Automatica 44 (2008) 2607–2613

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Localized adaptive bounds for approximation-based backstepping☆

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ARTICLE INFO

Article history:
Received 24 October 2006
Received in revised form
20 November 2007
Accepted 20 February 2008
Available online 19 September 2008

Keywords:
Adaptive control
Localized learning
Nonlinear systems
Adaptive bounds

ABSTRACT

Recent research has established the utility of adaptive bounds on model uncertainty in adaptive approximation-based control. Such bounds have utility both for robust control law design and for self-organizing approximators that could adjust the number of basis elements N by adding additional approximation resources in the regions where the approximation error bound is large. Existing adaptive bounding methods utilize algorithms with global forgetting. In this article, we investigate methods to develop bounds on approximation accuracy that involve local forgetting. The importance of local versus global forgetting is motivated in the text and illustrated with an example.

1. Introduction

In this article, we consider a general class of nth order scalar input nonlinear systems described by

$$\dot{x}_i = \left(f_i^o(x) + f_i(x)\right) + \left(g_i^o(x) + g_i(x)\right) x_{i+1}, \tag{1}$$

$$\dot{x}_n = \left(f_n^o(x) + f_n(x)\right) + \left(g_n^o(x) + g_n(x)\right) u \tag{2}$$

for $1 \leq i < n$ where $x = [x_1, \ldots, x_n]^\top \in \Re^n$ is the state vector and $u \in \Re^1$ is the scalar control signal. It is assumed that the uncertain system described by (1) and (2) is strictly feedback passive (see p. 46 in Krstic, Kanellakopoulos, and Kokotovic (1995)). The functions $f_i^o(x), g_i^o(x), i = 1, \ldots, n$ represent the known design models. The functions $f_i(x), g_i(x), i = 1, \ldots, n$ represent the unknown error between the true system and the design model. Both the known and unknown functions are assumed to be continuous in their arguments. For the system described above, we will employ the backstepping control design procedure, where adaptive approximation methods will be developed to address the unknown nonlinearities $f_i(x)$ and $g_i(x)$.

The objective of on-line approximation-based control is to achieve high performance reference input tracking, without using high control gains, in applications where the design model error is significant. In the literature (Chen & Khalil, 1995; Choi & Farrell, 2000; Jiang & Praly, 1998; Lewis, Yesildirek, & Liu, 1996; Ordonez & Passino, 2001; Polycarpou, 1996; Polycarpou & Mears, 1998; Sanner & Slotine, 1992), online approximation-based control methods are designed to achieve stability and accurate reference input tracking for systems with partially unknown nonlinearities, by implementing approximations to the unknown nonlinear functions during the operation of the system. Since online approximation-based control can never achieve exact modeling of unknown nonlinearities, *inherent approximation errors* could arise even if optimal approximator parameters were selected (see Section 3). Often, a restrictive assumption is made that a magnitude bound on the inherent approximation error is known. Articles (Jiang & Praly, 1998; Polycarpou, 1996) relax the assumption of a known bound on the inherent approximation errors. These articles estimate the bounding parameters and design adaptive robust controllers to guarantee global uniform ultimate boundedness.

As will be stated formally in Section 3, the approximator can be designed such that each approximator parameter θ_k only affects the function approximation over a local region S_k. The global features of the leakage modification in Jiang and Praly (1998) and Polycarpou (1996), see Eq. (31) herein, cause each parameter estimate θ_k to drift toward certain design parameters when the operating point x leaves S_k. Thus, both the approximated function and the bounding function will lose local accuracy on S_k while $x \notin S_k$. Therefore, any knowledge learned from past experience in S_k may not be retained for future use when the state later returns to S_k. This is referred to as *global forgetting*.

Fig. 1 indicates a region $\mathcal{D}$ over which the state is desired to operate. The two distant regions S_i and S_j ($j \neq i$) indicate the region of the influence of the parameters θ_i and θ_j, respectively. Herein, *localized adaptation* (Farrell & Polycarpou, 2006) implies that changing the parameter estimate θ_i affects the approximation

☆ This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Associate Editor Zhihua Qu under the direction of Editor Hassan K. Khalil.

