# Seminar 7 - MA

## I Lanțuri Markov

### Breviar teoretic - Seminar 6 - MA

**Ex. 1** Se consideră lanțul Markov $(X_k)_{k\geq 0}$ asociat tripletului $(S, p^{(0)}, \Pi)$, unde $S=\{1,2\}$, $p^{(0)}=(3a, 10a^2)$,
$\Pi = \begin{pmatrix} 3x^2 & x/2 \\ 1/3 & 2/3 \end{pmatrix}$. Să se determine:

1° $a$; $x$ ; 2° Probabilitatea ca lanțul să evolueze pe traiectoria $(2,1,1,2)$

3° $P(X_{730}=1, X_{728}=2, X_{727}=2 / X_{726}=1)$

4° a) Valorile proprii și vectorii proprii ale matricei $\Pi$; b) $\Pi^n$; $n\in\mathbb{N}$.

5° a) Probabilitățile absolute ale L.M. $(X_k)_{k\geq 0}$;
b) Repartiția de probabilitate a L.M. $(X_k)_{k\geq 0}$

6° Repartiția (distribuția) limită (invariantă, de echilibru) a L.M. $(X_n)_{n\geq 0}$

7° a) $P(X_{10}=2/X_7=2)$; b) $P(X_0=2/X_2=2)$;
c) $P(X_7=2/X_9=1)$

8° $P(X_{20}=1, X_{18}=2/X_{15}=2)$

9° a) $P(X_2=2/X_4=1, X_5=2)$; b) $P(X_{101}=2/X_{98}\neq 2)$

10° $P(X_3+X_5=2/X_0+X_1=4)$

### Rezolvare

1° $3a+10a^2=1$; $3x^2+x/2=1$; $3a\in[0,1]$, $10a^2\in[0,1]$; $3x^2\in[0,1]$; $\frac{x}{2}\in[0,1]$

$\Longleftrightarrow \begin{cases} 10a^2+3a-1=0; & a\in[0,1/3]\cap[-1/\sqrt{10}; 1/\sqrt{10}] = [0, 1/\sqrt{10}] \\ 6x^2+x-2=0; & x\in[-1/\sqrt{3}; 1/\sqrt{3}]\cap[0,2] = [0, 1/\sqrt{3}] \end{cases}$

$a_{1,2} = \frac{-3\pm 7}{20} \Rightarrow \underline{a=1/5}$ ; $x_{1,2} = \frac{-1\pm 7}{12} \Rightarrow \underline{x=1/2}$

Așadar; $p^{(0)} = \left(\frac{3}{5}, \frac{2}{5}\right)$ ; $\Pi = \begin{pmatrix} 3/4 & 1/4 \\ 1/3 & 2/3 \end{pmatrix}$.

2° $P(X_0=2, X_1=1, X_2=1, X_3=2) = P(X_0=2)\cdot P(X_1=1/X_0=2)\cdot$
$\cdot P(X_2=1/X_1=1, \cancel{X_0=2})\cdot P(X_3=2/X_2=1, \cancel{X_1=1, X_0=2}) =$
$= p_2^{(0)}\cdot p_{21}\cdot p_{11}\cdot p_{12} = \frac{2}{5}\cdot\frac{1}{3}\cdot\frac{3}{4}\cdot\frac{1}{4} = \frac{1}{40} = 2.5\%$

3° $P(X_{730}=1, X_{728}=2, X_{727}=2 / X_{726}=1) = P(A/B) = \frac{P(A\cap B)}{P(B)}$
$= \frac{P(X_{726}=1, X_{727}=2, X_{728}=2, X_{730}=1)}{P(X_{726}=1)} =$
$= \frac{\cancel{P(X_{726}=1)}\cdot P(X_{727}=2/X_{726}=1)\cdot P(X_{728}=2/X_{727}=2, \cancel{X_{726}=1})\cdot P(X_{730}=1/X_{728}=2, \cancel{X_{727}=2, X_{726}=1})}{\cancel{P(X_{726}=1)}}$ (2)

