Quarter Car Active Suspension Using Optimal and Robust Control Structures

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***Abstract*—To ensure the active suspension performs optimally across a range of conditions and uncertainties, advanced control techniques such as H_∞ and $\mu - Synthesis$ method is utilized. H_∞ control focuses on minimizing the worst-case peak gain from disturbances to outputs, aiming to maintain performance even in the presence of significant uncertainties. On the other hand, $\mu - Synthesis$, also known as structured singular value synthesis, extends this concept by considering structured uncertainties in the system, allowing for the design of controllers that are robust against a broader class of potential variations in the actuator model or other system parameters.**

***Index Terms*—Robust Control, Active Suspension, Mu-Synthesis, Quarter Car Model**

I. Introduction

Active suspension systems have become increasingly important in the automotive industry due to their ability to significantly improve ride comfort and handling performance. One common approach to studying and designing such systems is through the use of a quarter-car model, which simplifies the complex dynamics of a full vehicle suspension to a one-dimensional system consisting of a sprung mass (representing the vehicle body), an unsprung mass (representing the wheel and tire assembly), and various springs and dampers to simulate the suspension components' behavior. This model allows engineers to focus on the key dynamics involved in suspension performance, including how the suspension responds to road disturbances and how it can be controlled to achieve optimal ride quality and stability. In an active suspension system, actuators are integrated to apply forces based on real-time measurements and control algorithms, enabling the suspension to actively adjust its characteristics to match driving conditions.

II. Physical Model

This quarter-car model of the active suspension system is represented in Figure 1. The value m_b in kilograms represents the car chassis (body) and the value m_w in kilograms represents the wheel assembly. The spring and damper represent the passive spring and shock absorber placed between the car body and the wheel assembly. The spring models the compressibility of the pneumatic tire. The variables x_b, x_w and r (all in meters) are the body travel, wheel travel, and road disturbance, respectively. The force (in kiloNewtons) applied between the body and wheel assembly is controlled by feedback and represents the active component of the suspension system.

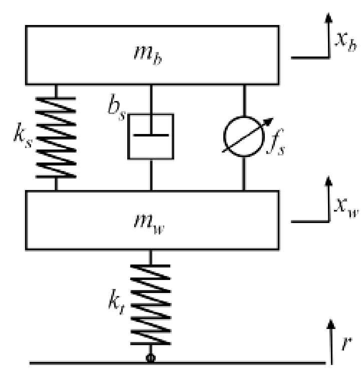

Fig. 1. Physical model of a quarter car active suspension

III. System

In Figure 2 is illustrated the entire structure of the system. There are two main signals that must be taken into consideration when designing the controller. The variable a_b represents the vehicle body acceleration and s_d is the suspension travel.

Based on these two signal, the controller should be able to adjust the hydraulic fluid force f_s which represents the active part of the process. This represents the feedback control problem illustrated in Figure 3

IV. Model Signals Weighting

When designing the controller for the process, there should be some differences between the signals, this being the reason why each signal is weighted as in Figure 4.

Each signal is weighted correspondingly, and, for example, based on W_{ab} and W_{sd} there can be implemented different modes for the suspension: *Comfort*, *Standard*, *Sport*.

V. Mathematical Model

The state-space model of the quarter car active suspension is represented in 1, 2, 3 and 4.

