Probability space

$(\Omega, \mathcal{F}, \mathbb{P})$ $\qquad \mathbb{P} : \mathcal{F} \longrightarrow [0,1]$

σ-algebra = { "measurable" subsets of Ω }

$\Omega = \{1, 2, 3, 4, 5, 6\}$

$\cup$

$\mathcal{F} = 2^{\Omega}$ $\qquad \{1, 2, 5\}$ $\qquad \sqcup$ - disjoint

$$\mathbb{P}[A \sqcup B] = \mathbb{P}[A] + \mathbb{P}[B] \qquad \begin{cases} A \cap B = \emptyset \\ C = A \cup B \end{cases} \iff C = A \sqcup B$$

$$\downarrow$$

$$\mathbb{P}\left[\bigsqcup_{i=1}^{\infty} A_i\right] = \sum_{i=1}^{\infty} \mathbb{P}[A_i]$$

$$A_1 \sqcup A_2 \sqcup A_3 \ldots$$

Counting

n objects

sample of size k

	with rep	w/o rep.
order	n^k	$\frac{n!}{(n-k)!}$
no order	$\binom{n+k-1}{k}$	$\binom{n}{k}$

$\binom{n}{k}$ " n choose k " $\qquad \binom{n}{k} = \frac{n!}{k!(n-k)!}$

C_n^k " це из n по k "

$$\{1, \ldots, n\} \xrightarrow{\binom{n}{k}} \{a_1, \ldots, a_k\} \xrightarrow{\text{add order}} k! \binom{n}{k}$$

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$k=2$

$s=4$

$t=8$

n = # types of pizza

no order + with rep.

$$\binom{n+k-1}{k} = \frac{(n+k-1)!}{k!\,(n-1)!}$$

mult. rule

$$n = \underbrace{s}_{\text{choose size}} \cdot \underbrace{2^t}_{\text{choose topping}}$$

$$\begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \hline 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{array}$$

count binary strings

$\{0, 1\}$

$N=2$

$K=8$

N^K

$$\underbrace{2 \cdot 2 \cdot \ldots \cdot 2}_{K}$$

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$k=5$

$n=10$

a) no rep. ord. $\rightarrow \frac{n!}{(n-k)!}$

$$\binom{n}{k} = \binom{n}{n-k}$$

$\cup$

$\cap$

$\{1, \ldots, n\} \rightarrow \{a_1, \ldots, a_k\}$

$\Updownarrow$

$\{1, \ldots, n\} \setminus \{a_1, \ldots, a_k\}$

$\cup$

$\cap$

$\bar{\ }$

$\emptyset$

Ω

^{c}

$\mathbb{P}[A \cap B] =$ " prob of A and B hold simultaneously"

AND

$\mathbb{P}[A \cup B] =$ " prob of at least A or B holding"

OR

$\mathbb{P}[A^c] =$ " prob that A doesn't hold"

Conditional prob

$\mathbb{P}[A \mid B] =$ " prob that A holds conditioned on B (i.e. B hold already)"

fair coin

$B = \{ 100 \text{ heads in a row} \}$

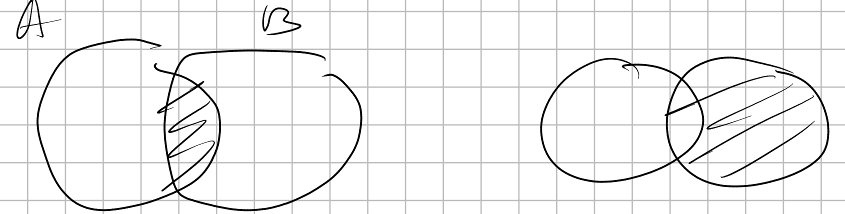
$A = \{ \text{tail} \}$

$\mathbb{P}[A \mid B] = \frac{1}{2}$

$\mathbb{P}[A] = \frac{1}{2}$

$$\boxed{\mathbb{P}[A \mid B] \overset{\text{def}}{:=} \frac{\mathbb{P}[A \cap B]}{\mathbb{P}[B]}}$$

$\mathbb{P}[A \cap B] \leq \mathbb{P}[B]$



Independent events

A and B indep. $\iff$ $\mathbb{P}[A \cap B] = \mathbb{P}[A] \cdot \mathbb{P}[B]$

Cor A and B indep. $\Rightarrow$ mult. rule works

Cor. Let A, B be indep. events

$$\boxed{\mathbb{P}[A | B] = \frac{\mathbb{P}[A \cap B]}{\mathbb{P}[B]}} = \mathbb{P}[A]$$

Product rule.

$$\mathbb{P}[A \cap B] = \mathbb{P}[A | B]\, \mathbb{P}[B]$$

$$\shortparallel$$

$$\mathbb{P}[B \cap A] = \mathbb{P}[B | A]\, \mathbb{P}[A]$$

Bayes formula

$$\boxed{\overbrace{\mathbb{P}[\theta | X]}^{\text{posterior}} = \frac{\mathbb{P}[\theta \cap X]}{\mathbb{P}[X]} = \frac{\mathbb{P}[X | \theta]\, \mathbb{P}[\theta]}{\mathbb{P}[X]}}$$

evidence

θ: learning param. (theta)

X: data

we know how to compute

$\boxed{\mathbb{P}[\theta] \text{ - prior}, \ \mathbb{P}[X | \theta] \text{ - likelihood}}$