Taguchi's Approach to Experimental Design

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Dr Genechi Taguchi (1924-2012)

Taguchi

- Born and raised at textile town, Tokamachi, Japan
- Studied textile engineering, later moved to Astronomical department of Navigational institute of Japanese Navy during world war II
- After world war II, moved to Ministry of Public Health and Welfare, where he was influenced by the great statistician Matosaburo Masuyama, a researcher in design of experiments. Also worked at Institute of Statistical Mathematics
- Visited Indian Statistical Institute, where he collaborated with C.R.Rao, R.A.Fisher, and W.A Shewhart
- Worked closely with Japanese industries and later with US industries, especially at Bell Labs and Ford Motor Company

Taguchi's Main Contributions

- New definition to quality
- Construction of linear graphs for fractional factorial experiments
- Introduce the concepts of Parametric Design and Tolerance Design, by exploring nonlinearity
- Off-line quality control concepts
- Developed signal-to-noise ratios as performance measure, depending on the response
- Mahalanobis- Taguchi Strategy

Taguchi's Definition of Quality

Loss to Society after the products shipped

This is a modern approach to quality

ORTHOGONAL ARRAYS

The concept of Orthogonal Arrays was first introduced by C. R. Rao (1947) .A N x K matrix A with entries from a set of S (>=2) elements is called an orthogonal array (OA) of size N, K constraints, S levels, strength d and index r if any N X d sub-matrix of A contains all possible N X d row vectors with the same frequency . Such an array is denoted symbolically by (N, K, S, d). N is also sometimes called the number of assemblies.

Linear Graphs

- G, Taguchi (I 959) introduced the concept of linear graph associated with orthogonal arrays and also developed various techniques for adapting orthogonal arrays to meet the requirements of different practical situations. The use of linear graph enables a scientist or an engineer to design and analyze complicated experiments without requiring a basic knowledge of the construction of designs using Galois fields.

