Seminar 5 - MA

Legi clasice de probabilitate

Breviar teoretic

1. Legea normală (Gauss): $N(m;\sigma^2)$; $m\in\mathbb{R}$; $\sigma\in(0,\infty)$

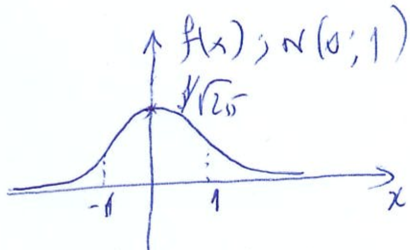
pdf $f(x)=\dfrac{1}{\sigma\sqrt{2\pi}}\,e^{-\frac{(x-m)^2}{2\sigma^2}}$

F. rep. $F(x)=\int_{-\infty}^{x}f(t)\,dt=\dfrac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{x}\exp\left(-\dfrac{(t-m)^2}{2\sigma^2}\right)dt$

$M(N(m;\sigma^2))=m$
$D^2(N(m;\sigma^2))=\sigma^2$
$\sigma(N(m;\sigma^2))=\sigma$

2. Legea normală redusă: $N(0;1)$

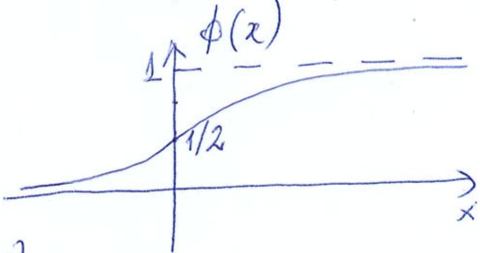
$f(x)=\dfrac{1}{\sqrt{2\pi}}\,e^{-x^2/2}$ $\quad m=0;\ \sigma=1$



3. Funcția lui Laplace

$\phi(x)=F_{N(0;1)}(x)=P(N(0;1)<x)$; $x\in\mathbb{R}$;

$\phi(x)=\dfrac{1}{\sqrt{2\pi}}\int_{-\infty}^{x}e^{-t^2/2}\,dt=\dfrac{1}{\sqrt{\pi}}\int_{-\infty}^{x/\sqrt{2}}e^{-t^2}\,dt$; $x\in\mathbb{R}$



Proprietăți: (i) $\phi(0)=\frac{1}{2}$; $\phi(-x)=1-\phi(x)$, $\forall x\in\mathbb{R}$

(ii) $X=N(m;\sigma^2)\Rightarrow \boxed{P(a\le X\le b)=\phi\left(\frac{b-m}{\sigma}\right)-\phi\left(\frac{a-m}{\sigma}\right)}$, $a,b\in\mathbb{R}$; $a\le b$

(iii) $\alpha>0$; $X=N(m;\sigma^2)\Rightarrow P(|X-m|\le\alpha)=2\phi\left(\frac{\alpha}{\sigma}\right)-1$

$\boxed{P(X\in[m-\alpha,m+\alpha])=2\phi\left(\frac{\alpha}{\sigma}\right)-1}$

4. Teorema limită centrală $(X_n)_{n\ge1}$ v.a. independente;

$\begin{cases} M(X_n)=m\in\mathbb{R},\ \forall n\ge1;\ D^2(X_n)=Var(X_n)=\sigma^2\ne0,\ \forall n\ge1 \Rightarrow \sigma(S_n)=\sigma \\ S_n=X_1+X_2+\dots+X_n;\ n\ge1;\ Y_n=\dfrac{S_n-mn}{\sigma\sqrt{n}}=\dfrac{S_n-M(S_n)}{\sigma(S_n)} \end{cases}$

Avem: $\lim\limits_{n\to\infty}F_{Y_n}(x)=\phi(x)=F_{N(0;1)}(x)$

OBS. Pentru n suficient de mare avem $Y_n\simeq N(0,1)$

$P(Y_n<x)\simeq\phi(x)$; $P(a\le Y_n\le b)\simeq\phi(b)-\phi(a)$.