$= p_{12}\cdot p_{22}\cdot p_{21}^{(2)} = p_{12}\cdot p_{22}\cdot p_{21}(\Pi^2) = \frac{1}{4}\cdot\frac{2}{3}\cdot(l_2\times c_1) =$
$= \frac{1}{6}\cdot\left(\frac{1}{3}, \frac{2}{3}\right)\cdot\left(\frac{3}{4}, \frac{1}{3}\right)^T = \frac{1}{6}\left(\frac{1}{\cancel{3}}\cdot\frac{\cancel{3}}{4} + \frac{2}{3}\cdot\frac{1}{3}\right) = \frac{1}{6}\left(\frac{1}{4} + \frac{2}{9}\right) = \frac{17}{216}$
$\approx 0.0787 = 7,87\%$

4° Vectori, valori proprii $\Pi x = \lambda x \Leftrightarrow (\Pi - \lambda I_2)x = 0$

ⓐ Valori proprii $\det(\Pi - \lambda I_2) = 0 \Leftrightarrow \begin{vmatrix} 3/4-\lambda & 1/4 \\ 1/3 & 2/3-\lambda \end{vmatrix} = 0 \Leftrightarrow$
$\Leftrightarrow \left(\frac{3}{4}-\lambda\right)\left(\frac{2}{3}-\lambda\right) - \frac{1}{12} = 0 \Leftrightarrow 12\lambda^2 - 17\lambda + 5 = 0 <$ $\lambda_1 = 1$, $\lambda_2 = 5/12$

Vectori proprii $\lambda_1 = 1 \Rightarrow v_1 = (1,1)^T = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\lambda_2 = 5/12 \Rightarrow \begin{pmatrix} 3/4 - 5/12 & 1/4 \\ 1/3 & 2/3 - 5/12 \end{pmatrix}\begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Leftrightarrow \begin{cases} \frac{1}{3}\alpha_1 + \frac{1}{4}\alpha_2 = 0 \\ \frac{1}{3}\alpha_1 + \frac{1}{4}\alpha_2 = 0 \end{cases}$

$\Leftrightarrow 4\alpha_1 + 3\alpha_2 = 0$ ; luăm $\alpha_1 = 3 \Rightarrow \alpha_2 = -4 \Rightarrow v_2 = \begin{pmatrix} 3 \\ -4 \end{pmatrix} = (3,-4)^T$

Matricea de pasaj $A = (v_1, v_2) = \begin{pmatrix} 1 & 3 \\ 1 & -4 \end{pmatrix}$

Matricea diagonală $D = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 5/12 \end{pmatrix}$

Are loc relația $\Pi = A\cdot D\cdot A^{-1}$ $\Rightarrow \Pi^2 = \Pi\cdot\Pi = (ADA^{-1})(ADA^{-1}) =$
$= (AD)(\underbrace{A^{-1}A}_{I_2})\cdot(DA^{-1}) = AD\cdot DA^{-1} = AD^2A^{-1}$

(b) În general $\boxed{\Pi^n = A\cdot D^n\cdot A^{-1}}$ ; $n\geq 1$ ; $\Pi^0 = I_2$

$$\Pi^n = A\cdot\begin{pmatrix}1^n & 0\\ 0 & (5/12)^n\end{pmatrix}\cdot A^{-1}$$

$$A^{-1} = \frac{1}{\det A}\cdot A^* = \left(-\frac{1}{7}\right)\cdot\begin{pmatrix}-4 & -3\\ -1 & 1\end{pmatrix} = \frac{1}{7}\begin{pmatrix}4 & 3\\ 1 & -1\end{pmatrix}, \text{ deci}$$

$$\Pi^n = \frac{1}{7}\begin{pmatrix}1 & 3\\ 1 & -4\end{pmatrix}\cdot\begin{pmatrix}1 & 0\\ 0 & (5/12)^n\end{pmatrix}\begin{pmatrix}4 & 3\\ 1 & -1\end{pmatrix}\Rightarrow$$

$$\Rightarrow \Pi^n = \frac{1}{7}\begin{pmatrix}4+3(5/12)^n & 3-3(5/12)^n\\ 4-4(5/12)^n & 3+4(5/12)^n\end{pmatrix} ;\ n\geq 1$$

5° (a) Probabilități absolute $p_1^{(k)} = P(X_k=1)$ ; $p_2^{(k)} = P(X_k=2)$.

(b) Repartiția de probabilitate $p^{(k)} = \left(p_1^{(k)}, p_2^{(k)}\right)$ ; $k\geq 0$.