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0005-1098/$ – see front matter
doi:10.1016/j.automatica.2008.02.013

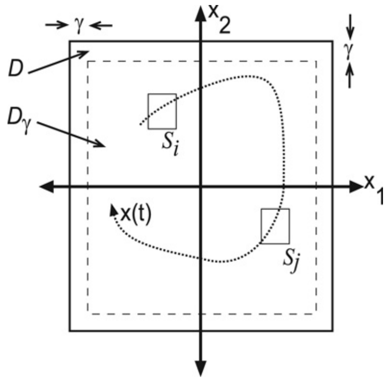

Fig. 1. The dotted line depicts a portion of a state trajectory. Two localized regions S_i and S_j are indicated by the small squares. The region $\mathcal{D}_\gamma$ is enclosed by the dashed square. The region $\mathcal{D}$ which contains $\mathcal{D}_\gamma$ is enclosed by the larger square.

accuracy only on S_i, which is specified such that $\mu(S_i) \ll \mu(\mathcal{D})$, where $\mu(A)$ is a function that measures the area of the set $A \subset \mathcal{D}$. For any other region $S_j \subset \mathcal{D}$ such that $S_i \bigcap S_j = \varnothing$, when the state x is in S_i, the function will be adapted locally on S_i to learn new information, without affecting the function approximation on S_j. In doing so, previously achieved approximation accuracy on S_j is retained and available if the state re-enters S_j in the future.

This issue of global forgetting was addressed in Zhao, Farrell, and Polycarpou (2004) by deriving localized leakage based adaptation algorithms for both the approximator and bounding parameters. The analysis of Zhao et al. (2004) focused on the scalar single-input–single-output system: $\dot{x} = f(x)+g(x)u$ with $g(x) = 1$ and $x \in \Re^1$. In Zhao and Farrell (2006), those results were extended to general, second order, triangular systems and localized adaptive bounding approaches utilizing σ-modification, a deadzone, and the ϵ-modification. Article Zhao and Farrell (2005) developed the robust adaptive control scheme for a wider class (not necessarily in triangular form) of nth order (i.e., $x \in \Re^n$, $n > 1$) single-input systems using the backstepping procedure proposed in Farrell, Polycarpou, and Sharma (2004); however, the robustness analysis of Zhao and Farrell (2005) focused on the approach utilizing a deadzone with projection.

The objective of this article is to achieve stability and accurate reference tracking performance for a general class of nth order nonlinear systems (as shown in Eqs. (1) and (2)) with unknown model errors. The approach builds on the ideas in Farrell et al. (2004) and Zhao et al. (2004). The controller design will incorporate both approximations to unknown nonlinearities and approximations to bounding functions for the residual function approximation errors. The main contribution of this article is the development of localized adaptation algorithms for approximator parameters and bounding parameters such that the issue of global forgetting is addressed and an incremental learning ability for the function approximation is achieved. The learning algorithms are incremental in the sense that increasingly more accurate approximations are achieved as experience is accumulated and knowledge learned from past experience is retained to improve future performance.

In the subsequent analysis, the problem statement is formulated in Section 2. The function approximator structure and properties are presented in Section 3 and a function approximation-based controller is developed in Section 4. Filtering techniques (Farrell & Polycarpou, 2006; Farrell et al., 2004) will be applied to handle the complexity of calculating time derivatives of intermediate state commands for the backstepping approach. The controller guarantees the convergence of the system state and the boundedness of all signals of the system and the approximator parameters, as shown in Theorem 1 for the ideal case where perfect function approximations are possible. The control design for the ideal case is very similar to that in Jiang and Praly (1998) and Polycarpou (1996). The purpose for discussing the ideal case is to establish notation and provide a point of reference for the subsequent analysis. Sliding components are included in the controller to ensure that if the state is outside of $\mathcal{D}$ at some time, then the state will enter $\mathcal{D}$ in finite time. That analysis is presented once in Appendix 7.3 and is not repeated in the following sections. Section 5 considers the more realistic situation where the function approximation errors are small but nonzero on $\mathcal{D}$. Section 5 only considers the σ-modification method, while two other localized modification methods (i.e., $\bar{x}$ modification and deadzone) will be discussed in Appendix 7.4 and 7.5, respectively. The modification approaches we propose herein are novel. In addition, they are shown to be efficient in computation and effective in eliminating global forgetting. The stability properties have shown that significantly smaller m.s.s. bounds will be achieved from our approaches than from the previously existing approaches. Due to space limitations, the Appendices are available online at http://www.ee.ucr.edu/~farrell.

