**This is a preliminary version of the slides that will be used for tutorials.**

**The slides will be revised to reflect recent studies and recommended improvements.**

**The final version may differ from this version.**

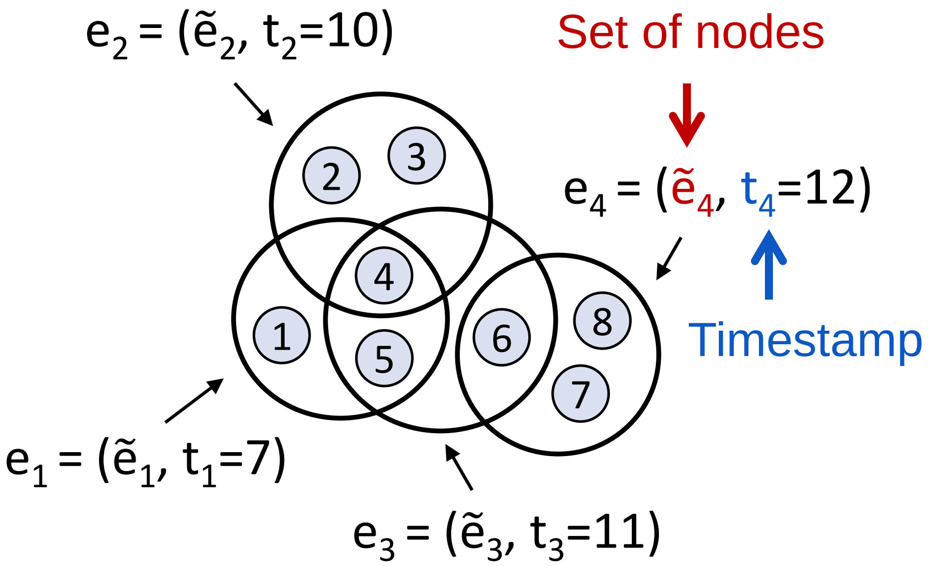
# Mining of Real-world Hypergraphs: Concepts, Patterns, and Generators

# Part III. Dynamic Structural Patterns

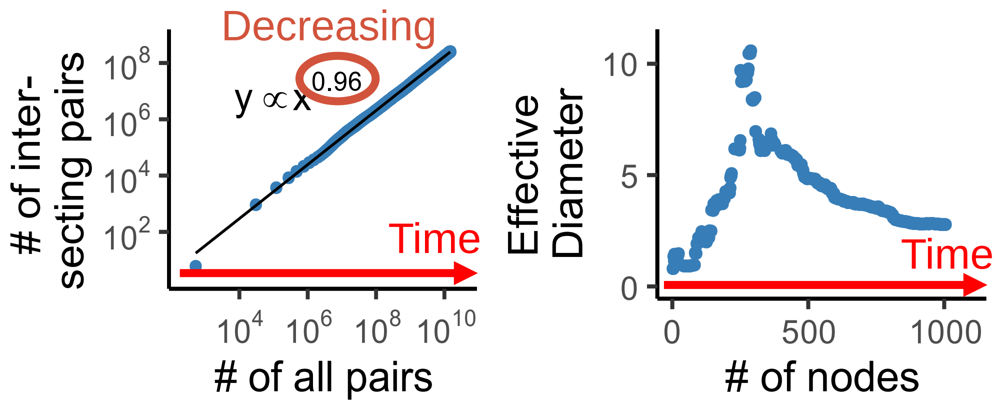

Geon Lee    Jaemin Yoo    Kijung Shin

# Part 2. Dynamic Structural Patterns

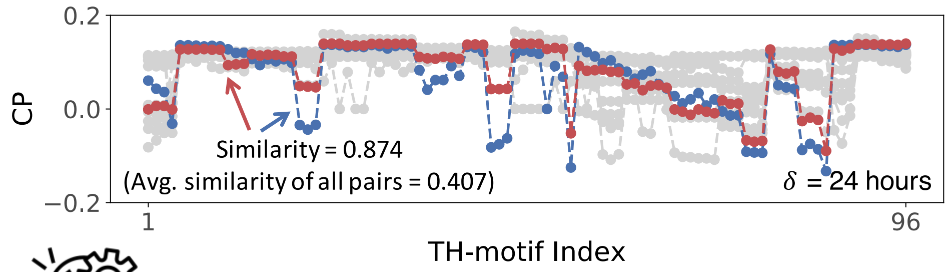
*“Given a **dynamic** hypergraph, how can we analyze its structure?”*




**Input Temporal Hypergraph**




**Basic Patterns (Part 2-1)**




**Advanced Patterns (Part 2-2)**

# Roadmap

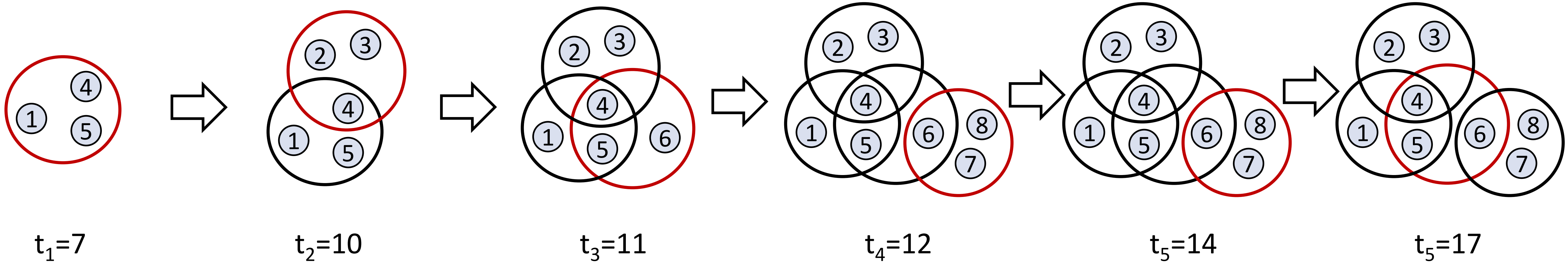
- **Part 1. Static Structural Patterns**
  - Basic Patterns
  - Advanced Patterns
- **Part 2. Dynamic Structural Patterns**
  - Basic Patterns **<<**
  - Advanced Patterns
- **Part 3. Generative Models**
  - Static Hypergraph Generator
  - Dynamic Hypergraph Generator

# Part 2-1. Basic Dynamic Structural Patterns

| | | Part 1. Static Patterns | Part 2. Dynamic Patterns |
|---|---|---|---|
| Basic Patterns | Node-Level | DYHS20, KKS20, LCS21 | BKT18, CS22 |
| | Hyperedge-Level | KKS20, LCS21 | BKT18, LS21, CBLK21 |
| | Hypergraph-Level | BASJK18, DYHS20, KKS20 | KKS20 |
| Advanced Patterns | Sub-hypergraph-Level | BASJK18, LMMB22, LKK20, LCS21 | BASJK18, CJ21, LS21 |

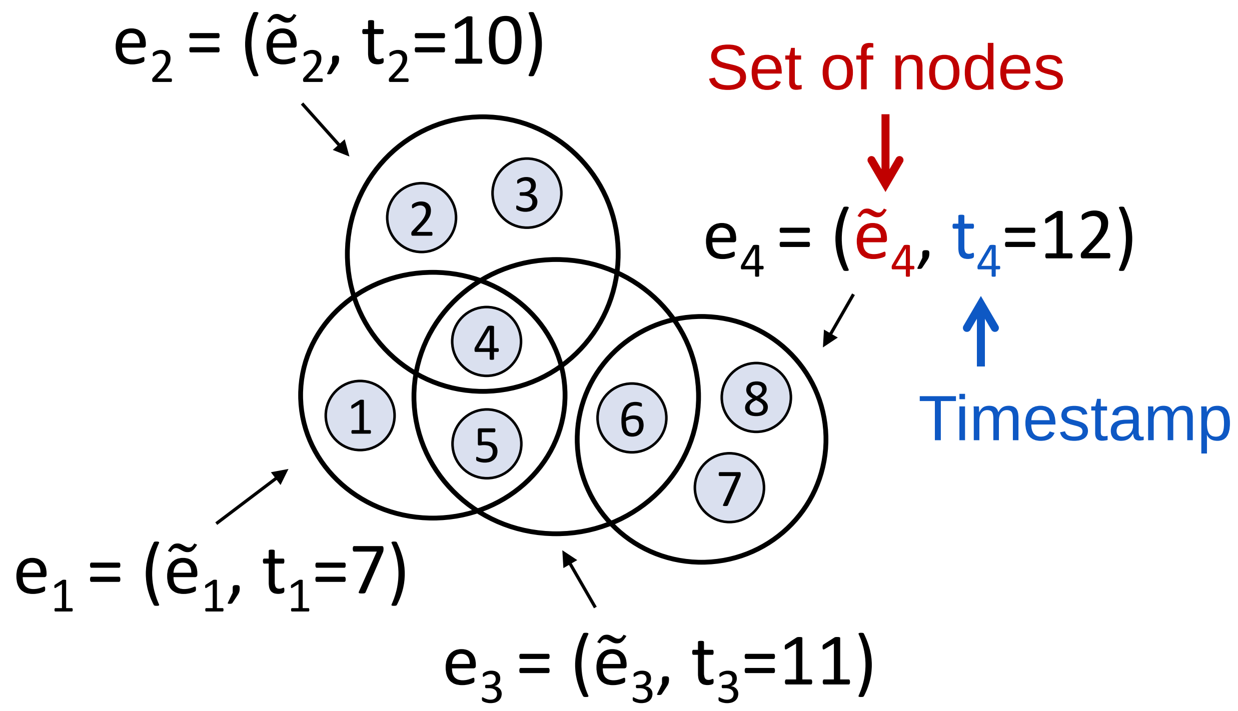
# Background

- **Temporal hypergraphs** consist of **temporal hyperedges**.
    - Temporal hyperedges consisting of the same set of nodes can appear at different timestamps.



# Background (cont.)

- Each **temporal hyperedge** consists of the set of nodes and the timestamp it arrived at.



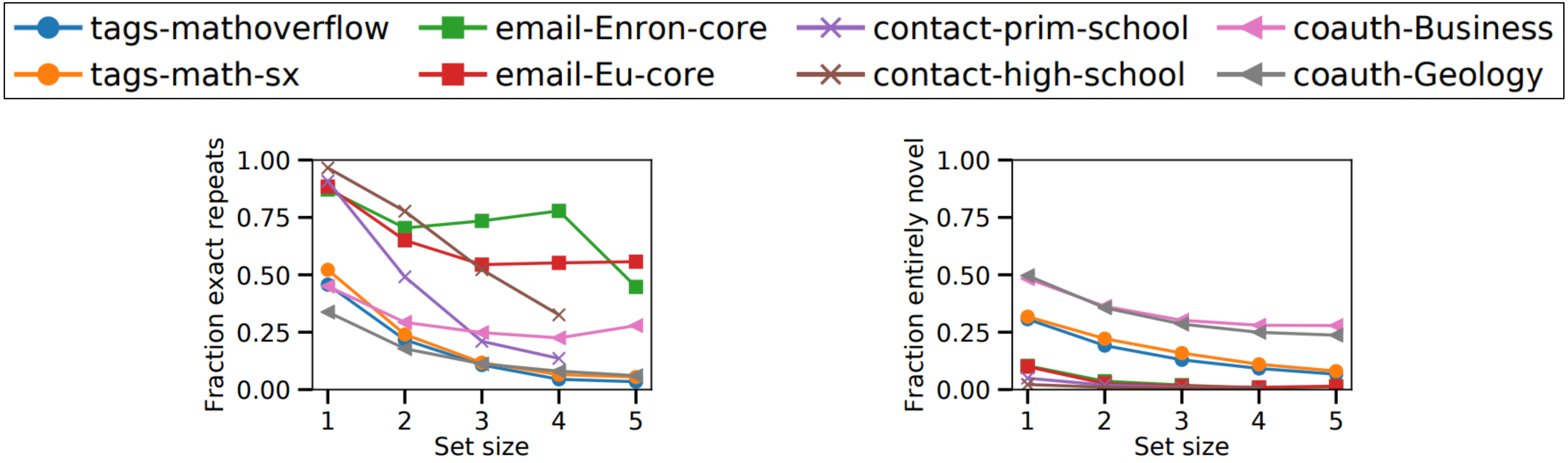
# BKT18: Three Basic Dynamic Patterns

- **P1.** Repeat behavior
- **P2.** Subset correlation
- **P3.** Recency bias

| | | | |
|---|---|---|---|
| | Node | | P2 |
| | Hyperedge | | P1, P3 |
| | Hypergraph | | |
| | Sub-hypergraph | | |

# Repeat Behavior

- Temporal hyperedges tend to **repeat** previous ones.




**Exactly repeated hyperedges** **Entirely novel hyperedges**

# Subset Correlation

- Subsets of nodes tend to be **correlated**.
  - Size-2 and size-3 subsets co-appear in multiple temporal hyperedges.

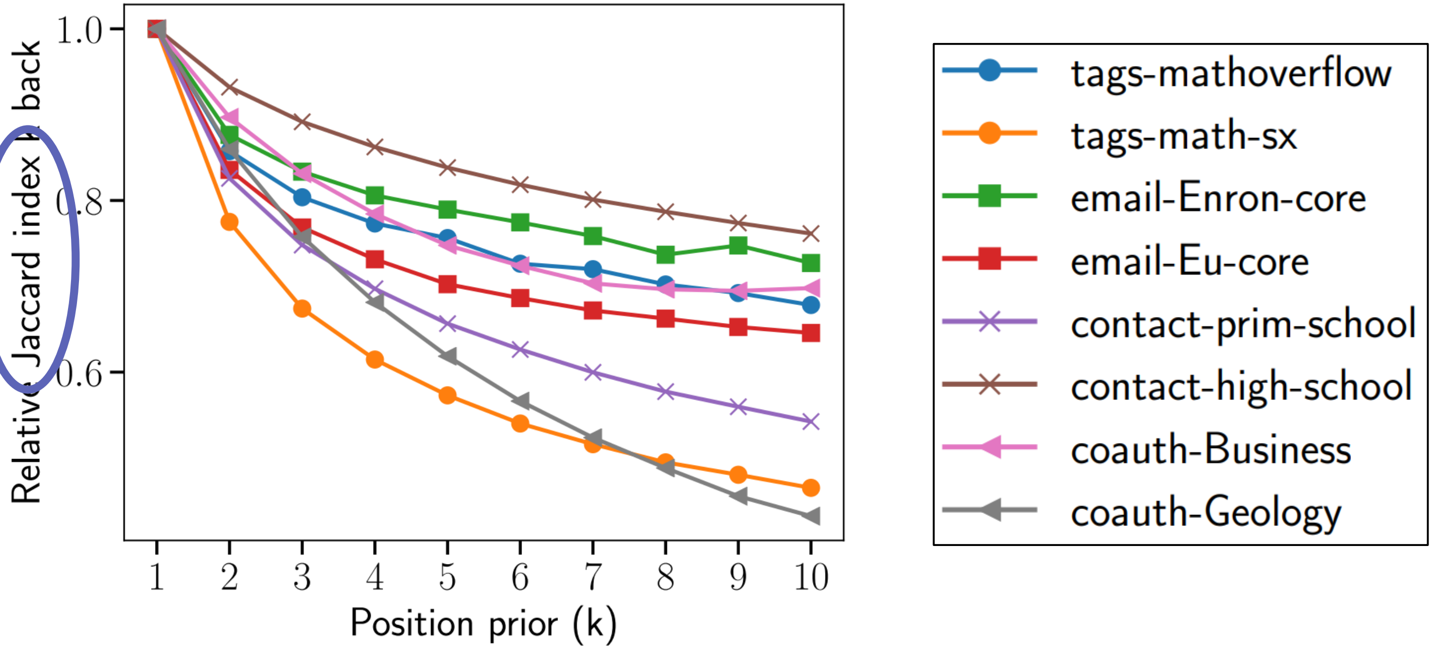
| | size-2 subset counts | | size-3 subset counts | |
|---|---|---|---|---|
| Dataset | data | null model | data | null model |
| email-Enron-core | 5.82 | 4.34 ± 0.043 | 4.23 | 2.67 ± 0.038 |
| email-Eu-core | 4.46 | 3.11 ± 0.008 | 3.23 | 2.08 ± 0.007 |
| contact-prim-school | 2.36 | 1.87 ± 0.003 | 1.35 | 1.09 ± 0.002 |
| contact-high-school | 4.49 | 3.26 ± 0.007 | 2.09 | 1.35 ± 0.004 |
| tags-mathoverflow | 1.49 | 1.41 ± 0.002 | 1.18 | 1.15 ± 0.002 |
| tags-math-sx | 1.49 | 1.31 ± 0.001 | 1.21 | 1.12 ± 0.001 |
| coauth-Business | 1.50 | 1.30 ± 0.001 | 1.40 | 1.24 ± 0.001 |
| coauth-Geology | 1.29 | 1.15 ± 0.000 | 1.15 | 1.07 ± 0.000 |

**Average number of times of size-2/3 subset appearance**

# Recency Bias

- Temporal hyperedges tend to be similar to **recent** ones.

