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Brief Paper

A common framework for anti-windup, bumpless transfer and reliable designs☆

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Abstract

In this paper, the $\mathscr{L}_2$ anti-windup approach introduced in Teel and Kapoor (Proceedings of the Fourth ECC, Brussels, Belgium, July 1997) is generalized and applied to the problem of bumpless transfer in (saturated) multi-controller systems, and to the problem of designing highly reliable saturated control systems exploiting hardware redundancy. We first illustrate the $\mathscr{L}_2$ anti-windup technique and apply it to a simple physical example and then demonstrate the performance of its extension to the above-mentioned problems using this example and an example taken from the literature. © 2002 Elsevier Science Ltd. All rights reserved.

Keywords: Windup; Saturation control; Multi-position controllers; System reliability; Control nonlinearities; Nonlinear control

1. Introduction

For linear plants with actuator saturation, if the constraints on the input are not accounted for in the control design, the results can be disastrous. Historically, the symptoms exhibited by a linear control design on a saturated plant have been called "controller windup" and early papers (see, e.g., Lozier, 1956) address the problem and give ad hoc solutions to avoid the associated undesired effects. Until recent years, numerous qualitative solutions to particular instances of the windup effect have been given in literature (for surveys of these approaches, see, e.g., Hanus, 1988; Kothare, Campo, Morari, & Nett, 1994). In the late 1980s, the inadequacy of such qualitative solutions has been pointed out (Doyle, Smith, & Enns, 1987) by means of simple examples associated with bad performance of the anti-windup solutions. More recently, a number of anti-windup schemes have been proposed, providing more rigorous stability and performance achievements (see, e.g., Gilbert, Kolmanovsky, & Tan, 1995; Miyamoto & Vinnicombe, 1996; Edwards & Postlethwaite, 1999; Shamma, 2000; Mulder, Kothare, & Morari, 2001; Grimm, Postlethwaite, Teel, Turner, & Zaccarian, 2001; Zaccarian & Teel, 2001). Without going into detail about these approaches, we focus on the $\mathscr{L}_2$ anti-windup definition (introduced in Teel & Kapoor, 1997), which is at the basis of the results given in this paper. In Teel and Kapoor (1997), the $\mathscr{L}_2$ anti-windup problem is formally defined in a very general nonlinear context and a solution is given for solving the problem globally for linear plants with no exponentially unstable modes. In Teel (1999) and, more recently in Barbu, Reginatto, Teel, and Zaccarian (2000), the $\mathscr{L}_2$ anti-windup solution is extended in a nonlocal way to exponentially unstable linear plants, explicitly addressing the problem of boundedness of the null controllability region for this class of systems. The effectiveness of the $\mathscr{L}_2$ anti-windup solution has been demonstrated in a number of case studies (see Barbu, Reginatto, Teel, & Zaccarian, 1999 and references therein) and in a patent pending industrial application (Zaccarian, Teel, & Marcinkowski, 2000).

In the literature, the anti-windup problem has been often addressed in conjunction with the so-called "bumpless transfer" problem (see Graebe & Ahlén, 1996 for a comparison of the two problems). Although the bumpless

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transfer problem has never been formally defined in the literature, already in early papers (such as Hanus, 1980; Hanus, Kinnaert, & Henrotte, 1987), the problem has been qualitatively characterized by the undesired large transients experienced at power-on or after switching among controllers in multi-controller schemes (especially in aggressive control designs). In the last decade, a number of bumpless transfer schemes have been proposed to systematically shape the switching transients (see, e.g., Zheng, Kothare, & Morari, 1994; Peng, Vrancic, & Hanus, 1996; Turner & Walker, 2000). More recently, bumpless transfer has also been addressed in the context of hybrid and switching systems (see Liberzon & Morse, 1999 for an accurate overview of the problem). Asymptotic stability of a switching linear system has been addressed from an analysis viewpoint (see Branicky, 1998; Liberzon, Hespanha, & Morse, 1999 where sufficient conditions for the asymptotic stability of the switching system are given) and from a more constructive viewpoint (Morse, 1996; Hespanha & Morse, 2000). The bumpless transfer scheme proposed in this paper employs the structure of the $\mathscr{L}_2$ anti-windup solution to be able to assign (by means of extra control action) a prescribed transient behavior to the system immediately after the switching, so as to guarantee the desired authority transfer (abrupt and fast, or gradual and slow) between the controllers. In addition, the proposed scheme guarantees asymptotic stability of the overall switching system for arbitrary switching [1] and in the presence of saturation at the plant's input.

A second problem addressed in this paper is the design of reliable control schemes via hardware redundancy. The need for reliable control schemes was pointed out already in the late 1960s (Kenny & Koppel, 1968); one of the first systematic design approaches, from the early 1980s (Šiljak, 1980), is based on multiple control systems in which a nonminimal controller realization stabilizes the plant even when it is only partially functioning. More recently, a reliable design based on factorization techniques is proposed in Vidyasagar and Viswanadham (1985) and analysis tools for testing the reliability of a control system are given in Birdwell, Castanon, and Athans (1986). Several other approaches are outlined and discussed from a robustness point of view in Braatz, Morari, and Skogestad (1994). Additional recent reliable control schemes can be found, e.g., in Zhao and Jiang (1998) and references therein. A number of reliable schemes are based on fault detection and subsequent control reconfiguration. Relative to this approach, numerous results on fault detection and classification are available in the literature (see, e.g., Willsky, 1976 for an early survey, Patton, Frank, & Clark, 1989 for a collection of design approaches and Chowdhury & Aravena, 1998 and references therein for more recent research results).

In this paper, we first recall and slightly extend the $\mathscr{L}_2$ anti-windup design formalized in Teel and Kapoor (1997) and Teel (1999) in Section 2. In Section 3 we propose extensions of the algorithm for the solution of the bumpless transfer problem and in Section 4 we also employ the proposed structure in a redundant control scheme for reliability improvement. Examples are given in the various contexts to illustrate the effectiveness of the proposed schemes.

[1] In particular, it is only assumed that the switching function is Lebesgue measurable.