5. Legea binomială (Bernoulli)

$$P(n;k) = C_n^k p^k q^{n-k}$$

$$X_n : \begin{pmatrix} 0 & 1 & 2 & 3 \dots & n \\ P(n;0) & P(n;1) & P(n;2) & P(n;3) \dots & P(n;n) \end{pmatrix} = \cancel{\begin{pmatrix} k \\ P(n;k) \end{pmatrix}_{0 \le k \le n}}$$

$$X_n : \begin{pmatrix} k \\ P(n;k) \end{pmatrix}_{0 \le k \le n}$$

Not. $X_n = B(p;n)$; $n \ge 1$; $p, q \in [0,1]$; $p + q = 1$.

$\underline{M(B(p;n)) = np}$, $\underline{D^2(B(p;n)) = Var(B(p;n)) = npq}$, $\underline{\sigma(B(p;n)) = \sqrt{npq}}$

6. Aproximarea $B(p;n)$ cu $N(m;\sigma^2)$

Fie $m = np$; $\sigma = \sqrt{npq}$; $X = B(p;n)$; $np \ge 18$; $n \ge 50 \Longrightarrow$

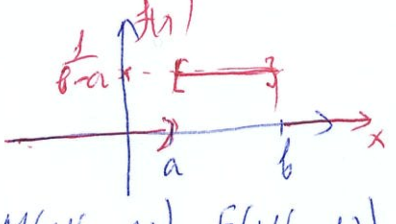
$$\Longrightarrow \begin{cases} P(a \le X \le b) = P(a \le B(p;n) \le b) \simeq \phi\left(\frac{b-m}{\sigma}\right) - \phi\left(\frac{a-m}{\sigma}\right) \\ P(a \le X \le b) = P(a \le B(p;n) \le b) \simeq \phi\left(\frac{b-np}{\sqrt{npq}}\right) - \phi\left(\frac{a-np}{\sqrt{npq}}\right) \\ -\infty \le a \le b \le +\infty \end{cases}$$

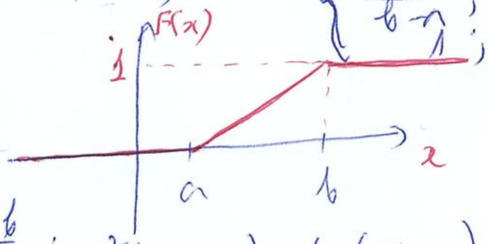
OBS. $P\left(a < \frac{X - np}{\sqrt{npq}} < b\right) \simeq \phi(b) - \phi(a)$; n ≥ a p.h.

7. Legea uniformă

p.d.f. $f : \mathbb{R} \to \mathbb{R}$, $f(x) = \begin{cases} \frac{1}{b-a}; & a \le x \le b \\ 0; & \text{în rest} \end{cases}$ Not. $X = U(a,b)$

F.rep. $F(x) = P(U(a,b) < x) = \begin{cases} 0; & x < a \\ \frac{x-a}{b-a}; & a \le x \le b \\ 1; & x > b \end{cases}$



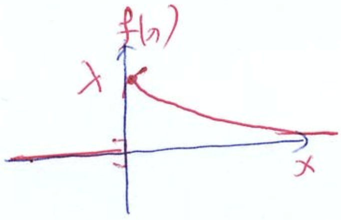


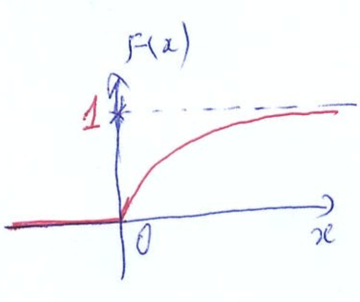
$M(U(a,b)) = E(U(a,b)) = \frac{a+b}{2}$; $D^2(U(a,b)) = Var(U(a,b)) = \frac{(b-a)^2}{12}$; $\sigma(U(a,b)) = \frac{b-a}{2\sqrt{3}}$

OBS. În cazul discret (simplu): $X = U_n : \begin{pmatrix} x_1 & x_2 & x_3 \dots & x_n \\ 1/n & 1/n & 1/n \dots & 1/n \end{pmatrix}$

8. Legea exponențială-negativă; $\lambda > 0$ dat

p.d.f. $f(x) = \begin{cases} 0; & x \le 0 \\ \lambda e^{-\lambda x}; & x > 0 \end{cases}$





F.rep. $F(x) = \begin{cases} 0; & x \le 0 \\ 1 - e^{-\lambda x}; & x > 0 \end{cases}$

