TRANSFORMATA FOURIER DISCRETA

SEMINAR

1) Breviar teoretic

; $N \in \mathbb{N}^*$

$$\boxed{w = w_N = \exp\left(\frac{2\pi j}{N}\right) = \cos\frac{2\pi}{N} + j\sin\frac{2\pi}{N}}$$

$$\Downarrow$$

$$\underline{w^N = 1} \quad ; \underline{|w| = 1}$$

$K^N = \{ x : \mathbb{Z} \to K \mid x(n+N) = x(n), \forall n \in \mathbb{Z} \}$
semnale finite de lungime N.

(1) $\underline{X = \mathcal{F}_d} \quad x \in K^N \Longrightarrow \underline{X(m) = (\mathcal{F}_d x)(m) = \sum_{n=0}^{N-1} x(n) w^{-mn} \;;\; m \in \mathbb{Z}}$

$X \in K^N$

(2) Forma matriceală $\quad X = Wx \;;\; x = (x(1), x(2), \dots, x(N-1))^T$

$X = (X(1), X(2), \dots, X(N-1))^T \;;\quad W = \begin{pmatrix} 1 & 1 & 1 \dots & 1 \\ 1 & w^{-1} & w^{-2} \dots & w^{-(N-1)} \\ 1 & w^{-2} & w^{-4} \dots & w^{-2(N-1)} \\ \hline 1 & w^{-(N-1)} & w^{-2(N-1)} & \dots w^{-(N-1)^2} \end{pmatrix}$

(3) $\underline{N = 4} \quad w = w_4 = \exp\left(\frac{2\pi j}{4}\right) = e^{\pi j/2} = j \Rightarrow$

$\Rightarrow w^{-1} = 1/j = -j \quad \Longrightarrow \quad W = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{pmatrix}$

(4) TFD inversă (TFDI) $\quad (\mathcal{F}_d^{-1} X)(n) = \frac{1}{N} \sum_{m=0}^{N-1} X(m) w^{mn}$

(5) Forma matriceală a TFDI $\quad x = \frac{1}{N} \overline{W} X$

$\begin{cases} |z| = 1 \\ \overline{z} = z^{-1} \\ \overline{z^{-1}} = z \end{cases}$

$\overline{W} = \begin{pmatrix} 1 & 1 & 1 \dots & 1 \\ 1 & w & w^2 \dots & w^{N-1} \\ 1 & w^2 & w^4 \dots & w^{2(N-1)} \\ \hline 1 & w^{N-1} & w^{2(N-1)} & w^{(N-1)^2} \end{pmatrix}$

(7) Energia $\quad E(x) = \sum_{n=0}^{N-1} |x(n)|^2$
$x \in K^N$

(6) $\underline{N = 4} \quad \overline{W} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{pmatrix}$

(8) Formula lui PARSEVAL
$E(X) = N\, E(x) \;;\; x \in K^N$

(2) $\boxed{\text{Ex. 1}}$ Fie $x \in K^{30}$; $x(n) = (-j)^n$, $0 \le n \le 29$. Se cere

1) $s = x(10) + x(30) + x(-101)$

2) $E(x) = ?$; 3) $X = F_d x$; 5) $E(X)$; 4) $X(15)$

R. $\begin{cases} 0 \le n \le 29 \Longrightarrow x(n) = (-j)^n \\ x(n+30) = x(n), \ \forall n \in \mathbb{Z} \iff x(n-30) = x(n), \ \forall n \in \mathbb{Z} \end{cases}$ $\boxed{w^{30} = 1}$ (*)

1) $x(10) = (-j)^{10} = j^{10} = -1$

$x(30) = x(30-30) = x(0) = (-j)^0 = 1$

$x(-101) = x(-101 + 4 \cdot 30) = x(19) = (-j)^{19} = -j^{19} = -j^3 = j$

$s = -1 + 1 + j = j$

2) $E(x) = \sum_{n=0}^{29} |x(n)|^2 = \sum_{n=0}^{29} |(-j)^n| = \sum_{n=0}^{29} |-j|^n = \sum_{n=0}^{29} 1^n = 30$

3) $X(m) = \sum_{n=0}^{29} x(n) w^{-mn}$; $w = \exp\left(\frac{2\pi j}{30}\right) = e^{\pi j/15} = \cos\frac{\pi}{15} + j \sin\frac{\pi}{15}$