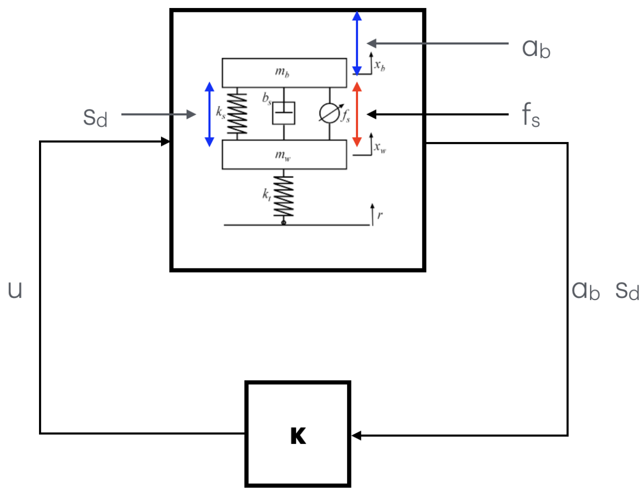

Fig. 2. The structure of the system

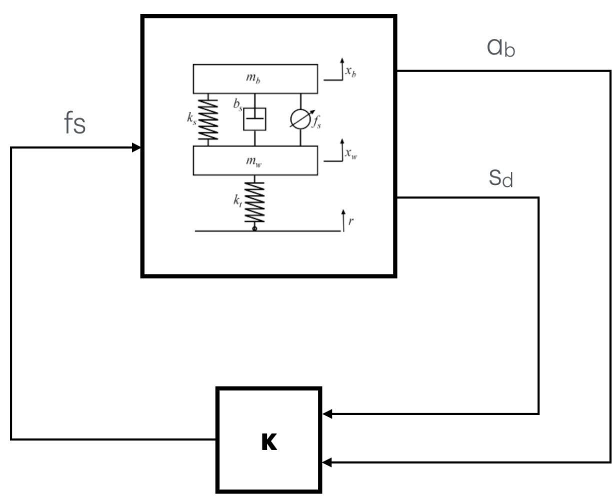

Fig. 3. Feedback control problem

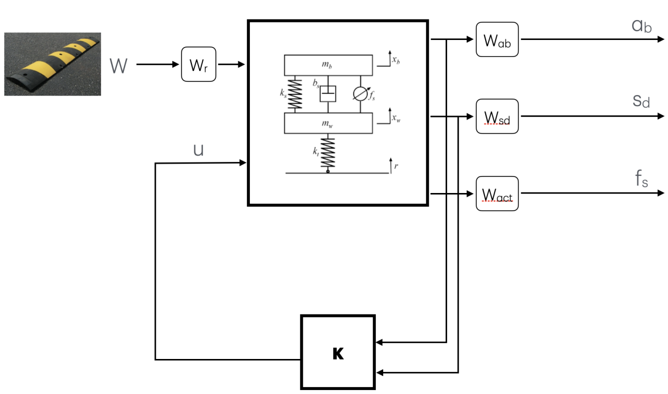

Fig. 4. Signals weighting

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-k_s}{m_b} & \frac{-b_s}{m_b} & \frac{k_s}{m_b} & \frac{b_s}{m_b} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_w} & \frac{b_s}{m_w} & \frac{-k_s-k_t}{m_w} & \frac{-b_s}{m_w} \end{bmatrix} \tag{1}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & \frac{1e3}{m_b} \\ 0 & 0 \\ \frac{k_t}{m_w} & \frac{-1e3}{m_w} \end{bmatrix} \tag{2}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ \frac{-k_s}{m_b} & \frac{-b_s}{m_b} & \frac{k_s}{m_b} & \frac{b_s}{m_b} \end{bmatrix} \tag{3}$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & \frac{1e3}{m_b} \end{bmatrix} \tag{4}$$

The values used are described by 5.

$$\begin{aligned} m_b &= 300Kg \\ m_w &= 60Kg \\ b_s &= 1000\frac{N}{m \cdot s} \\ k_s &= 16000\frac{N}{m} \\ k_t &= 190000\frac{N}{m} \end{aligned} \tag{5}$$

By applying a step reference to the model, it can be seen in Figure 5 that without any active component, the suspension oscillates much.

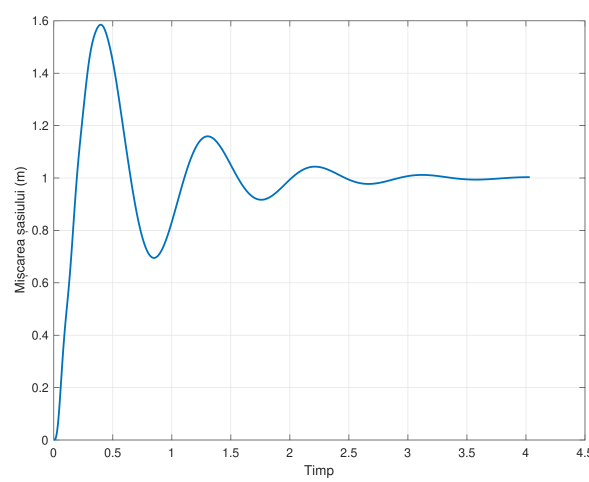

Fig. 5. The movement of the body of the car

In terms of the acceleration, Figure 6 shows that when the road disturbance is applied, the body acceleration rises quickly and, after a few oscillation, it reaches the value 0 and stays there.

