信号与系统第三章

3-3

(1) $f(t)$为偶函数

$a_0=\frac{4}{T}\int_0^{\frac{T}{2}} f(t)\cos(wt)dt$

$=\frac{4E}{T}\int_0^{\frac{\tau}{2}}\cos wt\,dt$

$=\frac{4E}{wT}\sin w\frac{\tau}{2}$

$=\frac{2E}{\pi}\sin w\frac{\tau}{2}$

$=\frac{2E\tau}{\pi}Sa(w\frac{\tau}{2})$

$=\frac{E\tau w_1}{\pi}Sa(w\frac{\tau}{2})$

$a_n=\frac{E\tau w}{\pi}Sa(nw\frac{\tau}{2})$

$b_n=0$

谱线间隔 $w_1=\frac{2\pi}{T}=\frac{2\pi}{10^{-6}}$ (s)

$f_1=\frac{1}{T}=10^6 HZ=10^3 KHZ$

第一零点

$n=1$ $Sa(w\frac{\tau}{2})=0$ 时

$w_1\frac{\tau}{2}=\pi$

$w_1=\frac{2\pi}{\tau}$

$\Delta f_1=\frac{w_1}{2\pi}=\frac{1}{\tau}=2\times10^6 HZ$

$=2\times10^3 KHZ$

(2) 同 (1)

$f_2=\frac{1}{T}=\frac{1000}{3}KHZ$

$\Delta f_2=\frac{1}{\tau_2}=\frac{2000}{3}KHZ$

(3) $A_{11}=\frac{2E_1}{\pi}\sin(w_1\frac{\tau_1}{2})=\frac{2\times1}{\pi}|\sin(\frac{\pi\tau_1}{T_1})|$

$A_{21}=\frac{2E_2}{\pi}\sin(w_2\frac{\tau_2}{2})=\frac{2\times3}{\pi}|\sin(\frac{\pi\tau_2}{T_2})|$

$\frac{A_{11}}{A_{12}}=\frac{1}{3}$

(4) $A_{11}=\frac{2\times1}{\pi}|\sin(\frac{\pi\tau_1}{T_1})|$ $A_{11}:A_{23}=1:1$

$A_{23}=\frac{2\times3}{3\pi}|\sin(\frac{3\pi\tau_2}{T_2})|$

3-6

$a_0=\frac{1}{T}\int_0^T E(1-\frac{t}{T})dt$

$=\frac{E}{2}$

$a_n=\frac{2}{T}\int_0^T E(1-\frac{t}{T})\cos(nwt)dt$

$=\frac{2E}{nwT}[E(1-\frac{t}{T})\sin(nwt)]\Big|_0^T+\frac{E}{T}\int_0^T\sin nwt\,dt$

$=0$

$b_n=\frac{2}{T}\int_0^T E(1-\frac{t}{T})\sin(nwt)dt$

$=\frac{2E}{T}[\int_0^T\sin(nwt)dt+\int_0^T\frac{t}{nwT}d[\cos(nwt)]]$

$=\frac{1}{nw}\frac{2E}{T}=\frac{E}{n\pi}$

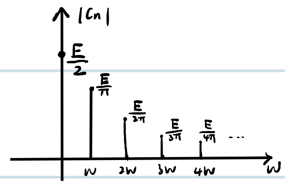
$f(t)=\frac{E}{2}+\frac{E}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin(nwt)$

$F(nw)=\frac{(a_n-b_nj)}{2}=\frac{-Ej}{2n\pi}$

$F_0=a_0$

$f(t)=\sum_{n=-\infty}^{\infty}F_n e^{jnwt}$

$=\frac{E}{2}+\sum_{n=-\infty, n\neq0}^{\infty}\frac{-E}{2\pi n}j e^{jnwt}$



3-8

(a) $a_0=\frac{1}{T}\int_0^T f(t)dt$

$=\frac{E}{2}$

$a_n=\frac{2}{T}[\int_{-\frac{T}{4}}^{\frac{T}{4}}(\frac{2E}{T}t+\frac{E}{2})\cos(nwt)dt+\int_{\frac{T}{4}}^{\frac{3T}{4}}(\frac{3E}{2}-\frac{2E}{T}t)\cos(nwt)dt]$

$=\frac{2}{nwT}[(\frac{2E}{T}t+\frac{E}{2})\sin(nwt)\Big|_{-\frac{T}{4}}^{\frac{T}{4}}-\int_{-\frac{T}{4}}^{\frac{T}{4}}\frac{2E}{T}\sin(nwt)dt]$