Avem: $\boxed{p^{(k)} = p^{(0)}\cdot\Pi^k} \Leftrightarrow \left(p_1^{(k)}, p_2^{(k)}\right) = \left(\frac{3}{5}, \frac{2}{5}\right)\cdot\frac{1}{7}\begin{pmatrix}4+3\cdot\left(\frac{5}{12}\right)^k & 3-3\left(\frac{5}{12}\right)^k\\ 4-4\left(\frac{5}{12}\right)^k & 3+4\left(\frac{5}{12}\right)^k\end{pmatrix}$

$$p^{(k)} = \frac{1}{7}\left(\frac{3}{5}\left(4+3\left(\frac{5}{12}\right)^k\right)+\frac{2}{5}\left(4-4\left(\frac{5}{12}\right)^k\right) ;\ \frac{3}{5}\left(3-3\cdot\left(\frac{5}{12}\right)^k\right)+\frac{2}{5}\left(3+4\cdot\left(\frac{5}{12}\right)^k\right)\right),$$

$$\begin{cases} p^{(k)} = \left(\frac{4}{7}+\frac{1}{35}\left(\frac{5}{12}\right)^k ;\ \frac{3}{7}-\frac{1}{35}\left(\frac{5}{12}\right)^k\right) ;\ k\geq 0\\ p_1^{(k)} = P(X_k=1) = \frac{4}{7}+\frac{1}{35}\left(\frac{5}{12}\right)^k\\ p_2^{(k)} = P(X_k=2) = \frac{3}{7}-\frac{1}{35}\left(\frac{5}{12}\right)^k\end{cases}$$

6° Repartiția limită

Metoda 1 $p^* = \lim\limits_{k\to\infty} p^{(k)} \overset{(5°)}{=} \left(\frac{4}{7}, \frac{3}{7}\right)$, deoarece $\left(\frac{5}{12}\right)^k\to 0$ pt. $k\to\infty$ ; $\left(\frac{5}{12}\in(-1,1)\right)$

Metoda 2 Fie $p^* = (p_1^*, p_2^*)$ ; $p_1^*, p_2^*\geq 0$ ; $p_1^*+p_2^*=1$.

$$p^*\Pi = p^* \Longleftrightarrow (p_1^*, p_2^*)\cdot\begin{pmatrix}3/4 & 1/4\\ 1/3 & 2/3\end{pmatrix} = (p_1^*, p_2^*) \Leftrightarrow$$

$$\Leftrightarrow \begin{cases}\frac{3}{4}p_1^*+\frac{1}{3}p_2^* = p_1^* \mid\cdot 12\\ \frac{1}{4}p_1^*+\frac{2}{3}p_2^* = p_2^* \mid\cdot 12 ;\ p_1^*+p_2^*=1\end{cases} \Leftrightarrow \begin{cases}-3p_1^*+4p_2^*=0\\ 3p_1^*-4p_2^*=0 ;\ p_1^*+p_2^*=1\end{cases}$$

Avem: $\begin{cases} 3p_1^* = 4p_2^* \\ p_1^* + p_2^* = 1 \end{cases} \Leftrightarrow \frac{p_1^*}{4} = \frac{p_2^*}{3} = \frac{p_1^* + p_2^*}{4+3} = \frac{1}{7} \Rightarrow$

$\Rightarrow p_1^* = \frac{4}{7}, \; p_2^* = \frac{3}{7} \Rightarrow p^* = \left(\frac{4}{7}, \frac{3}{7}\right).$

**7°** a) $P(X_{10} = 2 / X_7 = 2) = p_{22}^{(3)} = p_{22}(\Pi^3) \underset{n=3}{\overset{4b)}{=}} \left(3 + 4\left(\frac{5}{12}\right)^3\right)\frac{1}{7} =$

$= \frac{1}{7}\left(3 + 4 \cdot \frac{125}{1728}\right) = \frac{1}{7} \cdot \frac{5684}{1728} = \frac{812}{1728} = \frac{203}{432} \simeq 0.4699 = 46,99\%.$

b) $P(X_0 = 2 / X_2 = 2) = \frac{P(X_0 = 2, X_2 = 2)}{P(X_2 = 2)} \overset{Bayes}{=} \frac{P(X_0 = 2) \cdot P(X_2 = 2 / X_0 = 2)}{P(X_2 = 2)} =$