$\underline{M(X) = \sigma(X) = 1/\lambda}$; $D^2(X) = Var(X) = 1/\lambda^2$

Funcția de fiabilitate $R(t) = 1 - F(t) = \begin{cases} 1; & t \le 0 \\ e^{-\lambda t}; & t > 0 \end{cases}$; Rata de hazard $r(t) = \begin{cases} 0; & t < 0 \\ \lambda; & t > 0 \end{cases}$

OBS. $\underline{r(t) = \text{const.}}$

Ex. 1 În medie, 64% din semnalele recepționate într-un canal binar cu zgomot sunt simbolurile „1”, iar restul sunt simbolurile „0”. Se emit 5000 de simboluri „0” și „1” și admitem că variabila aleatoare care dă numărul de semnale „1” recepționate este de tip Bernoulli (binomială). Să se determine:

1° Probabilitatea ca numărul de simboluri „1” recepționate să fie situat între 3180 și 3248.

2° Un interval în care se află numărul de simboluri „0” recepționate, luând în considerare o eroare de 2% (probabilitate de 98%)

3° Un interval în care se află numărul de simboluri „1” recepționate, luând în calcul o eroare de 1% (probabilitate de 99%)

Rezolvare Fie X v.a. care reprezintă numărul de semnale „1” recepționate $\Rightarrow X = B(p; n)$; $n = 5000$; $p = 64\% = \frac{16}{25}$; $q = 1 - p = \frac{9}{25}$

Avem $m = np = 5000 \cdot \frac{64}{100} = 3200$; $\sigma = \sqrt{npq} = \sqrt{5000 \cdot \frac{64}{100} \cdot \frac{36}{100}} =$

$= \sqrt{32 \cdot 36} = \sqrt{16 \cdot 36 \cdot 2} = 24\sqrt{2} \simeq 33,94.$

1° $P(3180 \le X \le 3248) \simeq \phi\left(\frac{b-m}{\sigma}\right) - \phi\left(\frac{a-m}{\sigma}\right) = -\phi\left(\frac{3180-3200}{24\sqrt{2}}\right) +$

$\cancel{-\phi\left(\frac{-20}{24\sqrt{2}}\right)} + \phi\left(\frac{3248-3200}{24\sqrt{2}}\right) = -\phi\left(\underbrace{\frac{-20}{24\sqrt{2}}}_{-5/6\sqrt{2}}\right) + \phi\left(\frac{48^{2}}{24\sqrt{2}}\right) =$

$= \phi(\sqrt{2}) - \left(1 - \phi\left(\frac{5\sqrt{2}}{12}\right)\right) \simeq \phi(1.414) + \phi(0.589) - 1 =$

$\simeq 0.9214 + 0.7221 - 1 = 0.6435 \simeq 64,35\%$

2° $P(X \in [m-\alpha, m+\alpha]) = 2\phi\left(\frac{\alpha}{\sigma}\right) - 1 = 0.98 \Rightarrow$

$\Rightarrow \phi\left(\frac{\alpha}{\sigma}\right) = 0.99 \Rightarrow \frac{\alpha}{\sigma} = \phi^{-1}(0.99) \Rightarrow$

$\Rightarrow \frac{\alpha}{\sigma} = 2,33 \Rightarrow \alpha = 2,33 \times 33,94 = 79.08$

$X \in [m-\alpha, m+\alpha] = [3200 - 79.08; 3200 + 79.08] \Rightarrow$

$\Rightarrow X \in [3120; 3280] \rightarrow$ semnale „1”

$\bar{X} = \cancel{6}\, 5000 - X \in [1720; 1880] \rightarrow$ semnale „1”

(3°) $P(X \in [m-\alpha, m+\alpha]) = 2\phi\left(\frac{\alpha}{\sigma}\right) - 1 = 0.99 \Rightarrow$

$\Rightarrow \phi\left(\frac{\alpha}{\sigma}\right) = 0.995 \Rightarrow \frac{\alpha}{\sigma} = \phi^{-1}(0.995) = \cancel{2{,}5}\ 2{,}575 \Rightarrow$

$\alpha = 2{,}575 \cdot 33{,}94 \simeq 87.39 \Rightarrow X \in [m-\alpha, m+\alpha] =$