$+\frac{2}{nwT}[(\frac{3E}{2}-\frac{2E}{T}t)\sin(nwt)\Big|_{\frac{T}{4}}^{\frac{3T}{4}}+\int_{\frac{T}{4}}^{\frac{3T}{4}}\frac{2E}{T}\sin(nwt)dt]$

$=0$

$b_n=\frac{2}{T}[\int_{-\frac{T}{4}}^{\frac{T}{4}}(\frac{2E}{T}t+\frac{E}{2})\sin(nwt)dt+\int_{\frac{T}{4}}^{\frac{3T}{4}}(\frac{3E}{2}-\frac{2E}{T}t)\cos(nwt)dt]$

$=\frac{4E}{n^2\pi^2}\sin(\frac{n\pi}{2})$

$\Rightarrow f(t)=\frac{E}{2}+\sum_{n=1}^{\infty}[\frac{4E}{n^2\pi^2}\sin(\frac{n\pi}{2})\sin(nwt)]$

(b) 偶函数 $b_n=0$

$a_0=\frac{1}{T}\int_0^T f(t)dt$

$=\frac{3}{4}E$

$$f(t)=\begin{cases}\frac{4E}{T}t & 0\le t\le\frac{T}{4}\\ E & \frac{T}{4}\le t\le\frac{3T}{4}\\ 4E-(\frac{4E}{T})t & \frac{3T}{4}\le t\le T\end{cases}$$

$a_n=\frac{2}{T}\int_0^T f(t)\cos(nwt)dt$

$=\frac{2}{T}[\int_0^{\frac{T}{4}}\frac{4E}{T}t\cos(nwt)dt+\int_{\frac{3T}{4}}^{T}4E(1-\frac{t}{T})\cos(nwt)dt+\frac{-2E}{nw}]$

$=\frac{2}{T}[2\cdot\frac{4E}{T}\frac{t}{nw}\sin(nwt)\Big|_0^{\frac{T}{4}}+2\int_0^{\frac{T}{4}}\frac{4E}{Tn^2w^2}d\cos(nwt)-\frac{E}{nw}]$

$=\frac{4}{T}[\frac{4E}{n^2Tw^2}(\cos\frac{n\pi}{2}-1)]$

$=-\frac{4E}{n^2\pi^2}(1-\cos(\frac{n\pi}{2}))$

$f(t)=\frac{3E}{4}-\frac{4E}{\pi^2}\sum_{n=1}^{\infty}\frac{[1-\cos(\frac{n\pi}{2})]}{n^2}\cos(nwt)$.

3-9 $i(t)=I_m\frac{\cos(wt)-\cos\theta}{1-\cos\theta}$

(1) 为偶函数

$a_0=\frac{I_m}{T}\int_{-\theta/w}^{\theta/w}\frac{\cos(wt)-\cos\theta}{1-\cos\theta}dt$

$=\frac{I_m}{wT}(\frac{2\sin\theta}{1-\cos\theta}-\frac{2\cos\theta}{1-\cos\theta}\theta)=\frac{I_m}{\pi}(\frac{\sin\theta-\theta\cos\theta}{1-\cos\theta})$

$a_n=\frac{2}{T}\int_{-\theta/w}^{\theta/w}\frac{\cos(wt)-\cos\theta}{1-\cos\theta}\cos(nwt)dt\ I_m$

$=\frac{1}{T}\int_{-\theta/w}^{\theta/w}\frac{\cos[(n+1)wt]+\cos[(n-1)wt]}{1-\cos\theta}dt-\frac{2}{T}\int_{-\theta/w}^{\theta/w}\frac{\cos\theta\cos(nwt)}{1-\cos\theta}dt\ I_m$

$=\frac{2}{Tw(1-\cos\theta)}\{\frac{\sin[(n+1)\theta]}{n+1}+\frac{\sin[(n-1)\theta]}{n-1}\}-\frac{4}{wTn}\frac{\cos\theta\sin n\theta}{1-\cos\theta}I_m$

$=\frac{2}{2\pi(1-\cos\theta)}\{\frac{\sin[(n+1)\theta]}{n+1}-\frac{\sin[(n+1)\theta]}{n}-\frac{\cos[(n-1)\theta]}{n}+\frac{\sin[(n-1)\theta]}{n-1}\}I_m$