Taguchi Designs
Notation

Number of Factors

$$L_N\left(2^k\right)$$

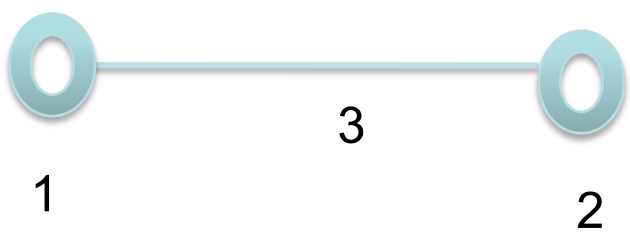
Total Number of Runs

Number of Levels per Factor

$L_4(2^3)$

$L_4(2^3)$

No.	1	2	3
1	1	1	1
2	1	2	2
3	2	1	2
4	2	2	1
	a	*b*	*a*
			b
	1	2	

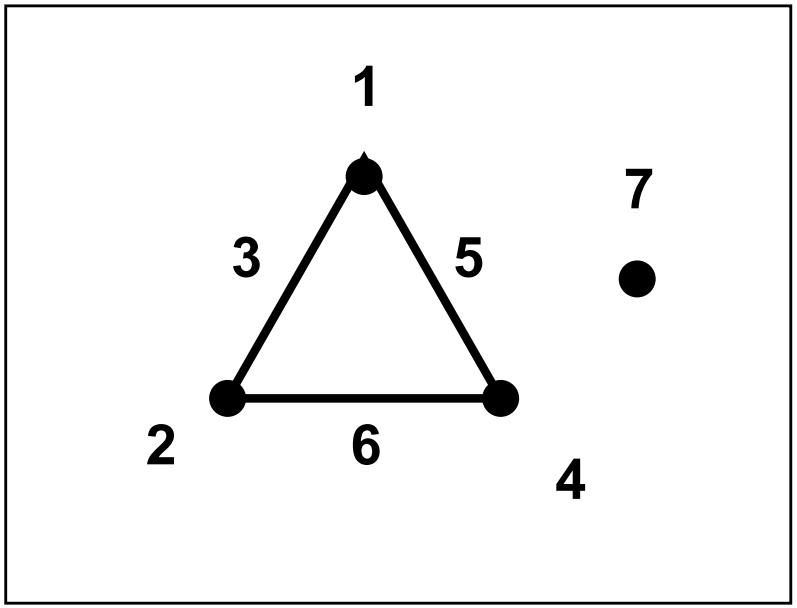


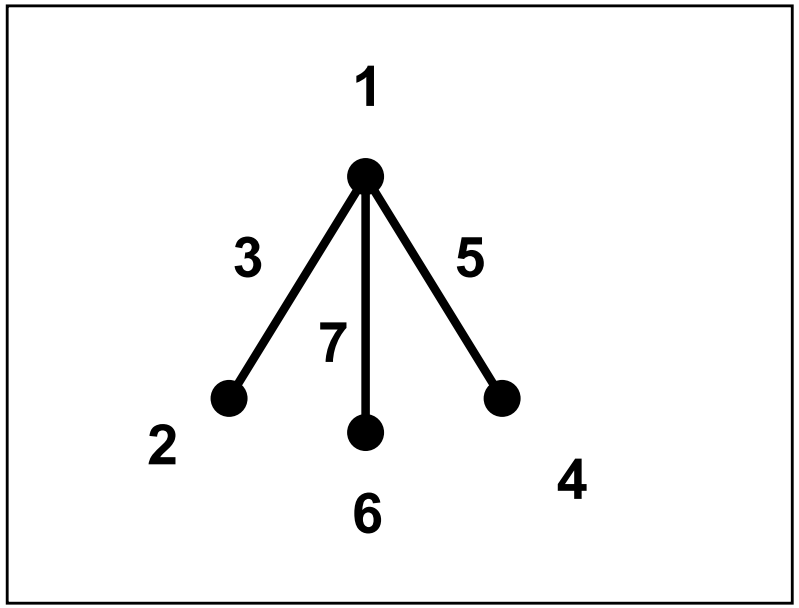
$L_8(2^7)$

$L_8(2^7)$

No.	1	2	3	4	5	6	7
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2
	a	*b*	*a*	*c*	*a*	*b*	*a*
			b		*c*	*c*	*b*
							c
	1	2			3		

Linear Graphs for L_8 Array





- **Main effects are assigned to columns at nodes in the plot.**
- **Interactions are assigned to the columns on the lines.**

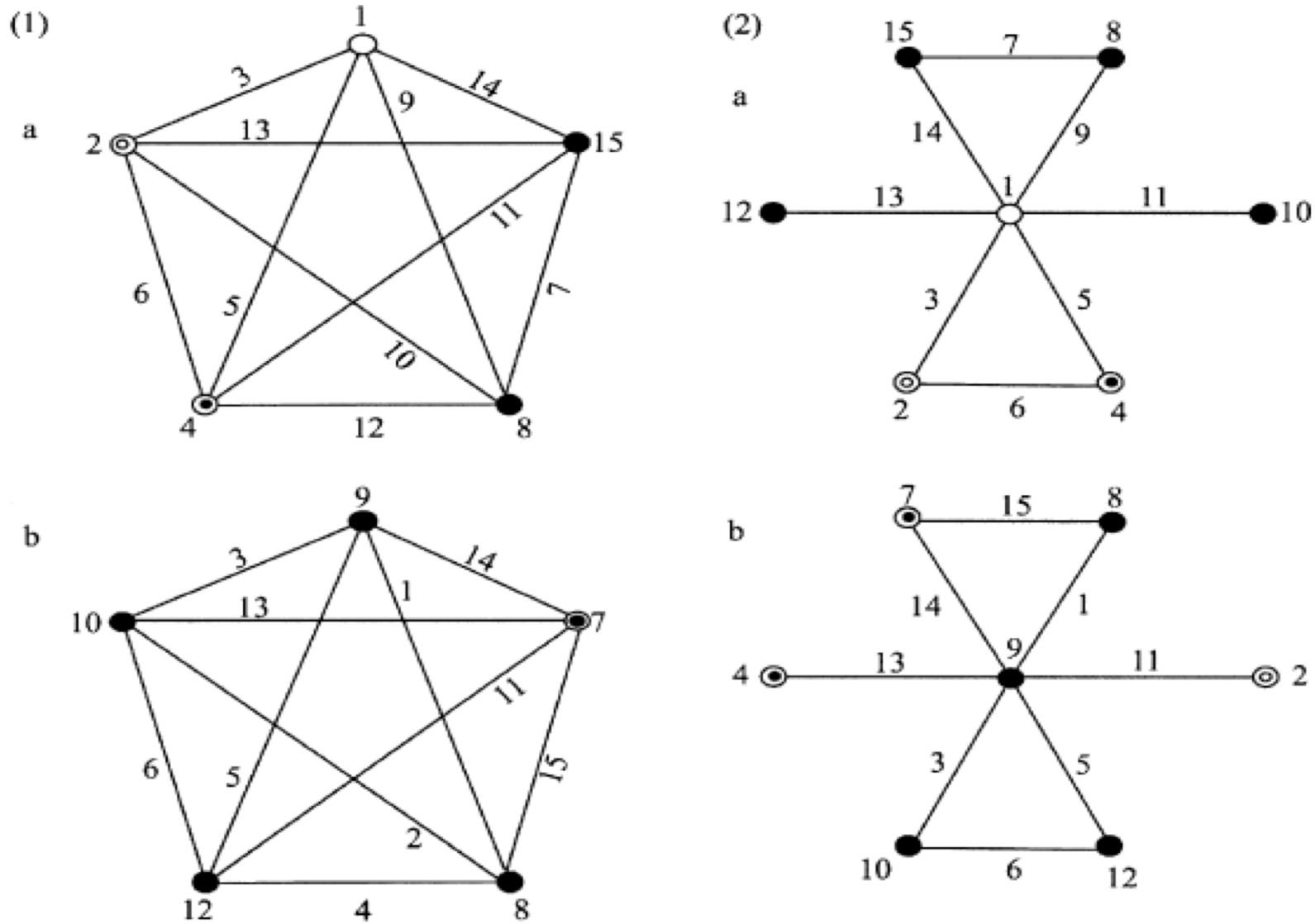
$L_{16}(2^{15})$

No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2
3	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2
4	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1
5	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2
6	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1
7	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1
8	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2
9	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2
10	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1
11	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1
12	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2
13	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1
14	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2
15	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2
16	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1