2. $\mathscr{L}_2$ anti-windup synthesis

In this section, we summarize and extend the $\mathscr{L}_2$ anti-windup construction of Teel and Kapoor (1997). Consider an asymptotically stable linear plant

$$\begin{aligned} \dot{x} &= Ax + B_1 d + B_2 u, \\ y &= Cx + D_1 d + D_2 u, \end{aligned} \tag{1}$$

where u is the control input and d is a disturbance input. Assume that a linear controller

$$\begin{aligned} \dot{x}_c &= A_c x_c + B_c u_c + B_r r \\ y_c &= C_c x_c + D_c u_c + D_r r \end{aligned} \tag{2}$$

has been designed in such a way that, based on the value of the reference signal r, its linear interconnection

$$u_c = y, \qquad u = y_c \tag{3}$$

with plant (1) guarantees internal stability and desirable closed-loop performance.

When saturation is accounted for at the plant's input, the desired response is no longer guaranteed for the nonlinear closed-loop system arising from (1) and (2) with the interconnection $u_c = y$, $u = \mathrm{sat}(y_c)$. As a matter of fact, as shown for instance in the following Example 1, even closed-loop stability can be lost due to the presence of saturation. The so-called *anti-windup* solution is to synthesize modifications to the control scheme (2) and (3), so that:

(1) for initial conditions and reference signals that do not cause actuator saturation, the linear response is retained;
(2) otherwise, the nonlinear response is close (as close as possible) to the linear response.

The anti-windup approach proposed in Teel and Kapoor (1997) is based on the introduction of the following filter, reproducing the plant dynamics (1):

$$\begin{aligned} \dot{\xi} &= A\xi + B_2(\mathrm{sat}(u) - y_c), \\ y_\xi &= C\xi + D_2(\mathrm{sat}(u) - y_c) \end{aligned} \tag{4}$$

with zero initial conditions, where y_c represents the output of controller (15) with the input $u_c = y - y_\xi$. This is consistent with the diagram in Fig. 1 where filter (4) plays the role of the "anti-windup compensator" block, $v_2 = -y_\xi$, the signal v_1 is to be determined, and the following anti-windup interconnections are used

$$u_c = y + v_2 = y - y_\xi, \qquad u = \mathrm{sat}(y_c + v_1). \tag{5}$$

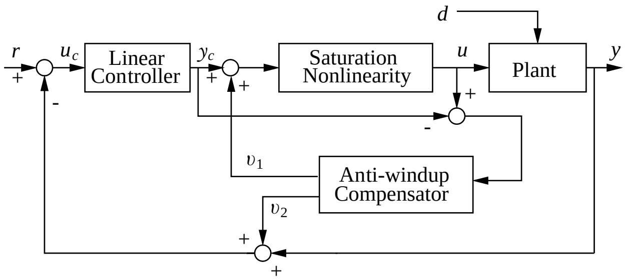

Fig. 1. The $\mathscr{L}_2$ anti-windup scheme.

Regardless of the presence of the disturbance input and of the choice for v_1 in (5), the introduction of the extra dynamics (4) causes (at least when the plant model is exact) the signal $y_c(t)$ to exactly match the control signal from the linear response, henceforth denoted u_ℓ. (Similarly, the controller input $u_c = y - y_\xi$ also matches the linear plant response y_ℓ.) This follows by defining $x_\ell := x - \xi$, $y_\ell := y - y_\xi$ and noting that dynamics (1),(2), (4) and (5) can be rewritten as

$$\begin{aligned} \dot{x}_\ell &= Ax_\ell + B_1 d + B_2 y_c, \\ y_\ell &= Cx_1 + D_1 d + D_2 y_c, \\ \dot{x}_c &= A_c x_c + B_c y_\ell + B_r r, \\ y_c &= C_c x_c + D_c y_\ell + D_r r, \end{aligned} \tag{6}$$

which coincide with the linear closed loop (1)–(3). Moreover, if we choose $\xi(0) = 0$, then $x_\ell(0) = x(0)$. Hence, by uniqueness, the solutions of (1)–(3) are coincident with the solutions of (6). It is also clear from (4) and (5) that, as long as $v_1 = 0$ when $\xi = 0$, if $u_\ell(t) \equiv \mathrm{sat}(u_\ell(t))$, then $\xi(t) \equiv 0$ and $v_1(t) \equiv 0$, $v_2(t) \equiv 0$, and the linear response is reproduced by the saturated system with anti-windup compensation. Consequently, the scheme in Fig. 1 constitutes a reasonable solution to the anti-windup problem described at the beginning of this section. Moreover, the stability and performance properties induced by filter (4) are robust with respect to parameter perturbations (see Teel & Kapoor, 1997 for a detailed proof of the robustness properties, which is based on a small gain argument).

By definition, we also have $y_\xi = y - y_\ell$, i.e., y_ξ quantifies the mismatch between the outputs in the saturated and linear cases. To satisfy the specifications for the anti-windup given in item 2, it is then desirable to keep y_ξ as small as possible. Using filter (4), the choice $u = \mathrm{sat}(v_1 + y_c)$ and based on the observation that $y_c(t) \equiv u_\ell(t)$, we have that y_ξ is determined by the dynamics

$$\begin{aligned} \dot{\xi} &= A\xi + B_2(\mathrm{sat}(v_1 + u_\ell) - u_\ell), \\ y_\xi &= C\xi + D_2(\mathrm{sat}(v_1 + u_\ell) - u_\ell). \end{aligned} \tag{7}$$

From these dynamics it becomes clear why, when the energy in $\mathrm{sat}(u_\ell) - u_\ell$ is small, it should not be difficult to keep the energy in y_ξ small or, equivalently, the mismatch between the outputs in the saturated and linear cases. It is clear that, when the plant is asymptotically stable, the choice $v_1 \equiv 0$ will at least cause the mismatch to decay exponentially to zero (since $\mathrm{sat}(u_\ell(t)) - u_\ell(t)$ converges to zero in finite time). However, other choices for v_1 might result more effective at keeping y_ξ small. For example, the stabilizing action performed by v_1 on dynamics (7) greatly improves the anti-windup performance when the plant contains slow modes (such as in the following Example 1). Indeed, if the plant has neutrally stable modes, asymptotic stability of the overall system cannot be guaranteed with $v_1 \equiv 0$ and typically, the closer the plant poles are to the imaginary axis, the slower ξ converges to zero (when $v_1 \equiv 0$), thus resulting in very poor performance. Nevertheless, the overall system can be made asymptotically stable via suitable designs for v_1, even in the case of marginally unstable plants.[2] In particular, it is shown in Teel and Kapoor (1997) that the (sub-optimal) anti-windup problem can always be solved by selecting the input v_1 as $v_1 = k(\xi)$, where $k(\cdot)$ is a suitably chosen static (nonlinear, in general) function (see Teel & Kapoor, 1997 for details). With the aim of designing alternative feedback laws for v_1, a large number of results on stabilization of linear systems with bounded inputs may be employed for the design of $k(\cdot)$ in addition to receding horizon/model predictive control, and a large amount of results on set invariance that have already been applied in the anti-windup context (Gilbert et al., 1995; Shamma, 2000).