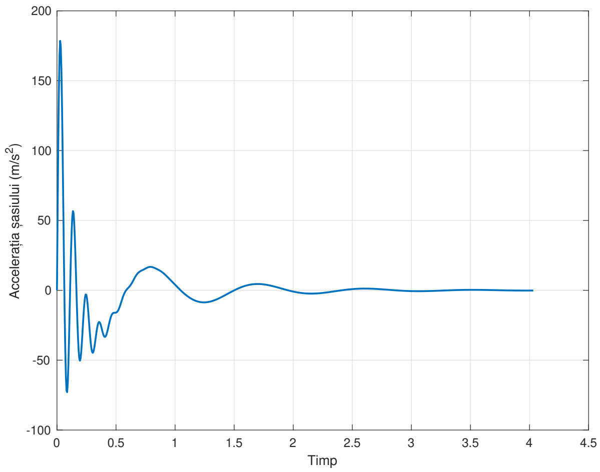

Fig. 6. The acceleration of the body of the car

VI. H_∞ Controller Design

The transfer function of the actuator is approximated to a first order system 6.

$$H_{act} = \frac{1}{0.01667s + 1} \tag{6}$$

The main idea here is to design a controller which takes as inputs two signals (a_b and s_d) and outputs the command signal u which is the input for the actuator. The actuator, then, is responsible for the actual controlled signal (f_s) sent to the suspension. The schematic is represented in Figure 7.

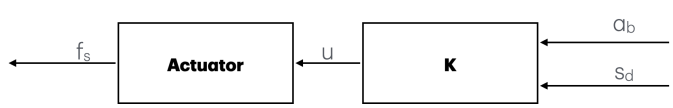

Fig. 7. The controller and actuator schematic

Thus, the closed loop can be defined as Figure 8.

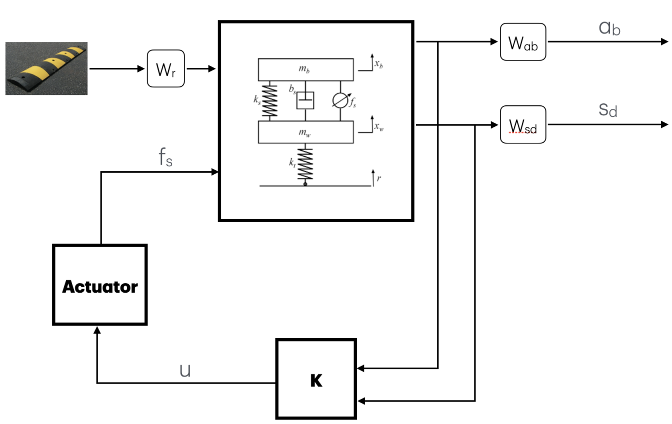

Fig. 8. The closed loop of the system

The controller K has been designed using the H_∞ method in MATLAB with the help of the command *hinfsyn* which takes as parameters:

1) The computed system
2) The number of measurable signals (a_b and s_d)
3) The number of controlled signals (f_s)

The disturbance from the road is a signal with the amplitude of 0.05 (which is translated as a speed bump with the height of 5 cm) an it is illustrated in Figure 9.

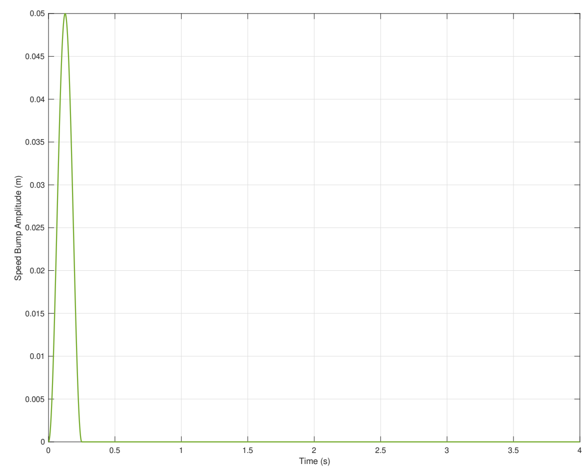

Fig. 9. Road Disturbance

The system responses for each mode (Standard, Comfort and Sport) and the open loop response are illustrated in Figures 10, 11, 12 and 13.