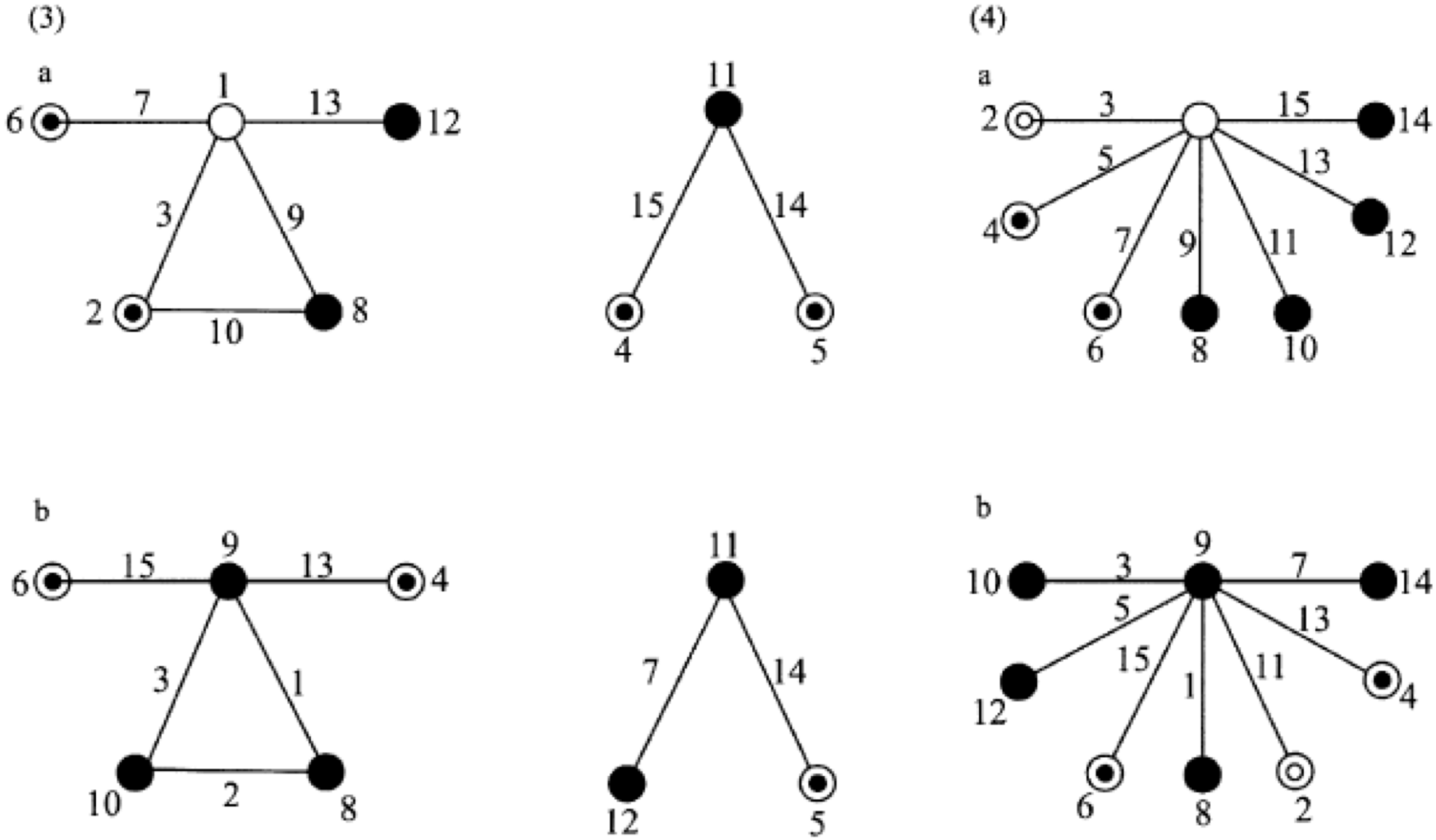
L_{16} (2^{15}) Interactions between Two Columns

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	(1)	3	2	5	4	7	6	9	8	11	10	13	12	15	14
		(2)	1	6	7	4	5	10	11	8	9	14	15	12	13
			(3)	7	6	5	4	11	10	9	8	15	14	13	12
				(4)	1	2	3	12	13	14	15	8	9	10	11
					(5)	3	2	13	12	15	14	9	8	11	10
						(6)	1	14	15	12	13	10	11	8	9
							(7)	15	14	13	12	11	10	9	8
								(8)	1	2	3	4	5	6	7
									(9)	3	2	5	4	7	6
										(10)	1	6	7	4	5
											(11)	7	6	5	4
												(12)	1	2	3
													(13)	3	2
														(14)	1

