## Higgs-portal dark matter models

Ref: Arcadi, Djouadi, Raidal, 1903.03616, Phys. Rept.

## Higgs-portal scalar dark matter

Real scalar field  $\chi$  for DM, SM Higgs field  $\Phi$ 

$$\mathcal{L} \supset -\frac{1}{2} M_{\chi}^2 \chi^2 - \frac{1}{4} \lambda_{\chi} \chi^4 - \frac{1}{4} \lambda_{H \chi \chi} \Phi^{\dagger} \Phi \chi^2$$

Unitary gauge 
$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix}$$
  $\Phi^{\dagger} \Phi = \frac{1}{2} (v + H)^2 = \frac{v^2}{2} + vH + \frac{H^2}{2}$ ,  $\langle \Phi^{\dagger} \Phi \rangle = \frac{v^2}{2}$ 

$$\text{Mass term } \mathcal{L} \supset -\frac{1}{2} M_{\chi}^2 \chi^2 - \frac{1}{4} \lambda_{H\chi\chi} \left\langle \Phi^{\dagger} \Phi \right\rangle \chi^2 = -\frac{1}{2} m_{\chi}^2 \chi^2, \quad m_{\chi}^2 = M_{\chi}^2 + \frac{1}{4} \lambda_{H\chi\chi} v^2$$

Higgs-portal coupling 
$$\mathcal{L} \supset -\frac{1}{4} \lambda_{H\chi\chi} v H \chi^2 - \frac{1}{8} \lambda_{H\chi\chi} H^2 \chi^2$$

$$H-\chi-\chi \text{ vertex } H-\left(\frac{\chi}{\chi}=-\frac{i}{2}\lambda_{H\chi\chi}v\right)$$
 SM  $H-f-f \text{ vertex } H-\frac{\chi}{\chi}f=-i\frac{m_f}{v}$ 

DM-quark scattering 
$$\chi(p_1) + q(k_1) \rightarrow \chi(p_2) + q(k_2)$$
,  $\chi \setminus \chi$   $|H|$   $q \nearrow \searrow q$ 

$$q \equiv p_1 - p_2 = k_2 - k_1, \quad t \equiv q^2$$

$$i\mathcal{M}_{\chi q} = -\frac{i}{2} \lambda_{H\chi\chi} v \frac{i}{t - m_H^2} \left( -i \frac{m_q}{v} \right) \overline{u}(k_2) u(k_1) \xrightarrow{t \to 0} \frac{i \lambda_{H\chi\chi} m_q}{2 m_H^2} \overline{u}(k_2) u(k_1)$$

Effective vertex 
$$\chi \chi \chi = iG_{\chi q}$$
,  $G_{\chi q} \equiv \frac{\lambda_{H\chi\chi} m_q}{2m_H^2}$ 

DM-quark effective interaction 
$$\mathcal{L}_{\chi q} = \sum_{q} \frac{G_{\chi q}}{2} \chi^2 \overline{q} q$$

DM-nucleon effective interaction 
$$\mathcal{L}_{\chi N} = \sum_{N=n}^{\infty} \frac{G_{\chi N}}{2} \chi^2 \overline{N} N$$

$$G_{\chi N} = m_N \sum_{q} \frac{G_{\chi q}}{m_q} f_q^N = \frac{\lambda_{H\chi\chi} m_N f_N}{2m_H^2}, \quad f_N \equiv \sum_{q} f_q^N, \quad f_c^N = f_b^N = f_t^N \equiv f_Q^N = \frac{2}{27} (1 - f_u^N - f_d^N - f_s^N)$$

$$f_u^p = 0.0208 \pm 0.0015, \quad f_d^p = 0.0411 \pm 0.0028$$

$$f_u^n = 0.0189 \pm 0.0014$$
,  $f_d^n = 0.0451 \pm 0.0027$  [Hoferichter, et al., 1506.04142, PRL]

$$f_s^p = f_s^n = 0.043 \pm 0.011$$
 [Junnarkar, Walker-Loud, 1301.1114, PRD]

$$\Rightarrow f_u^p + f_d^p + f_s^p = 0.105, \quad f_u^n + f_d^n + f_s^n = 0.107$$

$$\Rightarrow f_Q^p = 0.0663, \quad f_Q^n = 0.0661$$

$$\Rightarrow f_p = \sum_q f_q^p = 0.304, \quad f_n = \sum_q f_q^n = 0.305, \quad f_p \simeq f_n$$

Spin-independent DM-nucleon scattering cross section [Eq. (16) in Yu et al., 1112.6052, NPB]

$$\sigma_{\chi N}^{SI} = \frac{m_N^2 G_{\chi N}^2}{4\pi (m_\chi + m_N)^2} = \frac{\lambda_{H\chi\chi}^2 m_N^4 f_N^2}{16\pi m_H^4 (m_\chi + m_N)^2}$$

$$m_p \simeq m_n, \quad f_p \simeq f_n \quad \Rightarrow \quad \sigma_{\chi p}^{SI} \simeq \sigma_{\chi n}^{SI}$$