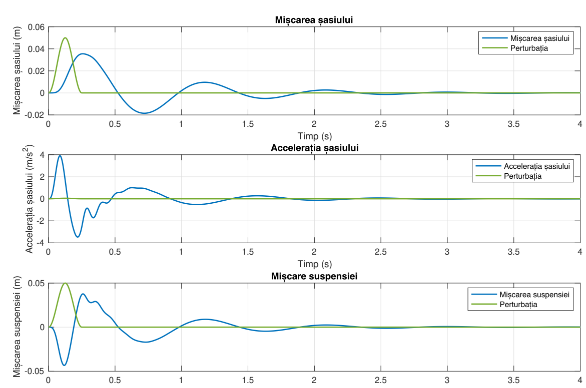

Fig. 10. Open loop response

A short comparison betweend each response can be seen in Figures 14, 15 and 16.

In Figure 14 can be seen that by applying a controller, the overall motion of the chassis is reduced compared to the open loop response (blue signal). The most aggressive response is from the *Sport* mode while the most convenient one, which responds very good to the disturbance is the *Comfort* one.

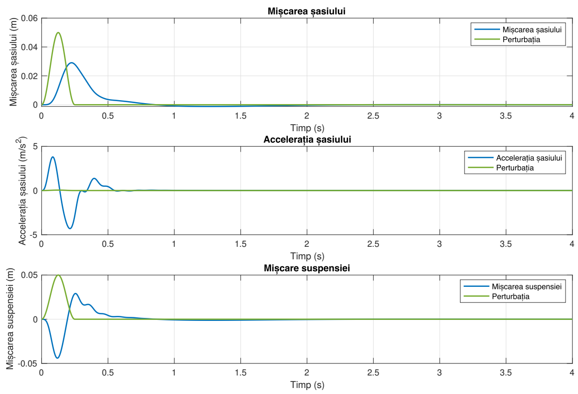

Fig. 11. Standard response

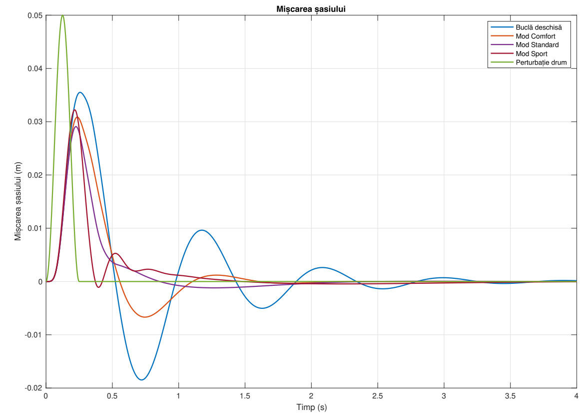

Fig. 14. Vehicle Body Travel

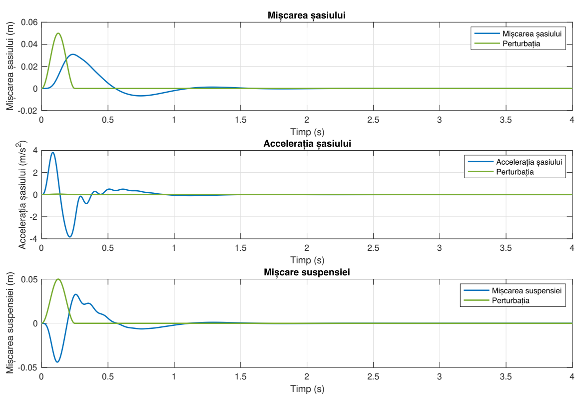

Fig. 12. Comfort response

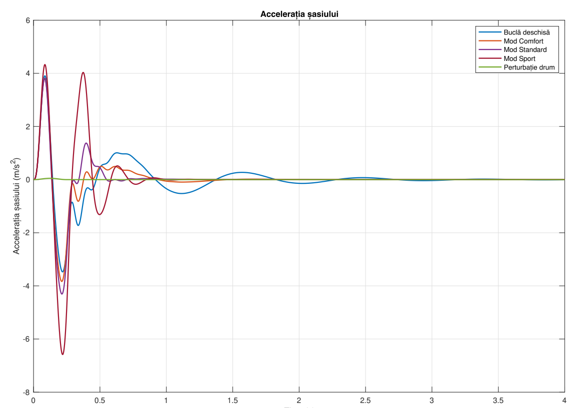

Fig. 15. Vehicle Body Acceleration

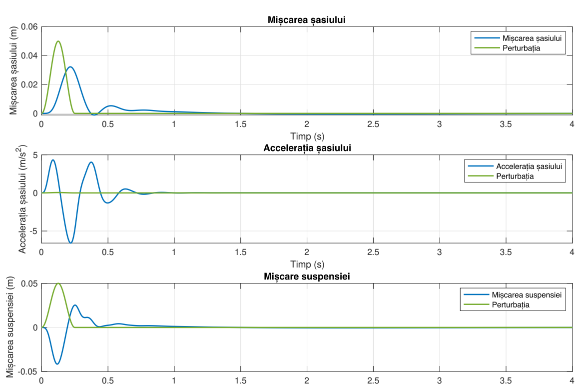

Fig. 13. Sport response

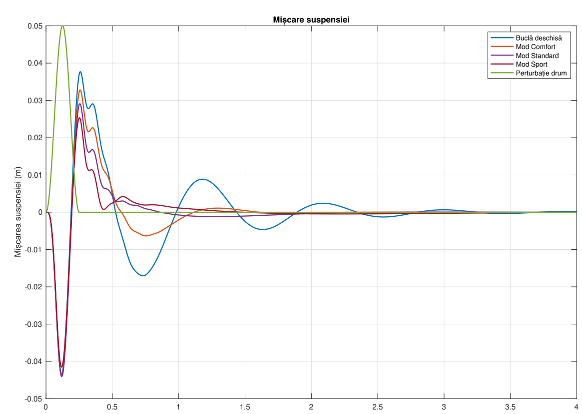

Fig. 16. Vehicle Suspension Travel

In terms of the Acceleration and the Suspension travel, the *Sport* mode reaches higher values because the suspension must be stiffer than the other modes, while the *Comfort* mode is the exact opposite. The *Standard* mode is somehow between them with sharp response to the disturbance, but not reaching so high values.

VII. $\mu - Synthesis$ CONTROLLER DESIGN