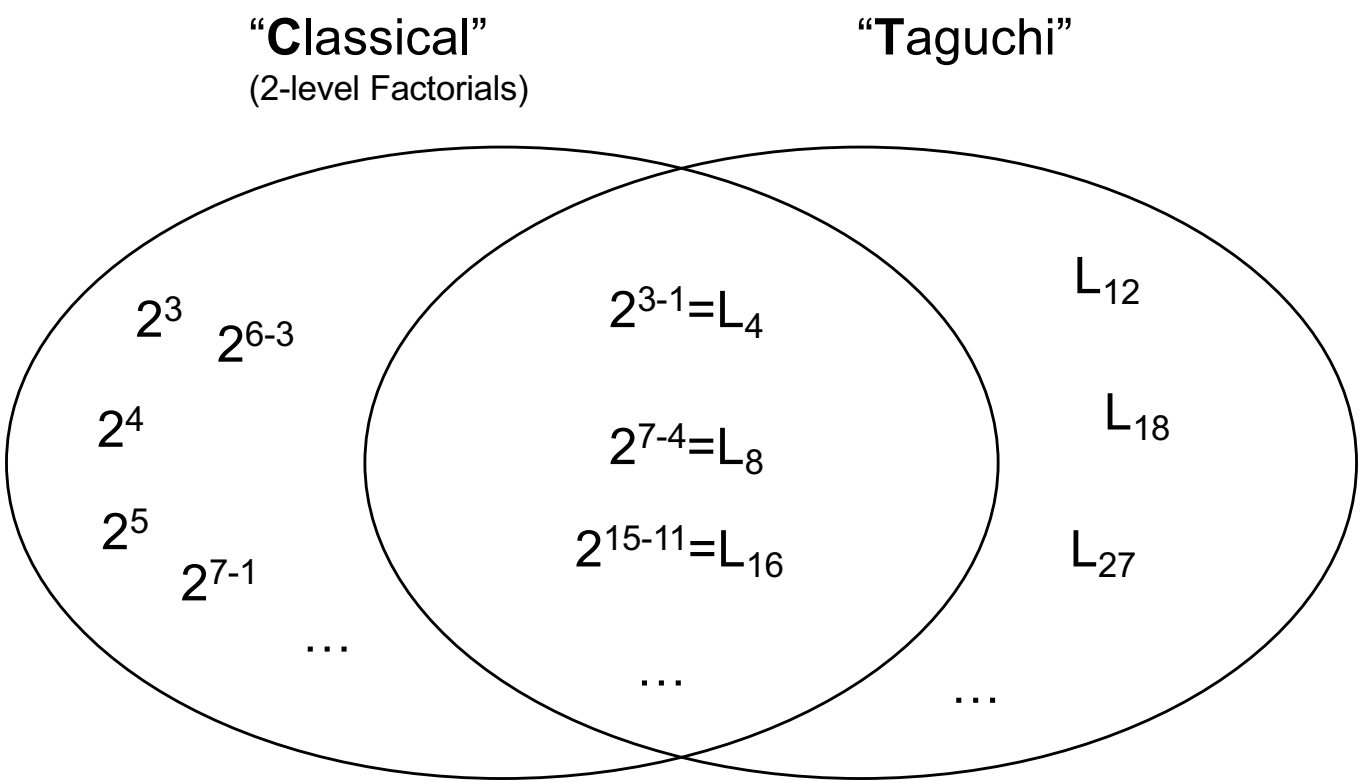
$L_{16}(2^{15})$


(1)
a
1
2
15
4
8
3
14
9
13
11
6
5
7
10
12
(2)
a
15
7
8
14
9
1
12
13
11
10
3
5
2
6
4
b
9
10
7
12
8
3
14
1
13
11
6
5
15
2
4
b
7
15
8
14
1
9
4
13
11
2
3
5
10
6
12

$L_{16}(2^{15})$


(3)
a
6
7
1
13
12
3
9
2
10
8
11
15
14
4
5
(4)
a
2
3
15
14
5
13
4
7
9
11
12
6
8
10
b
6
15
9
13
4
3
1
10
2
8
11
7
14
12
5
b
10
3
9
7
14
5
13
15
11
12
1
4
6
8
2

Orthogonal Designs



Construction of Designs

- Identify the factors and levels
- Identify the interactions suspected to be present

  (It is assumed that few two factors may be active and all higher order interactions are absent)
- Construct the required linear graphs
- Superimpose the required graph on the available linear graphs
- Assign the corresponding numbers to factors
- Select the columns corresponding to the factors from the design table as your design

Example 2 :

In an experiment, there are two kinds of raw materials A_1 and A_2, two annealing method B_1 and B_2, two temperatures C_1 and C_2, and two annealing times D_1 and D_2. It is required to get information on the main effects A, B, C, D interactions BxC and BxD. These requirements are shown by linear graph in figure 2.