Invisible Higgs decay  $H \rightarrow \chi \chi$ 

$$i\mathcal{M} = -\frac{i}{2}\lambda_{H\chi\chi}v, \quad |\mathcal{M}|^2 = \frac{1}{4}\lambda_{H\chi\chi}^2v^2$$

Invisible Higgs decay width 
$$\Gamma_{\text{inv}} = \frac{1}{2} \frac{|\mathcal{M}|^2}{16\pi m_H} \sqrt{1 - \frac{4m_\chi^2}{m_H^2}} = \frac{\lambda_{H\chi\chi}^2 v^2 \beta_\chi}{128\pi m_H}, \quad \beta_\chi \equiv \sqrt{1 - \frac{4m_\chi^2}{m_H^2}}$$

## Higgs-portal Majorana fermionic dark matter

Majorana spinor field  $\chi$  for DM, SM Higgs field  $\Phi$ 

$$\mathcal{L} \supset -\frac{1}{2} M_{\chi} \overline{\chi} \chi - \frac{\lambda_{H\chi\chi}}{4\Lambda} \Phi^{\dagger} \Phi \overline{\chi} \chi$$

Mass term 
$$\mathcal{L} \supset -\frac{1}{2} M_{\chi} \overline{\chi} \chi - \frac{\lambda_{H\chi\chi}}{4\Lambda} \frac{v^2}{2} \overline{\chi} \chi = -\frac{1}{2} m_{\chi} \overline{\chi} \chi, \quad m_{\chi} = M_{\chi} + \frac{\lambda_{H\chi\chi} v^2}{4\Lambda}$$

Higgs-portal coupling 
$$\mathcal{L} \supset -\frac{\lambda_{H\chi\chi}v}{4\Lambda}H\overline{\chi}\chi$$
,  $H-\chi-\chi$  vertex  $H-\left\langle \frac{\chi}{\chi} = -\frac{i\lambda_{H\chi\chi}v}{2\Lambda} \right\rangle$ 

DM-quark scattering 
$$\chi(p_1) + q(k_1) \rightarrow \chi(p_2) + q(k_2)$$
,  $\chi \setminus \chi$ 

$$|H|$$

$$q \nearrow \searrow q$$

$$i\mathcal{M}_{\chi q} = -\frac{i\lambda_{H\chi\chi}v}{2\Lambda}\frac{i}{t - m_H^2} \left(-i\frac{m_q}{v}\right) \overline{u}(k_2)u(k_1) \xrightarrow{t \to 0} \frac{i\lambda_{H\chi\chi}m_q}{2\Lambda m_H^2} \overline{u}(k_2)u(k_1)$$

Effective vertex 
$$\chi \chi \chi = iG_{\chi q}$$
,  $G_{\chi q} \equiv \frac{\lambda_{H\chi\chi} m_q}{2\Lambda m_H^2}$ 

DM-quark effective interaction 
$$\mathcal{L}_{\chi q} = \sum_{q} \frac{G_{\chi q}}{2} \overline{\chi} \chi \overline{q} q$$

DM-nucleon effective interaction  $\mathcal{L}_{\chi N} = \sum_{N=n,p} \frac{G_{\chi N}}{2} \, \overline{\chi} \chi \overline{N} N$ 

$$G_{\chi N} = m_N \sum_{q} \frac{G_{\chi q}}{m_q} f_q^N = \frac{\lambda_{H \chi \chi} m_N f_N}{2\Lambda m_H^2}$$

Spin-independent DM-nucleon scattering cross section [Eq. (44) in Zheng et al., 1012.2022, NPB]

$$\sigma_{\chi N}^{SI} = \frac{m_N^2 m_\chi^2 G_{\chi N}^2}{\pi (m_\chi + m_N)^2} = \frac{\lambda_{H\chi\chi}^2 m_N^4 m_\chi^2 f_N^2}{4\pi \Lambda^2 m_H^4 (m_\chi + m_N)^2}$$