$X(m) = \sum_{n=0}^{29} (-j)^n w^{-mn} = \sum_{n=0}^{29} (-j w^{-m})^n$; fie $q = -j w^{-m} \Longrightarrow$

$\Longrightarrow X(m) = \sum_{n=0}^{29} q^n = 1 + q + q^2 + q^3 + \dots + q^{29} = \begin{cases} \frac{1 - q^{30}}{1 - q}; & q \ne 1 \\ 30; & q = 1 \end{cases}$ (1)

~~Avem $q^{30} = w^{-30m} = (w^{30})^{-m} = \left(\left(e^{2\pi j/15}\right)^{30}\right)^{-m} = \left(e^{2\pi j}\right)^{-m} = 1^{-m} = 1$ (2)~~

~~$q = 1 \iff w^{-m} = 1 \iff e^{-2\pi j m/15} = 1 = e^{2k\pi j} \iff -2\pi m/15$~~

Avem: $q^{30} = (-j w^{-m})^{30} = (-j)^{30} w^{-30m} = -1 \cdot (w^{30})^{-m} \overset{(*)}{=} -1 \cdot 1^{-m} = -1$ (2)

$q = 1 \iff -j w^{-m} = -1 \iff w^{-m} = -j \iff \left(e^{\pi j/15}\right)^{-m} = e^{3\pi j/2} \iff$

$\iff -\frac{\pi j m}{15} = \frac{3\pi j}{2} + 2k\pi j \ \Big|\ \frac{30}{\pi j} \iff -2m = 45 + 60k \iff$

$\iff m = -30k + 45/2$, imposibil, deoarece $m \in \mathbb{Z}$.

Deci $q = -j w^{-m} \ne 1$, $\forall k \in \mathbb{Z}$ (3)

Din (1), (2), (3) $\Longrightarrow \begin{cases} X(m) = \frac{1+1}{1-q} = \frac{2}{1 + j w^{-m}}, \ w = e^{\pi j/15}; \ 0 \le m \le 29 \\ X(m) = X(m+30), \ \forall m \in \mathbb{Z} \end{cases}$

Deci $\begin{cases} X(m) = \dfrac{2}{1+j e^{-\pi j m/15}} ; 0 \le m \le 29 \\ X(m+30) = X(m), \forall m \in \mathbb{Z} \end{cases}$

4) $X(15) = \dfrac{2}{1+j e^{-\pi j \cdot 15/15}} = \dfrac{2}{1+j e^{-\pi j}} = \dfrac{2}{1-j} = 1+j$

5) $E(X) = 30\, E(x) \overset{2)}{=} 30 \cdot 30 = 900$ (PARSEVAL)

3 $\boxed{\text{Ex. 2}}$ Fie $x \in K^4$, $x = (0, 1+j, 2+j, 3+j)^T \in K^4$. Se cere:

1) $s = x(2) + x(-9) + x(727)$

2) $E(x)$; 3) $X = F_d\, x$; 4) $E(X)$ și să se verifice formula PARSEVAL.

R. $x(0) = 0$; $x(1) = 1+j$; $x(2) = 2+j$; $x(3) = 3+j$; $x(n+4) = x(n), \forall n \in \mathbb{Z}$

1) $\left.\begin{array}{l} x(2) = 2+j ; \; x(-9) = x(-9+3\cdot 4) = x(3) = 3+j \\ x(727) = x(27) = x(27-4\cdot 6) = x(3) = 3+j \end{array}\right\} \Rightarrow s = 8+3j$

2) $E(x) = \sum_{n=0}^{3} |x(n)|^2 = |x(0)|^2 + |x(1)|^2 + |x(2)|^2 + |x(3)|^2 =$