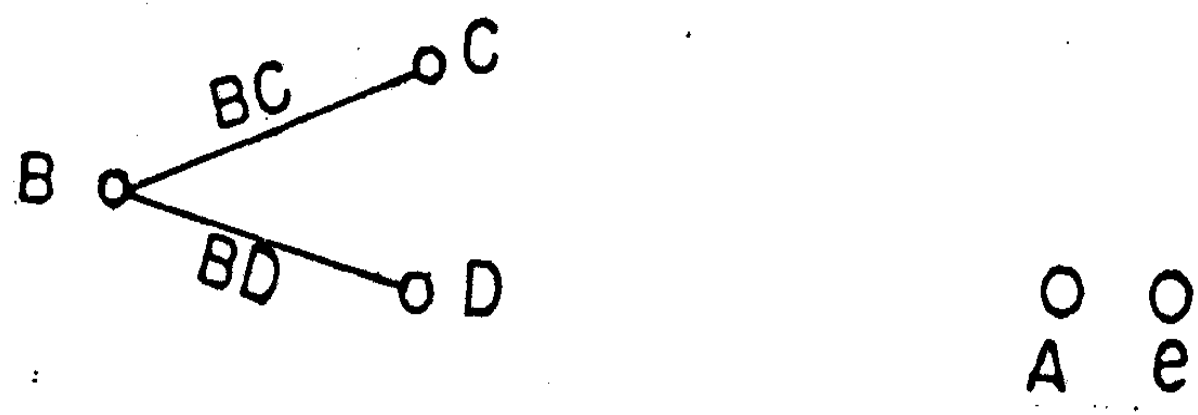


Figure-2: Linear graph for required information

Such layout is easily arranged using linear graph (2) of figure 1. That is use two lines out of the three radial lines in linear graph (2). The remaining edge is deleted from the graph and the associated numbers are now indicated as independent points as shown in figure-3.

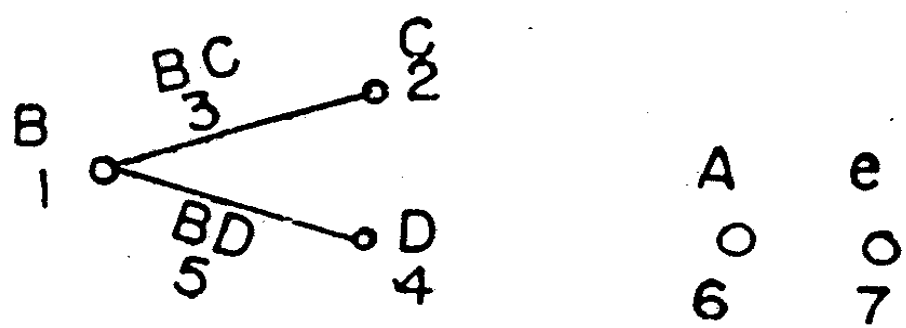


Figure-3: Layout of the experiment

An independent point is used for the assignment of a main effect; factor A is assigned to any of the remaining two points, since only the main effect is required for A. The layout shown as table 1 (Table II – OA Table)

Table 1 : Layout of the Design

Experiment	B 1	C 2	BxC 3	D 4	BxD 5	A 6	e 7
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2

Such layout can also be arranged from linear graph (I).
Using linear graph (I) remove column 6 which is the line connected by column 2 and 4, indicate it is an independent point like column 7, then assign factor A to 6.

Classification of Factors

- Control Factors–*Design factors* that are to be set at optimal levels to improve quality and reduce sensitivity to noise
  - Dimensions of parts, type of material, etc
- Noise Factors–Factors that represent the noise that is expected in production or in use
  - Dimensional variation
  - Operating Temperature

- Adjustment Factor – Affects the mean but not the variance of a response
  - Deposition time in silicon wafer fabrication
- Signal Factors – Set by the user to communicate desires of the user
  - Position of the gas pedal

Noise Factors – Causes of Variation

- External – Environment in which the product works
  Load to which product are subjected

- Unit-by-unit variation – The variation that is inevitable in a manufacturing process leads to variation in the product parameters from unit to unit.

- Deterioration – When a product is sold, all its functional characteristics may be on target. As time passes, the values of individual components may change leading to deterioration in product performance

Example – Noise Factors for Refrigerator

- External – The number of times the door is opened or closed, amount of food kept and the initial temperature of the food, variation in the ambient temperature, and the supply voltage variation
- Unit-to-unit variation – the tightness of the door closure and the amount of refrigerant used
- Deterioration - the leakage of refrigerant and mechanical wear of compressor parts.

Exploring the Nonlinearity

- Product quality characteristic is related to the various product parameters and noise factors through complicated, nonlinear functions.
- It is possible to identify the combination of product parameters to give the desired value of the product characteristics, under at exact values of noise variables.
- Due to nonlinearity, different parametric combinations may give quite different variation in QC
- Principal goal of robust design is to exploit the nonlinearity to find a combination of product parameter values that give smallest variation in QC

Exploring the Nonlinearity

- Let y- response, z – noise factors, x – control factors such that

$$y = f(x,z)$$

- The variation in z will transmitted to y, which suggests tightening the variation in x to reduce the variation in y.

- Alternatively, if x and z interact in their joint effect on y, then variation in y can also reduced by the setting of x since different setting of x may have different effects on the relationship between y and z

Exploring Nonlinearity

- For example, let the response model is

$$y = \alpha + \beta x_1 + \gamma z + \delta x_2 z + \varepsilon$$

$$= \alpha + \beta x_1 + (\gamma + \delta x_2) z + \varepsilon$$

Where x's are control factors and z is noise factor

We assume that the model has significant x_1 main effect, and significant interaction between x_2 and z.