$= \frac{p_2^{(0)} \cdot p_{22}^{(2)}}{p_2^{(2)}} = \frac{p_2^{(0)} \cdot p_{22}(\Pi^2)}{p_2^{(2)}} \underset{5b)}{\overset{4b)\, n=2}{=}} \frac{\frac{2}{5} \cdot \frac{1}{7}\left(3 + 4 \cdot \frac{25}{144}\right)}{\frac{3}{7} - \frac{1}{35}\left(\frac{5}{12}\right)^2} =$

$= \frac{2 \cdot 532}{5 \cdot 427} = \frac{1064}{2135} \simeq 0.4983 = 49,83\%.$

c) $P(X_7 = 2 / X_9 = 1) \overset{Bayes}{=} \frac{P(X_7 = 2) \cdot P(X_9 = 1 / X_7 = 2)}{P(X_9 = 1)} =$

$= \frac{p_2^{(7)} \, p_{21}^{(2)}}{p_1^{(9)}} \underset{5b)}{\overset{4b)}{=}} \frac{\left(\frac{3}{7} - \frac{1}{35}\left(\frac{5}{12}\right)^7\right)\left(1 - (5/12)^2\right) \cdot \frac{4}{7}}{\frac{4}{7} + \frac{1}{35}\left(\frac{5}{12}\right)^9} =$

$= \frac{\frac{1}{35}\left(15 - \left(\frac{5}{12}\right)^7\right) \cdot \frac{4}{7} \cdot \frac{119}{144}}{\frac{1}{35}\left(20 + \left(\frac{5}{12}\right)^9\right)} = \frac{17}{36} \cdot \left(15 - \left(\frac{5}{12}\right)^7\right)\left(20 + \left(\frac{5}{12}\right)^9\right)^{-1}.$

**8°** $P(X_{20} = 1, X_{18} = 2 / X_{15} = 2) = \frac{P(X_{15} = 2, X_{18} = 2, X_{20} = 1)}{P(X_{15} = 2)} =$

$= \frac{\cancel{P(X_{15} = 2)} \cdot P(X_{18} = 2 / X_{15} = 2) \cdot P(X_{20} = 1 / X_{18} = 2, \cancel{X_{15} = 2})}{\cancel{P(X_{15} = 2)}}$

$= p_{22}^{(3)} \cdot p_{21}^{(2)} \overset{4b)}{=} \frac{1}{7}\left(3 + 4 \cdot \frac{125}{1728}\right) \cdot (e_2 \times c_1) =$

$= \frac{1}{7} \cdot \frac{5684}{1728} \cdot \frac{4}{7}\left(1 - \frac{25}{144}\right) = \frac{4}{49} \cdot \frac{5684}{1728} \cdot \frac{119}{144} = \frac{3451}{15552} \simeq 0.2219 = 22,19\%$

Seminar 7 - MA

**9°** a) $P(X_2=2 / X_4=1, X_5=2) = \dfrac{P(X_2=2, X_4=1, X_5=2)}{P(X_4=1, X_5=2)} =$

$= \dfrac{P(X_2=2)\cdot P(X_4=1 / X_2=2)\cdot P(X_5=2 / X_4=1, \cancel{X_2=2})}{P(X_4=1)\cdot P(X_5=2/X_4=1)} =$

$= \dfrac{p_2^{(2)} \cdot p_{21}^{(2)} \cdot \cancel{p_{12}}}{p_1^{(4)} \cdot \cancel{p_{12}}} \quad \dfrac{5\ 1)}{4\ 6)} \quad \frac{1}{7}\cdot\left(4 - 4\cdot\frac{25}{144}\right)\cdot\frac{1}{35}\left(15-\frac{25}{144}\right)\cdot\left[\frac{1}{35}\left(20+\left(\frac{5}{12}\right)^4\right)\right]^{-1} =$

$= \dfrac{119}{144}\cdot\dfrac{12^4}{20\cdot 12^4+625} = \dfrac{119\cdot 144}{20\cdot 144^2+625}$

b) $P(X_{101}=2 / X_{98}\neq 2) = P(X_{101}=2/X_{98}=1) = p_{12}^{(3)} = p_{12}(17^3) =$

$= \frac{3}{7}\left(1-\left(\frac{5}{12}\right)^3\right) = \frac{3}{7}\left(1-\frac{125}{1728}\right) = \frac{3}{7}\cdot\dfrac{\cancel{1603}\ 229}{\cancel{1728}\ 576} = \dfrac{229}{576} \simeq 0.3976 = 39.76\%$

