LP is phrased as following:

$$x_{i,j} = \begin{cases} 1 & \text{path goes from } i \text{ to } j \text{ at some point} \\ 0 & \text{therwise} \end{cases}$$

$\#1 \wedge \#2 \Rightarrow |\delta(v)| = 2 \ \forall v$

$$\min \sum_{1 \le i < j \le n} c(i,j) x_{i,j} \quad \text{s.t}$$

Constraint #1 $\sum_{\substack{i=1 \\ i \neq j}}^{n} x_{i,j} \quad \forall\, 1 \le j \le n$

Constraint #2 $\sum_{j=1, j \neq i}^{n} x_{i,j} \quad \forall\, 1 \le i \le n$

Constraint #3 $u_i - u_j + n x_{i,j} \le n-1 \quad \forall\, 2 \le i \neq j \le n$

Constraint #4 $0 \le u_i \le n-1$

Dummy Variable

$u_i \ \forall\, 2 \le i \le n$

$u_i \in \mathbb{Z}$

$u_i = t$
means vertex i has been
visited at step i of the algorithm

v_1 : the vertex that starts and
end the tour with.

proof (or at least I tried)

prove that a valid solution, a tour satisfies the constraint.

Consider the solution G, which is a cycle, and any path $P \subseteq G$ and P contains v_1.

Consider: $\forall e=(i,j) \in P : u_i - u_j + n x_{i,j} \leq n-1$

* Then suming up all of them, assuming p has k edges

$$\sum_{i \neq 1} (e=(i,j) \in P : u_i - u_j) + n\left(\sum_{e \in p} x_e\right) \leq k(n-1)$$

then it's equal to:

$$u_2 - u_k + n(k) \leq k(n-1)$$

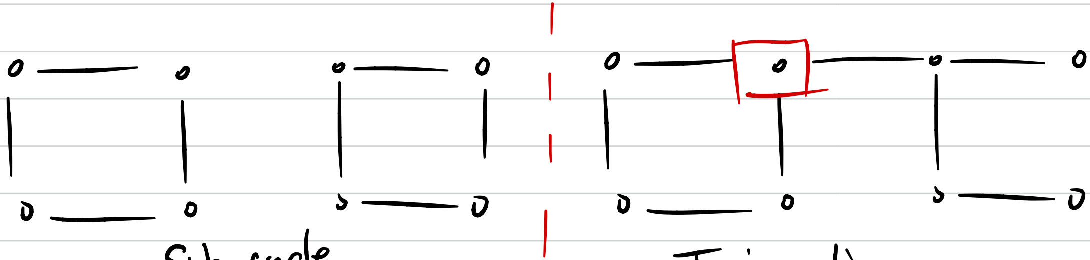
which is true for all k,

inductively, $u_i - u_{i+1} = -1$, hence the solution for dummy variable will be: $u_2 - u_k = (k-1)$

then $k(n-1) \leq k(n-1)$ $\square$

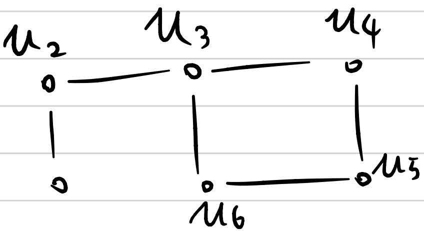
If a solution is not valid (Not a tour) then the constraint is not satisfied.

Assume constraint #1, #2 are satisfied, here is some non-trivial examples that are not a tour and satisfies the constraints:



Sub cycle

T junction



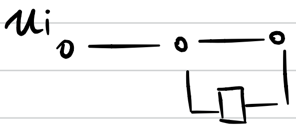
$u_3 - u_2 = 1$

but edges involve is 5

then $-1 + 6*5 \leq (5)*5$

$29 \leq 25$

Constraints #3 prevents Tea spoon configuration (Results will always be a path)



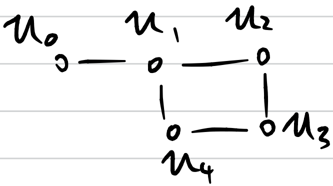
this config is prevented.

Say path length is k

$P_i, P_{i+1} \cdots P_{i+k}$

but one of the vertex repeated.

let's see a non-trivial example, which can be generalized easily.



$(u_0-u_1)+(u_1-u_2)+(u_2-u_3)+(u_3-u_4)+(u_4-u_1)$

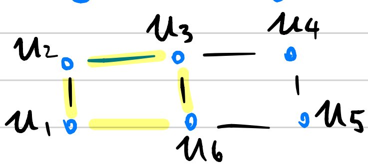
$\Rightarrow u_0-u_1=-1$

but then $k(X_{i,j})$ will be $5*5$

and $k(n-1)$ is $5*(5-1)=20$

$24 \leq 20$ is FALSE

Double cycle configuration

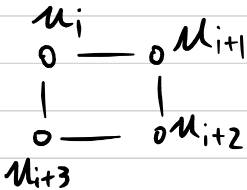


$u_2-u_2=0$

oh No...

kinda hard.

if any sub cycle exists, say this one:



$u_i - u_i = 0$ cycle sum

$n \sum_{e \in P} x_e = 4 * 4 = 16$

$n(n-1) = 4 * 3 = 12$

$16 \leq 12$ is false