Invisible Higgs decay  $H(p) \rightarrow \chi(k_1) + \chi(k_2)$ 

$$\begin{split} &i\mathcal{M} = -\frac{i\lambda_{H\chi\chi}v}{2\Lambda}\overline{u}(k_{1})v(k_{2}), \quad (i\mathcal{M})^{*} = \frac{i\lambda_{H\chi\chi}v}{2\Lambda}\overline{v}(k_{2})u(k_{1}) \\ &|\overline{\mathcal{M}}|^{2} = \sum_{\text{spins}}|\mathcal{M}|^{2} = \frac{\lambda_{H\chi\chi}^{2}v^{2}}{4\Lambda^{2}}\sum_{\text{spins}}\text{tr}[u(k_{1})\overline{u}(k_{1})v(k_{2})\overline{v}(k_{2})] = \frac{\lambda_{H\chi\chi}^{2}v^{2}}{4\Lambda^{2}}\text{tr}[(k_{1}+m_{\chi})(k_{2}-m_{\chi})] \\ &= \frac{\lambda_{H\chi\chi}^{2}v^{2}}{4\Lambda^{2}}[\text{tr}(k_{1}k_{2}) - 4m_{\chi}^{2}\text{tr}(1)] = \frac{\lambda_{H\chi\chi}^{2}v^{2}}{\Lambda^{2}}(k_{1}\cdot k_{2} - m_{\chi}^{2}) \\ &m_{H}^{2} = (k_{1}+k_{2})^{2} = 2m_{\chi}^{2} + 2k_{1}\cdot k_{2}, \quad k_{1}\cdot k_{2} = \frac{m_{H}^{2}}{2} - m_{\chi}^{2} \\ &k_{1}\cdot k_{2} - m_{\chi}^{2} = \frac{m_{H}^{2}}{2} - 2m_{\chi}^{2} = \frac{m_{H}^{2}}{2} \left(1 - \frac{4m_{\chi}^{2}}{m_{H}^{2}}\right) = \frac{m_{H}^{2}\beta_{\chi}^{2}}{2} \end{split}$$
Invisible Higgs decay width  $\Gamma_{\text{inv}} = \frac{1}{2}\frac{|\overline{\mathcal{M}}|^{2}}{16\pi m_{H}} \sqrt{1 - \frac{4m_{\chi}^{2}}{m_{H}^{2}}} = \frac{\lambda_{H\chi\chi}^{2}v^{2}m_{H}\beta_{\chi}^{3}}{64\pi\Lambda^{2}} \end{split}$ 

## Higgs-portal vector dark matter

Real vector field  $\chi^{\mu}$  for DM, SM Higgs field  $\Phi$ 

$$\mathcal{L} \supset \frac{1}{2} M_{\chi}^{2} \chi_{\mu} \chi^{\mu} + \frac{1}{4} \lambda_{\chi} (\chi_{\mu} \chi^{\mu})^{2} + \frac{1}{4} \lambda_{H \chi \chi} \Phi^{\dagger} \Phi \chi_{\mu} \chi^{\mu}$$

Mass term 
$$\mathcal{L} \supset \frac{1}{2} M_{\chi}^2 \chi_{\mu} \chi^{\mu} + \frac{1}{4} \lambda_{H\chi\chi} \frac{v^2}{2} \chi_{\mu} \chi^{\mu} = \frac{1}{2} m_{\chi}^2 \chi_{\mu} \chi^{\mu}, \quad m_{\chi}^2 = M_{\chi}^2 + \frac{1}{4} \lambda_{H\chi\chi} v^2$$

Higgs-portal coupling 
$$\mathcal{L} \supset \frac{1}{4} \lambda_{H\chi\chi} v H \chi_{\mu} \chi^{\mu}$$
,  $H - \chi - \chi$  vertex  $H - \left(\frac{\chi_{\mu}}{\chi_{\nu}} = \frac{i}{2} \lambda_{H\chi\chi} v g^{\mu\nu}\right)$ 

$$q \equiv p_1 - p_2 = k_2 - k_1, \quad t \equiv q^2$$

$$i\mathcal{M}_{\chi q} = \varepsilon_{\mu}(p_1)\varepsilon_{\nu}^*(p_2)\frac{i}{2}\lambda_{H\chi\chi}\nu\frac{i}{t-m_H^2}\left(-i\frac{m_q}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_q}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\varepsilon_{\mu}(p_1)\varepsilon_{\nu}^*(p_2)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_q}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_q}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_q}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_q}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_q}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_q}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_q}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_q}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_q}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_q}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_q}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_q}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_q}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_q}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_q}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_q}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_q}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_q}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_q}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_q}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_q}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_q}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_q}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_q}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_q}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_q}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_q}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_q}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_q}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_q}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_q}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_q}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_q}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_q}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_q}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_q}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_q}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_q}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_q}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_H^2}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_H^2}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_H^2}{\nu}\right)\overline{u}(k_2)u(k_1)\frac{i}{t-m_H^2}\left(-i\frac{m_H^2}{\nu}\right)\overline{u}(k_1)\frac{i}{t$$

Effective vertex 
$$\chi / \chi = iG_{\chi q}$$
,  $G_{\chi q} \equiv -\frac{\lambda_H \chi \chi}{2m_H^2}$ 

