1.6

$n = 20$

"у Эшка"

$\boxed{\# \text{pairs} = 10}$

matchings ?

$$\frac{2^{10} \cdot \underbrace{\binom{20}{2}}_{\text{choose } 1^{st} \text{ pair}} \cdot \underbrace{\binom{18}{2}}_{2nd \text{ pair}} \cdot \binom{16}{2} \cdot \ldots \cdot \underbrace{\binom{2}{2}}_{10^{th} \text{ pair}}}{10!} =$$

$$= \frac{2^{10} \cdot \frac{20!}{2!\,18!} \cdot \frac{18!}{2!\,16!} \cdot \frac{16!}{2!\,14!} \cdots \frac{4!}{2!\,2!} \cdot \frac{2!}{0!\,2!}}{10!} =$$

$$= \frac{2^{10} \cdot \frac{20!}{(2!)^{10}}}{10!} = \frac{20!}{10!} = 20 \cdot 19 \cdot 18 \cdot \ldots \cdot 11$$

I	II			X
1 2	1 2	1 2	...	1 2
1 2	3 4	..		19 20

3

1^{st}	2^{nd}	3^{rd}	
1 5	4 2	3 6	..
4 2	1 5	3 6	

2.6 $\quad n = 100$

1 double H

$A = $ 7 flip 7H

$B = $ double H

$\mathbb{P}[A|B] = 1$

$\mathbb{P}[B] = \frac{1}{100}$

$\mathbb{P}[A] \underset{\text{LOTP}}{=} \mathbb{P}[$

$$\overbrace{\mathbb{P}[B|A]}^{\text{posterior}} = \frac{\overbrace{\mathbb{P}[A|B]}^{\text{likelihood}}\ \overbrace{\mathbb{P}[B]}^{\text{prior}}}{\underbrace{\mathbb{P}[A]}_{\text{evidence}}} =$$

Law of total prob. (LOTP)

A partition

$$\boxed{\Omega = \bigsqcup_{i=1}^{n} B_i} \qquad B_i \cap B_j = \emptyset$$

$$\boxed{A \subseteq \Omega}$$

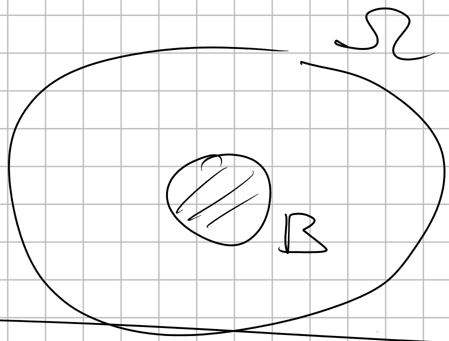
$$\mathbb{P}[A] = \sum_{i=1}^{n} \mathbb{P}[A \cap B_i]$$

$$\mathbb{P}[A] = \mathbb{P}[\underbrace{A \cap \Omega}_{}] = \mathbb{P}\left[A \cap \left(\bigsqcup_{i=1}^{n} B_i\right)\right] =$$

$$= \sum_{i=1}^{n} \mathbb{P}[A \cap B_i]$$

$A = (A \cap B_1) \cup (A \cap B_2) \cup (A \cap B_3)$

$B_1 \quad B_2 \quad B_3$



$$\Omega = B \sqcup B^c.$$

$n=2$ $\quad B_1 = B$

$B_2 = B^c$

$$\mathbb{P}[A] \overset{\text{LOTP}}{=} \mathbb{P}[A \cap B] + \mathbb{P}[A \cap B^c] =$$

A – 7 flip 7 H

B – double H

$$\mathbb{P}[A \cap B] \overset{\text{product}}{=} \mathbb{P}[A|B]\,\mathbb{P}[B]$$

$$\mathbb{P}[A|B] = 1$$

$$= \underbrace{\mathbb{P}[A|B]}_{=1}\,\underbrace{\mathbb{P}[B]}_{\frac{1}{100}} + \underbrace{\mathbb{P}[A|B^c]}_{\left(\frac{1}{2}\right)^7}\,\underbrace{\mathbb{P}[B^c]}_{\frac{99}{100}}$$

$$\frac{\frac{1}{100}}{\frac{1}{100} + \frac{1}{128} \cdot \frac{99}{100}} \approx \frac{1}{2}$$

2.13

D - disease

$\mathbb{P}[D] = 0.01$ prior

sens. $\mathbb{P}[+ \mid D] = 0.95$

A

spec. $\mathbb{P}[- \mid D^c] = 0.95$

$\mathbb{P}[A \mid B] = \frac{\mathbb{P}[A \cap B]}{\mathbb{P}[B]}$

B

$\mathbb{P}[+ \mid D] = 0$

$\mathbb{P}[- \mid D^c] = 1$

$\mathbb{P}[suc] = \mathbb{P}[TP \cup TN] =$

$= \mathbb{P}[+ \cap D \sqcup - \cap D^c] =$

$= \mathbb{P}[+ \cap D] \cup \mathbb{P}[- \cap D^c] =$

$= \mathbb{P}[+ \mid D]\, \underline{\underline{\mathbb{P}[D]}} + \mathbb{P}[- \mid D^c]\, \underline{\underline{\mathbb{P}[D^c]}}$

$\mathbb{P}[suc_A] = \underline{0.95} \cdot \underline{0.01} + \underline{0.95} \cdot \underline{0.99} = 0.55$

$\mathbb{P}[suc_B] = 0 \cdot \underline{0.01} + \underline{1} \cdot 0.99 = 0.99$

Cond. prob.

LOTP

Bayes formula