An alternative design strategy is based on the linear selection

$$v_1 = K\xi + L(\mathrm{sat}(v_1 + y_c) - y_c), \tag{8}$$

where the gains K and L are chosen among a set of possible values that, based on the sector properties of the saturation function, preserve the asymptotic stability and the well posedness of (4) and (8). Note that well posedness of the interconnection (4) and (8) needs to be addressed because we have allowed $L \neq 0$. This choice is motivated by the fact that using $L \neq 0$ can improve the resulting anti-windup performance (for a similar reasoning, see, e.g., Mulder et al., 2001, Section 5, where it is shown by an example that the presence of the algebraic loop around the saturation can lead to performance improvement). The set of the feasible selections for K and L is characterized by way of a suitable linear matrix inequality (LMI) as stated in the following theorem.

Theorem 1. *Suppose that the linear (unsaturated) closed loop (1)–(3) is internally stable. If there exist $P = P^{\mathrm{T}} > 0$, $W = W^{\mathrm{T}} > 0$ (diagonal) such that*

$$\begin{bmatrix} A^{\mathrm{T}}P + PA & PB_2 + K^{\mathrm{T}}W \\ \star & L^{\mathrm{T}}W + WL - 2W \end{bmatrix} < 0 \tag{9}$$

then, the anti-windup closed-loop system (1), (2), (4), (8) and (5) is well posed and guarantees $\mathscr{L}_p$ stability from (d, r) to the overall system state for all $p \in [1, \infty]$.

[2] By marginally unstable we mean linear plants with poles in the closed left-half plane.

Proof. See Appendix A. □

Remark 1. Condition (9) is an LMI in P and W, the feasibility of which can be checked easily with available software tools. Condition (9) can also be viewed as an LMI for synthesis of K and L by defining $Z = WK$, $Y = WL$ so that (9) becomes an LMI in P, W, Z and Y. This LMI is feasible if and only if A is Hurwitz. Once a solution is found, the compensation gains can be determined as $K = W^{-1}Z$, $L = W^{-1}Y$. Condition (9) is also equivalent to the condition

$$\begin{bmatrix} QA^{\mathrm{T}} + AQ & B_2U + X_1^{\mathrm{T}} \\ UB_2^{\mathrm{T}} + X_1 & X_2^{\mathrm{T}} + X_2 - 2U \end{bmatrix} < 0, \tag{10}$$

where $Q:=P^{-1}$, $U:=W^{-1}$, $X_1:=KQ$ and $X_2:=LU$. Neither the synthesis version of condition (9), nor (10), addresses performance. However, performance can be incorporated into the design by combining (10), say, with additional LMIs containing a performance index to be optimized. For example, the optimization problem

$$\begin{aligned} &\min \gamma \\ &\text{s.t. } (10), \\ &\quad \begin{bmatrix} \gamma I & I \\ I & Q \end{bmatrix} > 0, \\ &\quad \begin{bmatrix} QA^{\mathrm{T}} + AQ + B_2X_1 + X_1^{\mathrm{T}}B_2^{\mathrm{T}} & Q & X_1^{\mathrm{T}} \\ Q & -Q_P^{-1} & 0 \\ X_1 & 0 & -R_P^{-1} \end{bmatrix} < 0 \end{aligned} \tag{11}$$

in the variables γ, Q, U, X_1 and X_2, picks from among all the gains (K, L) that satisfy (10) the selection that minimizes $J := \int_0^\infty (\xi^{\mathrm{T}} Q_P \xi + v_1^{\mathrm{T}} R_P v_1)\,\mathrm{d}t$ for the system $\dot{\xi} = A\xi + B_2 v_1$, $v_1 = K\xi$, where Q_P and R_P are suitable positive-definite matrices constituting free parameters of the optimization problem. See also (Mulder et al., 2001; Grimm et al., 2001; Zaccarian & Teel, 2001) for additional results about LMIs for anti-windup construction with guaranteed stability and performance.

Example 1. Consider a damped mass-spring system, whose equations of motion are given by

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -k/m & -f/m \end{bmatrix} x + \begin{bmatrix} 0 \\ 1/m \end{bmatrix} (u + d),$$

$$y = [\,1 \;\; 0\,]x, \tag{12}$$

$$m_0 = 0.1\, kg, \quad k_0 = 1\,\frac{kg}{s^2}, \quad f_0 = 0.001\,\frac{kg}{s}, \tag{13}$$

$$m = 1.2 m_0, \quad k = 0.8 k_0, \quad f = 0.8 f_0. \tag{14}$$

where $x := [q \;\; \dot{q}]^{\mathrm{T}}$ represents position and speed of the body connected to the spring, m is the mass of the body, k is the elastic constant of the spring, f is the damping coefficient, u represents a force exerted on the mass and d represents an unmeasured disturbance input. Eq. (13) represents the nominal parameter values, and the simulations will be carried out according to the perturbed selections (14). The linear controller (2) is chosen as follows:

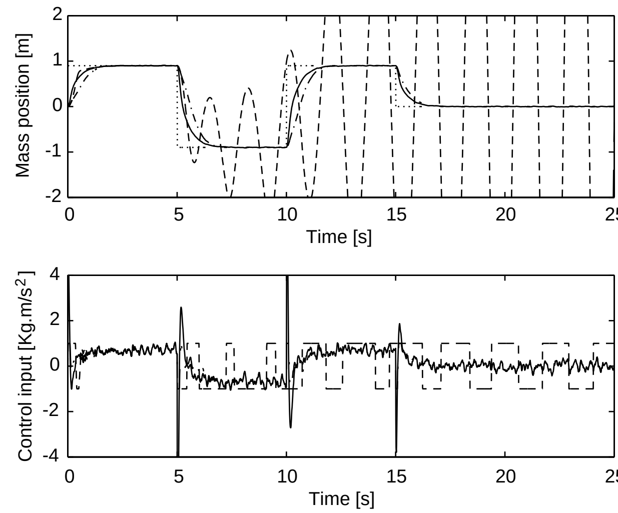

Fig. 2. Time responses of the saturated (dashed), unsaturated (solid) and anti-windup (dash-dotted) system to a double pulse reference (dotted curve in the upper plot).

$$\begin{aligned} &y_c = C_{fb}(s)(C_{ff}(s)r - u_c), \\ &C_{fb}(s) := 200\,\frac{(s+5)^2}{s(s+80)}, \quad C_{ff}(s) := \frac{5}{2s+5}. \end{aligned} \tag{15}$$

This controller, interconnected to the unsaturated plant (12) via the linear interconnection Eq. (3) guarantees a fast response of the mass-spring system to step references and zero steady-state error, in addition to a suitable disturbance rejection level.