$= |0|^2 + |1+j|^2 + |2+j|^2 + |3+j|^2 = 0 + (1+1) + (4+1) + (9+1) = 17$

$\boxed{|a \pm bi|^2 = a^2 + b^2}$

3) Utilizăm forma matriceală a TFD

$$X = Wx = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{pmatrix} \begin{pmatrix} 0 \\ 1+j \\ 2+j \\ 3+j \end{pmatrix} = \begin{pmatrix} 0+1+j+2+j+3+j \\ 0-j+\cancel{1}-2-j+3j-\cancel{1} \\ 0-1-j+2+j-3-j \\ 0+j-\cancel{1}-2-\cancel{j}-3j+\cancel{1} \end{pmatrix} = \begin{pmatrix} 6+3j \\ -2+j \\ -2-j \\ -2-3j \end{pmatrix},$$

deci $X(0) = 6+3j$; $X(1) = -2+j$; $X(2) = -2-j$; $X(3) = -2-3j$,

$X(m+4) = X(m), \forall m \in \mathbb{Z}$

4) $E(X) = |6+3j|^2 + |-2+j|^2 + |-2-j|^2 + |-2-3j|^2 = 45+5+5+13 = 68$

Verificarea formulei lui PARSEVAL: $E(X) = 4\, E(x) \Leftrightarrow$

$\Leftrightarrow 68 = 4 \cdot 17$ „A"

(4) $\boxed{\text{Ex. 3.}}$ Fie $X \in K^4$; $X = (1+j;\ 0;\ j;\ 3+5j)^T \in K^4$. Se cere:

1) $S = X(3) + X(-33) + X(333)$

2) $E(X)$; 3) $x = \mathcal{F}_d^{-1} X$; 4) $E(x)$ şi verificarea formulei lui PARSEVAL

R. $X(0) = 1+j$; $X(1) = 0$; $X(2) = j$; $X(3) = 3+5j$; $X(m+4) = X(m)$, $\forall m \in \mathbb{Z}$.

1) $X(3) = 3+5j$; $X(-33) = X(-33+36) = X(3) = 3+5j$;
$X(333) = X(33) = X(33-32) = X(1) = 0 \Rightarrow S = 6+10j$

2) $E(X) = |1+j|^2 + |0|^2 + |j|^2 + |3+5j|^2 = 2+0+1+34 = 37$

3) Utilizăm forma matriceală a TFDI

$$x = \frac{1}{4}\overline{W}X = \frac{1}{4}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{pmatrix}\begin{pmatrix} 1+j \\ 0 \\ j \\ 3+5j \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 1+j+0+j+3+5j \\ 1+j+0-j-3j+5 \\ 1+j+0+j-3-5j \\ 1+j+0-j+3j-5 \end{pmatrix}$$

$$x = \frac{1}{4}\begin{pmatrix} 4+7j \\ 6-3j \\ -2-3j \\ -4+3j \end{pmatrix}$$, deci $x(0) = 1+\frac{7}{4}j$; $x(1) = \frac{3}{2} - \frac{3}{4}j$; $x(2) = -1-\frac{3j}{4}$; ~~x(3)~~
$x(3) = -1+\frac{3}{4}j$

4) $E(x) = |x(0)|^2 + |x(1)|^2 + |x(2)|^2 + |x(3)|^2 = \frac{1}{4^2}\left(|4+7j|^2 + |6-3j|^2 + |-2-3j|^2 + |-4+3j|^2\right) = \frac{1}{16}(65+45+13+25) = \frac{1}{16}\,148 = \frac{1}{4}\cdot 37 =$
$= 9{,}25$.

Verificarea formulei lui PARSEVAL: $E(X) = 4E(x) \Leftrightarrow 37 = 4\cdot\frac{37}{4}$, „A"

(5) $\boxed{\text{Ex. 4.}}$ Temă $x \in K^{18}$; $x(n) = j^{n}$; $0 \le n \le 17$. Se cere

1) $s = x(18) + x(100) + x(-100)$; 2) $E(x)$; 3) $X = \mathcal{F}_d x$; 4) $X(9)$

$\boxed{\text{Ex. 5}}$ Temă $x \in K^4$; $x = (j;\ 1,\ 2j;\ 4+5j)^T \in K^4$. Se cere:

1) $s = x(1) + x(11) + x(-111)$; 2) $E(x)$; 3) $X = \mathcal{F}_d x$;
4) $E(X)$ şi să se verifice formula lui PARSEVAL

TEMA DIN CURS pag. 196–197 $\boxed{\text{MS}}$ Ex. 2, (vi), vii;
Ex. 5, (iii); Ex. 6 (integral); Ex. 7 (integral)