$\simeq [3200 - 87.39; 3200 + 87.39] \simeq [3112; 3288]$

$\boxed{\text{Ex. 2}}$ Fie $X = N(3, 6)$. Să se calculeze:

(i) $P(0 \leq X \leq 7)$; (ii) $P(|X| < 2)$; (iii) $P(|X| \geq 4)$;

(iv) $P(|X| < 3 / |X| > 2)$

Rezolvare $m = 3$, $\sigma^2 = 6 \Rightarrow \sigma = \sqrt{6} \simeq 2.45$

$$P(a \leq X \leq b) = \phi\left(\frac{b-m}{\sigma}\right) - \phi\left(\frac{a-m}{\sigma}\right)$$

(i) $P(0 \leq X \leq 7) = \phi\left(\frac{7-3}{\sqrt{6}}\right) - \phi\left(\frac{0-3}{\sqrt{6}}\right) = \phi\left(\frac{2\sqrt{6}}{3}\right) - \phi\left(-\frac{\sqrt{6}}{2}\right) \simeq$

$\simeq \phi(1.63) - (1 - \phi(1.22)) \simeq \phi(1.63) + \phi(1.22) - 1 \simeq$

$\simeq 0.9484 + 0.8888 - 1 = 0.8372 = 83{,}72\%$

(ii) $P(|X| < 2) = P(-2 < X < 2) = \phi\left(\frac{2-3}{\sqrt{6}}\right) - \phi\left(\frac{-2-3}{\sqrt{6}}\right) =$

$= \phi\left(-\frac{1}{\sqrt{6}}\right) - \phi\left(-\frac{5}{\sqrt{6}}\right) = 1 - \phi\left(\frac{1}{\sqrt{6}}\right) - \left(1 - \phi\left(-\frac{5}{\sqrt{6}}\right)\right) =$

$= \phi(5/\sqrt{6}) - \phi(1/\sqrt{6}) \simeq \phi(2.04) - \phi(0.41) =$

$= 0.9793 - 0.6591 = \cancel{0.2929 = 29.29\%}$ $\cancel{364}$

$= 0.3182 = 31{,}32\%$

(iii) $P(|X|\geq 4)=1-P(|X|<4)=1-P(-4<X<4)=$

$=1-\left(\phi\left(\frac{4-3}{\sqrt6}\right)+\phi\left(\frac{-4+3}{\sqrt6}\right)\right)=1-\phi\left(\frac{1}{\sqrt6}\right)+1-\phi\left(\frac{7}{\sqrt6}\right)\simeq$

$\simeq 2-\phi(0.41)-\phi(2.86)=2-0.6591-0.9979=2-1.6570=$

$=0.3430=34,3\%$

(iv) $P(|X|<3\,/\,|X|>2)=\dfrac{P((|X|<3)\cap(|X|>2))}{P(|X|>2)}=$

$=\dfrac{P(2<|X|<3)}{1-P(|X|<2)}=\dfrac{P(X\in(-3,-2)\cup(2,3))}{1-P(X\in(-2,2))}=$

$=\dfrac{P(X\in(-3,-2))+P(X\in(2,3))}{1-P(X\in(-2,2))}=\dfrac{\phi\left(\frac{-2-3}{\sqrt6}\right)-\phi\left(\frac{-3-3}{\sqrt6}\right)}{\cancel{1-\phi(}\ \underbrace{1-P(|X|<2)}_{(ii)}}=$

$=\dfrac{\phi\left(-\frac{5}{\sqrt6}\right)-\phi(-\sqrt6)}{1-0.3132}=\dfrac{\cancel{1}-\phi\left(\frac{5}{\sqrt6}\right)\cancel{-1}+\phi(\sqrt6)}{0.6868}\overset{(ii)}{=}\dfrac{0.9929-0.9773}{0.6868}\simeq$

$\simeq 0.023=2,3\%$

$\boxed{\text{Ex. 3}}$ Într-un sistem de așteptare, timpul de servire a unui client este descris de o v.a. $B(p;n)$, având media $m=8$ (secunde), iar deviația standard este $\sigma=2$.