For all our simulations, the disturbance d is chosen as a band-limited white noise of power 0.001 passed through a zero order holder with sampling time 0.001 s. The response of the linear closed-loop system (12), (15), (14) and (3), starting from the rest position and with the reference switching between ±0.9 meters every 5 s and going back to zero permanently after 15 s, is shown by the solid curve in Fig. 2. While the linear controller (15) induces on the unsaturated perturbed plant the highly desirable linear response corresponding to the solid curve, such is not the case when the force exerted at the plant's input u is limited between ±1 kg m/s^2. The corresponding closed-loop response, represented by the dashed curve in Fig. 2, converges to a stable limit cycle where the output persistently oscillates between positive and negative peaks $q_{\mathrm{PEAK}} \approx 370$ m.

Following the $\mathscr{L}_2$ anti-windup approach, we augment the control system as shown in Fig. 1, by implementing in filter (4) the plant dynamics (12) (using the nominal parameters (13)) and by selecting the linear stabilizing law (8) with the gain selection

$$K = [\,-0.0147 \quad -1.0107\,], \qquad L = [\,-0.8687\,], \tag{16}$$

which has been determined[3] by following the approach in Remark 1 with $Q_P = \mathrm{diag}(0.1, 10)$ and $R_P = 10$. The resulting response of the overall anti-windup closed-loop system (12), (15), (4), (8), (16) and (5) is represented by the dash dotted curve in Fig. 2, which almost coincides with the linear response.

3. Bumpless transfer

A problem closely linked to the anti-windup problem is the so-called bumpless transfer problem (see, e.g., Peng et al., 1996; Graebe & Ahlén, 1996). In the bumpless transfer problem, a supervising system oversees multiple controllers designed for the same linear system and switches among the controllers based on various scenarios. In such a situation it is desirable to perform a "bumpless" switching between the different controllers, i.e., a switching that does not induce a large transient because of incompatible "initial conditions" of the controller connected to the plant and of the plant itself. Moreover, to be realistic, this needs to be accomplished even in the presence of input saturation.

One global solution to this problem, at least for asymptotically stable plants, comes as a direct extension of the anti-windup strategy described in Section 2. In particular, as in Fig. 3, each controller of the switching scheme is augmented with dynamics that allow the states of the associated controller to evolve in a natural way even when that controller is not connected to the plant. These dynamics are analogous to the dynamics in the anti-windup filter (4). If the plant is described by equations (1), the extra dynamics and the rest of the bumpless transfer scheme take the form (see Fig. 3)

$$
\begin{aligned}
\dot{\xi}_i &= A\xi_i + B_2(\mathrm{sat}(u) - y_{ci}) \\
v_{1i} &= K_i\xi_i + L_i(\mathrm{sat}(u) - y_{ci}) \\
v_{2i} &= -C\xi_i - D_2(\mathrm{sat}(u) - y_{ci}) \\
y_{ci} &= \mathscr{K}_i(s)[r - y - v_{2i}] \quad i = 1, \ldots, n,
\end{aligned}
\tag{17}
$$

$$
u = y_{c\sigma} + v_{1\sigma}, \tag{18}
$$

where Eq. (17) corresponds to the dynamics of each one of the n modules and $\sigma \in \{1, \ldots, n\}$ denotes the position of the multi-plexer (namely, it indicates which controller is connected to the plant). With this construction, the ith controller is constantly fed by the output of the "fictitious" dynamics $x_{\ell i} := x - \xi_i$, which obey the dynamics

$$
\begin{aligned}
\dot{x}_{\ell i} &= Ax_{\ell i} + B_1 d + B_2 y_{ci}, \\
y_{\ell i} &= Cx_{\ell i} + D_1 d + D_2 y_{ci},
\end{aligned}
\tag{19}
$$

[3] To improve the numerical conditioning of the LMI solver, the value of L has been constrained between ± 2 by adding the two extra LMIs $-2U < X_2 < 2U$.

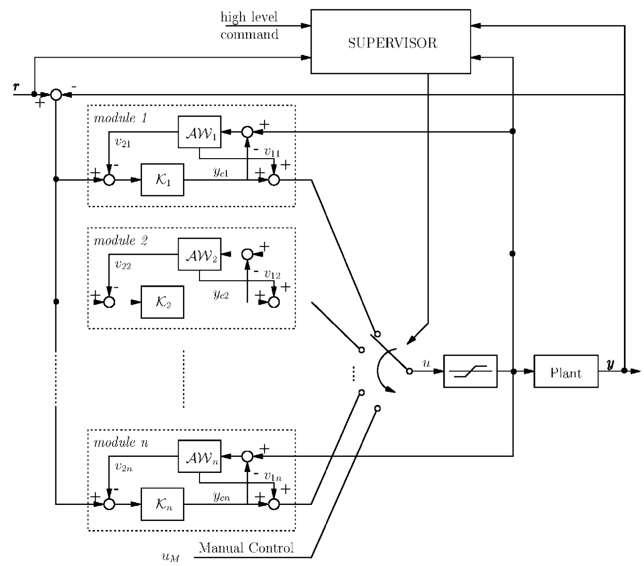

Fig. 3. Block diagram of the proposed bumpless transfer scheme.

i.e., the same dynamics as the linear unsaturated plant. It follows that, regardless of which controller is actually connected to the plant, for each i, y_{ci} evolves as it would if the related controller was constantly connected in closed loop with the unsaturated plant. Assume now that the output of the module j is connected to the plant. If the supervisory system switches the plant input to the output of the module k, the stabilizing action performed by $v_{1k}(\cdot)$ causes the plant's state x to smoothly converge to the kth fictitious state $x_{\ell k}$ by driving ξ_k to zero, thereby effecting a smooth transfer of authority from the jth controller to the kth controller. Since all of the additional compensators in (17) (corresponding to the state variables ξ_i) have the same dynamics, all of the K_i and L_i can be taken to be the same, if desired. However, the desired authority transfer might be different for each controller, hence different K_i and L_i might be desirable for the multi-controller system. The selection for K_i and L_i must also account for input saturation, as in the anti-windup solution. Guidelines for this selection come from the following result which generalizes Theorem 1.