2. Problem formulation

For the system described by (1) and (2), we are interested in tracking problems. There is a desired trajectory $x_d(t)$ with derivative $\dot{x}_d(t)$, each of which is available and bounded for any $t \geq 0$. Note that existing control design methods in the literature (e.g., Khalil (2002) and Krstic et al. (1995)) would require knowledge of the first n derivatives of $x_d(t)$. The approach herein only requires knowledge of $x_d(t)$ and its first derivative. This will become clear in the course of the control derivation using the command filtering technique for backstepping, as presented in Section 4.

The tracking error vector is defined as

$$\tilde{x}(t) = x(t) - x_c(t) \tag{3}$$

where $x_c(t) = [x_d, x_{2c}, \ldots, x_{nc}]^\top$ for all $t \geq 0$. The x_{ic} ($2 \leq i \leq n$) terms are virtual control inputs or intermediate control variables that will be defined by the backstepping control design procedure in Section 4. Our goal is to design the control signal u such that $x(t)$ converges to and tracks the reference signal (i.e., $\|\tilde{x}(t)\|$ sufficiently small) and all signals of the closed-loop system are bounded.

The following assumptions are necessary for the subsequent analysis:

Assumption 1. There exists a known compact region $\mathcal{D} \subset \Re^n$ that specifies the domain of operation for the state x.

Typically, $\mathcal{D}$ is determined by the physical limitations of the designed system. Consider motor control as an example. The range of motor voltage and the maximum current specified by the manufacturer determine the range of speed and torque over which the motor is designed to operate. This ensures the existence of a known compact operation region $\mathcal{D}$ for the state (e.g., motor speed and current).

The control designer ensures safe operation by keeping $x \in \mathcal{D}$ and attains the desired performance by keeping $\tilde{x}$ small, assuming that $x_c(t) \in \mathcal{D}$. Therefore, all trajectories $x_c(t)$ that the system is expected to track are assumed to be contained within the region $\mathcal{D}$. This assumption is stated explicitly as follows:

Assumption 2. For a fixed $\gamma > 0$, we define the region $\mathcal{D}_\gamma$ as $\mathcal{D}_\gamma = \{x \in \mathcal{D} \,|\, \|x - y\| > \gamma, \forall y \notin \mathcal{D}\}$. We assume that $x_c(t) \in \mathcal{D}_\gamma$ for any $t \geq 0$.

Fig. 1 indicates the region $\mathcal{D}$ enclosed by a solid line and the region $\mathcal{D}_\gamma$ enclosed by a dashed line. Assumption 2 states that the desired trajectory $x_c(t)$ is at least a distance γ from the boundary of $\mathcal{D}$, i.e., $\gamma \le \min_{y \in \{\Re^n - \mathcal{D}\}}(\|x_c(t) - y\|)$, for any $t \ge 0$.

To ensure safe system operation, if $x(t) \notin \mathcal{D}$, then the controller should return the state to $\mathcal{D}$ in finite time. This can be achieved by sliding mode design as is proven in Appendix 7.3. The body of this article will focus on $x(t) \in \mathcal{D}$. Since $\mathcal{D}$ defines the region of state space over which accurate reference input tracking is desired, the region $\mathcal{D}$ will also define the largest region over which f_i and g_i approximators will be maintained. For $x(t) \notin \mathcal{D}$ adaptation of the function approximators will stop.

To ensure controllability, it is necessary to assume that all $g_i^o(x) + g_i(x)$ terms are bounded away from zero and of known sign. Therefore, we will invoke the following assumption.

Assumption 3. There exists a positive constant g_l, such that for $i = 1, \dots, n$, $g_i^o(x) + g_i(x) \ge g_l > 0$ for all $x \in \Re^n$.

3. Approximator definition

This section provides various technical details related to the definition of the function approximation.