Să se determine:

1° Probabilitatea ca durata de servire a primilor 50 de clienți să depășească 410 secunde

2° Probabilitatea ca durata de servire a primilor 200 clienți să fie situată în intervalul (1590 s; 1625 s]

3° Numărul de clienți serviți a.î. cu probabilitate de 95%, timpul de servire să depășească 500 secunde.

Rezolvare: $X=B(p;n)\Rightarrow P(a\leq X\leq b)\simeq\phi\left(\frac{b-M(X)}{\sigma(X)}\right)-\phi\left(\frac{a-M(X)}{\sigma(X)}\right)$

Rezolvare Fie X_k v.a. care descrie timpul de service a clientului de rangul k și $S_N = X_1 + X_2 + X_3 + \dots + X_N$; $N \geq 1$.

Avem: $M(S_N) = M(X_1) + M(X_2) + \dots + M(X_N) = mN = 8N$;

$\cancel{D^2(S_n) = Var(S_n) = D^2(X_1) + \dots + D^2(X_n)}$ ~~v.a. indep~~

$D^2(S_N) = Var(S_N) \xlongequal{\text{v.a. indep}} D^2(X_1) + D^2(X_2) + \dots + D^2(X_N) = N\sigma^2 = 4N$

Deoarece $X_1, X_2, X_3, \dots, X_n$ sunt v.a. de tip $B(p;n) \Longrightarrow$

$\Rightarrow S_N = \sum_{k=1}^{N} X_k$ este o v.a. de tip $B(10; mN)$, având

$M(S_N) = 8N$, și dispersia standard (abaterea medie pătratică)

$\sigma(S_N) = \sqrt{D^2(S_N)} = 2\sqrt{N}$.

1° $P(S_{50} > 410) = P(410 < S_{50} < \infty) = \phi\left(\frac{\infty - M(S_{50})}{\sigma(S_{50})}\right) - \phi\left(\frac{410 - M(S_{50})}{\sigma(S_{50})}\right)$

$= \phi(\infty) - \phi\left(\frac{410 - 8 \cdot 50}{2\sqrt{50}}\right) = 1 - \phi\left(\frac{10}{10\sqrt{2}}\right) = 1 - \phi\left(\frac{\sqrt{2}}{2}\right) \simeq$

$\simeq 1 - \phi\left(\frac{1.414}{2}\right) = \cancel{\phi(1=} \ 1 - \phi(0.707) \simeq 1 - 0.7602 = 0.2398 = 23,98\%$.

2° $P(1590 < S_{200} < 1625) \simeq \phi\left(\frac{1625 - M(S_{200})}{\sigma(S_{200})}\right) - \phi\left(\frac{1590 - M(S_{200})}{\sigma(S_{200})}\right)$

$= \phi\left(\frac{1625 - 200 \cdot 8}{2\sqrt{200}}\right) - \phi\left(\frac{1590 - 200 \cdot 8}{2\sqrt{200}}\right) = \phi\left(\frac{25}{20\sqrt{2}}\right) - \phi\left(\frac{-10}{20\sqrt{2}}\right) =$

$= \phi\left(\frac{5\sqrt{2}}{8}\right) - \phi\left(-\frac{\sqrt{2}}{2}\right) \simeq \phi(0.884) - (1 - \phi(0.707)) =$

$\simeq 0.8117 + 0.7602 - 1 = 0.5719 = 57,19\%$

3° $P(S_N > 500) = 0.95 \Leftrightarrow \cancel{1 - P(S_N < 500) = 0.95} \Leftrightarrow$

$\cancel{\Leftrightarrow 1 - \phi} \quad \Leftrightarrow P(500 < S_N < \infty) = 0.95 \Leftrightarrow$

$\Leftrightarrow \phi\left(\frac{\infty - M(S_N)}{\sigma(S_N)}\right) - \phi\left(\frac{500 - M(S_N)}{\sigma(S_N)}\right) = 0.95 \Leftrightarrow$

$\Leftrightarrow 1 - \phi\left(\frac{500 - 8N}{2\sqrt{N}}\right) = 0.95 \Leftrightarrow \phi\left(\frac{8N - 500}{2\sqrt{N}}\right) = 0.95 \Leftrightarrow$

$1 - \phi(x) = \phi(-x)$.