Recent ⟷ Old




Similarity between two hyperedges

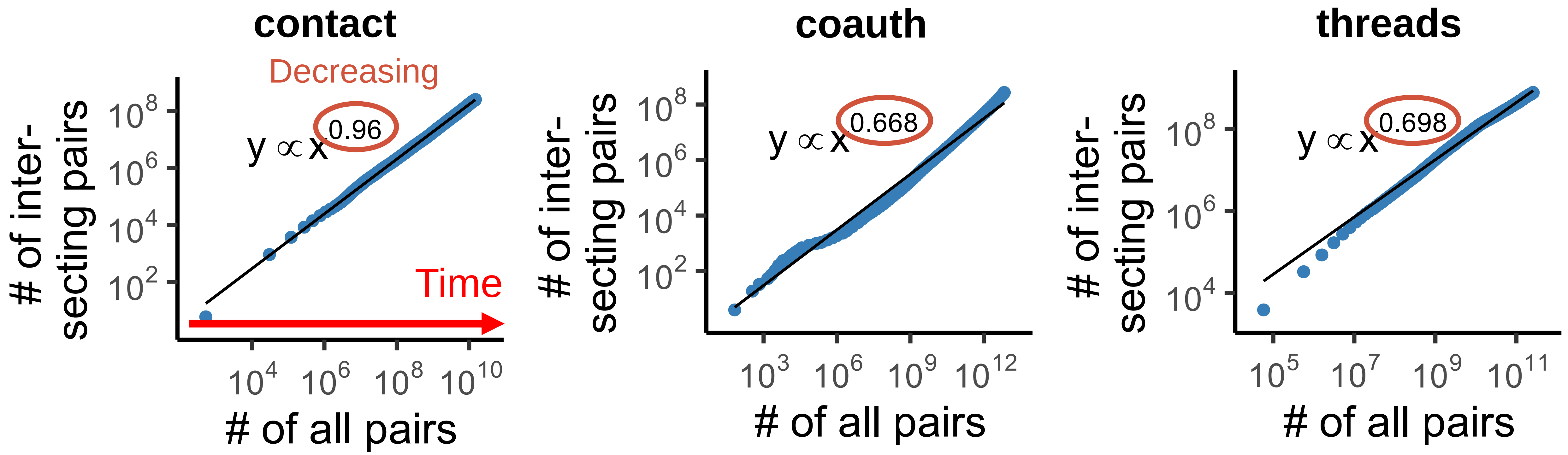
# KKS20: Three Basic Dynamic Patterns

- **P1.** Diminishing overlaps
- **P2.** Densification
- **P3.** Shrinking diameter

| | | | |
|---|---|---|---|
| | **Node** | | |
| | **Hyperedge** | | |
| | **Hypergraph** | | P1-P3 |
| | **Sub-hypergraph** | | |

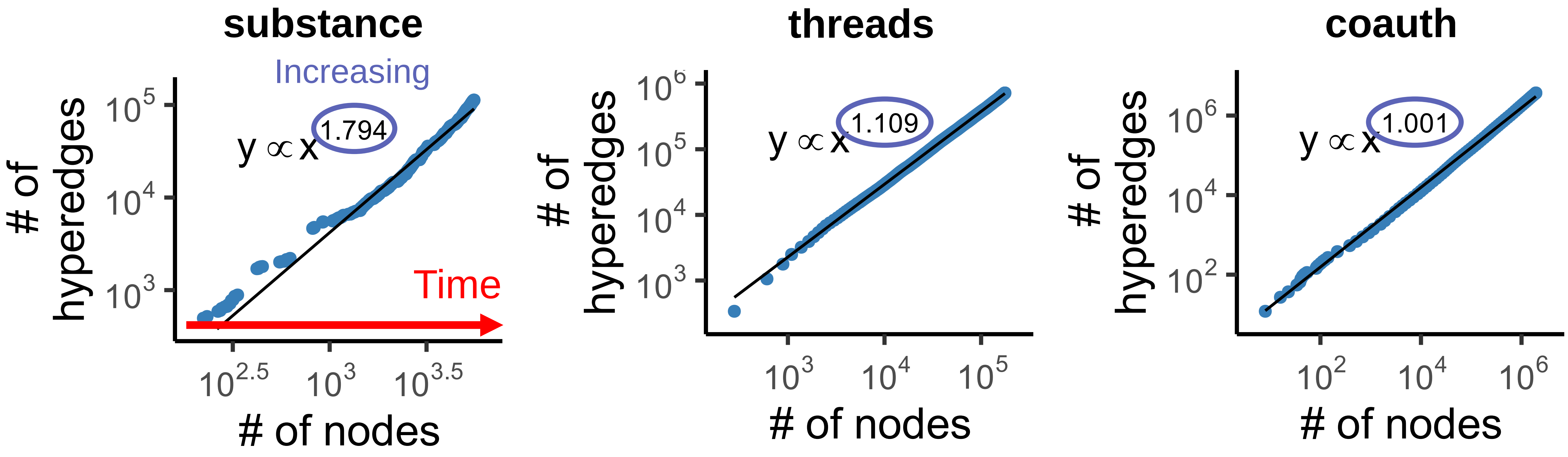
# Diminishing Overlaps

- Density of hyperedge interactions **decreases** over time.



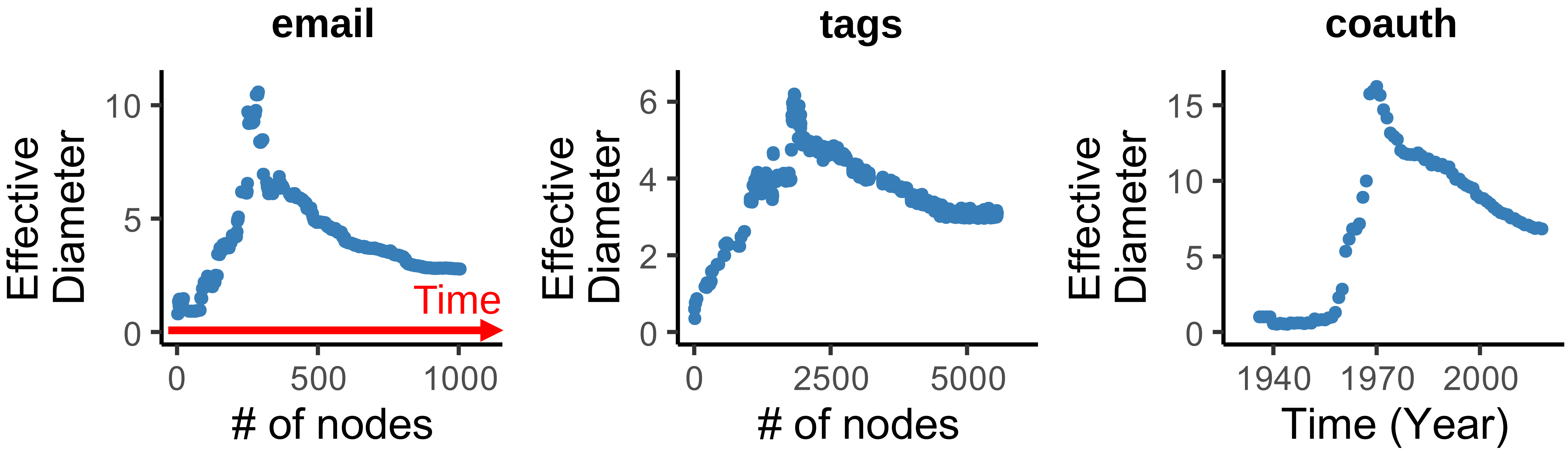
# Densification

- Average node degree of the hypergraph **increases** over time.



# Shrinking Diameter

- Effective diameters eventually **decrease** over time.



# LS21: Two Basic Dynamic Patterns

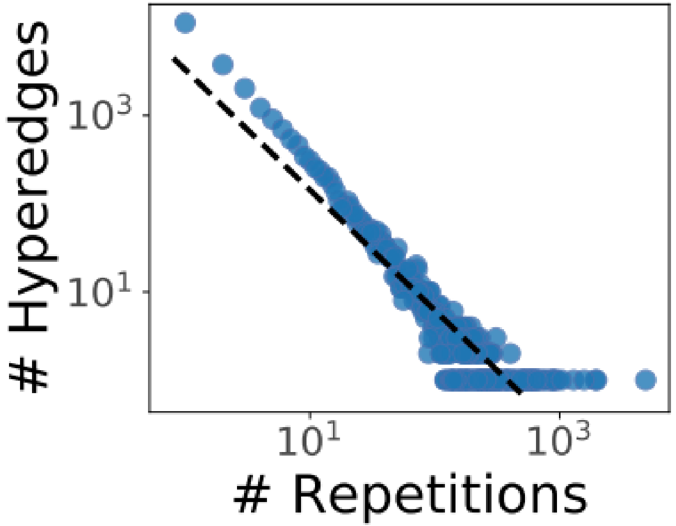
- **P1.** Hyperedge repetition
- **P2.** Temporal locality

| | | | |
|---|---|---|---|
| | Node | | |
| | Hyperedge | | P1, P2 |
| | Hypergraph | | |
| | Sub-hypergraph | | |

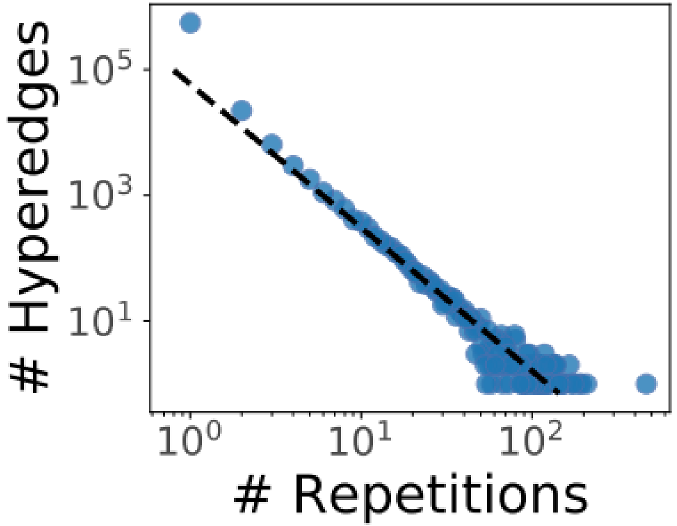
# Hyperedge Repetition

- Temporal hyperedges in real-world hypergraphs are **repetitive**.
- The number of repetitions is **heavy-tailed**.

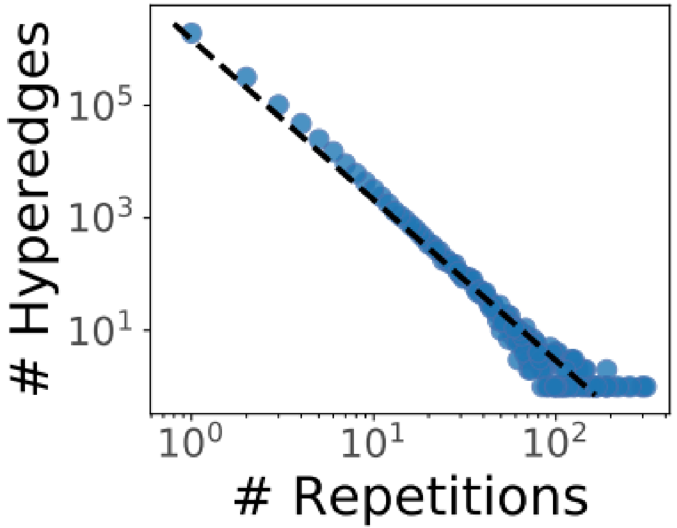
**email-EU**




**threads-math**

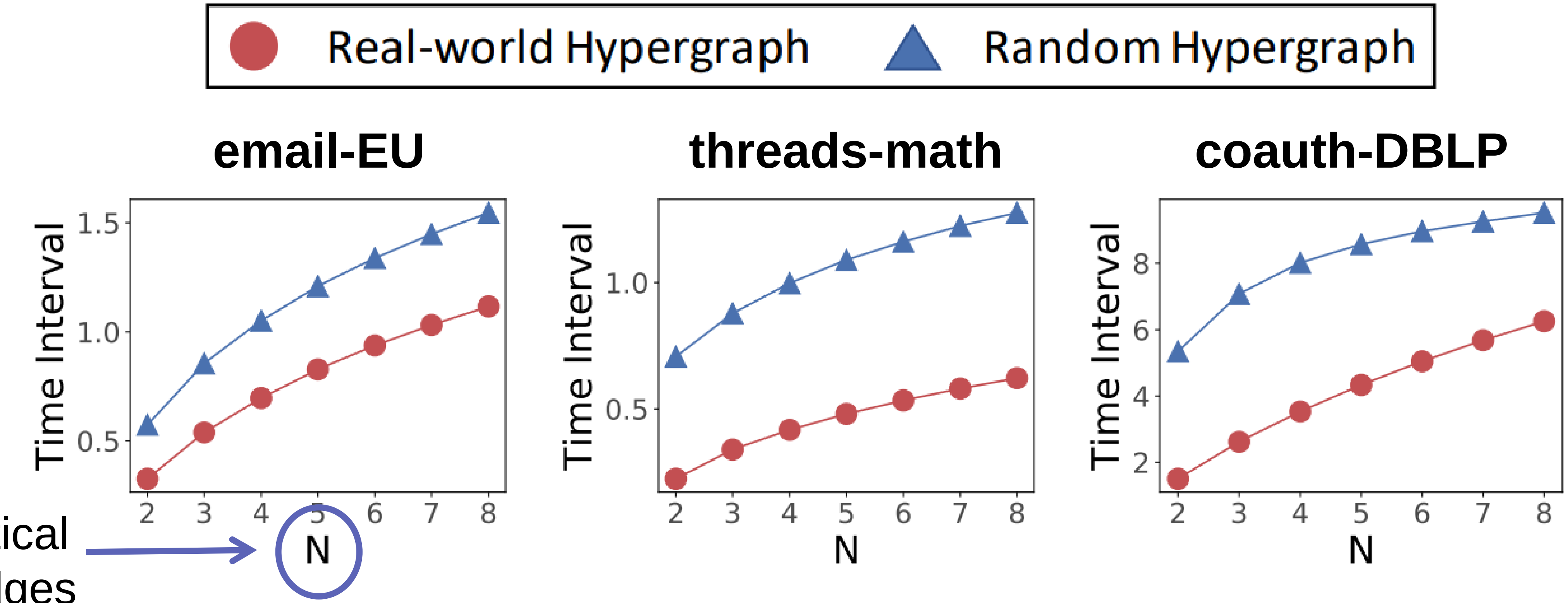



**coauth-DBLP**



# Temporal Locality

- Future temporal hyperedges are more likely to **repeat recent** ones.




Number of consecutive identical temporal hyperedges

# CBLK21: Three Basic Dynamic Patterns

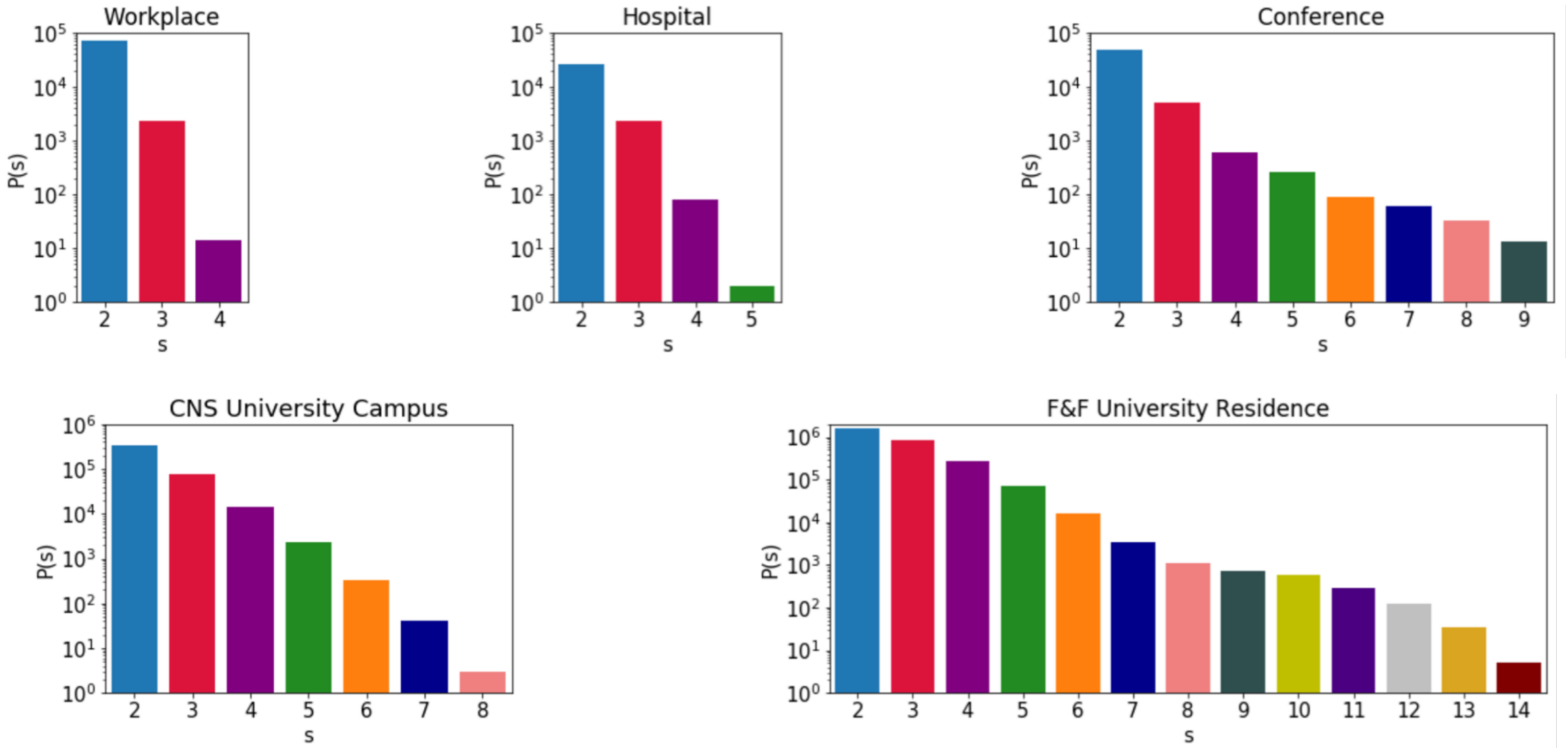
- **P1.** Hyperedge sizes and frequencies
- **P2.** Temporal heterogeneity of hyperedges
- **P3.** Bursty behavior of hyperedges
- **P4.** Temporal reinforcement of hyperedges

| | | | |
|---|---|---|---|
| | Node | | |
| | Hyperedge | | P1, P2, P3, P4 |
| | Hypergraph | | |
| | Sub-hypergraph | | |