All the simulations above are for a specific case, while in practice the components of the car can vary from one to another. Moreover, in time, the quality of the parts reduces, but the controller should be able to compensate these kind of problems and deliver the same experience to the user.

In this case just a quarter of a car was taken into consideration, but the car has four active suspensions and also the actuator parameters from the transfer function can vary a lot. In order to design a controller to overcome these problems, a family of responses were simulated in Figure 17 for *Standard* mode using the H_∞ .

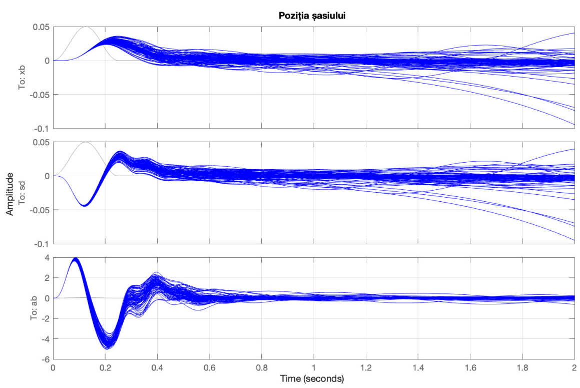

Fig. 17. Mulitple representation of H_∞ responses

There are a few cases when the H_∞ controller leads to an unstable system represented in figure by lines going all the way down.

To design the controller which can overcome the problems, the MATLAB command *musyn* was used and calculate the optimal controller taken into account the uncertainties from the system. The responses of the system with the $\mu - Synthesis$ controller are in Figure 18.

Now the responses of the system are all stable. A comparison between the responses can be seen in Figure 19. The red signals are from the controller using the $\mu - Synthesis$ method and the blue ones are from the H_∞ controller.