Theorem 2. *Suppose each of the linear (unsaturated) closed-loop interconnections is internally stable. If there exist* $P = P^{\mathrm{T}} > 0$, $W = W^{\mathrm{T}} > 0$ *(diagonal) such that*

$$
\begin{bmatrix} A^{\mathrm{T}}P + PA & PB_2 + K_i^{\mathrm{T}}W \\ \star & L_i^{\mathrm{T}}W + WL_i - 2W \end{bmatrix} < 0 \quad \forall i \tag{20}
$$

then, for any switching strategy, the bumpless transfer scheme (17) and (18) *is well posed and guarantees* $\mathscr{L}_p$ *stability from* (d, r) *to the overall system state for all* $p \in [1, \infty]$.

Proof. See Appendix A. □

The inclusion of a manual control input is allowed in the scheme (see Fig. 3). In addition, safety specifications can be met by the multi-controller scheme by enforcing the action of one controller (e.g., the ith controller) if the manual input is active and drives the plant outside of some prescribed safety region. The corresponding stabilizing action performed by $v_{1i}(\cdot)$ will then drive the plant states back into the safety region, toward the state response $x_{\ell i}(\cdot)$ of the ith closed loop, given by (19). Note that the proposed bumpless transfer scheme incorporates the $\mathscr{L}_2$ anti-windup solution described in Section 2. Therefore, it provides automatically the anti-windup feature when the plant has input saturation and, as before, it does not require any extra state measurements from the plant. To illustrate our idea, we employ it on an example taken from Peng et al. (1996).

Example 2. Consider the linear plant

$$G(s) = \frac{1}{(1+8s)(1+4s)} \tag{21}$$

with input saturation between ± 1 in a control scheme as in Fig. 3 with a manual control input and one controller module ($n = 1$). The desired bumpless transfer occurs between the manual and the automatic control action. As in Peng et al. (1996), the nominal controller is a linear approximated PID controller $C(s) = C_P(s) + C_I(s) + C_D(s)$, corresponding to the following transfer functions:

$$C_P(s) = 20, \quad C_I(s) = \frac{20}{30s}, \quad C_D(s) = \frac{19s}{0.095s+1}.$$

The dynamics of the anti-windup compensator (17) have been chosen as a state-space realization of the plant transfer function (21) and the matrices K_1 and L_1, chosen as follows:

$$\dot{\xi}_1 = \begin{bmatrix} -0.375 & -0.0625 \\ 0.5 & 0 \end{bmatrix} \xi_1 + \begin{bmatrix} 0.25 \\ 0 \end{bmatrix} (\mathrm{sat}(u) - y_{c1}),$$

$$v_{11} = -[\,8.1 \quad 11.27\,]\xi_1 - 0.538(\mathrm{sat}(u) - y_{c1}),$$

$$v_{21} = [\,0 \ \ 0.25\,]\xi_1$$

satisfy condition (20) with the selection $W = 8.8\mathrm{e}8$ and

$$P = \begin{bmatrix} 3.3\mathrm{e}7 & -2.5\mathrm{e}7 \\ -2.5\mathrm{e}7 & 4.2\mathrm{e}7 \end{bmatrix}.$$

In the responses in Fig. 4, the system is under the action of manual control until time $t = 22$ s. After this time, the automatic control loop is closed. The reference r is zero for this particular example. The dotted curve shows the response of the saturated system without any bumpless strategy, the dashed curve shows the response using the conditioning technique proposed in Peng et al. (1996) and the solid curve shows the response using the proposed method. The response shows successful bumpless transfer in the presence of input saturation with an improvement relative to the conditioning technique (which has been shown to be largely superior to other bumpless transfer strategies in Peng et al., 1996).

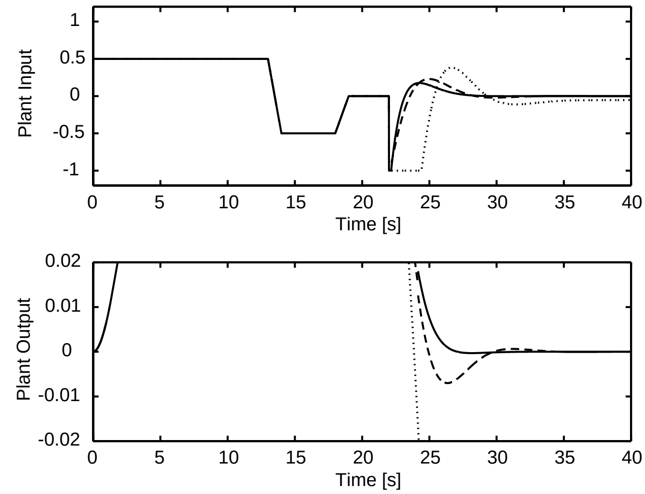

Fig. 4. Transfer responses: without bumpless technique (dotted line), with conditioning technique (Peng et al., 1996) (dashed line) and with the proposed method (solid line).

4. Reliable control via hardware redundancy

Following the discussion on bumpless transfer, a natural idea to consider is the possibility of assigning the n controllers in Fig. 3 as multiple copies of the same controller. This may be desirable as a means for increasing the reliability of control schemes subject to actuator, sensor or other hardware failures.

When all of the controllers in Fig. 3 reproduce the same dynamics, all linear closed loops (represented by the dynamics of the states $(x_{\ell i}, x_{ci})$) coincide. In this case, all of the controller outputs y_{ci} coincide at all times and switching among the different modules does not affect the system response. However, if each controller has a dedicated sensor located on the plant for the measurement of the output y, the differences among the state responses x_{ci} of the controllers can be caused by differences among measurement noise associated to each sensor, or by controller failures.