G. Cencetti, F. Battiston, B. Lepri, M. Karsai, "Temporal Properties of Higher-order Interactions in Social Networks", **Scientific Report (2021)**

# Hyperedge Sizes and Frequencies

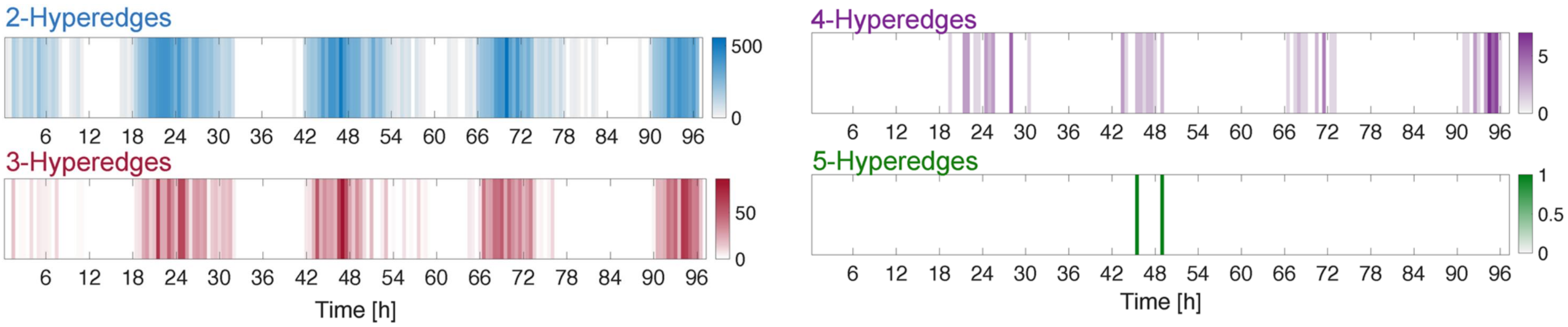
- **Smaller hyperedges** are more numerous in real-world dataset.




G. Cencetti, F. Battiston, B. Lepri, M. Karsai, “Temporal Properties of Higher-order Interactions in Social Networks”, **Scientific Report (2021)**

# Temporal Heterogeneity of Hyperedges

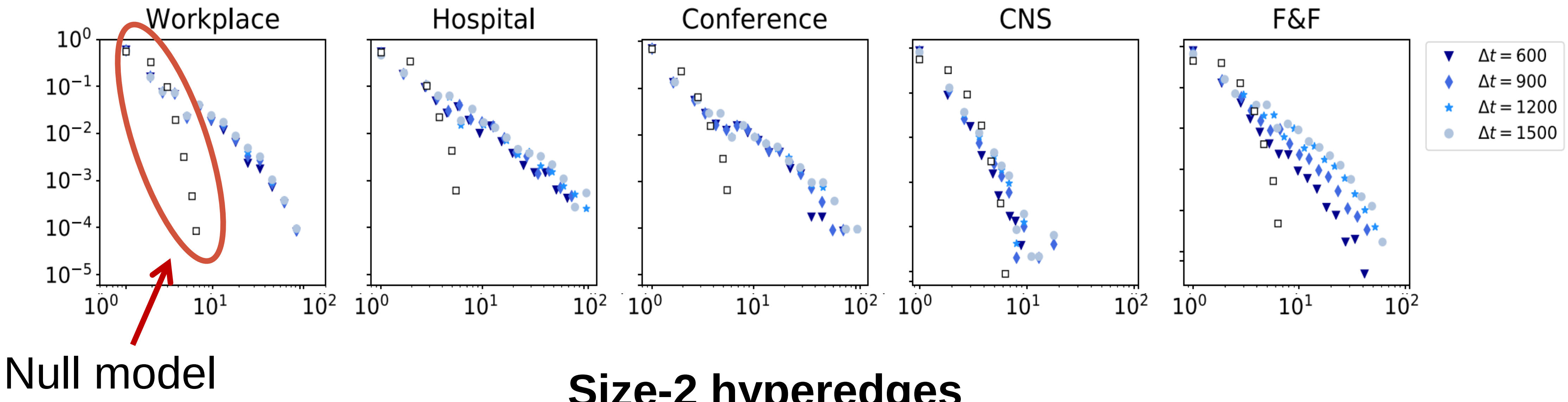
- Emergence of hyperedges is strongly **heterogeneous in time**.
  - Bursty patterns of hyperedges are not independent across sizes.




G. Cencetti, F. Battiston, B. Lepri, M. Karsai, "Temporal Properties of Higher-order Interactions in Social Networks", **Scientific Report (2021)**

# Bursty Behavior of Hyperedges

- Hyperedges in real-world hypergraphs are likely to **repeatedly appear within a short time (Δ$t$)** and the distribution of their numbers of appearances are **heavy-tailed**.

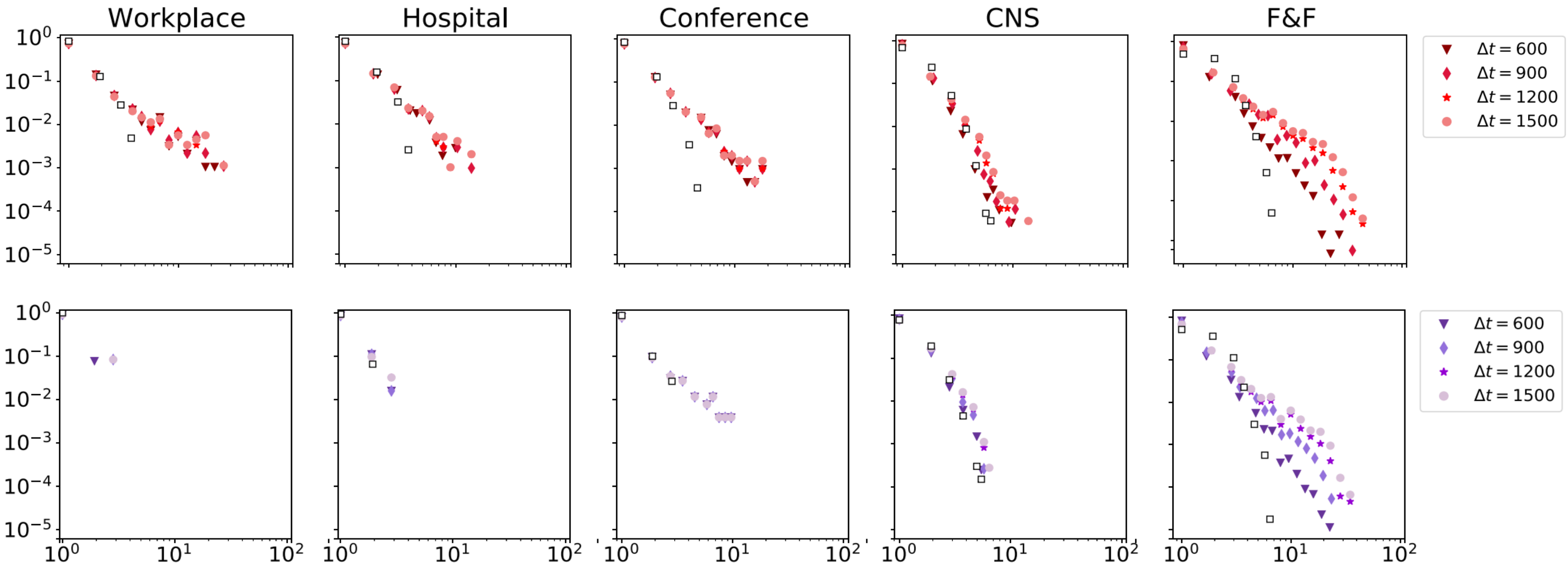



**Size-2 hyperedges**

G. Cencetti, F. Battiston, B. Lepri, M. Karsai, “Temporal Properties of Higher-order Interactions in Social Networks”, **Scientific Report (2021)**

# Bursty Behavior of Hyperedges (cont.)

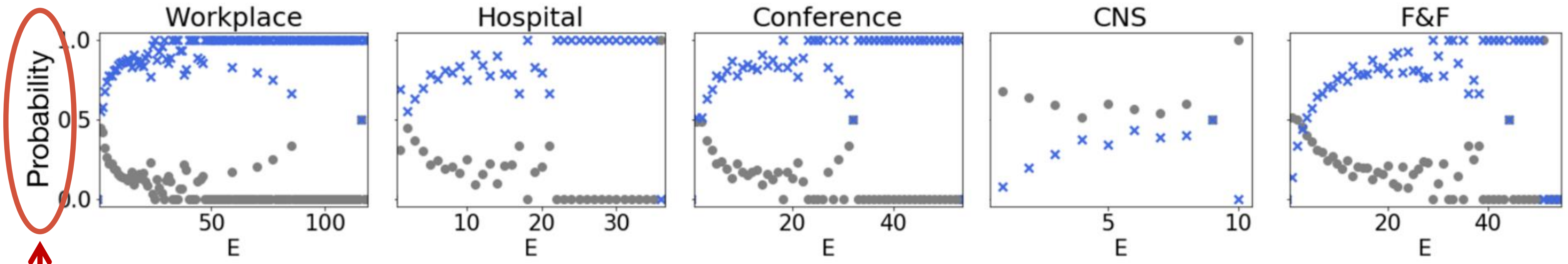
- Larger hyperedges (i.e., size-2 & 3) also follow the same pattern.




**Size-3 & size-4 hyperedges**

# Temporal Reinforcement of Hyperedges

- There exist **temporal reinforcement** in real-world hypergraphs, i.e., the longer the length of an interaction, the higher chances the interaction will not break down.



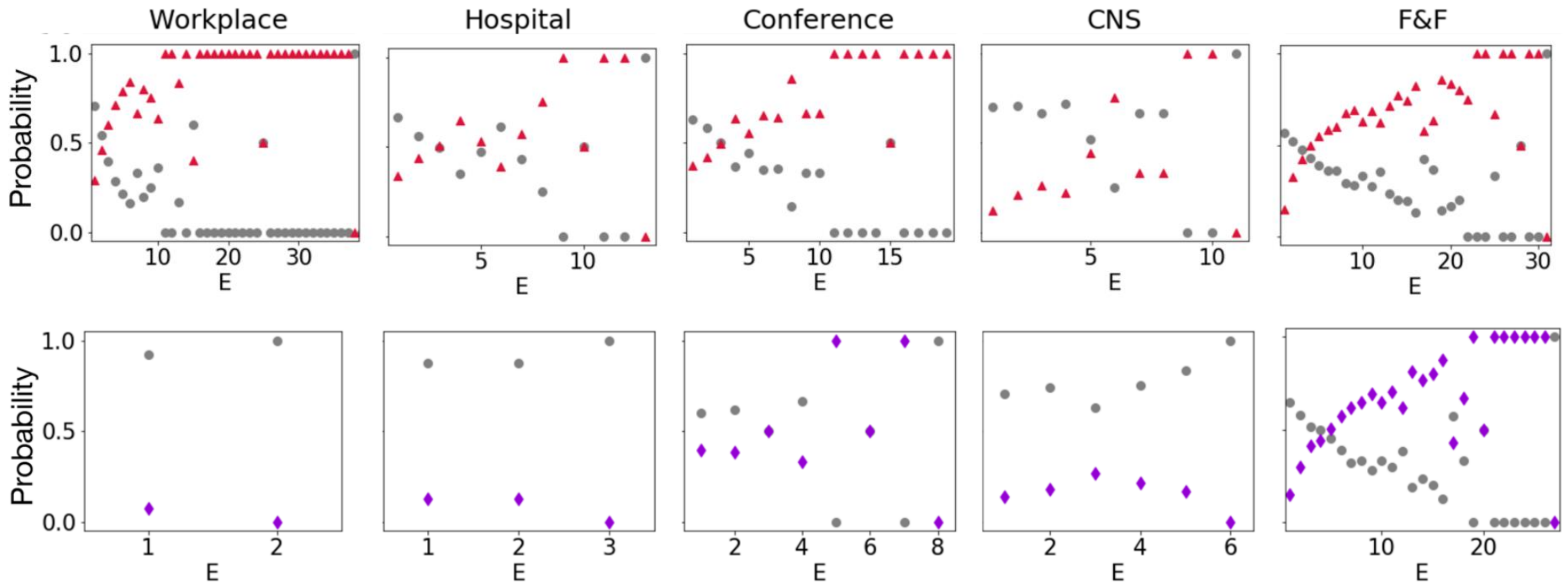

Prob. that after interactions at least E times, the hyperedge to appear within $\Delta t$

**Size-2 hyperedges**

G. Cencetti, F. Battiston, B. Lepri, M. Karsai, “Temporal Properties of Higher-order Interactions in Social Networks”, **Scientific Report (2021)**

# Temporal Reinforcement of Hyperedges (cont.)

- **Temporal reinforcement** exist in larger hyperedges as well.




**Size-3 & size-4 hyperedges**

# Roadmap

- **Part 1. Static Structural Patterns**
  - Basic Patterns
  - Advanced Patterns
- **Part 2. Dynamic Structural Patterns**
  - Basic Patterns
  - Advanced Patterns **<<**
- **Part 3. Generative Models**
  - Static Hypergraph Generator
  - Dynamic Hypergraph Generator

# Part 2-2. Advanced Dynamic Structural Patterns

| | | Part 1. Static Patterns | Part 2. Dynamic Patterns |
|---|---|---|---|
| Basic Patterns | Node-Level | DYHS20, KKS20, LCS21 | BKT18, CS22 |
| | Hyperedge-Level | KKS20, LCS21 | BKT18, LS21, CBLK21 |
| | Hypergraph-Level | BASJK18, DYHS20, KKS20 | KKS20 |
| Advanced Patterns | Sub-hypergraph-Level | BASJK18, LMMB22, LKK20, LCS21 | BASJK18, CJ21, LS21 |

# BASJK18: One Advanced Dynamic Pattern

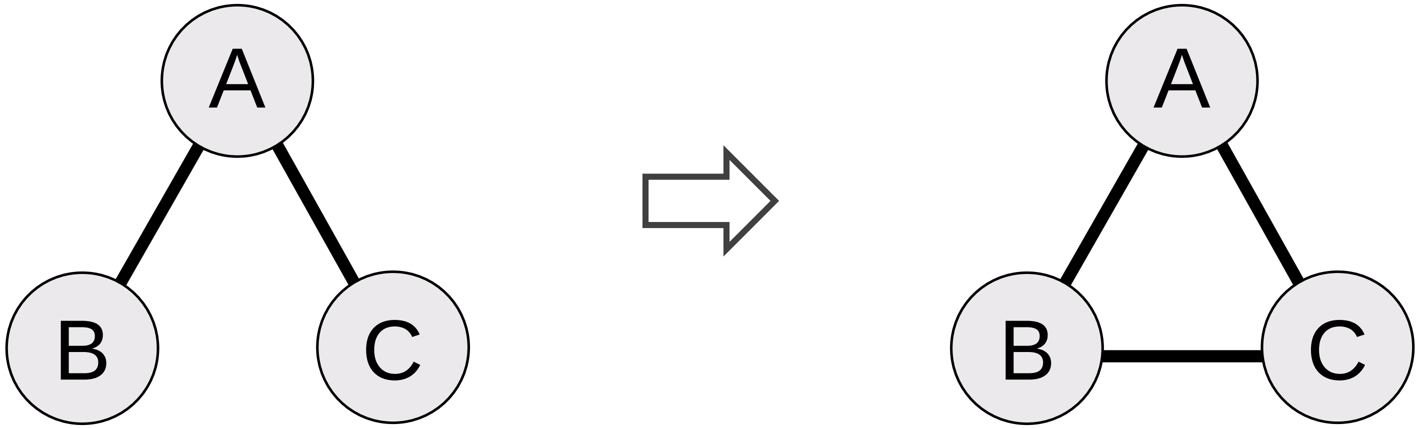
- **P1.** Simplicial closure

| | | | |
|---|---|---|---|
| | | | |
| | Node | | |
| | Hyperedge | | |
| | Hypergraph | | |
| | Sub-hypergraph | | P1 |

A. R. Benson, R. Abebe, M. T. Schaub, A. Jadbabaie, J. Kleinberg, "Simplicial Closure and Higher-order Link Prediction", **PNAS (2018)**