DM-quark effective interaction 
$$\mathcal{L}_{\chi q} = \sum_{q} \frac{G_{\chi q}}{2} \chi_{\mu} \chi^{\mu} \overline{q} q$$

DM-nucleon effective interaction  $\mathcal{L}_{\chi N} = \sum_{N=n,p} \frac{G_{\chi N}}{2} \chi_{\mu} \chi^{\mu} \overline{N} N$ 

$$G_{\chi N} = m_N \sum_q \frac{G_{\chi q}}{m_q} f_q^N = -\frac{\lambda_{H\chi\chi} m_N f_N}{2m_H^2}$$

Spin-independent DM-nucleon scattering cross section [Eq. (50) in Yu et al., 1112.6052, NPB]

$$\sigma_{\chi N}^{SI} = \frac{m_N^2 G_{\chi N}^2}{4\pi (m_{\chi} + m_N)^2} = \frac{\lambda_{H\chi\chi}^2 m_N^4 f_N^2}{16\pi m_H^4 (m_{\chi} + m_N)^2}$$

Invisible Higgs decay  $H(p) \rightarrow \chi(k_1) + \chi(k_2)$ 

$$i\mathcal{M} = \frac{i}{2} \lambda_{H\chi\chi} v g^{\mu\nu} \mathcal{E}_{\mu}^*(k_1) \mathcal{E}_{\nu}^*(k_2), \quad (i\mathcal{M})^* = -\frac{i}{2} \lambda_{H\chi\chi} v g^{\rho\sigma} \mathcal{E}_{\rho}(k_1) \mathcal{E}_{\sigma}(k_2)$$
$$k_1^2 = k_2^2 = m_{\chi}^2, \quad \frac{k_1 \cdot k_2}{m_{\chi}^2} = \frac{m_H^2}{2m_{\chi}^2} - 1$$

$$\begin{split} \overline{|\mathcal{M}|^{2}} &= \sum_{\text{spins}} |\mathcal{M}|^{2} = \frac{\lambda_{H\chi\chi}^{2} v^{2}}{4} \sum_{\text{spins}} g^{\mu\nu} g^{\rho\sigma} \mathcal{E}_{\mu}^{*}(k_{1}) \mathcal{E}_{\rho}(k_{1}) \mathcal{E}_{\nu}^{*}(k_{2}) \mathcal{E}_{\sigma}(k_{2}) \\ &= \frac{\lambda_{H\chi\chi}^{2} v^{2}}{4} g^{\mu\nu} g^{\rho\sigma} \left( -g_{\mu\rho} + \frac{k_{1\mu} k_{1\rho}}{m_{\chi}^{2}} \right) \left( -g_{\nu\sigma} + \frac{k_{2\nu} k_{2\sigma}}{m_{\chi}^{2}} \right) = \frac{\lambda_{H\chi\chi}^{2} v^{2}}{4} \left( \delta^{\nu}_{\rho} - \frac{k_{1}^{\nu} k_{1\rho}}{m_{\chi}^{2}} \right) \delta^{\rho}_{\nu} - \frac{k_{2\nu} k_{2}^{\rho}}{m_{\chi}^{2}} \right) \\ &= \frac{\lambda_{H\chi\chi}^{2} v^{2}}{4} \left[ 4 - \frac{k_{1}^{2}}{m_{\chi}^{2}} - \frac{k_{2}^{2}}{m_{\chi}^{2}} + \frac{(k_{1} \cdot k_{2})^{2}}{m_{\chi}^{4}} \right] = \frac{\lambda_{H\chi\chi}^{2} v^{2}}{4} \left[ 2 + \left( \frac{m_{H}^{2}}{2m_{\chi}^{2}} - 1 \right)^{2} \right] \\ &= \frac{\lambda_{H\chi\chi}^{2} v^{2}}{4} \left( 3 + \frac{m_{H}^{4}}{4m_{\chi}^{4}} - \frac{m_{H}^{2}}{m_{\chi}^{2}} \right) = \frac{\lambda_{H\chi\chi}^{2} v^{2} m_{H}^{4}}{16m_{\chi}^{4}} \left( 1 - \frac{4m_{\chi}^{2}}{m_{H}^{2}} + \frac{12m_{\chi}^{4}}{m_{H}^{4}} \right) \end{split}$$

Invisible Higgs decay width 
$$\Gamma_{\text{inv}} = \frac{1}{2} \frac{\overline{|\mathcal{M}|^2}}{16\pi m_H} \sqrt{1 - \frac{4m_{\chi}^2}{m_H^2}} = \frac{\lambda_{H\chi\chi}^2 v^2 m_H^3 \beta_{\chi}}{512\pi m_{\chi}^4} \left(1 - \frac{4m_{\chi}^2}{m_H^2} + \frac{12m_{\chi}^4}{m_H^4}\right)$$