(1) $X :=$ # people to obtain match

$$\mathbb{P}[X = 1] = 0$$

$$\mathbb{P}[X = 3] = \frac{365 \cdot 364}{365^2} \cdot \underbrace{\frac{2}{365}}$$

$$\mathbb{P}[X = k] = \frac{\frac{365!}{(365-k)!}}{365^{k-1}} \cdot \frac{k-1}{365}$$

(2) Geometric dist. p - success prob.

a) $\mathbb{P}[X = k] = (1-p)^{k-1} p$ $\qquad q = 1 - p$

b) $\mathbb{P}[X = k] =$

FS

$$\underbrace{\underbrace{F\ F}_{k-1}\ S}_{k}$$

$$\boxed{\left(\frac{1}{2}\right)^{k-1} \cdot \frac{1}{2}}$$

$$\underbrace{\underbrace{S\,S}_{k-1}\,F}_{k} \qquad \left(\frac{1}{2}\right)^{k-1} \cdot \frac{1}{2}$$

OR

$$\{X = k\} = \{\overbrace{\underbrace{F F}_{k-1} S}^{k}\} \sqcup \{\overbrace{\underbrace{S S}_{k-1} F}^{k}\}$$

$$\mathbb{P}[X = k] = \mathbb{P}[\underbrace{F F}_{k-1} S] + \mathbb{P}[\underbrace{S S}_{k-1} F] =$$

$$= \begin{cases} p = \frac{1}{2} & \left(\frac{1}{2}\right)^{k-1} \cdot \frac{1}{2} \cdot 2 \end{cases}$$

$$= (1-p)^{k-1} p + p^{k-1}(1-p)$$

5 $\quad p_n = \left(\frac{1}{2}\right)^{n+1}$

a) 1 V $\quad \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} = \frac{1}{2} \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n} = \frac{1}{2} \cdot \frac{1}{1 - \frac{1}{2}}$

$q \in (0,1)$ $\qquad \sum_{n=0}^{\infty} q^n = \frac{1}{1-q} \Big| = 1$

b)

$q \quad q^2 \quad \ldots \quad q^m$

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$n = 100$

$k = 110$

$\mathbb{P}[S] = p = 0.9$

$1 \quad 2 \quad \ldots \quad K$

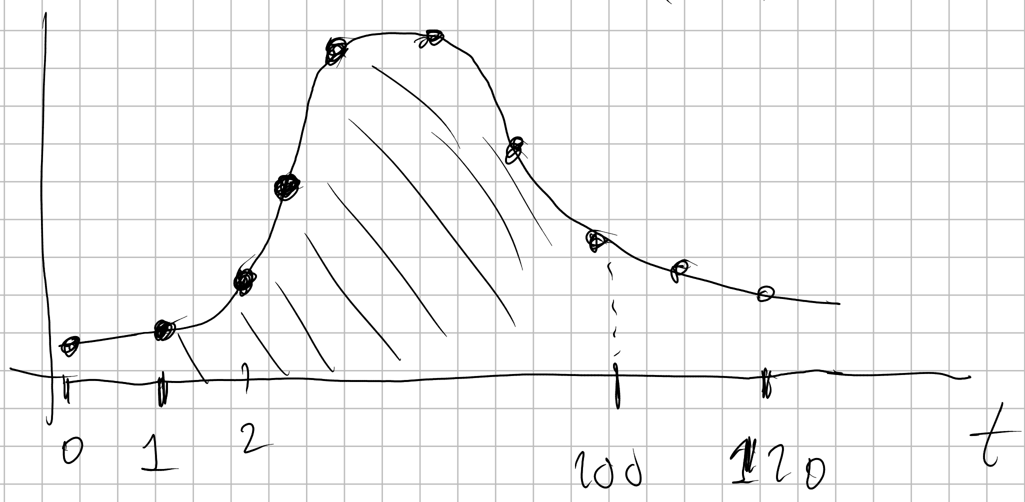
$p \swarrow \; \searrow 1-p \qquad \ldots \qquad p \swarrow \; \searrow 1-p$