# Background

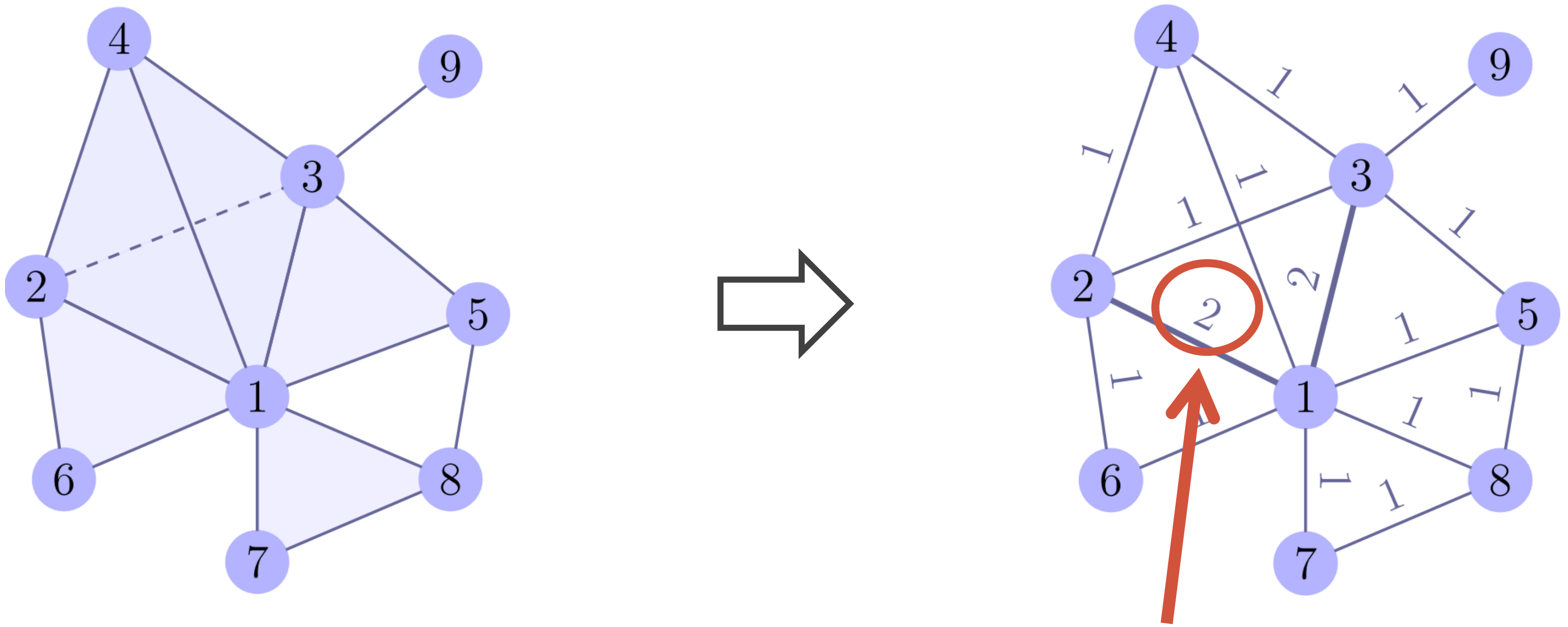
- **Triadic closure** in networks is a tendency of forming a triangle.




A. R. Benson, R. Abebe, M. T. Schaub, A. Jadbabaie, J. Kleinberg, "Simplicial Closure and Higher-order Link Prediction", **PNAS (2018)**

# Weighted Projected Graph

- Hypergraphs can be transformed into **weighted projected graphs**.

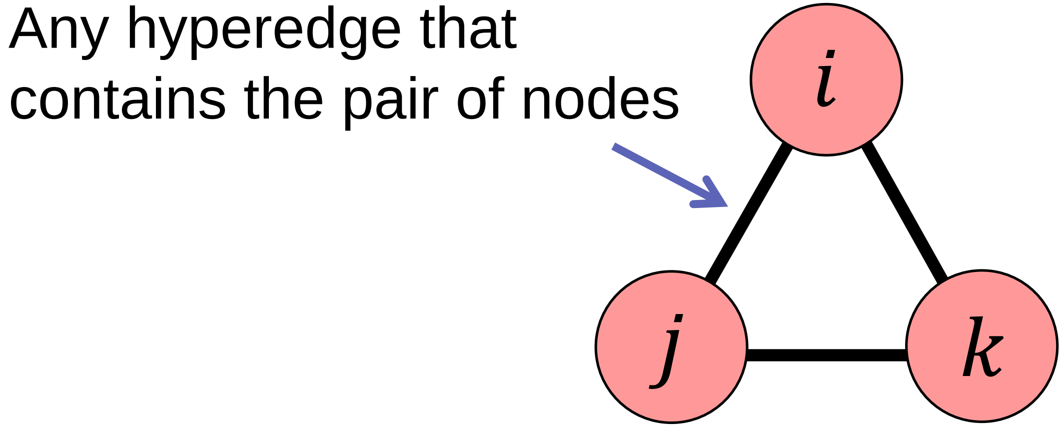


$\boldsymbol{W_{ij}}$: Number of hyperedges overlapping a pair of node $\boldsymbol{i}$ & $\boldsymbol{j}$.
An edge $(i, j)$ is a weak tie if $W_{ij} = 1$ and a strong tie if $W_{ij} \geq 2$.

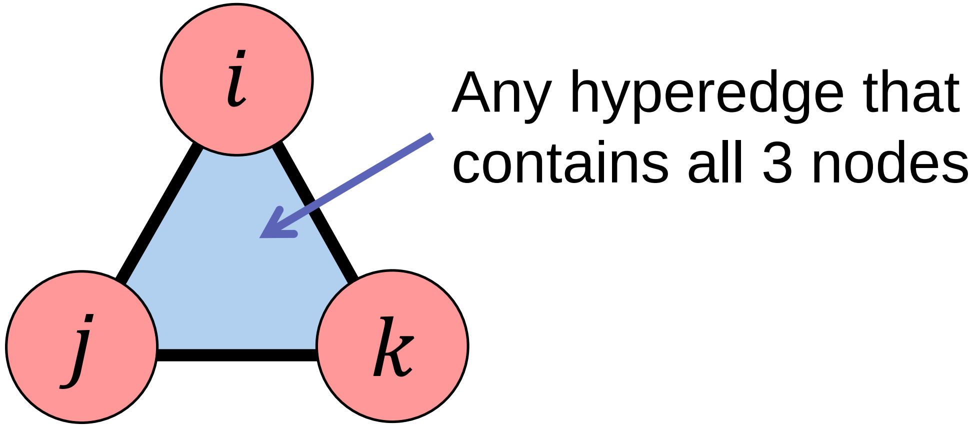
A. R. Benson, R. Abebe, M. T. Schaub, A. Jadbabaie, J. Kleinberg, “Simplicial Closure and Higher-order Link Prediction”, **PNAS (2018)**

# Recap: Open and Closed Triangles

- There are two types of **triangles** in hypergraphs.
  - Static observations can be found at Part II.




**Open Triangle**

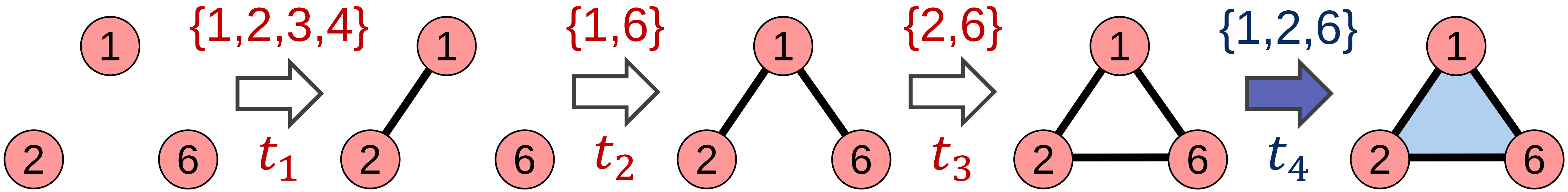



**Closed Triangle**

A. R. Benson, R. Abebe, M. T. Schaub, A. Jadbabaie, J. Kleinberg, "Simplicial Closure and Higher-order Link Prediction", **PNAS (2018)**

# Simplicial Closure

- **Simplicial closure** is an event of forming a **closed triangle**.



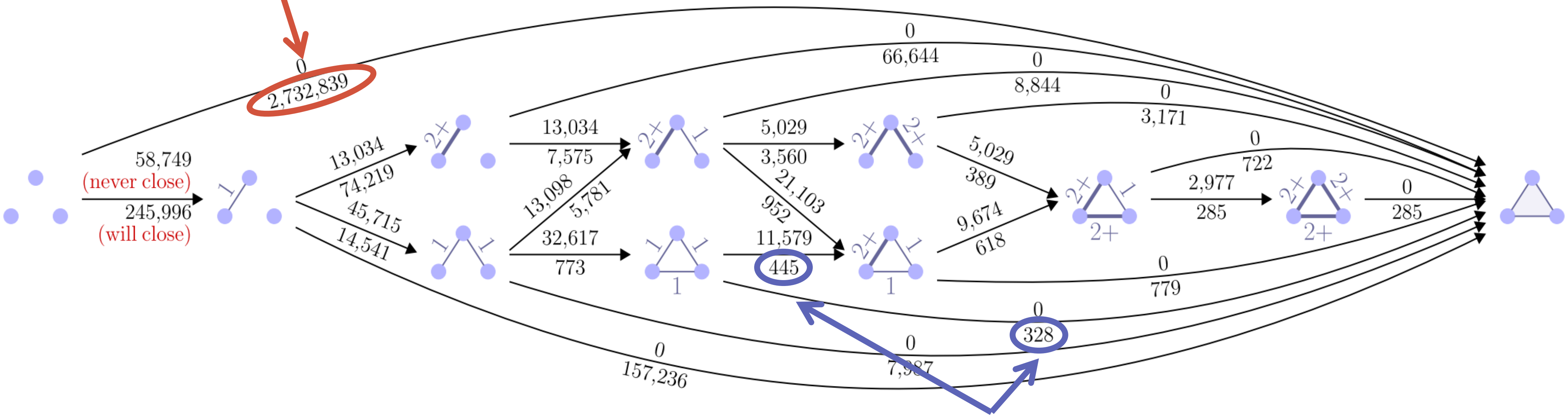

***Simplicial closure***

A. R. Benson, R. Abebe, M. T. Schaub, A. Jadbabaie, J. Kleinberg, "Simplicial Closure and Higher-order Link Prediction", **PNAS (2018)**

# Trajectories of Simplicial Closure

- Example: Co-authorship hypergraph

Most groups formed have no previous interaction.




Open triangles of all weak ties are more likely to form a strong tie before closing.

A. R. Benson, R. Abebe, M. T. Schaub, A. Jadbabaie, J. Kleinberg, “Simplicial Closure and Higher-order Link Prediction”, **PNAS (2018)**

# Structural Factors of Simplicial Closure

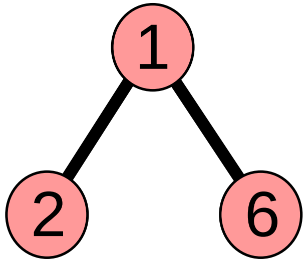
**Questions:**

How does the simplicial closure occur?

Can we predict it from the structure of the projected graph?

**Approach:**

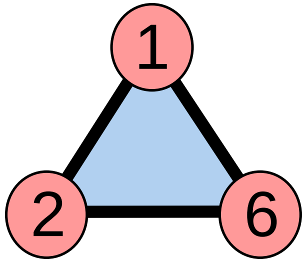
**First 80%** of the data (in time)




**Triplets not in closed triangle**

Compute the ratio

**Last 20%** of the data (in time)

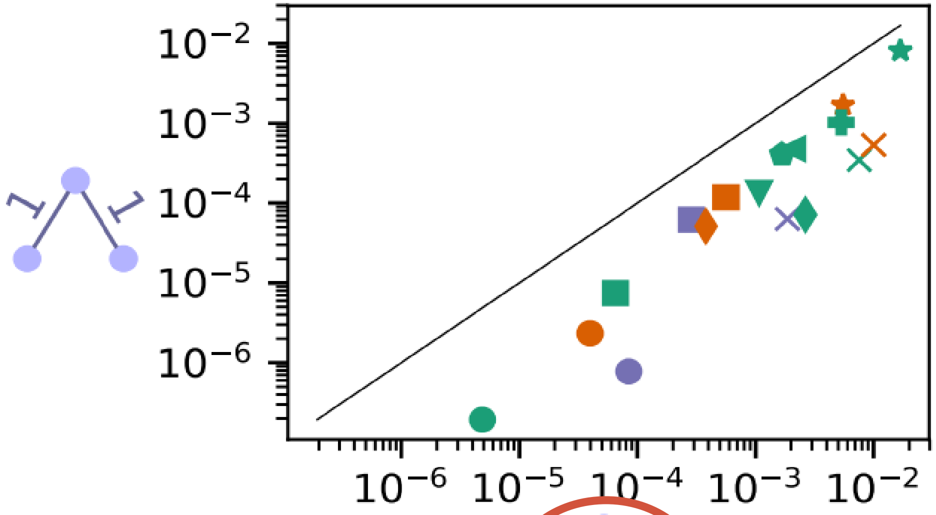



**Closed Triangle**

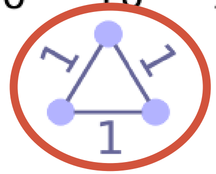
A. R. Benson, R. Abebe, M. T. Schaub, A. Jadbabaie, J. Kleinberg, “Simplicial Closure and Higher-order Link Prediction”, **PNAS (2018)**

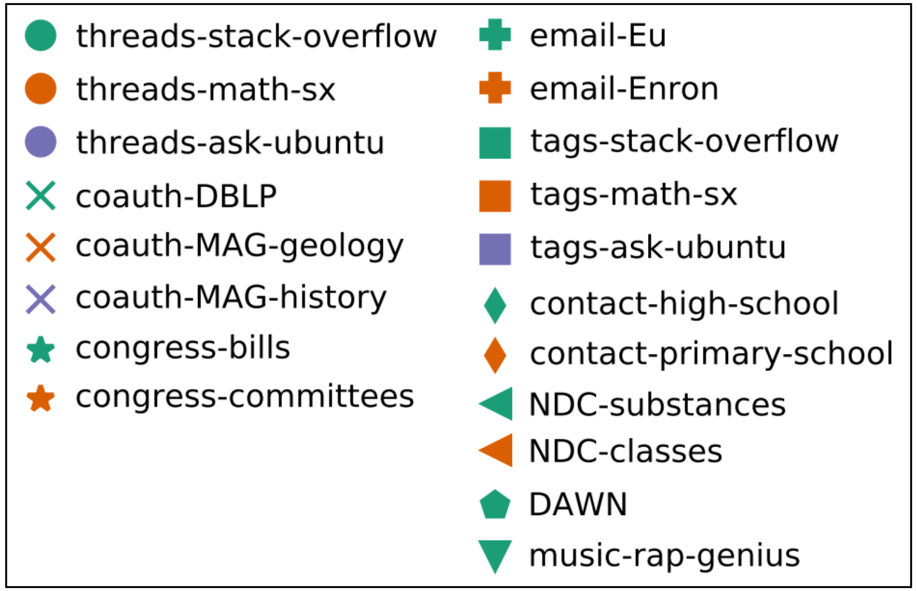
# Edge Density vs. Simplicial Closure

- Increased **edge density** increases simplicial closure probability.




Higher edge density

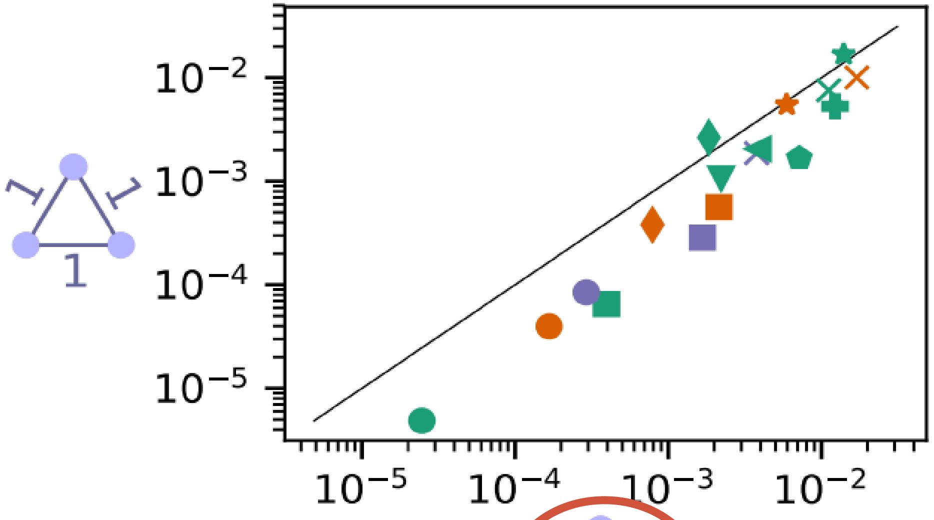


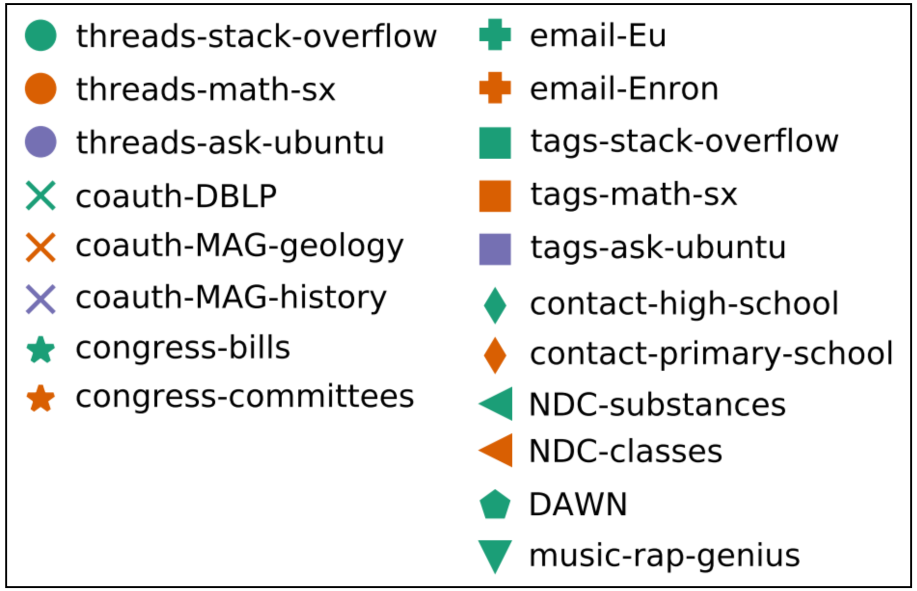




A. R. Benson, R. Abebe, M. T. Schaub, A. Jadbabaie, J. Kleinberg, “Simplicial Closure and Higher-order Link Prediction”, **PNAS (2018)**

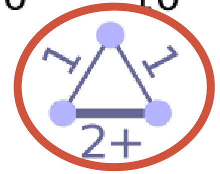
# Tie Strength vs. Simplicial Closure

- Increased **tie strength** increases simplicial closure probability.