TRANSFORMAREA „Z"

SEMINAR

① BREVIAR TEORETIC

$$S_d = \{x : \mathbb{Z} \to K\} = \{(x_n)_{n\in\mathbb{Z}}\} ; \ n\in\mathbb{Z} \mapsto x(n) = x_n \in K$$

$$S_d^+ = \{x \in \mathbb{Z} \to K \ / \ x(n) = 0, \ \forall n<0\} = \{(x_n)_{n\geq 0}\}$$

(1) **Def.** $x \in S_d$; $X(z) = \sum_{n=-\infty}^{\infty} \frac{x(n)}{z^n}$; $z \in U(x)$

$$X(z) = \dots + \frac{x(-n)}{z^{-n}} + \dots + \frac{x(-2)}{z^{-2}} + \frac{x(-1)}{z^{-1}} + x(0) + \frac{x(1)}{z} + \frac{x(2)}{z^2} + \dots + \frac{x(n)}{z^n} + \dots$$

(2) **Def.** $x \in S_d^+$; $X(z) = \sum_{n=0}^{\infty} \frac{x(n)}{z^n} = x(0) + \frac{x(1)}{z} + \frac{x(2)}{z^2} + \dots + \frac{x(n)}{z^n} + \dots$

Funcția $X(z)$ din (1) sau (2) se numește transformata „z" asociată semnalului $x^{(n)} \in S_d$, respectiv $x(n) \in S_d^+$

NOTAȚIE
$X(z) = \mathcal{Z}\{x(n)\}(z)$

(3) $\delta_k : \mathbb{Z} \to K$; $\delta_k(n) = \begin{cases} 0; & n \neq k \\ 1; & n = k \end{cases}$

$\delta : \mathbb{Z} \to K$, $\delta(n) = \begin{cases} 0; & n \neq 0 \\ 1; & n = 0 \end{cases} = \delta_0(n)$

semnale (impulsuri) discrete Dirac

$u : \mathbb{Z} \to K$, $u(n) = \begin{cases} 0; & n < 0 \\ 1; & n \geq 0 \end{cases}$ semnalul discret Heaviside

(4) $\mathcal{Z}\{\delta_k(n)\} = z^{-k}$; $\mathcal{Z}\{\delta(n)\} = 1$

(5) $\mathcal{Z}\{a^n u(n)\}(z) = \frac{z}{z-a}$; $a \in K^*$; $|z| > |a|$

(6) $\mathcal{Z}\{n a^n u(n)\}(z) = \frac{az}{(z-a)^2}$; $a \in K^*$; $|z| > |a|$

(7) Teorema derivării semnalului (Teorema înmulțirii cu n)

$x \in S_d \Longrightarrow \mathcal{Z}\{n x(n)\}(z) = -z \cdot \mathcal{Z}'\{x(z)\}(n) = -z \cdot X'(z)$

OBS. $x(n) \xmapsto{\mathcal{Z}} X(z) \Rightarrow n x(n) \xmapsto{\mathcal{Z}} -z \cdot X'(z)$

(8) Transformata Z inversă

$X(z) \in \mathcal{H}(U(0; r; R)) \Longrightarrow x(n) = \mathcal{Z}^{-1}\{X(z)\}(n) = \sum_{k=1}^{m} \operatorname{Rez}[z^{n-1} X(z); z_k]$,

unde $z_1, z_2, z_3, \dots, z_m \in \mathbb{C}$ sunt punctele singulare pt. $g(z) = z^{n-1} X(z)$

(9) $X(z)=\mathcal{Z}\{x(n)\}(z) \iff x(n)=\mathcal{Z}^{-1}\{X(z)\}(n)$ | p.248

(10) $\begin{cases} \mathcal{Z}\{\delta_k(n)\}(z)=z^{-k} \iff \mathcal{Z}^{-1}\{z^{-k}\}(n)=\delta_{k}(n) \\ \mathcal{Z}\{a^n u(n)\}(z)=\dfrac{z}{z-a} \iff \mathcal{Z}^{-1}\left\{\dfrac{z}{z-a}\right\}(z)=a^n u(n) \\ \mathcal{Z}\{na^n u(n)\}(z)=\dfrac{az}{(z-a)^2} \iff \mathcal{Z}^{-1}\left\{\dfrac{z}{(z-a)^2}\right\}(z)=na^{n-1}u(n) \end{cases}$

