2.5

3

52

$B = A_s\, 8_c$

$A = A$

$$\mathbb{P}[A|B] = \frac{\mathbb{P}[A \cap B]}{\mathbb{P}[B]} =$$

$$P[A|B] = \frac{3}{50}$$

$$= \frac{\cancel{\frac{1}{52} \cdot \frac{1}{51}} \cdot \frac{3}{50}}{\cancel{\frac{1}{52} \cdot \frac{1}{51}}} = \frac{3}{50}$$

---

a) $\mathbb{P}[X|A] = 1 - \mathbb{P}[X^c|A]$

$\mathbb{P}[\text{rec } 1 \mid \text{send } 0] = 0.05$ $\qquad \mathbb{P}[\text{send } 0] = \frac{1}{2}$

$\mathbb{P}[\text{rec } 0 \mid \text{send } 1] = 0.1$ $\qquad \mathbb{P}[\text{rec } 1 \mid \text{send } 1] = 0.9$

$$\mathbb{P}[\text{send } 1 \mid \text{rec } 1] = \frac{\mathbb{P}[\text{rec } 1 \mid \text{send } 1]\, \mathbb{P}[\text{send } 1]}{\mathbb{P}[\text{rec } 1]}$$

$$\mathbb{P}[\text{rec } 1] \overset{\text{LOTP}}{=} \mathbb{P}[\text{rec } 1 \mid \text{send } 1]\, \mathbb{P}[\text{send } 1] +$$

$$+ \mathbb{P}[\text{rec } 1 \mid \text{send } 0]\, \mathbb{P}[\text{send } 0]$$

$$= \frac{1}{2}(0.9 + 0.05) = \frac{0.95}{2} = 0.475$$

$$\mathbb{P}[\text{send } 1 \mid \text{rec } 1] = \frac{\frac{0.9}{2}}{\frac{0.95}{2}} = \frac{0.9}{0.95}$$

$$\mathbb{P}[\text{send } 1 \mid \text{rec } 110] = \frac{\mathbb{P}[\text{rec } 110 \mid \text{send } 1]\,\mathbb{P}[\text{send } 1]}{\mathbb{P}[\text{rec } 110]} =$$

$$\mathbb{P}[\text{send } 1] = \frac{1}{2}$$

$$\mathbb{P}[\text{rec } 110 \mid \text{send } 1] = 0.9 \cdot 0.9 \cdot 0.1$$

$$1 \longrightarrow 111$$

$$\mathbb{P}[\text{rec } \underline{1}\,\underline{1}\,\underline{1} \mid \underline{\text{send } 1}] = 0.9 \cdot 0.9 \cdot 0.9$$

111

110

000

$$\mathbb{P}[\text{rec } 110] \overset{\text{LOTP}}{=} \mathbb{P}[\text{rec } 110 \mid \text{send } 1]\,\mathbb{P}[\text{send } 1] +$$

$$+ \mathbb{P}[\text{rec } 110 \mid \text{send } 0]\,\mathbb{P}[\text{send } 0] =$$

$$\mathbb{P}[\text{rec } 110 \mid \text{send } 0] = 0.05 \cdot 0.05 \cdot 0.95$$

$$\Leftrightarrow \frac{1}{2}\left(0.9^2 \cdot 0.1 + 0.95 \cdot 0.05^2\right) =$$

$$= \frac{\frac{0.9^2 \cdot 0.1}{2}}{\frac{1}{2}\left(0.9^2 \cdot 0.1 + 0.95 \cdot 0.05^2\right)} =$$

$$= \frac{0.9^2 \cdot 0.1}{0.9^2 \cdot 0.1 + 0.95 \cdot 0.05^2} \approx 0.971$$

$$\mathbb{P}[\text{send } 1 \mid \text{rec } 1] \approx 0.95$$

$$\wedge$$

$$\mathbb{P}[\text{send } 1 \mid \text{rec } 110] \approx 0.971$$

$$0.95 < 0.971$$

$$1 \longrightarrow 1$$

$$1 \longrightarrow \boxed{111}$$

30 A, B, C

$E_1 = \{A > B\}$

$E_2 = \{A > C\}$

Ann 30

Bob 25

Charlie ???

E_1 indep. E_2

$\Updownarrow$ def

$$\boxed{\mathbb{P}[E_1 \cap E_2] = \mathbb{P}[E_1]\,\mathbb{P}[E_2]}$$

$\mathbb{P}[E_1] = \frac{1}{2}$, $\mathbb{P}[E_2] = \frac{1}{2}$

$\mathbb{P}[E_1 \cap E_2] = \frac{1}{4}$

Sample space

Ω

---

32 a) $\mathbb{P}[A > B] \overset{\text{LOTP}}{=} \mathbb{P}[A > B \mid A = 4]\,\mathbb{P}[A = 4]$

$+ \mathbb{P}[A > B \mid A = 0]\,\mathbb{P}[A = 0] =$

$= 1 \cdot \frac{4}{6} + 0 \cdot \frac{2}{6} = \frac{2}{3}$

$\mathbb{P}[B > C] = \mathbb{P}[3 > C] = \frac{4}{6} = \frac{2}{3}$

$A > B$, $B > C$

$$\mathbb{P}[A>B, B>C] = \mathbb{P}[A>3, C<3] =$$

1	2	3	4	5	6
4	4	4	4	0	0

1	2	3	4	5	6
6	6	2	2	2	2

(i, j) $\quad i \in \{1, \dots, 6\}$

$j \in \{1, \dots, 6\}$

$\Omega = \{(i, j)\}$, $|\Omega| = 6^2$

$A>3, C<3$

$$= \frac{|A>3, C<3|}{|\Omega|} =$$

$$= \frac{16}{36} = \frac{4}{9}.$$