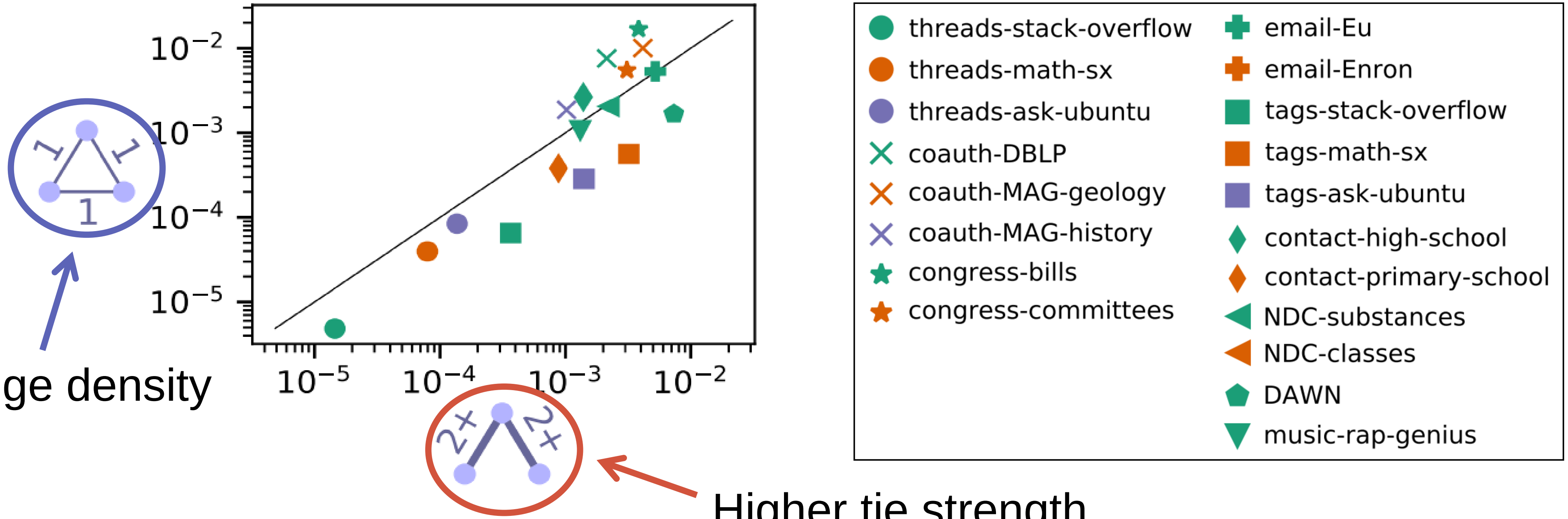




Higher tie strength →



# Edge Density vs. Tie Strength

- Relative importance of **edge density** and **tie strength** depends on the dataset.




Higher edge density

Higher tie strength

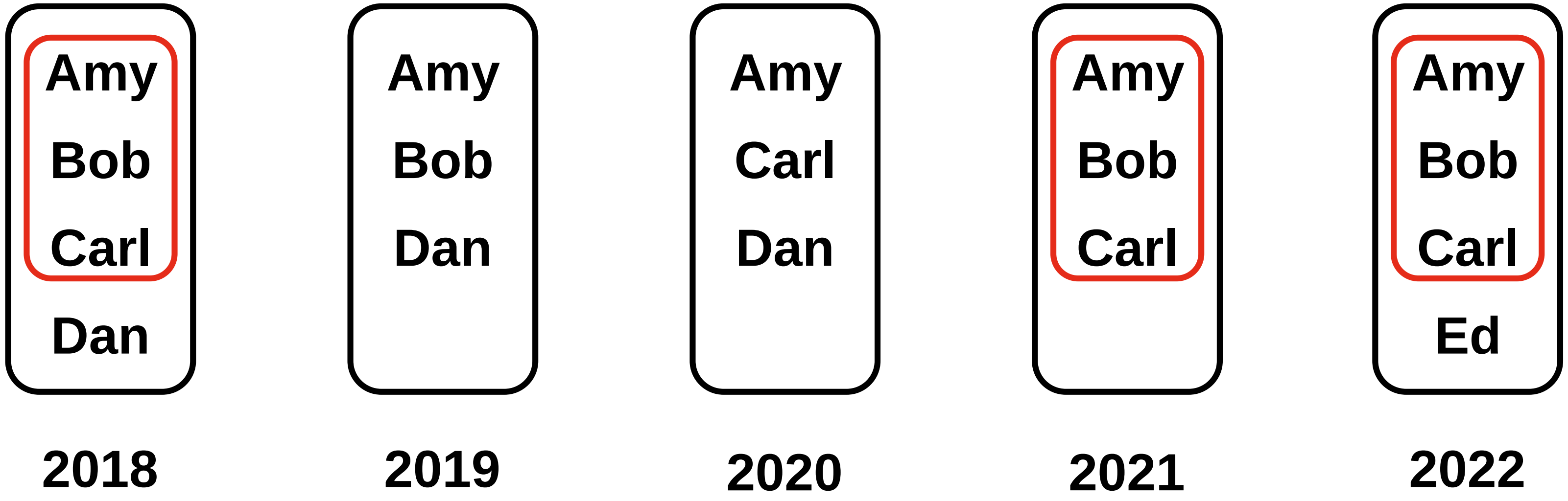
# CS22: One Advanced Dynamic Pattern

- **P1.** Persistence of higher-order interactions (HOIs)

| | | | |
|---|---|---|---|
| | Node | | |
| | Hyperedge | | |
| | Hypergraph | | |
| | Sub-hypergraph | | P1 |

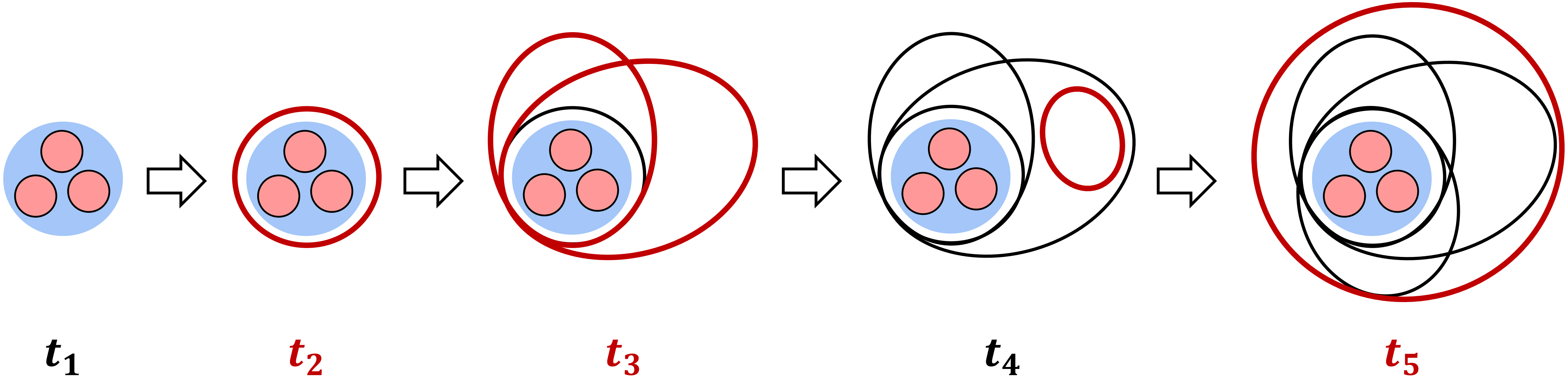
# Higher-Order Interaction

- A **higher-order interaction (HOI)** is the co-appearance of a set of nodes in any hyperedge.
- HOIs can appear **repeatedly** over time.



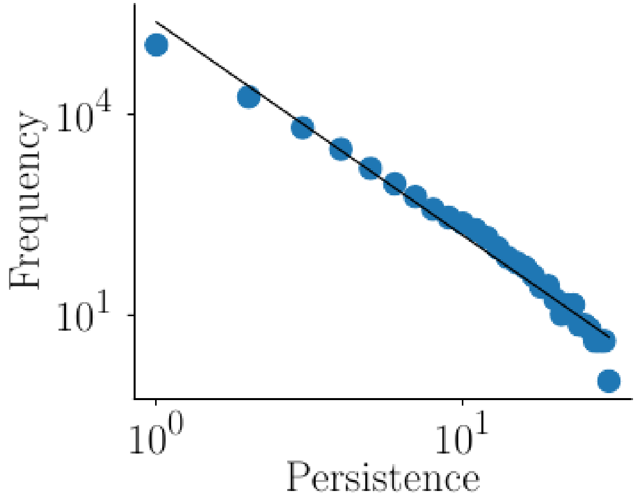
# Persistence of HOIs

- **Persistence** of a HOI $\boldsymbol{S}$ over time range $\boldsymbol{T}$ is the number of time units in $\boldsymbol{T}$ when $\boldsymbol{S}$ co-appear in any hyperedge.
- Example: **Persistence = 3**

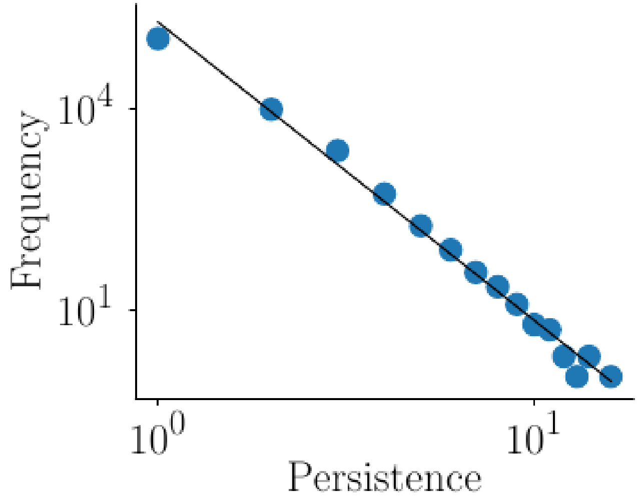


# Frequency of Persistence

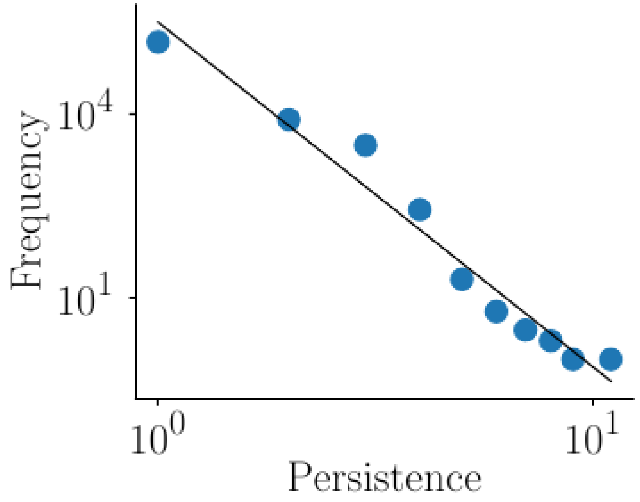
- Frequency of HOI persistence tends to follow a **power-law**.




**HOI size = 2**




**HOI size = 3**




**HOI size = 4**

H. Choo, K. Shin, "On the Persistence of Higher-order Interactions in Real-world Hypergraphs", **SDM 2022**

# Group Features and Persistence

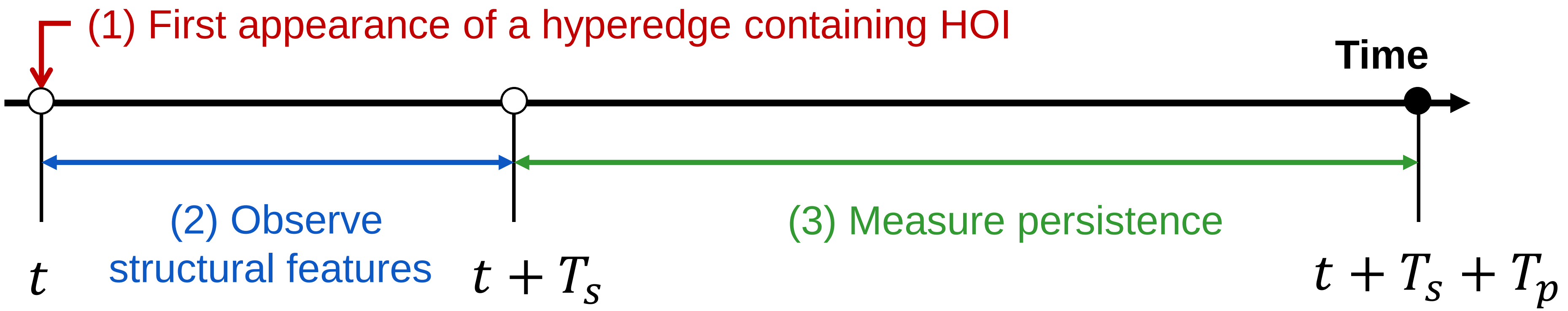
?

**Questions:**

What structural features are related to HOI persistence?

**Answer:**

- We examine relations between various structural features and HOI persistence with the following setting:




!

# Group Features and Persistence (cont.)

- Basic structural features of each HOI $S$:
  - #: number of hyperedges including $S$
  - $\Sigma$: sum of sizes of hyperedges containing $S$
  - $\cup$: number of hyperedges overlapping $S$
  - $\Sigma\cup$: sum of sizes of hyperedges overlapping $S$
  - $\cap$: number of common neighbors of $S$
  - $\mathcal{H}$: entropy in sizes of hyperedges containing $S$
- Group structural features of each HOI $S$:

(1) $\#$, (2) $\#/\cup$, (3) $\Sigma/(\Sigma\cup)$, (4) $\cap$, (5) $\#/\cap$, (6) $\Sigma/\cap$, (7) $\Sigma/\#$, (8) $\mathcal{H}$

# Group Features and Persistence (cont.)

- HOI persistence is **positively correlated** with:
  - The **number of hyperedges** containing the HOI
  - The **entropy of the sizes of hyperedges** containing the HOI

| | Size of HOIs | # | $\frac{\#}{\cup}$ | $\frac{\Sigma}{\Sigma\cup}$ | $\cap$ | $\frac{\#}{\cap}$ | $\frac{\Sigma}{\cap}$ | $\frac{\Sigma}{\#}$ | $\mathcal{H}$ |
|---|---|---|---|---|---|---|---|---|---|
| Normalized mutual information → MI | 2 | 0.13 | 0.11 | <u>0.14</u> | 0.05 | 0.10 | 0.12 | 0.10 | **0.15** |
| | 3 | <u>0.11</u> | 0.06 | 0.08 | 0.05 | 0.08 | 0.09 | 0.08 | **0.12** |
| | 4 | <u>0.11</u> | 0.05 | 0.07 | 0.06 | 0.07 | 0.10 | 0.07 | **0.12** |
| | Avg. | <u>0.12</u> | 0.08 | 0.10 | 0.05 | 0.08 | 0.11 | 0.08 | **0.13** |
| Pearson correlation coefficient → CC | 2 | **0.36** | 0.09 | 0.09 | 0.17 | 0.19 | 0.26 | -0.08 | <u>0.32</u> |
| | 3 | **0.31** | 0.10 | 0.10 | 0.05 | 0.16 | 0.20 | -0.09 | <u>0.25</u> |
| | 4 | **0.30** | 0.13 | 0.13 | -0.01 | 0.17 | 0.20 | -0.10 | <u>0.24</u> |
| | Avg. | **0.32** | 0.10 | 0.11 | 0.07 | 0.17 | 0.22 | -0.09 | <u>0.27</u> |

# LS21: One Advanced Dynamic Pattern

- **P1.** Temporal hypergraph motifs (TH-motifs)

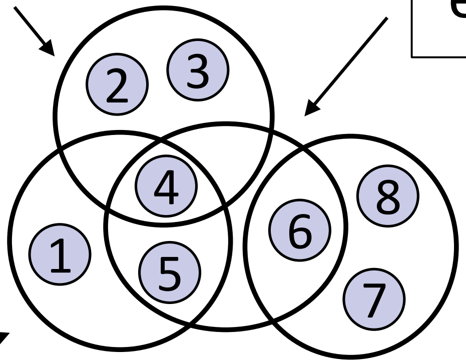
| | | | |
|---|---|---|---|
| | Node | | |
| | Hyperedge | | |
| | Hypergraph | | |
| | Sub-hypergraph | | P1 |

# Temporal Hypergraph Motifs: Definition

*"How can we define **motifs** in **temporal hypergraphs**?"*

$e_2 = (\tilde{e}_2=\{2,3,4\}, t_2=10)$

$e_3 = (\tilde{e}_3=\{4,5,6\}, t_3=11)$
$e_6 = (\tilde{e}_6=\{4,5,6\}, t_3=17)$




$e_4 = (\tilde{e}_4=\{6,7,8\}, t_4=12)$
$e_5 = (\tilde{e}_5=\{6,7,8\}, t_5=14)$

$e_1 = (\tilde{e}_1=\{1,4,5\}, t_1=7)$

# Temporal Hypergraph Motifs: Definition (cont.)

?