(2) Ex. 1. Utilizând metoda transformării „z", să se calculeze suma seriei numerice $s=\sum_{n=-1}^{\infty}\dfrac{3n^2(-1)^n+2n\cos^2\left(\frac{n\pi}{3}\right)}{2^n}$

R. Avem: $\mathcal{Z}\{x(n)\}(z)=\sum_{n=0}^{\infty}\dfrac{x(n)}{z^n}$; $x\in S_d^+$

$$s=\sum_{n=0}^{\infty}\frac{3n^2(-1)^n+2n\cos^2\left(\frac{n\pi}{3}\right)}{2^n}+\left.\frac{3n^2(-1)^n+2n\cos^2\left(\frac{n\pi}{3}\right)}{2^n}\right|_{n=-1}$$

$$s=3\sum_{n=0}^{\infty}\frac{n^2(-1)^n}{2^n}+2\sum_{n=0}^{\infty}\frac{n\cos^2\left(\frac{n\pi}{3}\right)}{2^n}+\frac{3\cdot1(-1)-2\cos^2(-\pi/3)}{2^{-1}}$$

$$s=3\sum_{n=0}^{\infty}\frac{n^2}{(-2)^n}+2\sum_{n=0}^{\infty}\frac{n\cos^2\left(\frac{n\pi}{3}\right)}{2^n}-7=3s_1+2s_2-7 \quad (1)$$

$$3s_1=\sum_{n=0}^{\infty}\frac{\overset{x(n)}{n^2}}{\underset{z}{(-2)}^n}=3\mathcal{Z}\{n^2\}(-2)=3\cdot\left.\frac{z(z+1)}{(z-1)^3}\right|_{z=-2}=3\frac{(-2)(-1)}{(-3)^3}=-\frac{2}{9} \quad (2)$$

Utilizând egalitatea $\cos^2\alpha=\frac{1}{2}(1+\cos 2\alpha)$, avem:

$$2s_2=\cancel{2}\sum_{n=0}^{\infty}\frac{n}{2^n}\cdot\frac{1+\cos\frac{2n\pi}{3}}{\cancel{2}}=\sum_{n=0}^{\infty}\frac{n}{2^n}+\sum_{n=0}^{\infty}\frac{\overset{x(n)}{n\cdot\cos\frac{2n\pi}{3}}}{\underset{z}{2}^n}=s_3+s_4 \quad (3)$$

$$s_3=\sum_{n=0}^{\infty}\frac{\overset{x(n)}{n}}{\underset{z}{2}^n}=\mathcal{Z}\{n\}(2)=\left.\frac{z}{(z-1)^2}\right|_{z=2}=2 \quad (4)$$

Pentru s_4, utilizăm Teorema derivării (1), (7), cu $x(n)=\left(\cos\frac{2n\pi}{3}\right)u(n)$ și $z=2$ $\Rightarrow s_4=\mathcal{Z}\{n\cos\frac{2n\pi}{3}\}(2)=-z\cdot\mathcal{Z}'\{\cos\frac{2n\pi}{3}\}(z)\Big|_{z=2}$

Deoarece $\boxed{\mathcal{Z}\{\cos(an)u(n)\}(z)=\dfrac{z(z-\cos a)}{z^2-2z\cos a+1},\ a=\frac{2\pi}{3}}$ (p.248),

primim: $s_4=-2\cdot\left(\dfrac{z(z-\cos\frac{2\pi}{3})}{z^2-2z\cos\frac{2\pi}{3}+1}\right)'_{z=2}=-2\cdot\left(\dfrac{z^2+\frac{z}{2}}{z^2+z+1}\right)'_{z=2}=$

$$= -\left(\frac{2z^2+z}{z^2+z+1}\right)'_{z=2} = -\left.\frac{(4z+1)(z^2+z+1)-(2z+1)(2z^2+z)}{(z^2+z+1)^2}\right|_{z=2} =$$

$$= -\frac{9\cdot 7-5\cdot 10}{7^2} = -\frac{13}{49} \quad (5)$$

Din (1) – (5) $\Longrightarrow s = -\frac{2}{9}+2-\frac{13}{49}-7 = -5-\frac{98+117}{49\cdot 9} = -5-\frac{215}{441} \Rightarrow$