$1 \quad 0 \qquad \ldots \qquad 1 \quad 0$

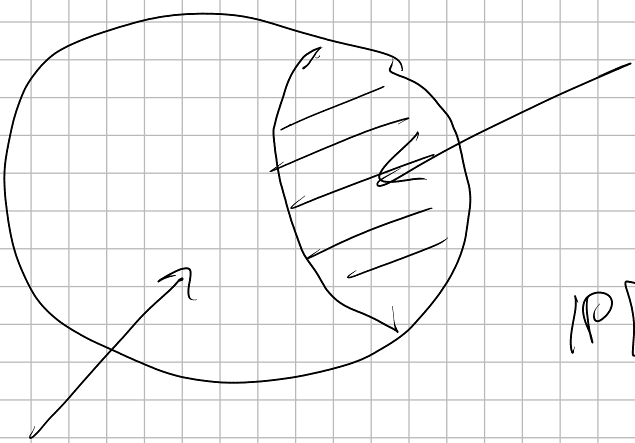
$X \sim Bin(K, p)$

our observable value

$$\mathbb{P}[X = t] = \binom{k}{t} p^t (1-p)^{k-t}$$



$$\overbrace{\mathbb{P}[X \le 100]}^{\text{cdf}} = \sum_{t=0}^{100} \binom{k}{t} p^t (1-p)^{k-t}$$



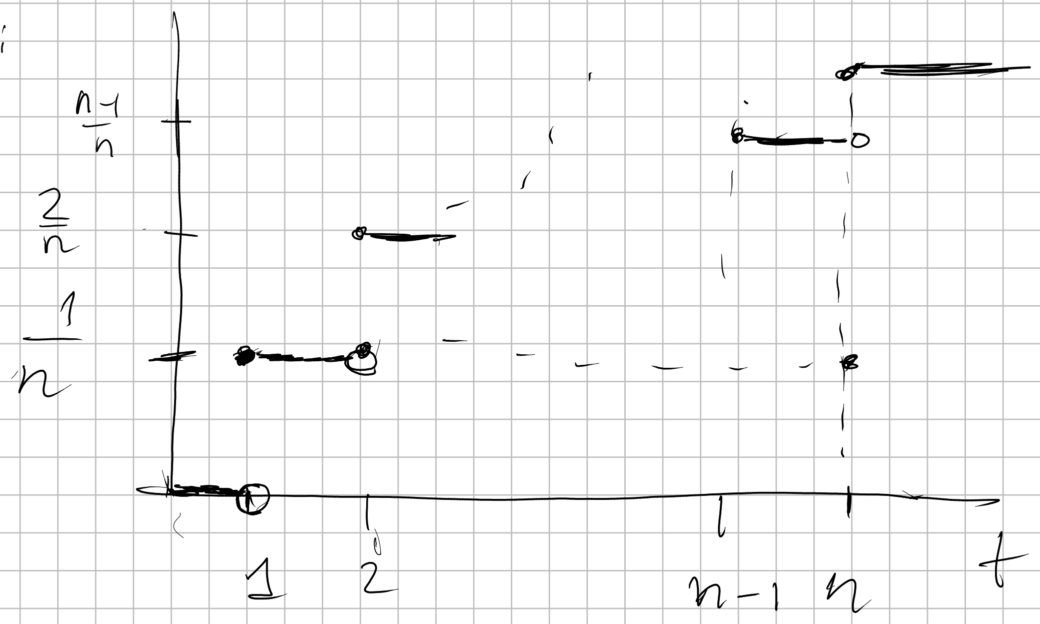
$$A = \{X \le 100\}$$

$$\mathbb{P}[B] = \mathbb{P}[X > 100] = \sum_{101}^{110} \binom{k}{t} p^t (1-p)^{k-t}$$

$$B = A^c = \{X > 100\}$$

$$A = B^c$$

$$\mathbb{P}[A] = \mathbb{P}[B^c] = 1 - \mathbb{P}[B]$$



$\mathbb{P}[X \le t]$

③ $Y = \underline{\mu + \sigma X}$ – known

$G(t) = \mathbb{P}[Y \le t]$ – ?

$F(s) = \mathbb{P}[X \le s]$ – known

reparametrization

$X.\text{cdf}(\ldots) \Leftrightarrow Y.\text{cdf}(t)$

we don't have Y.cdf

$$G(t) = \mathbb{P}[Y \le t] = \mathbb{P}[\mu + \sigma X \le t] = $$

$$\{\mu + \sigma X \le t\} \iff \{1 \cdot X \le s\}$$

$$\mu + \sigma X \le t$$

$$\sigma X \le t - \mu$$

$$X \le \frac{t-\mu}{\sigma}$$

$$= \mathbb{P}\left[X \le \frac{t-\mu}{\sigma}\right] =$$

$$= F\left(\frac{t-\mu}{\sigma}\right)$$

$$Y.\mathrm{cdf}(t) \iff X.\mathrm{cdf}\left(\frac{t-\mu}{\sigma}\right)$$

$$\mathcal{N}(\mu, \sigma^2)$$

$$\mathcal{N}(0, 1)$$