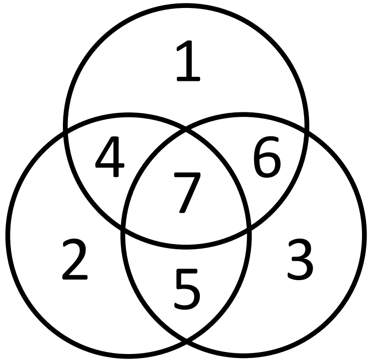
**Question:**

How can we capture **temporal** properties of hypergraphs?

**Answer 2:**

The three temporal hyperedges should **arrive within $\delta$ time**.

$e_1 = (\tilde{e}_1, t_1=7)$




$e_2 = (\tilde{e}_2, t_2=10)$ $e_3 = (\tilde{e}_3, t_3=11)$

$$\mathbf{max}(\mathbf{t_1}, \mathbf{t_2}, \mathbf{t_3}) - \mathbf{min}(\mathbf{t_1}, \mathbf{t_2}, \mathbf{t_3}) \leq \boldsymbol{\delta}$$

!

# Temporal Hypergraph Motifs: Definition (cont.)

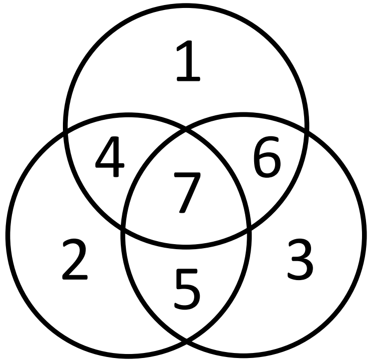
**Question:**

How can we capture **temporal** properties of hypergraphs?

**Answer 2:**

The **order** of the three temporal hyperedges is considered.

$e_1 = (\tilde{e}_1, t_1{=}7)$

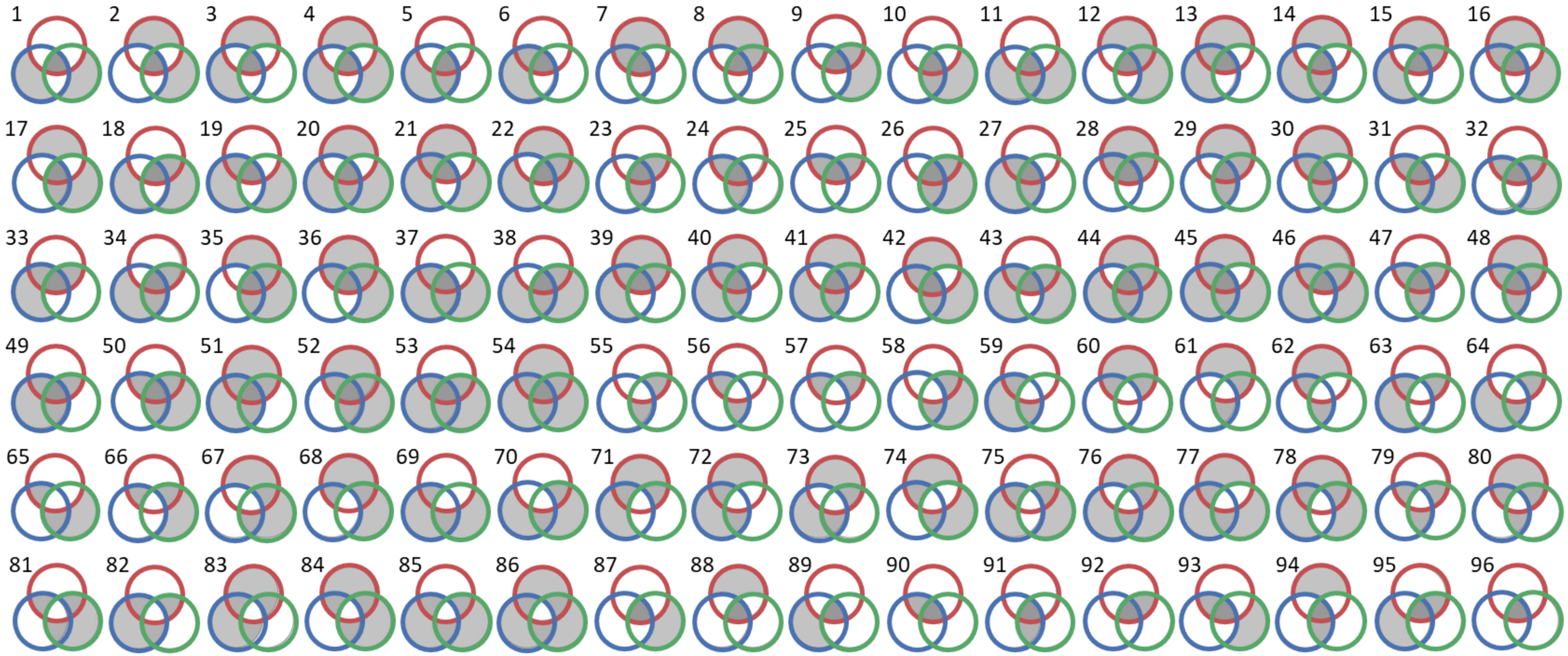



$e_2 = (\tilde{e}_2, t_2{=}10)$ $e_3 = (\tilde{e}_3, t_3{=}11)$

$\mathbf{e_1 \rightarrow e_2 \rightarrow e_3}$

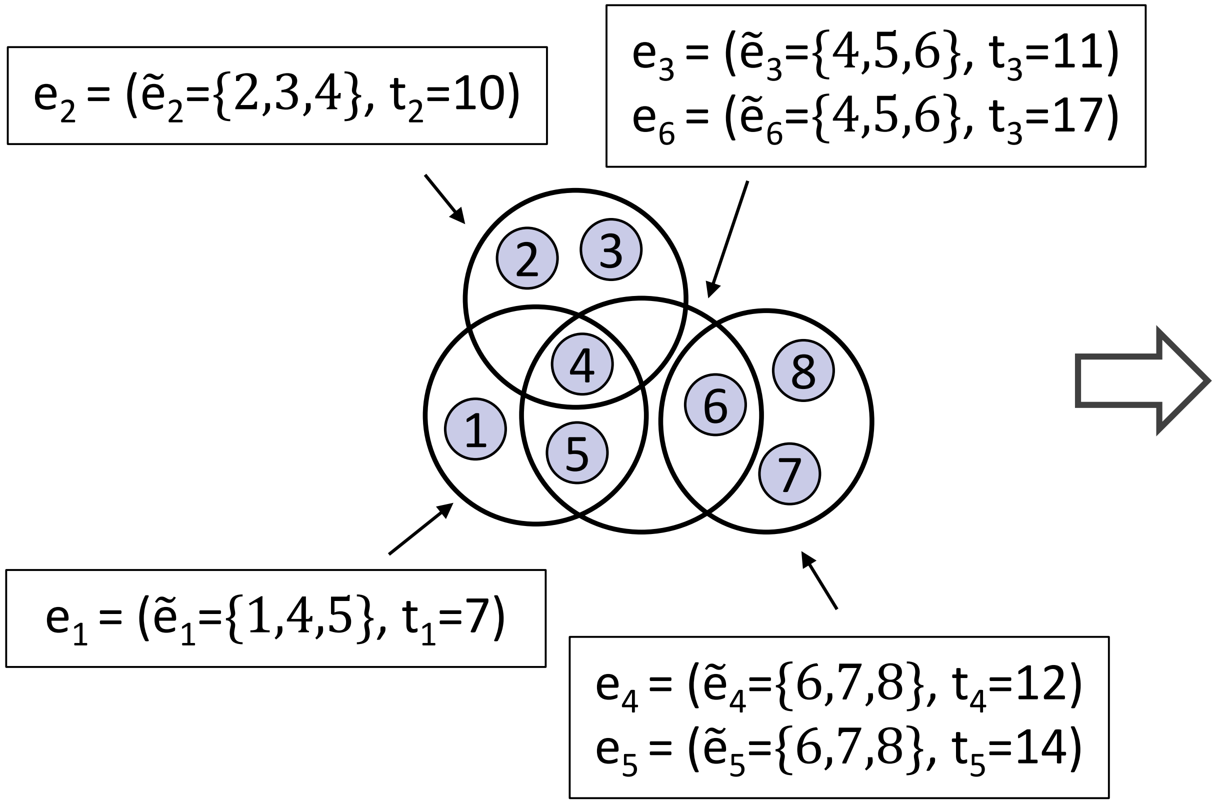
# Temporal Hypergraph Motifs: Definition (cont.)

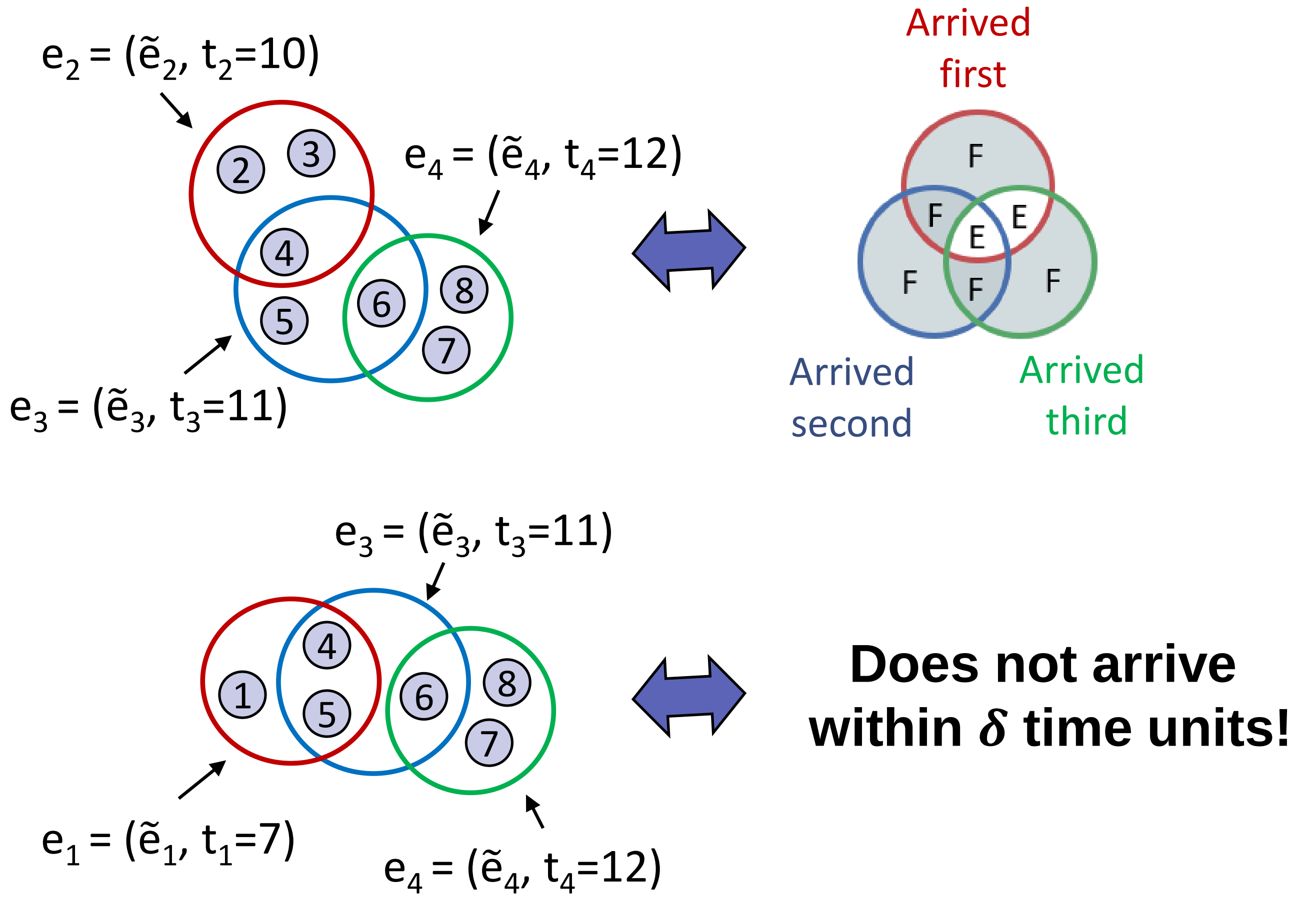
- There exist **96 TH-motifs**.



# Temporal Hypergraph Motifs: Example

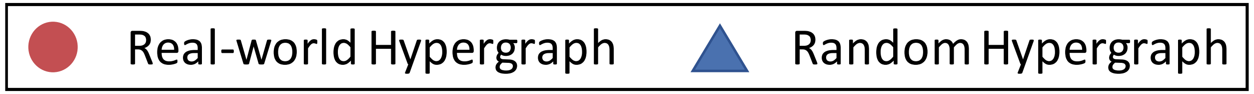
- For example, let $\delta = 3$.

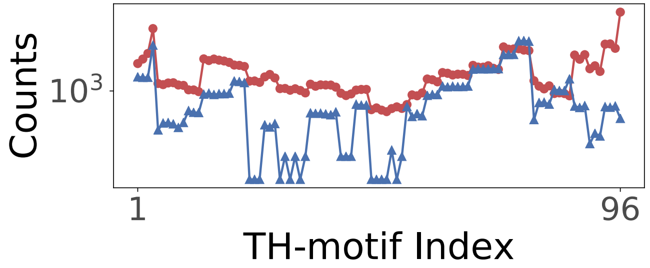


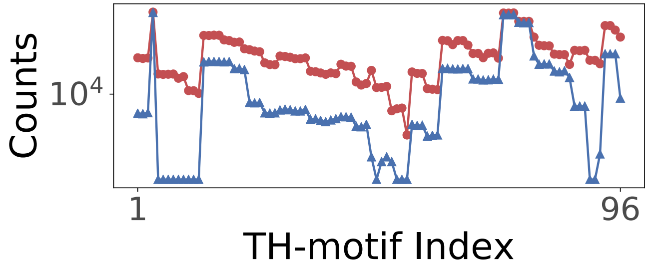
# Real vs. Random

- Real hypergraphs are clearly **distinguished** from randomized ones.

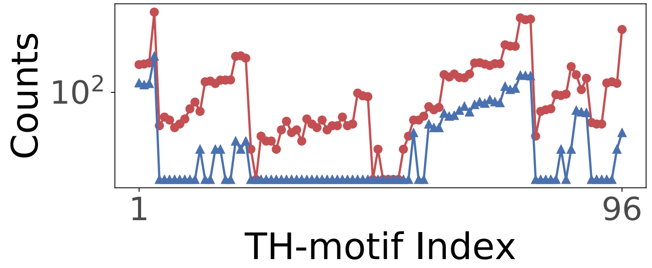


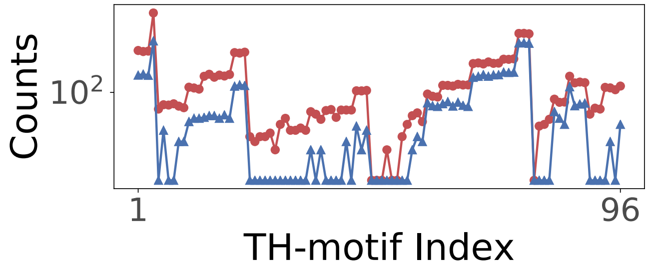

**email-Eu**



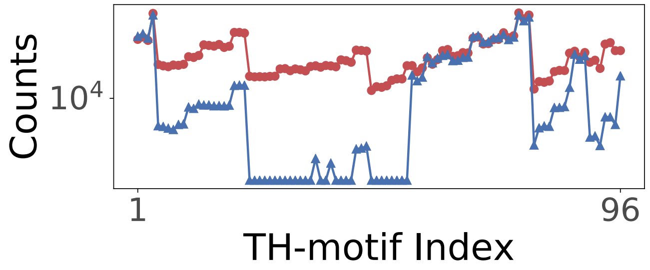

**contact-primary**




**threads-math**




**tags-ubuntu**

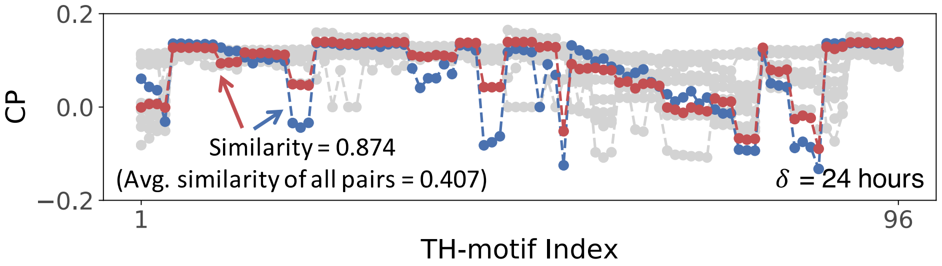



**coauth-DBLP**

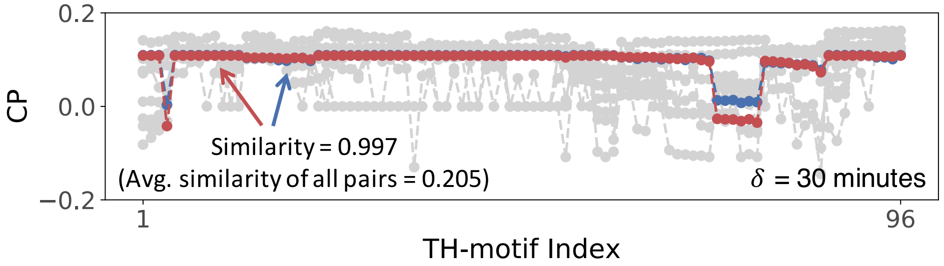
# Comparison across Domains

- TH-motifs play a key role in capturing **structural** & **temporal** patterns.