In this case, one can choose the value of x_2 to make the coefficient $(\gamma + \delta x_2)$ of z as small as possible to dampen the effect of z on y

* In the data analysis, we can explore this using control – by-noise interaction plots.

Exploring Nonlinearity

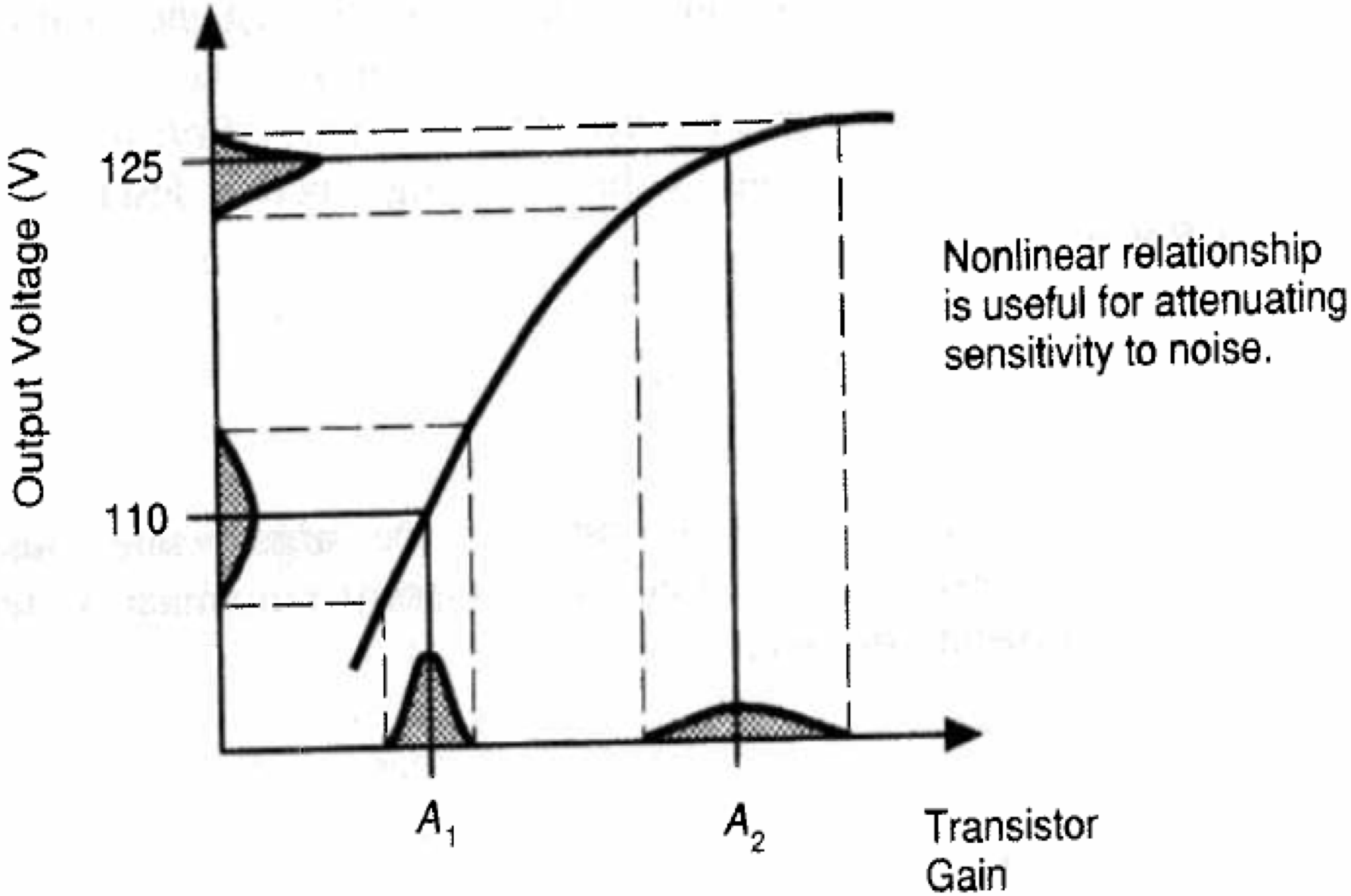
- Robustness can also achieved by exploiting the nonlinearity in the relationship if the control factors are the nominal values of the components of the products and noise factors are the tolerances
- Let x denote the component values and the relationship is represented as, y=f(x). Assuming that each component of x has a distribution with mean μ_i and variance σ_ι^2, then variance of y can be approximated as