$\Leftrightarrow \frac{8N-500}{2\sqrt{N}} = \phi^{-1}(0.95) \Leftrightarrow \frac{4N-250}{\sqrt{N}} = 1.645$

Fie $x = \sqrt{N} > 0 \Rightarrow 4x^2 - 1.645x - 250 = 0$

$D = 1.645^2 + 2000 \Rightarrow x_{12} = \frac{1.645 \pm 44.752}{8} \Rightarrow$

$\Rightarrow x \cong 5.799 \Rightarrow x^2 \simeq 33,64$

Deci $N = x^2 = 34$.

$\boxed{Ex. 4}$ Semiaxele unei elipse sunt v.a. aleatoare independente, notate X, Y, repartizate uniform în intervalul $[0,1]$. Să se determine:

(1°) Funcția de repartiție, p.d.f, momentul de ordin n, valoarea medie și dispersia pentru v.a. Z care reprezintă aria elipsei

(2°) Densitatea de probabilitate (p.d.f) pentru $U = X+Y$

(3°) p.d.f. pentru v.a. $W = XY^{-1} = X/Y$

Rezolvare

(1°) Fie $f_1 = f_X = $ p.d.f(X); $f_2 = f_Y = $ pdf$(Y) \Rightarrow f_1(x) = f_2(x) = \begin{cases} 1; & x \in [0,1] \\ 0; & \text{în rest} \end{cases}$

Avem $Z = \pi XY$. Determinăm $g(x) = g_Z(x)$. Se știe că

Se știe că $g(x) = g_{XY}(x) = \int_{-\infty}^{\infty} \frac{1}{|t|} f(t, \frac{x}{t})dt$, unde $f(x,y) = $ pdf$(X \cdot Y)$

Dacă V.a. X, Y sunt indep. $\Rightarrow f(x,y) = f_{X \cdot Y}(x,y) = f_X(x) f_Y(x) = f_1(x) f_2(y)$

Deci $g(x) = \int_{-\infty}^{\infty} \frac{1}{|t|} f_1(t) f_2(\frac{x}{t}) dt = \int_0^1 \frac{1}{t} \cdot 1 \cdot f_2(\frac{x}{t}) dt$ (p. 49-50)

$f_2(\frac{x}{t}) = \begin{cases} 1; & \frac{x}{t} \in [0,1] \Leftrightarrow 0 \le \frac{x}{t} \le 1 \Leftrightarrow x \le t \le 1 \\ 0; & \text{în rest} \end{cases}$, deci

$g(x) = \int_x^1 \frac{1}{t} \cdot 1 \cdot dt = \ln t \Big|_x^1 = -\ln x$; $x \in (0,1)$

$g(x) = \begin{cases} -\ln x; & x \in (0,1) \\ 0; & \text{în rest} \end{cases} \Rightarrow G(x) = G_{X \cdot Y}(x) = \int_{-\infty}^{x} g(t)dt = \begin{cases} 0; & x \le 0 \\ \int_0^x \ln t \, dt; & 0 < x < 1 \\ \int_0^1 \ln t \, dt; & x \ge 1 \end{cases}$

$$G'(x) = \begin{cases} 0; & x \le 0 \\ x(1-\ln x); & x \in (0,1) \\ 1; & x \ge 1 \end{cases} \quad ; \int_0^x \ln t\,dt = \int_0^x t' \ln t\,dt = t\ln t \Big|_0^x - \int_0^x t\cdot\frac{1}{t}dt$$
$$= x\ln x - \int_0^x 1\,dx = x\ln x - x$$

Atunci $F_Z(x) = P(Z<x) = P(\pi X\cdot Y < x) = P(XY < \frac{x}{\pi}) =$

$$= F_{XY}\left(\frac{x}{\pi}\right) = G'(x/\pi) = \begin{cases} 0; & \frac{x}{\pi} \le 0 \Leftrightarrow x \le 0 \\ \frac{x}{\pi}\left(1-\ln\frac{x}{\pi}\right); & \frac{x}{\pi} \in (0,1) \Leftrightarrow x \in (0,\pi) \\ 1; & x/\pi \ge 1 \Leftrightarrow x \ge \pi \end{cases}$$