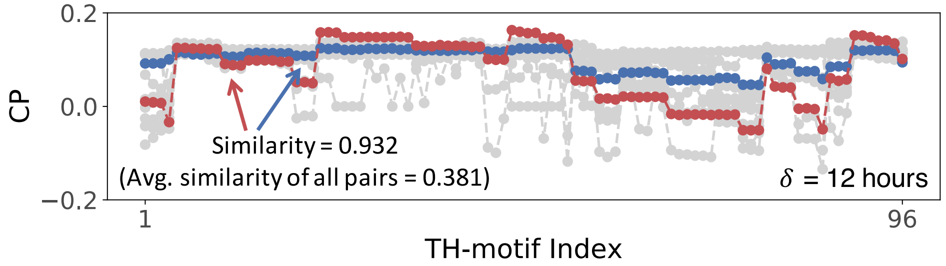
**Email domain**



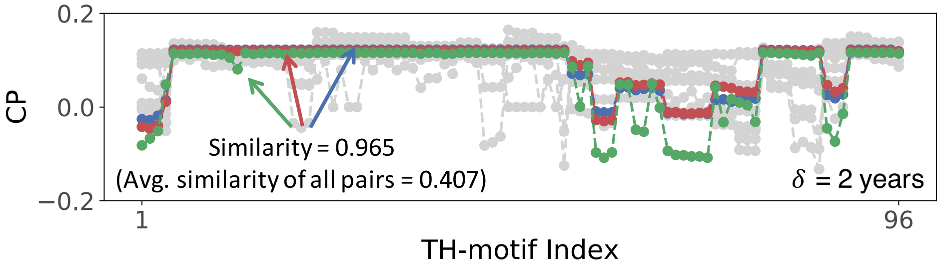

**Contact domain**



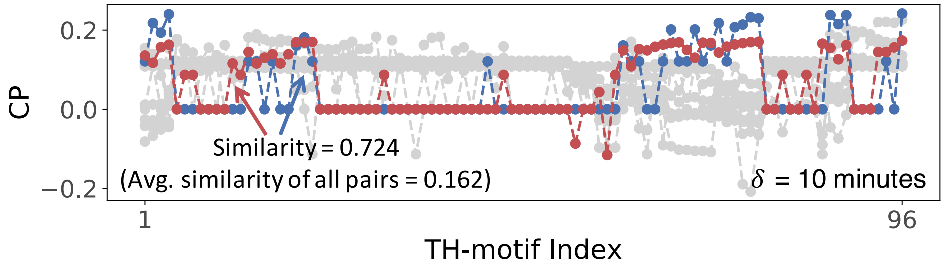

**Tags domain**




**Coauthorship domain**

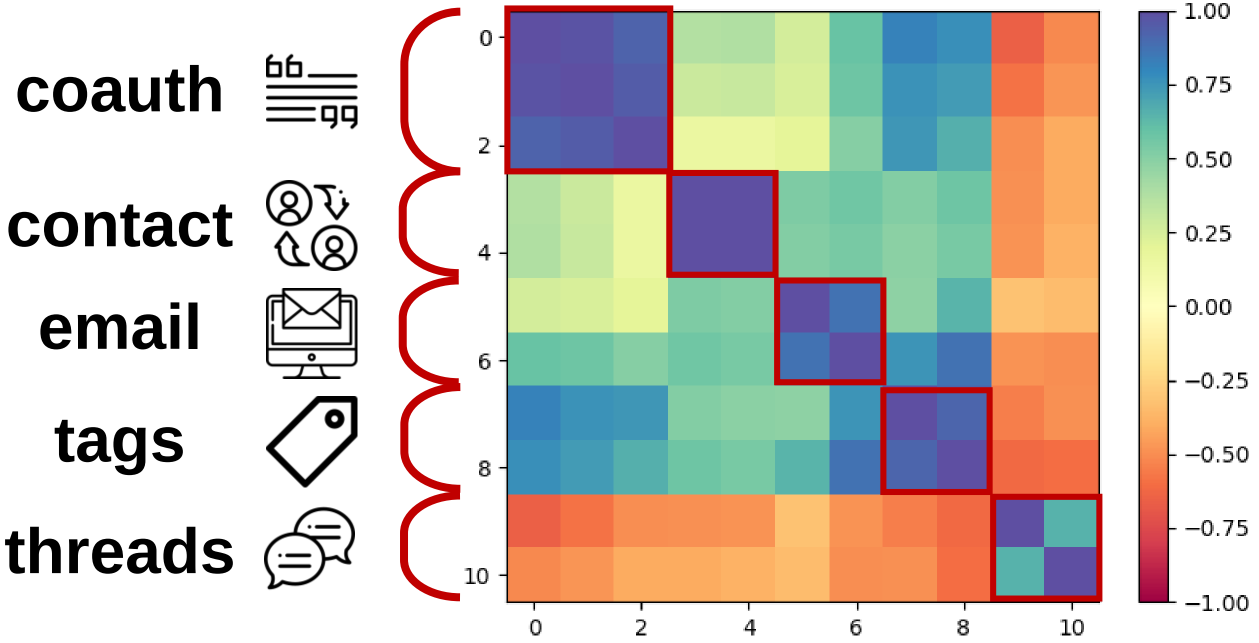



**Threads domain**



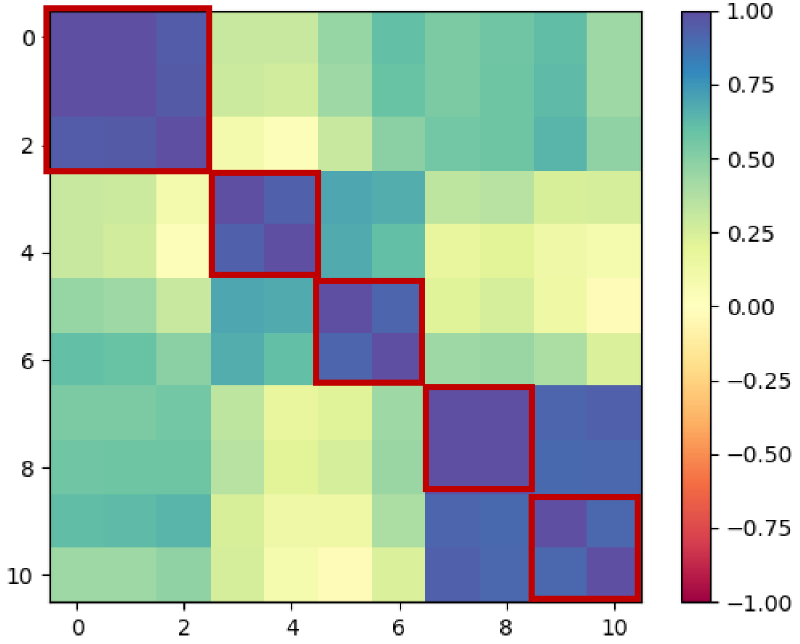
# Comparison across Domains (cont.)

- TH-motifs play a key role in capturing **structural** & **temporal** patterns.




**Temporal Hypergraph Motif**

- Within-domain: 0.900 Gap: **0.759**
- Between-domain: 0.141 Times: **6.38X**




**Static Hypergraph Motif**

- Within-domain: 0.951 Gap: 0.517
- Between-domain: 0.434 Times: 2.19X

# CK21: One Advanced Dynamic Pattern

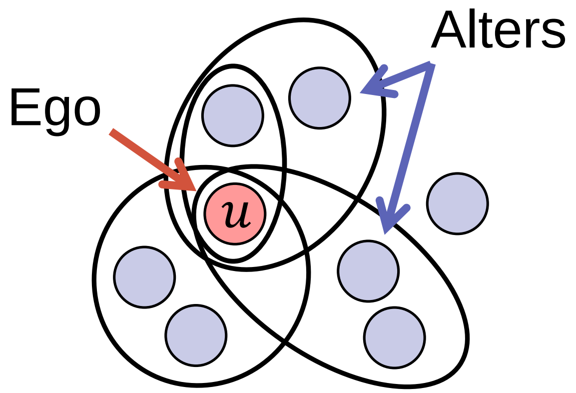
- **P1.** Intersection size of ego-networks
- **P2.** Spread of alter-networks
- **P3.** Anthropic principles of ego-networks
- **P4.** Novelty rate of ego-networks

| | | | |
|---|---|---|---|
| | Node | | |
| | Hyperedge | | |
| | Hypergraph | | |
| | Sub-hypergraph | | P1, P2, P3, P4 |

# Hypergraph Ego-networks

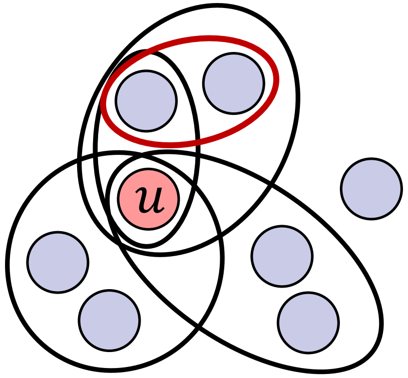
- There are three types of **hypergraph ego-networks**.

**Star ego-network**



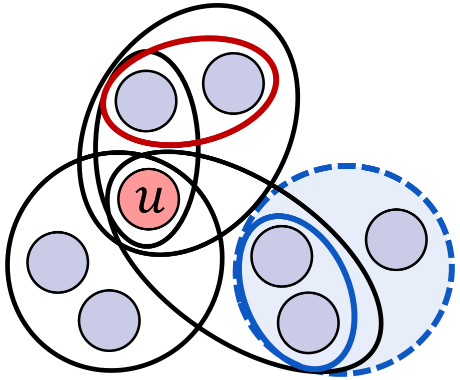

Hyperedges containing the ego-node.

**Radial ego-network**




Hyperedges containing ego-node **or** its neighbors.

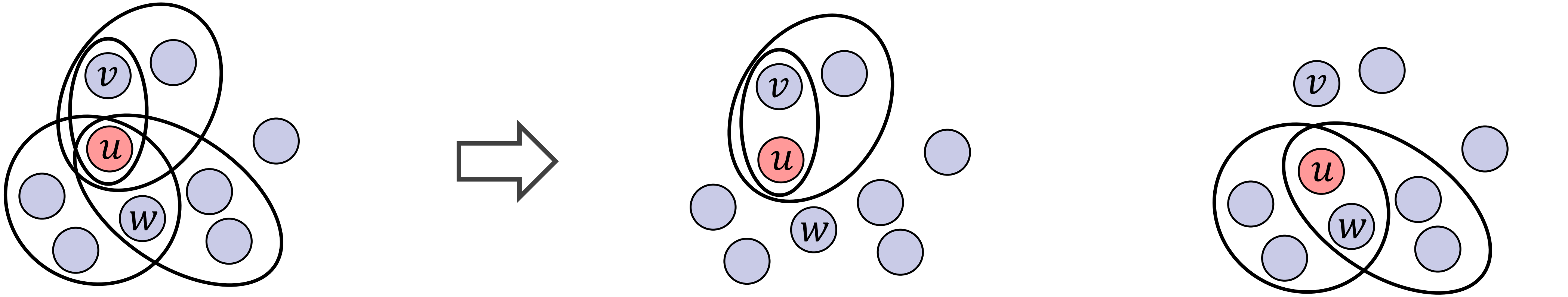
**Contracted ego-network**




Hyperedges containing ego-node or its neighbors **+ sub-hyperedges** containing them

# Hypergraph Alter-networks

- An **alter-network** of node $\boldsymbol{v}$ in **ego-network** of node $\boldsymbol{u}$ is the collection of hyperedges in the ego-network that contain $\boldsymbol{v}$.



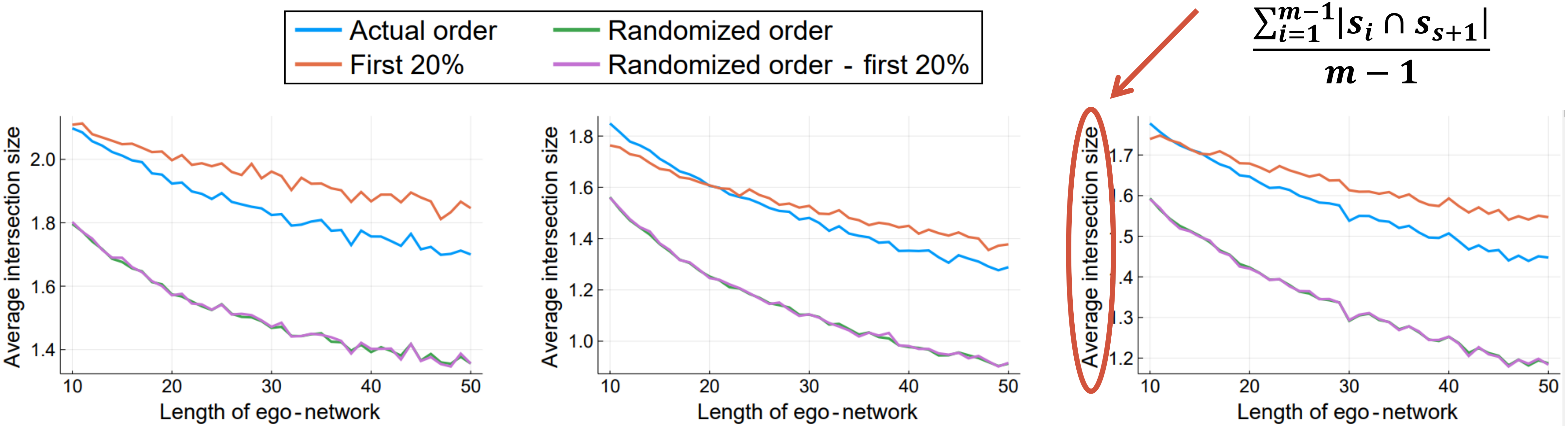

**Ego-network of $\boldsymbol{u}$** **Alter-network of $\boldsymbol{v}$** **Alter-network of $\boldsymbol{w}$**

# Intersection Size of Ego-networks

- **Temporally adjacent** hyperedges are similar.

Average intersection size of ego-network $\{s_1, \ldots, s_m\}$:

$$\frac{\sum_{i=1}^{m-1} |s_i \cap s_{s+1}|}{m-1}$$



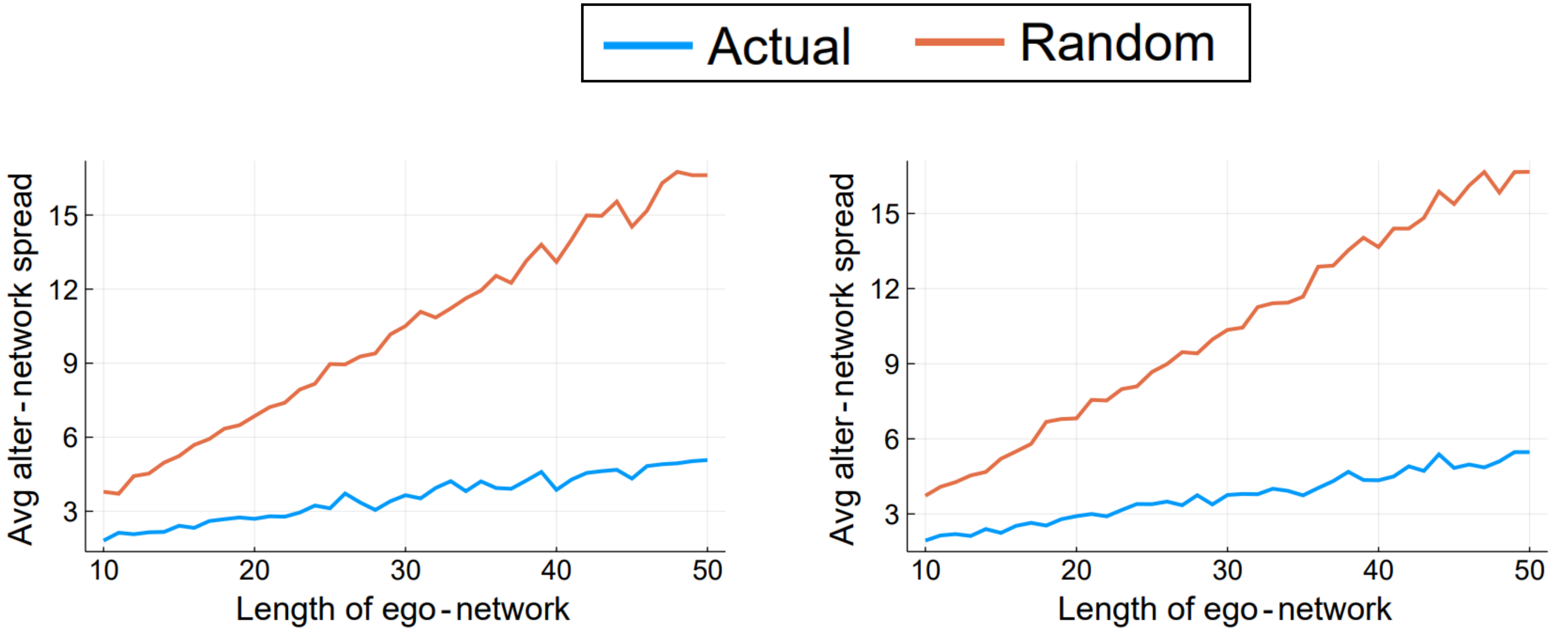

**Star ego-network** **Radial ego-network** **Contracted ego-network**

# Spread of Alter-networks

- Spread of **alter networks** are temporally local.