  $\sigma^2 = \Sigma_\iota (df/dx_i \mid \mu_i)^2 \sigma_\iota^2$

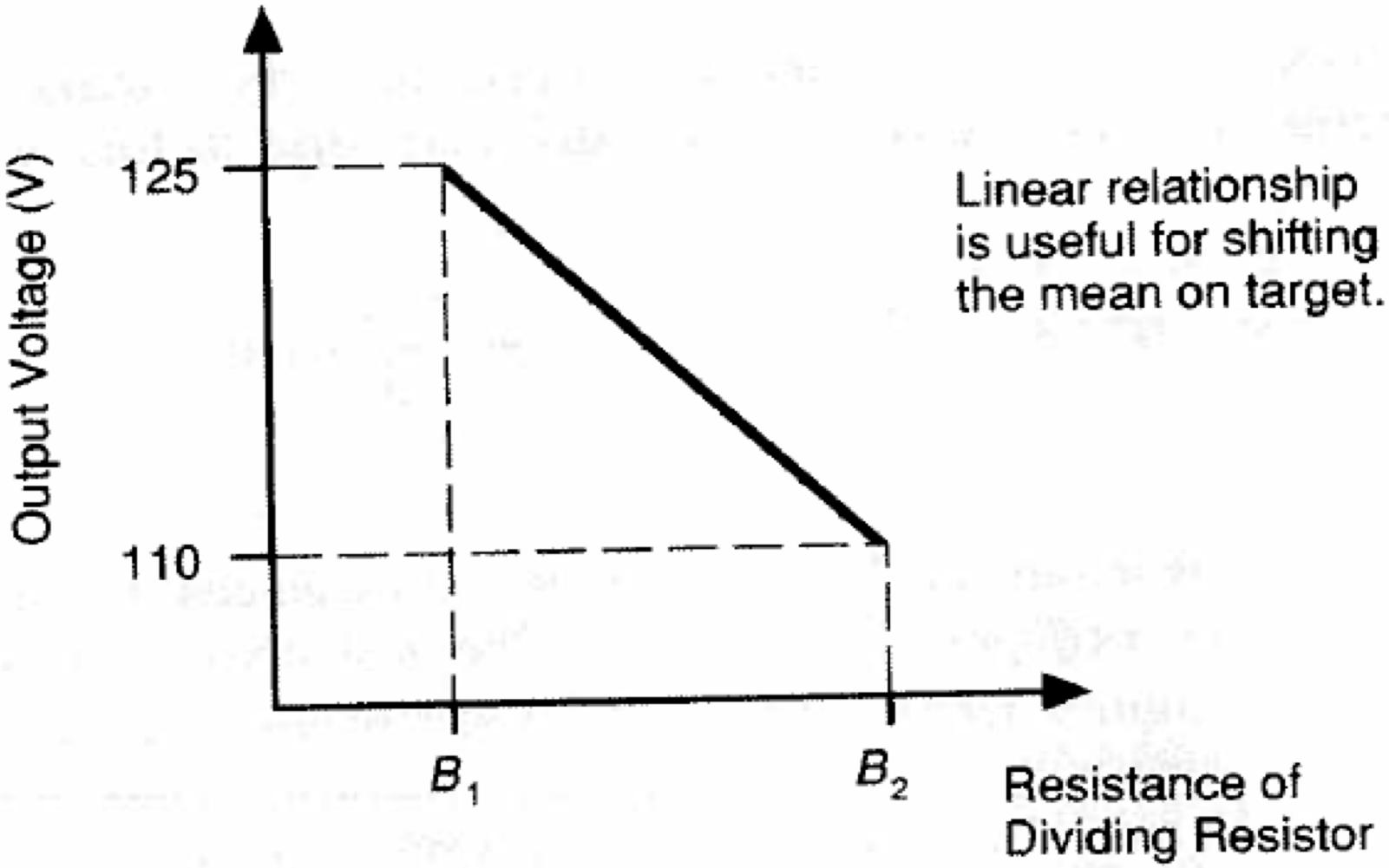
if f is nonlinear, then the nominal value with a flatter slope is preferred (small df/dx – sensitivity coefficients)

To distinguish between these two strategies, the first one is referred as Parameter Design and the later is called Tolerance design

Exploring Nonlinearity



Adjusting the Response to Target



Exploiting Interaction between Noise and Control Factors

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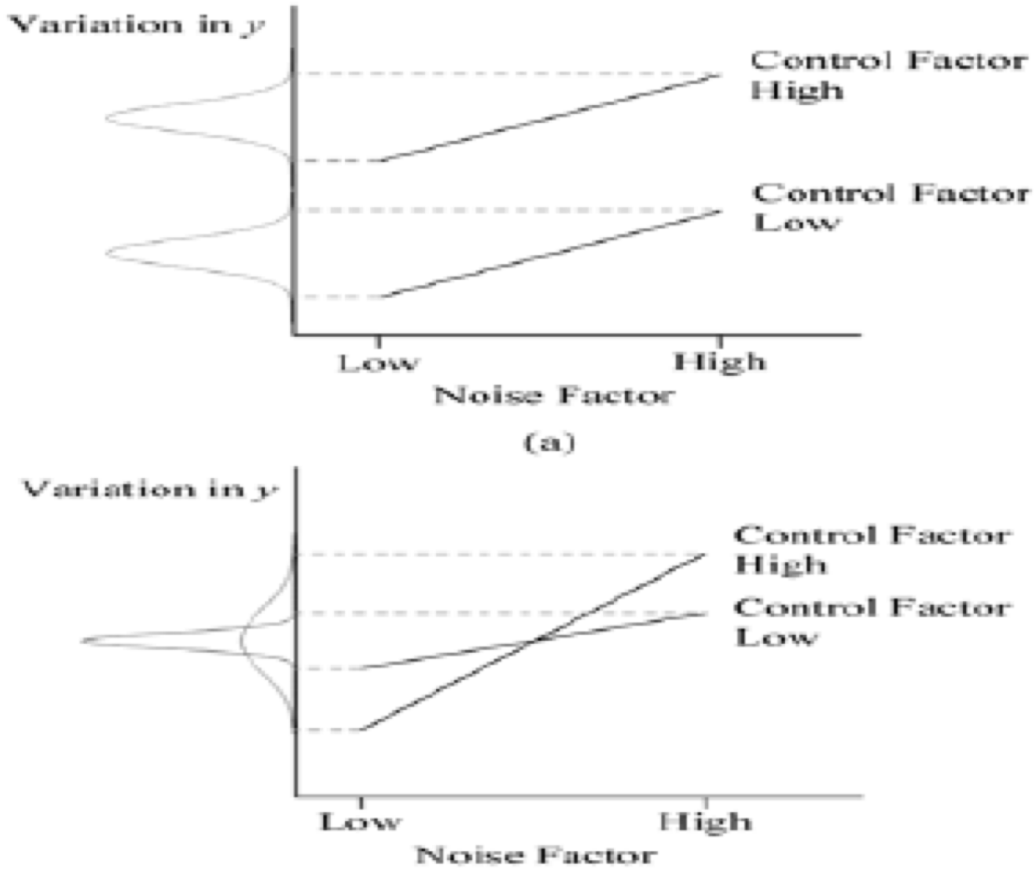

