructed images. To improve the quality of reconstructed images, we can use more sensors, and more projection slices, scan the subject more than 180^{o} and replace all damaged sensors.

Even DFR is efficient, filtered backprojection is preferred in clinics application. The procedure of this method is only composed by two steps. The first one is to filter the projection and the second one is to backproject this filtered projection. This reconstruction needs only 1D operation, so it required less calculation and hence, the time execution becomes faster, that's why this method is preferably used

References

1. Forrest Sheng Bao, 2008, "FT, STFT, DTFT, DFT and FFT, revisited", Forrest Sheng Bao, http://narnia.cs.ttu.edu/drupal/node/46