Hypothesis testing (demo)

$\boxed{X \sim Bin(n, \theta)}$

$H_0 : \theta = \theta_0$

$H_1 : \theta > \theta_0$

coin

r.v.

Observe : number of heads $\leftrightarrow X$

$$\mathbb{P}[X = k \mid H_0] = \binom{n}{k}\left(\frac{1}{2}\right)^k \left(1 - \frac{1}{2}\right)^{n-k}$$

realization (what we saw)

100 $\to m_1, s_1 \leftarrow m_0$

101 $\to m_2, s_2 \leftarrow m < m_0$

$\mathbb{E}$, Var

$\downarrow$

Random forest

Prob. theory $\to$ Linear regression $\leftarrow$ Linear alg. Calc.

indep.

$$A \perp B \iff \mathbb{P}[AB] = \mathbb{P}[A]\,\mathbb{P}[B]$$

$$\boxed{\mathbb{P}[AB] = \mathbb{P}[A \mid B]\,\mathbb{P}[B]}$$

2.30 $E_1 = A > B$, $E_2 = A > C$

$$E_1 \longrightarrow E_2$$

$A, B, C \sim U\{1, \dots, n\}$

$(a, b, c) \in \{1, \dots, n\}^3$

$$\boxed{\mathbb{P}[A = k \mid B = l] = \mathbb{P}[A = k]}$$

$$\mathbb{P}[A \geq C \mid A \geq B] \neq \mathbb{P}[A \geq C]$$

1 2 3

$\boxed{A \geq B \geq C}$ F_1

$\boxed{A \geq C \geq B}$ F_2

$B \geq \cancel{A \geq C}$ F_3

3! $B \geq \cancel{C \geq A}$ F_4

$\boxed{C \geq A \geq B}$ F_5

$C \geq \cancel{B \geq A}$ F_6

$\mathbb{P}[F_i] = \frac{1}{6}$

$A \geq C = \{F_1, F_2, F_3\}$

$A \geq B = \{F_1, F_2, F_5\}$

$\mathbb{P}[\{F_1, F_2\}] = \frac{2}{3}$

$$\mathbb{P}[A \geq C \mid \cancel{A} \geq B] = \frac{\mathbb{P}[A \geq C, A \geq B]}{\mathbb{P}[A \geq B]} = \frac{\mathbb{P}[\{F_1, F_2\}]}{\mathbb{P}[\{F_1, F_2, F_5\}]} = \frac{\frac{2}{6}}{\frac{3}{6}} = \frac{2}{3}$$

$$\mathbb{P}[A \geq C] = \mathbb{P}[\{F_1, F_2, F_3\}] = \frac{3}{6} = \frac{1}{2}$$

$A \geq B$ $\xrightarrow{\text{e.g. A isn't last}}$ A — already know sth about A — $\boxed{A \geq C}$

$\neq$ <u>w/o knowledge about A</u>

$$A \perp A \iff \mathbb{P}[A \cap A] = \mathbb{P}[A]\,\mathbb{P}[A]$$

$$\overbrace{\mathbb{P}[A]}^{x} = \overbrace{\mathbb{P}[A]}^{x}\,\overbrace{\mathbb{P}[A]}^{x}$$

$$x(x-1) = 0$$

$$\boxed{x = x^2} \Rightarrow \boxed{x = 1},\ x = 0$$

holds only in discrete space

$$\begin{array}{lcl} \mathbb{P}[A] = 0 & \cancel{\Longrightarrow} & A = \emptyset \\ \mathbb{P}[A] = 1 & & A = \Omega \end{array}$$

$$\Longleftarrow$$

$$\begin{tikzcd} & \theta \arrow[dl] \arrow[dr] \arrow[dr, bend left] & \\ X \sim \mathrm{Bin}(m, \theta) \arrow[ur, bend left] & & Y \sim \mathrm{Bin}(n, \theta^2) \end{tikzcd}$$
