BENCHMARK SYSTEMS FOR PID CONTROL

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Abstract   This paper describes a collection of systems that are suitable for
testing PID controllers. The systems are collected from a wide range of sources.
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INTRODUCTION

When evaluating PID controllers it is of interest
to have a batch of test examples so that differ-
ent schemes can be evaluated. Over the years
we have collected a large number of test exam-
ples which we have used for research and for
evaluation of commercial systems. This paper
describes a collection of systems that we have
found useful. The material is based on many
different sources. The systems 1-5 are standard
systems that are well suited to parametric stud-
ies. Their properties can easily be changed by
varying a parameter. The systems 6-10 are more
specialized. They illustrate systems with vari-
ous difficulties of control. PID control is not well
suited for all of them. Parameters for PID con-
trollers for many of these systems are found
in (Åström and Hägglund (1995)), (Åström
et al. (1998)) (Panagopoulos *et al.* (1999)).

1. MULTIPLE EQUAL POLES

Transfer function

$$G(s) = \frac{1}{(s+1)^n} \qquad n = 1,2,3,4,8 \qquad (1)$$

These systems are very common. For $n = 1$
and 2 anything can be achieved by PI or PID
control respectively. For large values of n the
system behaves like systems with long dead
times. The systems have been used by controller
manufacturers as test cases for a long time. We
have obtained this information from Eurotherm
and Foxboro.

2. FOURTH ORDER SYSTEM

Transfer function

$$G(s) = \frac{1}{(s+1)(1+\alpha s)(1+\alpha^2 s)(1+\alpha^3 s)} \quad (2)$$
$$\alpha = 0.1, 0.2, 0.5, 1.0$$

This system has four poles whose spacing is
determined by parameter α. For small values
there are drastic improvements when going
from PI to PID control. For $\alpha = 1$ the system is
identical to the system (1) for n=4.

3. RIGHT HALF PLANE ZERO

Transfer function

$$G(s) = \frac{1-\alpha s}{(s+1)^3} \qquad (3)$$
$$\alpha = 0.1, 0.2, 0.5, 1, 2, 5$$

This system has three equal poles and a right
half plane zero. The achievable performance is
determined by parameter α. The difficulty of
control increases with increasing α.

4. TIME DELAY AND LAG

Transfer function

$$G(s) = \frac{1}{1+sT}e^{-s} \qquad (4)$$
$$T = 0, 0.1, 0.2, 0.5, 2, 5, 10$$

This is the classic system which has been used
in many investigations of PID control. The

system reduces to a pure time delay for $T = 0$ and represents lag dominated systems for large T. Many of the early tuning rules were derived based on this model. A drawback with the model is that it has slow roll-off at high frequencies.

5. TIME DELAY AND DOUBLE LAG

Transfer function

$$G(s) = \frac{1}{(1+sT)^2}e^{-s} \tag{5}$$
$$T = 0, 0.1, 0.2, 0.5, 2, 5, 10$$

This system is similar to (4) but it has more high frequency roll off. The system reduces to a pure time delay for $T = 0$.

6. HEAT CONDUCTION

Transfer function

$$G(s) = e^{-\sqrt{s}} \tag{6}$$

This system represents the dynamics of one dimensional heat conduction. The ultimate gain is $k_u = e^\pi$. Analog implementations of this system has been used by Eurotherm to test temperature controllers. Other common industrial examples are heat exchangers.

7. FAST AND SLOW MODES

Transfer function

$$G(s) = \frac{100}{(s+10)^2}\left(\frac{1}{s+1} + \frac{0.5}{s+0.05}\right) \tag{7}$$

The essential dynamics of this system has a fast mode with time constant 1 and moderate gain (1) and a slow pole with time constant 20 and a large gain (10). Simple tuning rules based on the step response do normally not give good tuning for systems of this type because it is difficult to get a good estimate of the gain and the time constant.

8. CONDITIONALLY STABLE SYSTEM

Transfer function

$$G(s) = \frac{(s+6)^2}{s(s+1)^2(s+36)} \tag{8}$$

This system is conditionally stable. The stability region under PI control consists of two disjoint sets.

9. OSCILLATORY SYSTEM

Transfer function

$$G(s) = \frac{\omega_0^2}{(s+1)(s^2 + 2\zeta\omega_0 s + \omega_0^2)}, \tag{9}$$
$$\zeta = 0.1, \quad \omega_0 = 1, 2, 5, 10.$$

Systems of this type with small damping ζ are not good candidates for PID control. The system is easy to control if ω_0 is large. The performance can often be improved drastically by more general controller structures.

10. UNSTABLE POLE

Transfer function

$$G(s) = \frac{1}{s^2 - 1} \tag{10}$$

This is a simple model of an inverted pendulum. An unstable batch reactors is an example from industry. Notice that particular care must be taken with saturating actuators in this case.

11. SYSTEMS WITH INTEGRAL ACTION

It is very useful to also have systems with integral action. A good collection is obtaind by adding an integrator to the systems 1-5.

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