Under the assumption that A is Hurwitz so that each anti-windup compensated loop is input-to-state stable, we can conclude that bounded measurement disturbances correspond to bounded differences among the controllers' responses. On the other hand, if a failure occurs in the ith controller, typically the control objective is no longer met by the ith linear closed-loop system. In this case, differences will eventually be observed between the input (or, similarly, the output) of the ith controller and the inputs (or, respectively, the outputs) of the other controllers. Based on these differences, the "supervisor" block in Fig. 3 can detect the failures and switch off the defective controller guaranteeing correct functioning of the overall control system. As an example, a possible supervising strategy is to define a max-

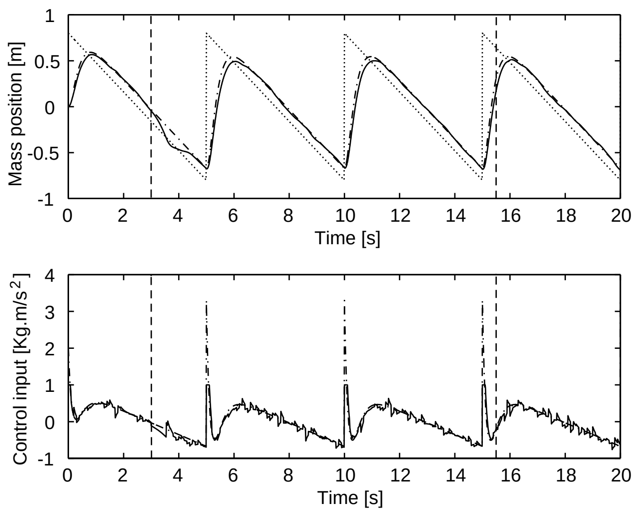

Fig. 5. Input and output of the plant in the reliable control scheme: reference (dotted line), nominal unsaturated response (dash–dotted line) and reliable response (solid line) with sensor failures. The dashed vertical lines correspond to the sensor failures times.

imum tolerance relating the differences among inputs and outputs of the controllers and to detect the malfunctioning controller by majority comparison (provided that a majority of controllers are operating correctly).

A controller failure can occur in two main situations. If the failing controller *is not connected* to the plant, then the controller can be shut down and repaired without affecting the control performance. Indeed, from Fig. 3 it is clear that any disconnected controller is driven by the plant but does not drive the plant itself. After the controller is repaired, its state x_{ci} converges to the values of the other controllers' states thus re-constituting its normal operation. If the failing controller *is connected* to the plant, the state of the plant could be driven far from the nominal trajectory before the failure is detected. However, due to the bumpless scheme, once the control input is switched to a functioning controller, the state of the plant is driven back to the nominal trajectory, while the malfunctioning controller can be repaired as previously described.

Example 3. Consider the nominal control design for the mass-spring system (12) with the nominal parameters (13) and assume that three copies of the controller in Eqs. (15) are combined in a reliable scheme. Assume that the reference signal is a saw-toothed wave of period 5 s and amplitude 1.6 m. Assume that the measurement of each sensor is affected by band limited white noise with noise power 10^{-4}.

Assume that the sensor of the first controller fails at time $t_{F1} = 3$ s giving a zero output from time t_{F1} until its repair time $t_{R1} = 10$ s. Assume also that the sensor of the second controller fails at time $t_{F2} = 15.5$ s, giving a large constant output value after that time. Fig. 5 represents the response of the system when the supervisor block only compares the various controller outputs y_{ci}, $i = 1, 2, 3$ with tolerance value 0.4. The dotted curve represents the reference signal, the dash–dotted curve represents the nominal response (in the absence of measurement noise) and the solid curve represents the reliable control scheme response. Sensor failures occur at the vertical dashed lines. The reliable response confirms the effectiveness of the scheme in detecting the failures and performing the bumpless transfer. Note that the first sensor failure could be detected earlier by comparing the controller inputs u_{ci}, $i = 1, 2, 3$ in the supervisor block. However, it is of interest to show how the bumpless scheme is able to steer the plant output back to the nominal trajectory even when the fault detection is delayed.

5. Conclusions

In this paper, the $\mathscr{L}_2$ anti-windup design technique proposed in Teel and Kapoor (1997) has been first summarized. Then, a new scheme, based on an extension of this technique, has been proposed to achieve bumpless transfer among different controllers in a saturated multi-controller scheme. The resulting performance has been compared to previous solutions on an example taken from the literature. The bumpless transfer scheme has also been employed to propose a reliable design, based on hardware redundancy, that guarantees sensor/actuator fault detection and performance recovery, in addition to the anti-windup action.

Appendix A. Proof of the main theorems

Since Theorem 1 is a special case of Theorem 2, we only need to prove the latter. For $i = 1, \ldots, n$, define the following change of coordinates:

$$X_i := \begin{bmatrix} x_{1i} \\ x_{ci} \end{bmatrix} := \begin{bmatrix} x - \xi_i \\ x_{ci} \end{bmatrix}, \tag{A.1}$$

where x_{ci} is the state of the ith controller $\mathscr{K}_i$. Then, the bumpless transfer scheme in Fig. 3 can be rewritten in the coordinates $(X_1, X_2, \ldots, X_n, x)$ as follows:

$$\begin{aligned} \dot{X}_i &= A_{\mathrm{cl}_i} X_i + B_{\mathrm{cl}_{ri}} r + B_{\mathrm{cl}_{di}} d, \\ y_{ci} &= C_{\mathrm{cl}_i} X_i + D_{\mathrm{cl}_{ri}} r + D_{\mathrm{cl}_{di}} d, \quad i = 1, \ldots, n, \end{aligned} \tag{A.2}$$

$$\begin{aligned} \dot{x} &= Ax + B_1 d + B_2\,\mathrm{sat}(u), \\ y &= Cx + D_1 d + D_2\,\mathrm{sat}(u), \end{aligned} \tag{A.3}$$

$$\begin{aligned} u &= y_{c\sigma(t)} + v_{1\sigma(t)}, \\ v_{1\sigma(t)} &= K_{\sigma(t)}(x - x_{1\sigma(t)}) + L_{\sigma(t)}(\mathrm{sat}(u) - y_{c\sigma(t)}), \end{aligned} \tag{A.4}$$

where (A.2) represent the linear closed-loops, (A.3) represents the plant dynamics and (A.4) represents the input selection, based on $\sigma(\cdot) : \mathbb{R}_{\geqslant 0} \to \{1, \ldots, n\}$, which is an arbitrary, measurable function that indicates which controller is connected at a given time. From linearity, the assumption that the linear closed loops (A.2) are internally stable, and since n is a finite number, given any $p\in[1,\infty]$, there exist positive constants γ_0, γ_d and γ_r such that the following bound holds for the linear responses:

$$\|X_i\|_p \leqslant \frac{\gamma_d}{n}\|d\|_p + \frac{\gamma_r}{n}\|r\|_p + \gamma_0|X_i(0)|, \quad i=1,\dots,n.$$