**10°** $P(X_3+X_5=2 / X_0+X_1=4) = P(X_3=X_5=1 / X_0=X_1=2) =$

$= \dfrac{P(X_0=X_1=2, X_3=X_5=1)}{P(X_0=2, X_1=2)} = \cancel{P(X_0=2)P(X_1=2/X_0=2)\cdot P(X_3=1}$

$= \dfrac{\cancel{P(X_0=2)}\, P(X_1=2/X_0=2)\cdot P(X_3=1/X_1=2, \cancel{X_0=2})\cdot P(X_5=1/X_3=1, \cancel{X_1=X_0=2})}{\cancel{P(X_0=2)}\cdot P(X_1=2/X_0=2)} =$

$= \cancel{p_{22}}\, p_{21}^{(2)}\cdot p_{11}^{(2)}\cdot\dfrac{1}{\cancel{p_{22}}} = (\ell_2\times C_1)\cdot(\ell_1\times C_1) = \left(\frac{1}{3}, \frac{2}{3}\right)\cdot\left(\frac{3}{4}, \frac{1}{3}\right)^T\cdot$

$\cdot\left(\frac{3}{4}, \frac{1}{4}\right)\cdot\left(\frac{3}{4}, \frac{1}{3}\right)^T = \left(\frac{1}{4}+\frac{2}{9}\right)\left(\frac{\overset{3}{9}}{16}+\frac{\overset{4}{1}}{12}\right) = \frac{17}{36}\cdot\frac{31}{48} \simeq 0.3050 = 30.5\%$

**Ex. 2** Se consideră lanțul Markov $(X_n)_{n\ge 0}$ asociat tripletului $(S, p^{(0)}, \Pi)$, unde $S=\{1,2,3\}$, $p^{(0)}=(a, 2a, 3a)$, $\Pi = \begin{pmatrix} x^2 & x & \frac{1}{4} \\ 0 & 1/3 & 2/3 \\ 1/5 & 2/5 & 2/5 \end{pmatrix}$.

Să se determine:

1° $a$; $x$ ; 2° Probabilitatea ca lanțul să evolueze pe traiectoria $(1,3,2,2)$

3° $P(X_{891}=3, X_{889}=3, X_{888}=1 / X_{887}=2)$

4° $P(X_0 \neq 1 / X_2 = 1)$ ; 5° Repartiția limită

6° $P(X_0=2 / X_2=1, X_3=2)$

7° $P(X_1+X_2=2 / X_2+X_3=3)$

8° $P(X_0+X_2=6 / X_1+X_2+X_3=5)$

9° $P(X_1+X_2=2 / X_3+X_4=2)$

10° $P(X_1-X_0=2 / X_1+X_2+X_3=5)$

## II Studiul filtrelor digitale (numerice) utilizând transformata „z"

**Def.** Un filtru digital este un SCD $(S_d, S_d, L)$, cu proprietatea că $\forall x \in S_d$, eşantioanele $y(n)$ ale semnalului de ieşire $y = L(x)$ verifică o relaţie de forma

$$a_s y(n-s) + a_{s-1} y(n-s+1) + \dots + a_2 y(n-2) + a_1 y(n-1) + a_0 y(n) =$$
$$= b_p x(n-p) + b_{p-1} x(n-p+1) + \dots + b_2 x(n-2) + b_1 x(n-1) + b_0 x(n) \iff$$
$$\iff \sum_{k=0}^{s} a_k\, y(n-k) = \sum_{i=0}^{p} b_i\, x(n-i), \quad \forall n \in \mathbb{Z},$$

unde $a_k \in K$, $\forall k \in \{0, 1, 2, \dots, s\}$, $a_0 \neq 0$ şi $b_i \in K$, $\forall i \in \{0, 1, 2, \dots, p\}$; $s, p \in \mathbb{N}$.

Rezolvare $x(n) \in S_d \xrightarrow{L} y(n) = (L(x))(n) \in S_d$ $\begin{cases} x(n) = \text{„intrare"} \\ y(n) = \text{„ieşire"} \end{cases}$

Se aplică transformarea „z" şi se utilizează egalitatea

$$\mathcal{Z}\{x(n-m)\}(z) = z^{-m} X(z) \;; \text{ unde } X(z) = \mathcal{Z}\{x(n)\}(z) \;;\; m \in \mathbb{Z}$$

**Ex. 1** Se consideră un filtru digital $(S_d, S_d, L)$, care verifică relaţia

(1) $8y(n-3) + 4y(n-2) - 2y(n-1) - y(n) = x(n) + 2x(n-1)$, $\forall n \in \mathbb{Z}$.