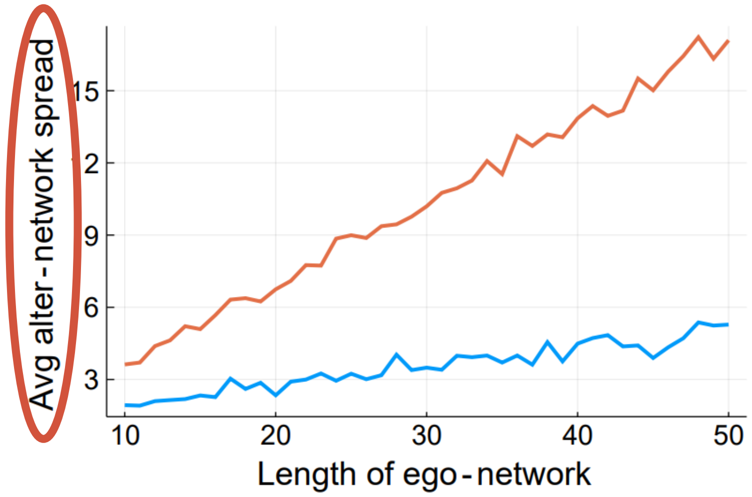
Average spread time of alter-network $\{s_i, \ldots, s_m\}$:

$$\frac{t(s_m) - t(s_1)}{m - 1}$$




**Star ego-network**

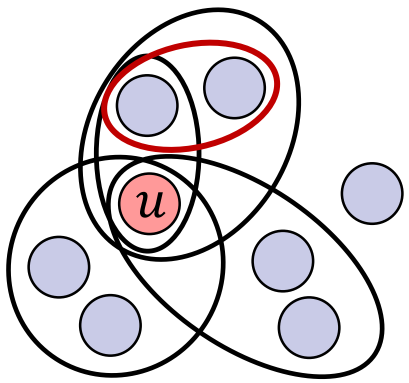
**Radial ego-network**



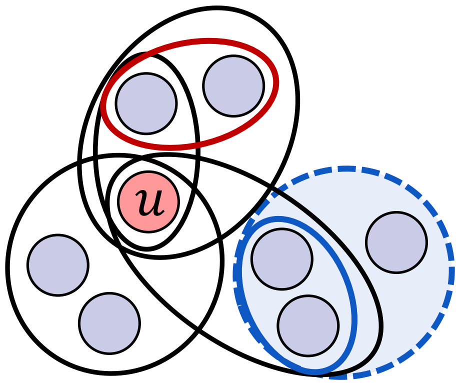

**Contracted ego-network**

# Anthropic Principle of Ego-networks

- **Radial & contracted ego-networks** may include hyperedges without ego-node $u$.
- These hyperedges may arrive **before** the ego-node entered it.




**Radial ego-network**




**Contracted ego-network**

# Anthropic Principle of Ego-networks (cont.)

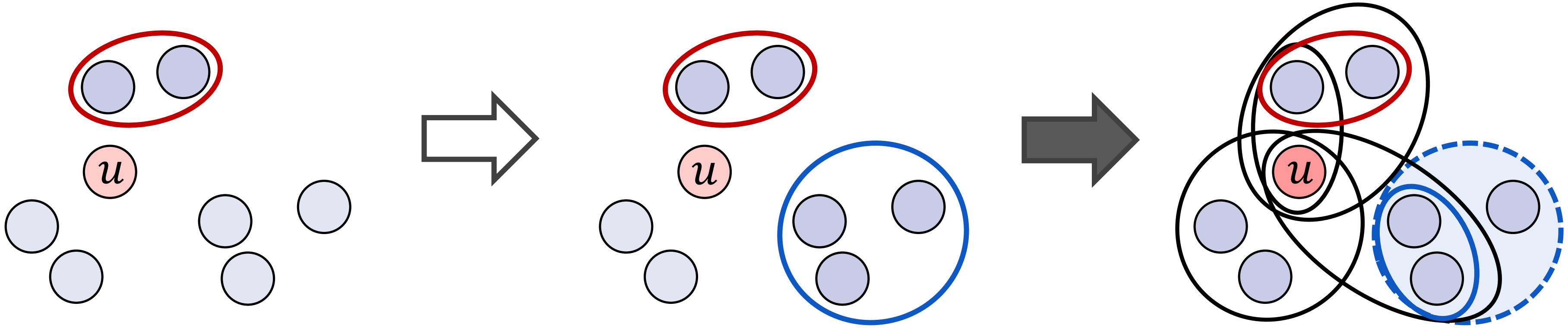
?

**Question:**

Across real-world radial & contracted ego-networks,
at what timestamp does the ego-node arrive?

**Answer:**

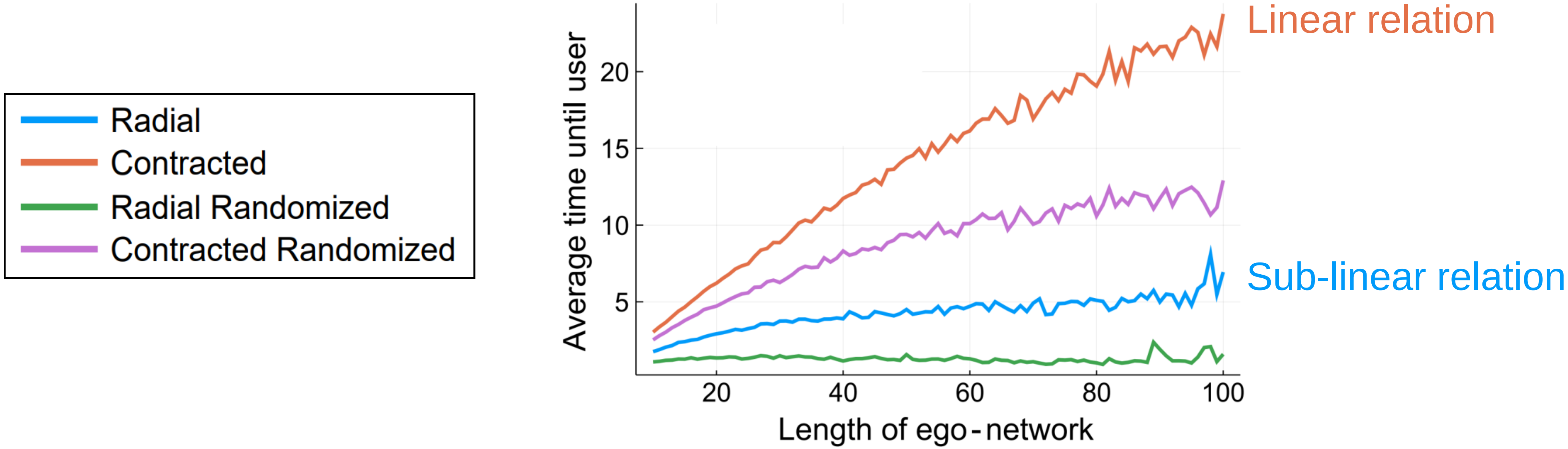
Timestamps of the ego-node arrival are observed.




!

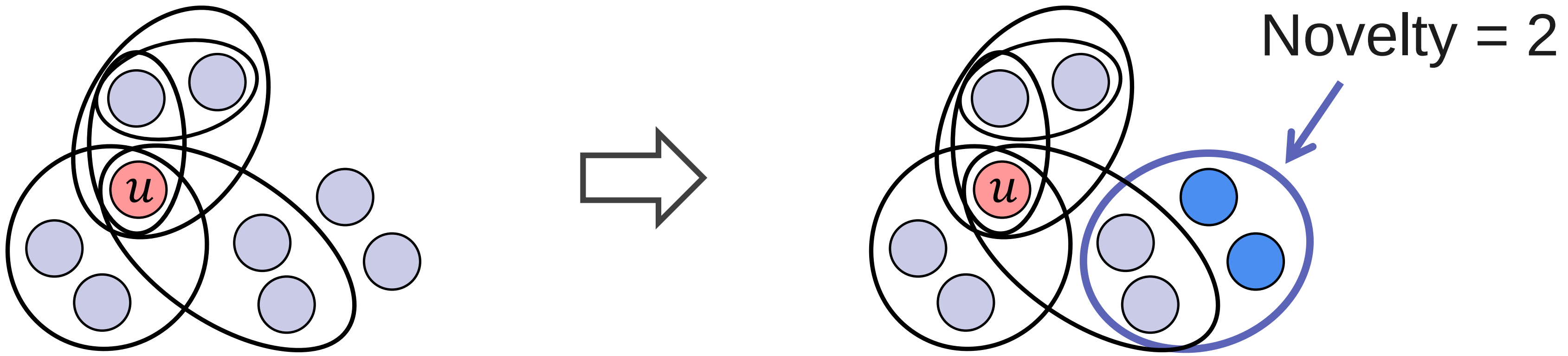
# Anthropic Principle of Ego-networks (cont.)

- The **ego-node** tends to arrive…
  - before or around the fifth timestamp in **radial ego-networks**,
  - at linear timestamp to the size in **contracted ego-networks**.



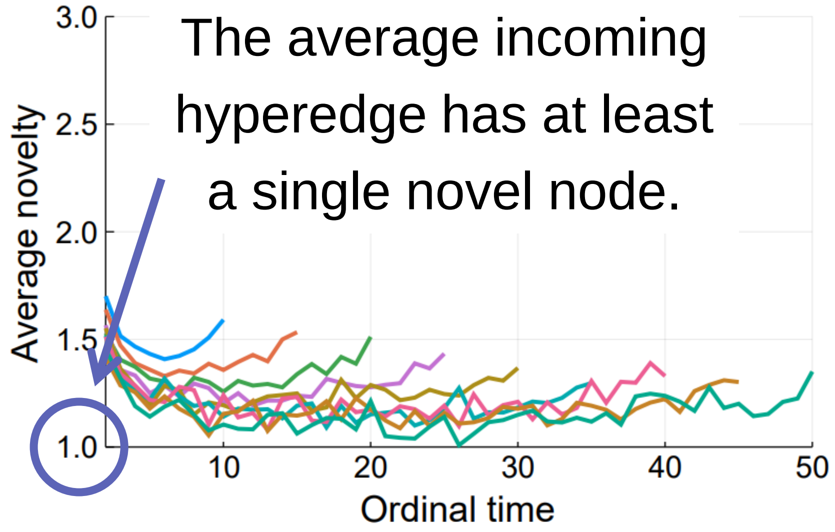
# Novelty Rate of Ego-networks

- **Novelty** of a hyperedge is the number of nodes in the hyperedge that have never been contained in any previous hyperedges.

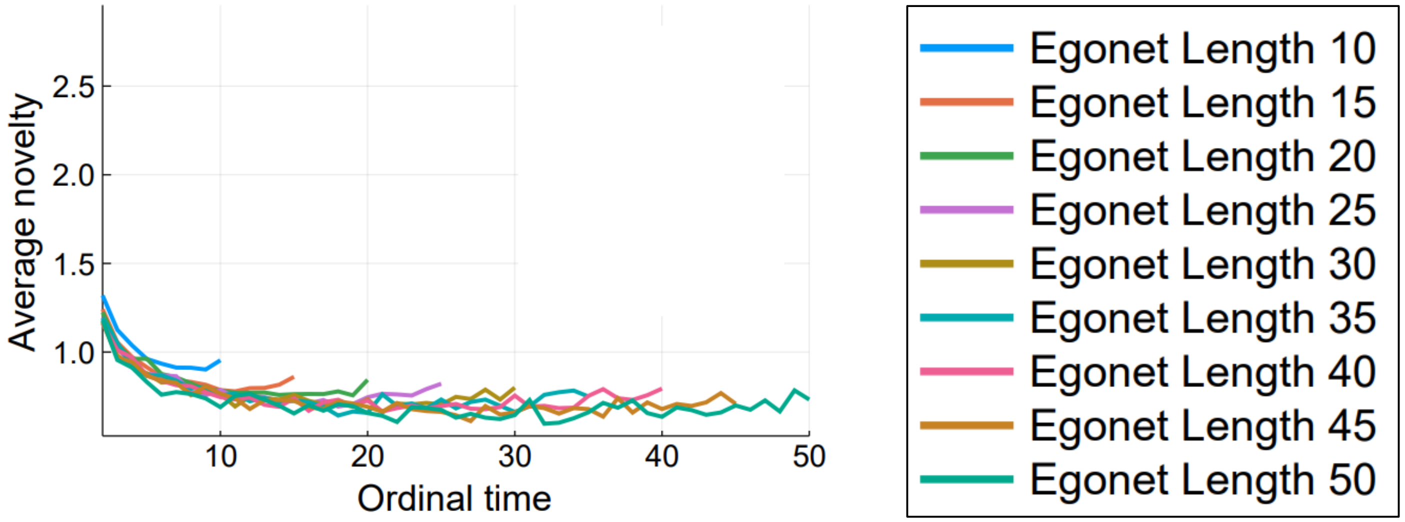


# Novelty Rate of Ego-networks (cont.)

- Novelty slowly & gradually **decreases** in **star & radial ego-networks**.



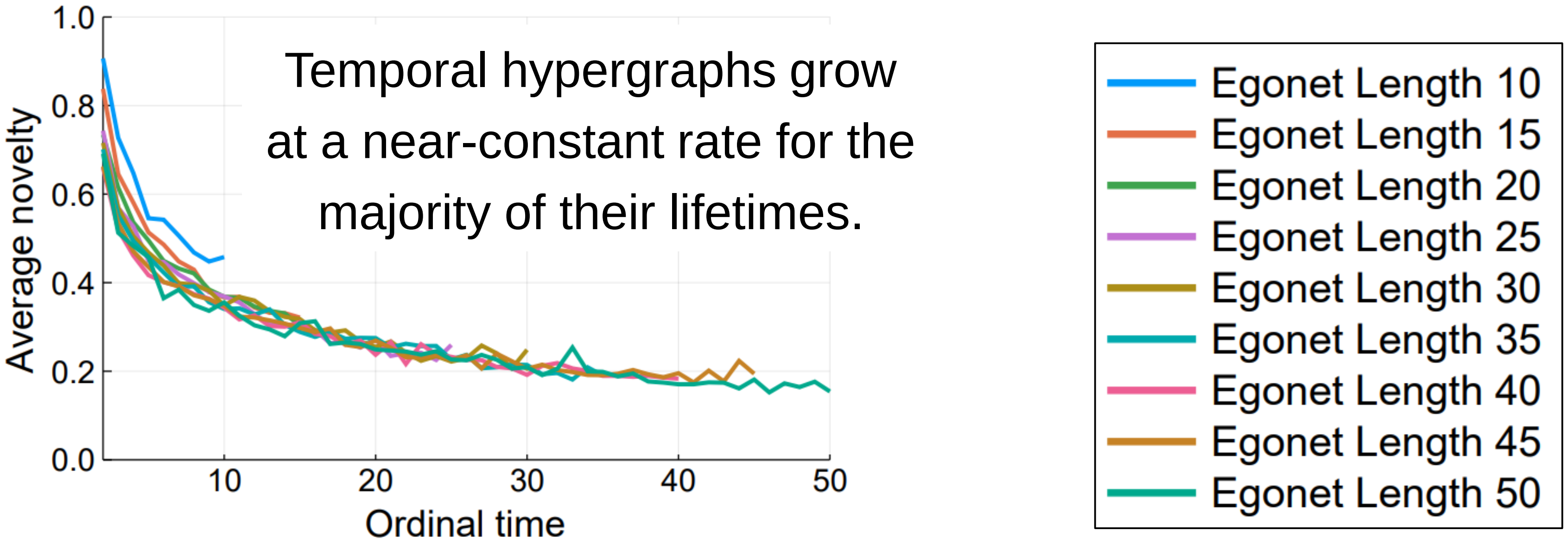

**Star ego-network**




**Radial ego-network**

# Novelty Rate of Ego-networks (cont.)

- Novelty always **decreases** in **contracted ego-networks**.




**Contracted ego-network**

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7. [LS21] Lee, Geon, and Kijung Shin. “THyMe+: Temporal Hypergraph Motifs and Fast Algorithms for Exact Counting.” ICDM 2021.