3.1. Function approximation to model errors

For $x \in \mathcal{D}$, we define approximations to the unknown functions $f_i(x)$ and $g_i(x)$ as $\hat{f}_i(x) = \theta_{f_i}^\top \Phi_{f_i}(x)$ and $\hat{g}_i(x) = \theta_{g_i}^\top \Phi_{g_i}(x)$ for $i = 1, \dots, n$, where the parameter vectors θ_{f_i} and θ_{g_i} will be estimated online. For $x \notin \mathcal{D}$, $\hat{f}_i(x) = 0$ and $\hat{g}_i(x) = g_l$. The vector $\Phi_{f_i}(x)$ is a designer specified regressor vector containing the basis functions for the approximation. Denote the support of the kth basis function of the $\Phi_{f_i}(x)$ vector by $S_{f_i,k} = \{x \in \mathcal{D} \,|\, \Phi_{f_i,k}(x) \ne 0\}$. Let $\bar{S}_{f_i,k}$ denote the closure of $S_{f_i,k}$. Note that each $\bar{S}_{f_i,k}$ is a compact set. For each i, the $\Phi_{f_i}(x)$ vector is defined as a set of positive, locally supported[1] functions $\Phi_{f_i,k}(x)$ for $k = 1, \dots, N$ such that each set $S_{f_i,k}$ is connected with $\mathcal{D} = \bigcup_{k=1}^N S_{f_i,k}$ where N is a finite integer. This ensures that for any $x \in \mathcal{D}$, there exists at least one k such that $\Phi_{f_i,k}(x) \ne 0$. Therefore, $\{S_{f_i,k}\}_{k=1}^N$ forms a finite cover for $\mathcal{D}$. Similarly, we define the support of the kth basis function of $\Phi_{g_i}(x)$ as $S_{g_i,k}$ with closure $\bar{S}_{g_i,k}$. The sets $\bar{S}_{g_i,k}$, $k = 1, \dots, N$ also form a finite cover of region $\mathcal{D}$.

We define a set of parameters $\theta_{f_i}^*$ that are optimal in the sense: $\theta_{f_i}^* = \arg\min_\theta \left(\max_{x \in \mathcal{D}} \left|f_i(x) - \theta^\top \Phi_{f_i}(x)\right|\right)$. Note that these optimal parameters are unknown. They are not used in the implemented control law, but are useful for the analysis that follows. Since $\mathcal{D}$ is compact and each f_i is continuous, the vector $\theta_{f_i}^*$ exists and is well-defined. Define the parameter estimation error vector $\tilde{\theta}_{f_i} = \theta_{f_i} - \theta_{f_i}^*$. Let $\delta_{f_i}(x) = f_i(x) - (\theta_{f_i}^*)^\top \Phi_{f_i}(x)$ represent the *inherent* or *residual approximation error*. Note that by the definition of $\theta_{f_i}^*$ above, the maximum value of $\delta_{f_i}(x)$ on $\mathcal{D}$ is bounded. See Zhao (2007) and Zhao and Farrell (2005) for additional detail. The quantities $\theta_{g_i}^*$, $\tilde{\theta}_{g_i}$ and $\delta_{g_i}(x)$ are defined similarly.

3.2. Bounding function approximations

By the definition of δ_{f_i} and δ_{g_i}, the magnitude of these inherent approximation error functions are bounded on $\mathcal{D}$; however, the bound is not known. Our control approach will utilize an estimate of these upper bound functions. Therefore, we assume a form for the bounding functions with multiplicative parameters that will be estimated. To save computational effort, we reuse the same basis elements; however, the approach easily extends to the case of different basis elements.

By the above discussion, there exists a positive constant vector $\Psi_{f_i}^*$, $i = 1, \dots, n$, referred as the *optimal bounding parameter*, such that $\left|\delta_{f_i}\right| \le (\Psi_{f_i}^*)^\top \Phi_{f_i}$. A vector $\Psi_{g_i}^*$ yielding a bound on $\left|\delta_{g_i}\right|$ is defined similarly. Note that the optimal bounding parameter vectors $\Psi_{f_i}^*$ and $\Psi_{g_i}^*$ are unknown. Therefore, the approximated bounding functions are $\Psi_{f_i}^\top \Phi$ for $|\delta_{f_i}|$ and $\Psi_{g_i}^\top \Phi$ for $|\delta_{g_i}|$, where the vectors Ψ_{f_i} and Ψ_{g_i} will be estimated online. For the following analysis, we define bounding parameter estimation errors as $\tilde{\Psi}_{f_i} = \Psi_{f_i} - \Psi_{f_i}^M$ and $\tilde{\Psi}_{g_i} = \Psi_{g_i} - \Psi_{g_i}^M$, where each element of $\Psi_{f_i}^M$ is defined as $\psi_{f_i,k}^M = \max\{\psi_{f_i,k}^*, \psi_{f_i,k}^0\}$, $k = 1, \dots, N$ with the vector $\Psi_{f_i}^0 = [\psi_{f_i,1}^0, \dots, \psi_{f_i,N}^0]^\top$ selected in the design stage.