Based on $\sigma(\cdot)$, define the selection functions $t\mapsto s_i(t)$, $i=1,\dots,n$, as follows:

$$s_i(t)=\begin{cases}1 & \text{if } \sigma(t)=i,\\ 0 & \text{otherwise,}\end{cases} \quad i=1,\dots,n. \tag{A.5}$$

Then, by standard properties of $\mathscr{L}_p$ norms, the following inequalities hold:

$$\begin{aligned}\|X_{\sigma(t)}\|_p &= \left\|\sum_{i=1}^n s_i(t)X_i\right\|_p \leqslant \sum_{i=1}^n \|X_i\|_p\\ &\leqslant \gamma_d\|d\|_p + \gamma_r\|r\|_p + \gamma_0\sum_{i=1}^n|X_i(0)|.\end{aligned} \tag{A.6}$$

To prove the theorem, it is sufficient to show well posedness of the interconnection (A.4) and $\mathscr{L}_p$ stability of (A.3) and (A.4) from the input $(x_{l\sigma(t)}, y_{c\sigma(t)}, d)$ to the state x, or equivalently, from the inputs $(X_{\sigma(t)}, d, r)$ to x. This is shown in the rest of the proof.

Well posedness. The interconnection Eqs. (A.4) can be rewritten as follows:

$$F(u){:=}u - L_i\,\mathrm{sat}(u) = K_i x - K_i x_{li} + (I-L_i)y_{ci}, \tag{A.7}$$

where, at each given time, $i=\sigma(t)$. Consider the following claim, whose proof is reported in Appendix B.

Claim 1. *For each $i\in\{1,\dots,n\}$, Eq. (A.7) is uniquely solvable for u as $u=\zeta_i(x,x_{li},y_{ci})$, where $\zeta_i(\cdot,\cdot,\cdot)$ is globally Lipschitz.*

Based on Claim 1, Eq. (A.4) can be rewritten as follows:

$$u=\sum_{i=1}^n s_i(t)\zeta_i(x,x_{li},y_{ci}), \tag{A.8}$$

where the functions $s_i(\cdot)$, defined in (A.5), are measurable by assumption. Finally, since the representation (A.2), (A.3) and (A.8) for the overall bumpless scheme is globally Lipschitz in the state $(X_1,X_2,\dots,X_n,x)$ (and measurable in t), existence and uniqueness of solutions is guaranteed, as desired.

$\mathscr{L}_p$ stability. Consider the candidate Lyapunov function $V=x^{\mathrm{T}}Px$, where P is the solution to the set of LMIs (20). Define $\hat{u}{:=}\mathrm{sat}(u)$. By the sector $[0,I]$ property of the sat$(\cdot)$ function, we have

$$\hat{u}^{\mathrm{T}}W(u-\hat{u})\geqslant 0, \tag{A.9}$$

where W is any positive-definite matrix. Hence, based on (A.9), the derivative of V along the dynamics of system (A.3) and (A.4) satisfies the following inequality:

$$\begin{aligned}\dot V &= x^{\mathrm{T}}(A^{\mathrm{T}}P+PA)x + 2x^{\mathrm{T}}PB_1d + 2x^{\mathrm{T}}PB_2\hat{u}\\ &\leqslant x^{\mathrm{T}}(A^{\mathrm{T}}P+PA)x + 2x^{\mathrm{T}}P(B_1d+B_2\hat{u})\\ &\quad + 2\hat{u}^{\mathrm{T}}W(u-\hat{u}),\end{aligned}$$

which, substituting the expression for u from Eq. (A.4), can be rewritten as

$$\begin{aligned}\dot V &\leqslant [x^{\mathrm{T}}\ \ \hat{u}^{\mathrm{T}}]\begin{bmatrix}A^{\mathrm{T}}P+PA & PB_2+K_{\sigma(t)}^{\mathrm{T}}W\\ \star & L_{\sigma(t)}^{\mathrm{T}}W+WL_{\sigma(t)}-2W\end{bmatrix}\begin{bmatrix}x\\ \hat{u}\end{bmatrix}\\ &\quad + 2\hat{u}^{\mathrm{T}}W((I-L_{\sigma(t)})y_{c\sigma(t)} - K_{\sigma(t)}x_{l\sigma(t)})\\ &\quad + 2x^{\mathrm{T}}PB_1d.\end{aligned}$$

Finally, since n is finite and the n inequalities (20) are all strict, there exists a small enough number ε such that

$$\begin{aligned}\dot V + \varepsilon(|x|^2+|\hat{u}|^2) &\leqslant 2\hat{u}^{\mathrm{T}}W((I-L_{\sigma(t)})y_{c\sigma(t)} - K_{\sigma(t)}x_{l\sigma(t)})\\ &\quad + 2x^{\mathrm{T}}PB_1d,\end{aligned} \tag{A.10}$$

which, by completion of squares, and by the definition of $y_{c\sigma(t)}$ and $x_{l\sigma(t)}$, implies

$$\dot V + \frac{\varepsilon}{2}|x|^2 \leqslant \gamma_1|X_{\sigma(t)}|^2 + \gamma_2|d|^2 + \gamma_3|r|^2, \tag{A.11}$$

where γ_1, γ_2 and γ_3 are sufficiently large constants. Integrating Eq. (A.11) on both sides, based on the positive definiteness of V and considering Eq. (A.6), $\mathscr{L}_2$ stability from (d, r) to x follows.