$=\frac{2I_m}{\pi(1-\cos\theta)}\cdot\frac{\sin n\theta\cos\theta-n\cos(n\theta)\sin\theta}{(n^2-1)\cdot n}$ $(n\neq1)$ $a_1=\frac{I_m}{\pi}\frac{(\theta-\sin\theta\cos\theta)}{\pi(1-\cos\theta)}$

$I_0=a_0=\frac{I_m}{\pi(1-\cos\theta)}(\sin\theta-\theta\cos\theta)$

$I_1=a_1=\frac{I_m}{\pi(1-\cos\theta)}(\theta-\sin\theta\cos\theta)$

$I_K=a_K=\frac{2I_m}{\pi(1-\cos\theta)}\frac{\sin n\theta\cos\theta-n\cos n\theta\sin\theta}{(n^2-1)n}$ $K>2$

(2) $\theta = 60^\circ$

$$I_0 = a_0 = \frac{I_m}{\pi(1-\cos\theta)}(\sin\theta - \theta\cos\theta) = \frac{2I_m}{\pi}\left(\frac{\sqrt{3}}{2} - \frac{\pi}{3}\cdot\frac{1}{2}\right) \approx 2.18\, I_m$$

$$I_1 = a_1 = \frac{I_m}{\pi(1-\cos\theta)}(\theta - \sin\theta\cos\theta) = \frac{2I_m}{\pi}\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right) \approx 0.391\, I_m$$

$$I_k = a_k = \frac{2I_m}{\pi(1-\cos\theta)}\ \frac{\sin n\theta\cos\theta - n\cos n\theta\sin\theta}{(n^2-1)n} = \frac{2I_m}{\pi}\left(\frac{\sin k\frac{\pi}{3} - k\cos k\frac{\pi}{3}\sqrt{3}}{(k^2-1)k}\right)$$

(3) $I_0 = \left|\frac{I_m}{\pi}\right| = 0.318\, I_m$

$$I_1 = \frac{I_m}{2} = 0.5\, I_m$$

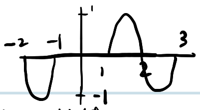
$$I_k = \frac{2I_m}{\pi(k^2-1)}\cos\frac{\pi}{2}k$$

$$= \frac{2\cos\frac{\pi}{2}k}{\pi(k^2-1)} I_m$$

3-11

$f(t-1)$ 为奇函数 $g(t)$

$g(t) = f(t+1)$



$w' = \frac{2\pi}{4} = \frac{\pi}{2}$

$a_0' = 0$

$a_n' = 0 \quad (n \neq 2)$

$$b_n' = \frac{4}{4}\int_0^2 g(t)\cdot\sin(nwt)\,dt$$

$$= \int_1^2 \sin[2w(t-1)]\sin(nwt)\,dt$$

$$= -\frac{1}{2}\int_1^2 \cos[(n+2)wt - 2w] - \cos[(n-2)wt + 2w]\,dt$$

$$= -\frac{1}{2}\frac{1}{w}\left\{\frac{\sin[(n+2)wt - 2w]}{n+2} - \frac{\sin[(n-2)wt+2w]}{n-2}\right\}\Big|_1^2$$

$$= -\frac{1}{2w}\left\{\left[\frac{\sin[(n+1)\pi]}{n+2} - \frac{\sin[n\frac{\pi}{2}]}{n+2}\right] - \left[\frac{\sin[(n-1)\pi]}{n-2} - \frac{\sin[n\frac{\pi}{2}]}{n-2}\right]\right\}$$

$$= -\frac{1}{2w}\left\{\frac{4}{n^2-4}\sin\left(n\frac{\pi}{2}\right)\right\}$$

$$= -\frac{4}{\pi(n^2-4)}\left|\sin\left(n\frac{\pi}{2}\right)\right| = \frac{4}{(4-n^2)\pi}\left|\sin\left(n\frac{\pi}{2}\right)\right|$$

$a_2' = -\frac{1}{2}$

$g(t) \to f(t) = g\left(t + \frac{7}{4}\right)$

$f(t) = -\frac{1}{2}\cos(2wt)$ 相位右移 $\frac{\pi}{2}$

$= \frac{1}{2}\sin(2wt)$

$b_2 = \frac{1}{2}$

$$a_n = \frac{4}{(4-n^2)\pi}\left|\sin\left(n\frac{\pi}{2}\right)\right| = \begin{cases} 0 & n\text{为偶} \\ \frac{4}{(4-n^2)\pi} & n\text{为奇} \end{cases}$$

$$f(t) = \frac{4}{\pi}\left[\frac{1}{3}\cos(wt) - \frac{1}{5}\cos(3wt) - \frac{1}{21}\cos(5wt)\cdots\right] + \frac{1}{2}\sin(2wt)$$