The same approach can be applied to *Comfort* mode. The responses are illustrated in Figures 20, **??** and 22.

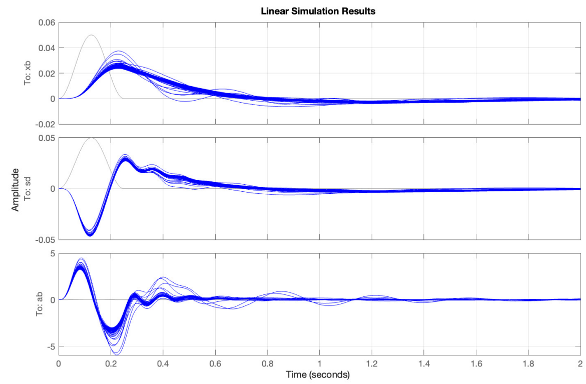

Fig. 18. System with $\mu - Synthesis$ controller

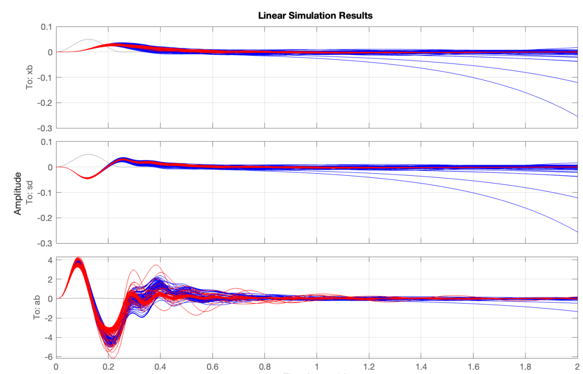

Fig. 19. Comparions between $\mu - Synthesis$ and H_∞ controller

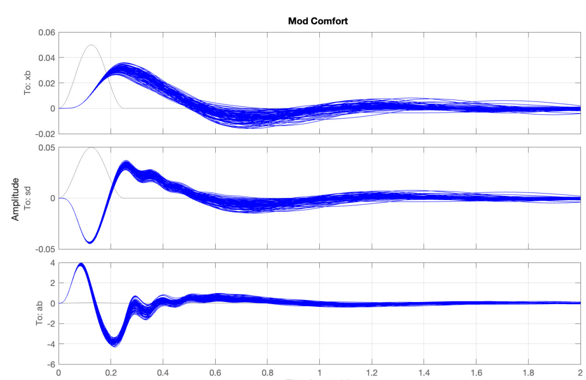

Fig. 20. Mulitple representation of H_∞ responses

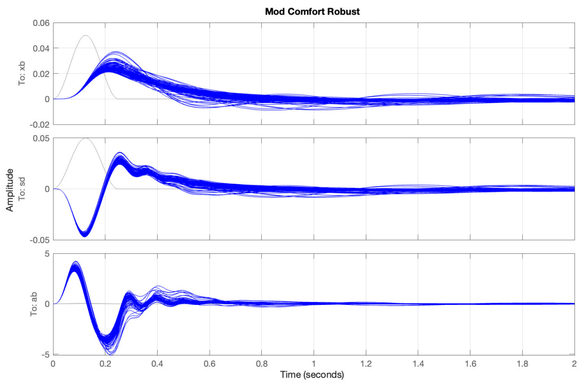

Fig. 21. System with $\mu - Synthesis$ controller

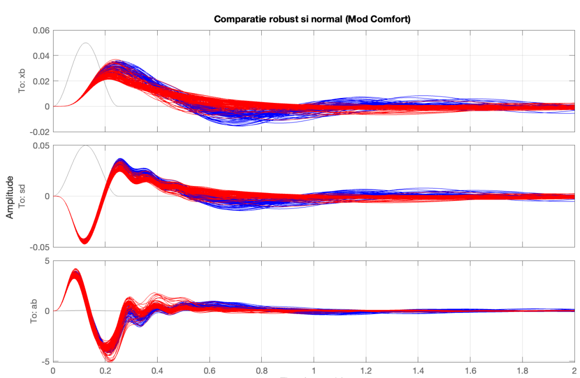

Fig. 22. Comparions between $\mu - Synthesis$ and H_∞ controller