Signal to Noise Ratios

- Taguchi explored the quality loss as a measure to identify the optimum factor levels.
- Arrived the quality loss after the adjustment of the process mean to the target, which is referred as signal to noise ratios
- For example, the quality loss for the nominal the better is defined as

$$Q = k\left((\mu - T)^2 + \sigma^2\right)$$

After adjusting mean by T/μ, the modified quality loss is

$$Q = k\, T^2 \left(\sigma^2/\mu^2\right)$$

$$\text{SN Ratio} = 10 \log 10 \left(\sigma^2/\mu^2\right)$$

We use the SN ratio as response and maximize it to select the factor levels.

Taguchi's Approach

- Factors that have a significant effect on SN ratio – Identify the best factor levels that maximize SN Ratio
- Factors that have a significant effect on Mean (μ) but practically no effect on SN Ratio.- Use any one of these to adjust the mean to target
- Factors that have no effect on SN ratio, but effect on Mean (μ), - neutral factors, choose the levels from other consideration such as operation or cost.

SN Ratios

- $SN_s = -10 \log(\Sigma y^2/n)$
  - What are the optimal values for y_i?
  - Used when "smaller is better"

- $SN_L = -10 \log(\Sigma(1/y^2)/n)$
  - What are the optimal values for y_i?
  - Used when "larger is better"

Note that we wish to maximize the SN ratios

Taguchi's Approach

- From every trial series we can obtain an average result level and a measure of the variation, si, i=1,2, … ,9. These values can then be used as a basis for choosing the combination of factor levels that provides the most robust design.

Experiment	Factor 1	2	3	4
1	-1	-1	-1	-1
2	-1	0	0	0
•	•	•	•	•
•	•	•	•	•
•	•	•	•	•
9	1	1	0	-1

Levels of disturbing factors

Individual results

-1	-1	-1	Y_{11}	$(\overline{Y}_1, s_1)$
-1	1	1	Y_{12}	
1	-1	1	Y_{13}	
1	1	-1	Y_{14}	

-1	-1	-1	Y_{21}	$(\overline{Y}_2, s_2)$
-1	1	1	Y_{22}	
1	-1	1	Y_{23}	
1	1	-1	Y_{24}	

Inner and Outer Array

				Outer Array			
			N1	1	2	1	2
Inner Array			N2	1	1	2	2
A	B	C					
1	1	1		y11	y12	y13	y14
2	1	1					
1	2	1					
2	2	1					
1	1	2					
2	1	2					
1	2	2					
2	2	2					

Taguchi's Experimental Plan

1. Problem Identification
2. Brainstorming Session
   (Identify: factors, factor settings, possible interactions, objectives)
3. Experimental Design
   (Choose orthogonal arrays, design experiment)

Initiating Steps

4. Run Experiment
5. Analyze Results
6. Confirmation Runs

Analysis using SN Ratio

- Even though, we have replications, use of SN ratio as performance measure reduced experiment to un-replicated experiments.
- Usual ANOVA method cannot be employed
- Significant of factor effects can be arrived using graphical tools (normal or half normal prob. plots), % contribution or ,Lenth's method.
- Can use the Jacknife method to estimate the error sum of squares and use it to test the significance (Variyath, Abraham and Chen (2005)).

Another Approach

- There are some criticism on the Taguchi's approaches.
- Wu and Hamada (2002) suggested the following:

For the nominal the best characteristics:

i) Select the levels of dispersion factors to minimize the dispersion – analyze log (s^2) first

ii) Select the levels of the adjustment factor to bring the location on target

For larger the better (or small the better) type characteristics:

i) Select the levels of location factors to maximize (minimize) the location – analyze mean first

ii) Select the levels of dispersion factors that are not location factors to minimize the dispersion