(2) $f_2(t) = f_1\left(t+\frac{1}{2}\right) - f_1\left(t-\frac{1}{2}\right)$

$$F_n = \frac{1}{4}\int_{-\frac{1}{2}}^{\frac{3}{2}} \sin\left(\pi\left(t+\frac{1}{2}\right)\right)e^{-jn\frac{\pi}{2}t}\,dt - \int_{-\frac{1}{2}}^{\frac{3}{2}} \sin\left(\pi\left(t-\frac{1}{2}\right)\right)e^{-jn\frac{\pi}{2}t}\,dt$$

$$= \frac{1}{4}\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos(\pi t)\, e^{-jn\frac{\pi}{2}t}\,dt - \int_{\frac{1}{2}}^{\frac{3}{2}} \cos(\pi t)\, e^{-jn\frac{\pi}{2}t}\,dt$$

$$= \frac{1}{4}\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{e^{j\pi t} + e^{-j\pi t}}{2} e^{-jn\frac{\pi}{2}t}\,dt - \int_{\frac{1}{2}}^{\frac{3}{2}} \frac{e^{j\pi t} + e^{-j\pi t}}{2} e^{-jn\frac{\pi}{2}t}\,dt$$

$$= \frac{1}{8}\left(\frac{e^{j\pi t(1-\frac{n}{2})}}{\pi j - n\frac{\pi}{2}j} - \frac{e^{-j\pi t(1+\frac{n}{2})}}{\pi j(1+\frac{n}{2})}\right)\Bigg|_{-\frac{1}{2}}^{\frac{1}{2}} + \Bigg|_{\frac{1}{2}}^{\frac{3}{2}}$$

$$= \frac{1}{8\pi j}\left(\frac{(1+\frac{n}{2})e^{j\pi t(1-\frac{n}{2})}}{(1-\frac{n^2}{4})} - \frac{(1-\frac{n}{2})e^{-j\pi t(1+\frac{n}{2})}}{(1-\frac{n^2}{4})}\right)\Bigg|_{-\frac{1}{2}}^{\frac{1}{2}} \Bigg|_{\frac{1}{2}}^{\frac{3}{2}}$$

$$= \frac{1}{2(8-2n^2)\pi j}\left((n+2)e^{j\pi\frac{3}{2}(1-\frac{n}{2})} + (n-2)e^{-j\pi\frac{3}{2}(1+\frac{n}{2})}\right)\times 2$$

$$- \frac{1}{2(8-2n^2)\pi j}\left((n+2)e^{j\frac{\pi}{2}(1-\frac{n}{2})} + (n-2)e^{j\frac{\pi}{2}(1+\frac{n}{2})}\right)$$

$$- \frac{1}{2(8-2n^2)\pi j}\left((n+2)e^{j\frac{1}{2}\pi(1-\frac{n}{2})} + (n-2)e^{-\frac{j\pi}{2}(1-\frac{n}{2})}\right)$$

$$= \frac{1}{2(8-2n^2)\pi j}\left[e^{-j\pi n\frac{3}{4}}((n+2)(-j) + (n-2)j)2 - e^{j\pi n\frac{1}{4}}((-j)(n+2) + j(n-2)) - e^{-j\pi n\frac{7}{4}}((n+2)j + (-j)(n-2))\right]$$

$$= \frac{-1}{2(8-2n^2)\pi j}\left[-8j\, e^{-j\pi n\frac{3}{4}} + 4j\, e^{j\pi n\frac{1}{4}} + 4j\, e^{-j\pi n\frac{7}{4}}\right]$$

$$= \frac{1}{(n^2-4)\pi j}\left[-2j\, e^{-j\pi n\frac{3}{4}} + j\, e^{j\pi n\frac{1}{4}} + j\, e^{-j\pi n\frac{7}{4}}\right]$$

$$= \frac{1}{\pi(n^2-4)}\left[-2e^{-j\pi n} + 1 + e^{j2\pi n}\right]e^{j\pi n\frac{1}{4}}$$

$$= \frac{1}{\pi(n^2-4)}\left[-2(\cos\pi n) + 1 + 1\right]e^{j\pi n\frac{1}{4}}$$

$$= \frac{-2}{\pi(n^2-4)}\left[\cos(\pi n) - 1\right]\left(\cos\frac{1}{4}n\pi + j\sin\frac{1}{4}n\pi\right)$$