$$\Rightarrow s = -\frac{2420}{441}$$

3 Ex. 2 Utilizând metoda (tehnica) transformării „z", să se rezolve ecuația cu diferențe finite:

$$x(n+2)-4x(n+1)+4x(n) = 3^n u(n) \; ; \; x \in S_d^+$$

$$x(0) = 0; \; x(1) = 2$$

R. Formulare echivalentă $x(n) = x_n$.

$\begin{cases} \text{Să se determine șirul } (x_n)_{n\ge 0} \text{ care verifică relația de} \\ \text{recurență liniară } x_{n+2}-4x_{n+1}+4x_n = 3^n; \; n\ge 0; \; x_0=0, x_1=2 \end{cases}$

Etapa 1 Determinăm $X(z) = Z\{x(n)\}(z)$ (1)

Din Teorema translației la stânga, avem:

$$(2)\begin{cases} Z\{x(n+1)\} = zX(z)-zx(0) \\ Z\{x(n+2)\} = z^2X(z)-z^2x(0)-zx(1) \end{cases}$$

Aplicând operatorul (transformata) z ecuației date, obținem

$$Z\{x(n+2)\}(z)-4Z\{x(n+1)\}(z)+4Z\{x(n)\}(z) = Z\{3^n u(n)\}(z)$$

Din (1) și (2), rezultă

$$z^2X(z)-\cancel{z^2 0}-z\cdot 2-4(zX(z)-\cancel{z0})+4X(z) = \frac{z}{z-3}$$

$$X(z)\cdot(z^2-4z+4) = 2z+\frac{z}{z-3} \iff X(z) = \frac{2z^2-5z}{(z-3)(z^2-4z+4)} \iff$$

$$X(z) = \frac{z(2z-5)}{(z-3)(z-2)^2} \quad (3)$$

Etapa 2 Determinăm $x(n) = \mathcal{Z}^{-1}\{X(z)\}(n)$

Din (3) $\Rightarrow \dfrac{X(z)}{z} = \dfrac{2z-5}{(z-3)(z-2)^2} = \dfrac{A}{z-3} + \dfrac{B}{z-2} + \dfrac{C}{(z-2)^2} \Leftrightarrow$

$\Leftrightarrow 2z-5 = A(z-2)^2 + B(z-2)(z-3) + C(z-3)$

$z=3 \Rightarrow A=1$; $z=2 \Rightarrow C=1$; z^2: $0 = A+B \Rightarrow B=-1$

Deci: $\dfrac{X(z)}{z} = \dfrac{1}{z-3} - \dfrac{1}{z-2} + \dfrac{1}{(z-2)^2}$

$X(z) = \dfrac{z}{z-3} - \dfrac{z}{z-2} + \dfrac{z}{(z-2)^2} \Big/ \mathcal{Z}^{-1}$

$x(n) = \mathcal{Z}^{-1}\left\{\dfrac{z}{z-3}\right\}(n) - \mathcal{Z}^{-1}\left\{\dfrac{z}{z-2}\right\}(n) + \mathcal{Z}^{-1}\left\{\dfrac{z}{(z-2)^2}\right\}(n)$

$x(n) = \left(3^n - 2^n + n\cdot 2^{n-1}\right)u(n)$

(4) Temă 1) $s = \displaystyle\sum_{n=1}^{\infty} \dfrac{3n(-1)^n + n\cdot \sin^2\left(\frac{n\pi}{4}\right) + \cos\frac{n\pi}{3}}{3^n}$

2) $s = \displaystyle\sum_{n=0}^{\infty} \dfrac{n^2 + 3n\cos\frac{4n\pi}{3} + (-4)^n}{5^n}$

(5) Temă Rezolvați ecuațiile cu diferențe finite:

1) $2x(n+2) - 5x(n+1) + 2x(n) = n(-1)^n$; $x(0) = x(1) = 0$

2) $x(n+2) - (2+j)x(n+1) + (1+j)x(n) = 1$; $x(0) = x(1) = 0$

3) $x(n+2) - 9x(n) = 3^n u(n)$; $x(0) = 1$; $x(1) = 2$