Să se determine ieşirea $y(n)$ dacă sistemul este în repaus până la momentul $n = 0$ (adică $y \in S_d^+$), iar semnalul de intrare este $x(n) = u(n)$.

Rezolvare Aplicăm $\mathcal{Z}$ ecuaţiei (1) şi notăm $X = \mathcal{Z}x$ şi $Y = \mathcal{Z}y \Rightarrow$

$$\Rightarrow 8\mathcal{Z}\{y(n-3)\}(z) + 4\mathcal{Z}\{y(n-2)\}(z) - 2\mathcal{Z}\{y(n-1)\}(z) - \mathcal{Z}\{y(n)\}(z) =$$
$$= \mathcal{Z}\{x(n)\}(z) + 2\mathcal{Z}\{x(n-1)\}(z) \Rightarrow$$
$$\Rightarrow 8 \cdot z^{-3} Y(z) + 4z^{-2} Y(z) - 2z^{-1} Y(z) - Y(z) = X(z) + 2 \cdot z^{-1} X(z) \Rightarrow$$
$$\Rightarrow \left(\frac{8}{z^3} + \frac{4}{z^2} - \frac{2}{z} - 1\right) Y(z) = \left(1 + \frac{2}{z}\right) X(z) \;;\; X(z) = \mathcal{Z}\{u(n)\}(z) = \frac{z}{z-1}$$
$$\Rightarrow \frac{8 + 4z - 2z^2 - z^3}{z^3} Y(z) = \frac{z+2}{\cancel{z}} \cdot \frac{\cancel{z}}{z-1} \Rightarrow Y(z) = \frac{z^3(z+2)}{(z-1)[4(2+z) - z^2(2+z)]}$$
$$\Rightarrow Y(z) = \frac{z^3(z+2)}{(z-1)(z+2)(4-z^2)} = \frac{z^3\cancel{(z-2)}^{\,-1}}{(z-1)(z+2)(2+z)\cancel{(2-z)}}$$
$$\Rightarrow Y(z) = \frac{-z^3}{(z-1)(z+2)^2}.$$

 

Mai departe, $y(n) = \mathcal{Z}^{-1}\{X(z)\}(n)$

Avem $\frac{Y(z)}{z} = \frac{-z^2}{(z-1)(z+2)^2} = \frac{A}{z-1} + \frac{B}{z+2} + \frac{C}{(z+2)^2} \Rightarrow$

$\Rightarrow -z^2 = A(z+2)^2 + B(z-1)(z+2) + C(z-1)$

$z=1 \Rightarrow A = -1/9$ ; $z=-2 \Rightarrow C = 4/3$ ; $z^2: -1 = A + B \Rightarrow B = -8/9$

Astfel, $Y(z) = -\frac{1}{9}\cdot\frac{z}{z-1} - \frac{8}{9}\cdot\frac{z}{z+2} + \frac{4}{3}\cdot\frac{z}{(z+2)^2}$

$\mathcal{Z}^{-1}\left\{\frac{z}{z-a}\right\}(n) = a^n u(n)$ ; $\mathcal{Z}^{-1}\left\{\frac{z}{(z-a)^2}\right\}(n) = na^{n-1}u(n)$ $\Bigg\} \Rightarrow$

$\Rightarrow y(n) = \mathcal{Z}^{-1}\{Y(z)\}(n) = \left(-\frac{1}{9} - \frac{8}{9}(-2)^n + \frac{4}{3}n(-2)^{n-1}\right)u(n) \Rightarrow$

$\Rightarrow y(n) = \frac{1}{9}\left[12n(-2)^{n-1} - 8(-2)^n - 1\right]u(n).$

Ex. 2. Analog Ex. 1 (Temă)

1° $8y(n-3) - 4y(n-2) - 2y(n-1) + y(n) = 4x(n-2) + 4x(n-1) + x(n)$

pentru a) $x(n) = u(n)$ ; b) $x(n) = \delta_{-2}(n)$

2° $8y(n-2) - 2y(n-1) - y(n) = x(n) + 2x(n-1)$

pentru: a) $x(n) = n\cdot 2^n u(n)$

b) $x(n) = \delta_{-1}(n) + \delta(n)$