$$f_Z(x) = F_Z'(x) = \begin{cases} -\frac{1}{\pi}\ln\frac{x}{\pi} = \frac{1}{\pi}\ln\frac{\pi}{x}; & x \in (0,\pi) \\ 0; & \text{în rest} \end{cases}$$

$$M_n(Z) = \int_{-\infty}^{\infty} x^n f_Z(x)\,dx = \int_0^{\pi} x^n \frac{1}{\pi}\ln\frac{\pi}{x}\,dx = \frac{1}{\pi}\int_0^{\pi}\left(\frac{x^{n+1}}{n+1}\right)' \underbrace{\ln\frac{\pi}{x}}_{\ln\pi - \ln x}dx =$$

$$= \frac{1}{\pi}\left[\frac{x^{n+1}}{n+1}\ln\frac{\pi}{x}\Big|_0^{\pi} - \int_0^{\pi}\frac{x^{n+1}}{n+1}\left(-\frac{1}{x}\right)dx\right] = \frac{1}{\pi}\left(\frac{\pi^{n+1}}{n+1}\ln 1 - \frac{1}{n+1}\underbrace{\lim_{x\to 0} x^{n+1}\ln\frac{\pi}{x}}_{=0;\ l'H}\right)$$

$$+ \frac{1}{\pi(n+1)}\int_0^{\pi} x^n\,dx = \frac{1}{\pi}\frac{x^{n+1}}{(n+1)^2}\Big|_0^{\pi} = \frac{\pi^n}{(n+1)^2} \quad ; \quad \boxed{M_n(Z) = \frac{\pi^n}{(n+1)^2}}$$

OBS $M_0(Z) = 1$ ✓

$$M(Z) = M_1(Z) = \frac{\pi}{4} \simeq 0.79; \quad D^2(Z) = M_2(Z) - M_1^2(Z) = \frac{\pi^2}{9} - \frac{\pi^2}{16} \Rightarrow$$
$$\Rightarrow D^2(Z) = Var(Z) = \frac{7\pi^2}{144} \simeq 0.48$$

2° Se știe că $f_{X+Y}(x) = \int_{-\infty}^{\infty} f(t, x-t)\,dt$.

X, Y indep $\Rightarrow f_{X+Y}(x) = \int_{-\infty}^{\infty} f_1(t) f_2(x-t)\,dt = (f_1 * f_2)(x)$

$$f_U(x) = f_{X+Y}(x) = \int_0^1 1\cdot f_2(x-t)\,dt = \int_{x-1}^{x} 1\,dt = \begin{cases} \int_0^x dt; & 0 \le x \le 1 \\ \int_{x-1}^1 dt; & 1 \le x \le 2 \end{cases}$$

$$f_2(x-t) = \begin{cases} 1; & x-t \in (0,1) \Leftrightarrow 0 \le x-t \le 1 \Leftrightarrow \underbrace{x-1 \le t \le x}_{} \Leftrightarrow t \in [0,x];\ 0 \le x \le 1 \\ 0; & \text{în rest} \end{cases}$$
$$t \in [x-1, 1];\ 1 \le x \le 2$$

$$f_U(x) = \begin{cases} x; & 0 \le x \le 1 \\ 2-x; & 1 \le x \le 2 \\ 0; & \text{în rest} \end{cases}$$

3° $f_W(x) = f_{X/Y}(x) = \int_{-\infty}^{\infty} |t| f(tx, t)\,dt \quad ; \ 0 < tx < 1 \Leftrightarrow 0 < t < \frac{1}{x}$

$$f_W(x) = \int_0^1 |t|\cdot f_1(tx)\underbrace{f_2(t)}_{1}\,dt = \cancel{\int_0^{1/x} t\,dt} \begin{cases} \int_0^1 t\,dt = 1/2; & x \in (0,1) \\ \int_0^{1/x} t\,dt = \frac{1}{2x^2}; & x \ge 1 \end{cases}$$

$$f_W(x) = \begin{cases} 0; & x \le 0 \\ 1/2; & x \in (0,1) \\ 1/(2x^2); & x \ge 1 \end{cases}$$

$\boxed{\text{Ex. 5}}$ Temă - Individual.

$\boxed{\text{5.1}}$ Ex. 1, 5, 6, 7, 8, 9, 10, 11, 12, 16 $\longrightarrow$ p. 182-185