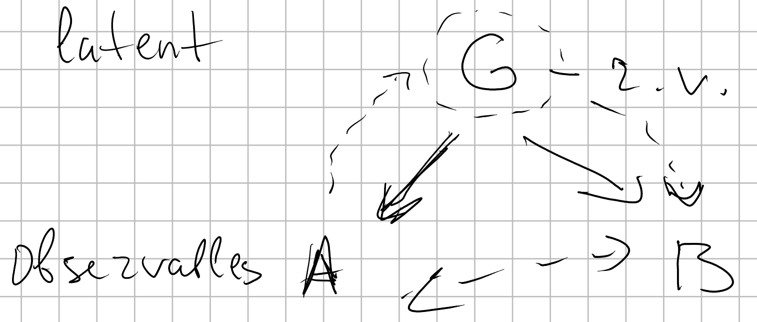

$A \not\perp B$

$A \perp B \mid G$

b) $\mathbb{P}[\underset{\substack{\uparrow \\ \text{type}}}{G} \mid \underset{\substack{\uparrow \\ \text{event}}}{A^c}] = \dfrac{\mathbb{P}[A^c \mid G]\,\mathbb{P}[G]}{\mathbb{P}[A^c]} \ldots$

c) $\mathbb{P}[B \mid A^c] =$

$\mathbb{P}[\cdot \mid A]$ is also a prob. measure

$\| $

$\mathbb{P}_A[\cdot]$ $\qquad \mathbb{P}[B \mid A] = \mathbb{P}_A[B]$

$T \in \{t_1, \ldots, t_n\}$

$$\mathbb{P}_A[B] \underset{\substack{\uparrow \\ \text{LOTP}}}{=} \sum_{i=1}^{n} \mathbb{P}_A[B \mid T = t_i]\, \mathbb{P}_A[T = t_i] =$$

$$= \sum_{i=1}^{n} \mathbb{P}[B \mid T = t_i, A]\, \mathbb{P}[T = t_i, A]$$

LOTV for cond. prob. $(\mathbb{P}[\cdot \mid A^c])$

$$\mathbb{P}[B \mid A^c] \stackrel{\text{LOTV}}{=} \mathbb{P}[B \mid A^c, G]\, \underline{\mathbb{P}[G \mid A^c]} +$$

$$+ \mathbb{P}[B \mid A^c, G^c]\, \underline{\mathbb{P}[G^c \mid A^c]} =$$

$$\mathbb{P}[B \mid A^c, G] = \overline{\mathbb{P}[B \mid G]}$$

$$\mathbb{P}[B \mid A^c, G^c] = \underline{\mathbb{P}[B \mid G^c]}$$

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$$\mathbb{P}[b] = \mathbb{P}[i] = \mathbb{P}[m] = \frac{1}{3}$$

$$\mathbb{P}[W_1 \mid b] = 0.9$$

a) $$\mathbb{P}[W_1] \underset{\text{LOTV}}{=} \mathbb{P}[W_1 \mid b]\,\mathbb{P}[b] + \mathbb{P}[W_1 \mid i]\,\mathbb{P}[i] + \mathbb{P}[W_1 \mid m]\,\mathbb{P}[m]$$

0.9

b) $$\mathbb{P}[W_2 \mid W_1] = \mathbb{P}[W_2 \mid B, W_1]\,\mathbb{P}[B \mid W_1] + \mathbb{P}[W_2 \mid i, W_1]\,\mathbb{P}[i \mid W_1] + \mathbb{P}[W_2 \mid m, W_1]\,\mathbb{P}[m \mid W_1] =$$

$$\mathbb{P}[W_2 \mid B, W_1] \overset{?}{=} \mathbb{P}[W_2 \mid B]$$

$$\begin{tikzcd} & T \arrow[dl] \arrow[dr] \arrow[dr, bend left] & \\ W_1 \arrow[ur, bend left] & & W_2 \end{tikzcd}$$

$$W_2 \perp W_1 \mid T$$

direct product

$$X \times Y \times Z = \{ (x,y,z) \mid x \in X,\ y \in Y,\ z \in Z \}$$

$$W_2 \not\perp W_1$$

$$X : \underbrace{\Omega}_{\text{sample space}} \longrightarrow \underbrace{\mathbb{R}}_{\text{results}}$$

$\mathrm{Bin}(10, \frac{1}{2})$ $\qquad X : \{0,1\}^{10} \longrightarrow \{0,1,2,\dots,10\}$

X^{-1} preimage

1	2	3	...	10
0	1	0	...	0

sample

$$X(010\,111000\,1) = 5 \qquad X^{-1}(k) := \{ \omega \in \Omega ;\ X(\omega) = k \}$$

$$\mathbb{P}[X = k] \overset{\text{def}}{:=} \mathbb{P}[X^{-1}(k)] =$$

$$\mathbb{P}[0\,1\,0\ldots] = \frac{1}{2^{10}}$$

$$\mathbb{P}[X = k] = \left\{ \binom{n}{k} = \frac{n!}{k!(n-k)!} \right\}$$

$\{1, \ldots, n\}$ choose k elements

$k = 5$

$n = 10$

$\binom{n}{k} :=$

$1\ 2\ \ldots\ n$

choose k slots for 1

0 0 1 0 0 1 0 0 1 0 0

$\binom{n}{k}$:= binary strings of len n with k 1s

$$\mathbb{P}[X = k] = \binom{n}{k}\left(\frac{1}{2}\right)^n \qquad \mathrm{Bin}\left(n, \frac{1}{2}\right)$$

$X \sim Bin(n, p)$ $\quad \mathbb{P}[X = k] - ?$ $\quad 1 - p = q$

$$\mathbb{P}[X = k] = \sum_{w \in \{0,1\}^n} \mathbb{P}[X = k \mid w]\, \mathbb{P}[w] =$$

$w \in \{0,1\}^n$ — binary string.

$$\mathbb{P}[X = k \mid w] = \begin{cases} 1, & w \text{ has exactly } k \text{ 1s} \\ 0, & \text{otherwise} \end{cases}$$

if w given has exacty k 1s (we know positions for 1s)

$$\mathbb{P}[w] = p^k \cdot q^{n-k}$$

$$\underset{p}{1} \quad \underset{q}{0} \quad \underset{p}{1} \quad \underset{q}{0} \quad 1 \quad 0 \quad 1 \quad \ldots$$

$$\underbrace{p \cdot p \cdot \ldots \cdot p}_{k \text{ times}} = p^k$$

$$= \sum_{w \text{ has exactly } k \text{ 1s}} 1 \cdot p^k q^{n-k} = \binom{n}{k} p^k q^{n-k}$$