The $\mathscr{L}_p$ stability property proven above for $p=2$, can be proven to hold for all $p\in[1,\infty]$ employing standard results on $\mathscr{L}_p$ stability of nonlinear systems. In particular, since representation (A.2), (A.3) and (A.8) for the overall system ensures that the right-hand side is globally Lipschitz in the state (and measurable in time), the assumptions of Khalil (1996), Theorem 6.1 are trivially satisfied by choosing the Lyapunov function $V=x^{\mathrm{T}}Px$, resulting from the solution to LMIs (20) and rewriting Eq. (A.10) with $(x_{l\sigma(t)}, y_{c\sigma(t)}, d)=0$. In fairness, in Khalil (1996) piecewise continuity of the right-hand side with respect to time is assumed. However, this property is only used in that proof to guarantee existence of solutions, which also holds when the right-hand side is only measurable in time (see, e.g., Coddington & Levinson, 1955, Theorem 1.1, Chapter 2). Hence, the extension of Khalil (1996), Theorem 6.1 to this case is straightforward.

Appendix B. Proof of Claim 1

The following propositions will be useful for the proof of Claim 1.

Proposition 1. *Given a diagonal positive-definite matrix W and a square matrix D, if $-2W+WD+D^{\mathrm{T}}W<0$, then $I-D\Delta$ is nonsingular for all matrices Δ belonging to the set $\mathscr{D}_{n_u}{:=}\{\Delta\colon\ \Delta=\mathrm{diag}(\delta_1,\delta_2,\dots,\delta_{n_u}),\ \delta_k\in[0,1]\ \forall k\}$.*

Proof. Assume that $I - D\Delta$ is singular for some $\Delta \in \mathscr{D}_{n_u}$, then there exists $z \neq 0$ such that

$$(I - D\Delta)z = 0. \tag{B.1}$$

Define $\bar{z} := \Delta z$ and observe that $\bar{z} \neq 0$ because if $\bar{z} = 0$, then $(I - D\Delta)z = z \neq 0$, which contradicts (B.1). From (B.1), we also obtain $\bar{z}^{\mathrm{T}} W (I - D\Delta) z = 0$, or equivalently

$$\bar{z}^{\mathrm{T}} W z - \tfrac{1}{2}\bar{z}^{\mathrm{T}} W D \bar{z} - \tfrac{1}{2}\bar{z}^{\mathrm{T}} D^{\mathrm{T}} W \bar{z} = 0. \tag{B.2}$$

Since W is diagonal ($W = \mathrm{diag}(w_1, w_2, \ldots, w_{n_u})$) and the entries of Δ are such that $\delta_i \in [0,1]$, then

$$\bar{z}^{\mathrm{T}} W z = \sum_{i=1}^{n_u} \delta_i w_i z_i^2 \geqslant \sum_{i=1}^{n_u} \delta_i^2 w_i z_i^2 = \bar{z}^{\mathrm{T}} W \bar{z},$$

hence, from (B.2)

$$-\bar{z}^{\mathrm{T}} W \bar{z} + \tfrac{1}{2}\bar{z}^{\mathrm{T}} W D \bar{z} + \tfrac{1}{2}\bar{z}^{\mathrm{T}} D^{\mathrm{T}} W \bar{z} \geqslant 0.$$

However, by assumption, $-2W + WD + D^{\mathrm{T}} W < 0$, thus, since $\bar{z} \neq 0$, we have a contradiction. □

Proposition 2. *Consider a locally Lipschitz function $F : \mathbb{R}^n \to \mathbb{R}^n$ and assume that the Jacobian of F satisfies*:

$$JF(x) \in \mathscr{M} \quad \textit{for almost all } x \in \mathbb{R}^n,$$

where the set $\mathscr{M} \subset \mathbb{R}^{n \times n}$ is compact, convex, and each matrix in $\mathscr{M}$ is nonsingular. Then there exists a (unique) globally Lipschitz function $G : \mathbb{R}^n \to \mathbb{R}^n$ such that $F(G(x)) = x$ for all $x \in \mathbb{R}^n$. Equivalently, F is a homeomorphism with globally Lipschitz inverse.

Proof. The proof is a straightforward extension of that used to prove the local version given in Clarke (1990), Theorem 7.1.1. That theorem is proved in "lemma-size steps". The only lemma that needs modification is the first one which, in our setting, becomes: there exists $\delta > 0$ such that, for each unit vector $v \in \mathbb{R}^n$, there exists a unit vector $w \in \mathbb{R}^n$ such that

$$\langle w, JF(x)v \rangle \geqslant \delta \quad \forall x \text{ where } JF(x) \text{ is defined.} \tag{B.3}$$

This follows from the fact that, for any $x \in \mathbb{R}^n$, $JF(x) \in \mathscr{M}$. Moreover, by assumption, $\mathscr{M}$ is compact and each matrix in $\mathscr{M}$ is nonsingular. Then, for any unit vector v, the set $\Omega_v := \{Av, A \in \mathscr{M}\}$ is distant at least δ from the origin of $\mathbb{R}^n$. Similar to the proof in Clarke (1990), Eq. (B.3) then follows from the separation theorem for convex sets. Based on Eq. (B.3), the proof of the proposition can be completed by following verbatim the rest of the proof of Theorem 7.1.1 in Clarke (1990). □

Proof of Claim 1. For any $i \in \{1, \ldots, n\}$, denote by $JF(u)$ the derivative of $F(u)$, wherever it exists. Since the derivative of the scalar saturation function is either 0 or 1 almost everywhere and $\mathrm{sat}(u)$ is decoupled by assumption, $JF(u)$ satisfies the following relationship for almost all u:

$$\begin{aligned} JF(u) &\in \overline{\mathrm{co}}(\{(I - L_i\, \mathrm{diag}\{\delta_1, \ldots, \delta_n\}), \delta_k \in \{0,1\}\ \forall k\}) \\ &= \{(I - L_i \Delta),\ \Delta \in \mathscr{D}_{n_u}\} =: \mathscr{M}, \end{aligned}$$

where $\overline{\mathrm{co}}(\mathscr{A})$ denotes the closed convex hull of the set $\mathscr{A}$, and $\mathscr{D}_{n_u}$ is defined in Proposition 1. If the LMIs (20) hold, necessarily, all the terms on the diagonal are negative definite. In particular, for any $i \in \{1, \ldots, n\}$, $L_i^{\mathrm{T}} W + W L_i - 2W < 0$. Hence, by Proposition 1, each matrix in the set $\mathscr{M}$ is nonsingular, and the claim follows from Proposition 2. □

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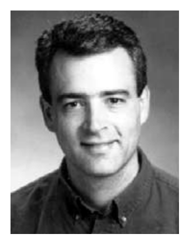

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