With these estimated upper bounds, we will select proper terms in the control signal or the intermediate state commands to properly handle the inherent approximation errors.

4. Approximation-based backstepping

For each $i = 2, \dots, n$, the signal x_{ic} and its derivative $\dot{x}_{ic}$ (used in Eqs. (6) and (10) are produced by using a command filter as defined in (36) of Appendix 7.2 with the input x_{ic}^0. The variable x_{ic}^0 is defined as

$$x_{ic}^0 = \alpha_{i-1} - \xi_i, \quad \text{for } i = 2, \dots, n$$

where α_{i-1} is the virtual control signal for the x_{i-1} subsystem defined in (4); ξ_i defined in (37) of Appendix 7.2 is a filtered version of the error $(x_{i+1,c} - x_{i+1,c}^0)$ imposed by the command filter. A block diagram for this x_{ic} calculation for one value of $i \in [2, n]$ is shown in Fig. 2. The design of α_i for $i \in [1, n-1]$ and $u = \alpha_n$ is presented in Section 4.1 and the procedure for the calculation of ξ_i and x_{ic} is given in Appendix 7.2 due to space limitations.

4.1. Controller design

We divide the control signal design into two cases:

Case 1. For $i \in [1, n-1]$, the virtual control signals α_i of the backstepping procedure are defined as

$$\alpha_i = \frac{u_{a_i}}{g_i^o + \hat{g}_i + \beta_{g_i}} \tag{4}$$

where, for $i = 1$,

$$u_{a_1} = -k_1 \tilde{x}_1 + \dot{x}_d - f_1^o - \hat{f}_1 - \beta_{f_1} + u_{s_1}, \tag{5}$$

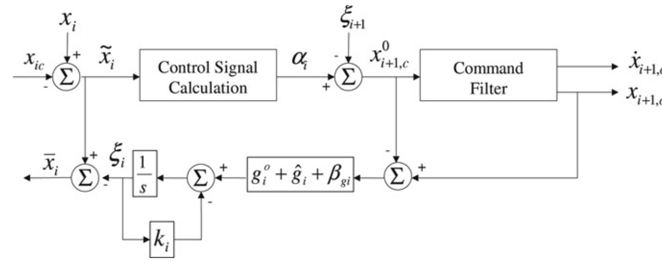

Fig. 2. Block diagram of the command filtered approximation-based backstepping implementation for $i \in [1, n-1]$. The inputs to the block diagram include x_i from the plant, and the outputs x_{ic}, $\dot{x}_{ic}$ from the previous block. The outputs are the commands $x_{i+1,c}$, $\dot{x}_{i+1,c}$ to the next block.

[1] 'Locally supported' means that $\rho(S_{f_i,k}) < \mu \ll \rho(\mathcal{D})$, where for set A, $\rho(A) = \max_{x,y \in A}(\|x - y\|)$.

and, for $i = 2, \ldots, (n-1)$,

$$u_{a_i} = -k_i\tilde{x}_i + \dot{x}_{ic} - f_i^o - \hat{f}_i - \beta_{f_i} - (g_{i-1}^o + \hat{g}_{i-1} + \beta_{g_{i-1}})\bar{x}_{i-1} + u_{s_i}. \quad (6)$$

The control gains k_i are designer specified positive constants that will determine the decay rate for disturbances and initial condition errors.

The signals β_{f_i} and β_{g_i} ($1 \le i < n$) are defined to obtain robustness to the inherent approximation errors. Because the magnitude of these errors are unknown, we are interested in developing adaptive bounds on the approximation errors and using those bounds in the definitions of β_{f_i} and β_{g_i} as

$$\beta_{f_i} = \Psi_{f_i}^\top \Phi_{f_i}\omega\left(\frac{\bar{x}_i}{\epsilon}\right), \qquad \beta_{g_i} = \Psi_{g_i}^\top \Phi_{g_i}\omega\left(\frac{\bar{x}_i x_{i+1}}{\epsilon}\right) \quad (7)$$

where $\bar{x}_i$, which represent compensated tracking errors, will be defined below in Eq. (12); the function $\omega(\cdot)$ is continuous and satisfies the Assumption 5 of Appendix 7.7.1; $\epsilon > 0$ is a small design constant. $\diamond$

Case 2. The control law is defined for the nth subsystem as

$$u = u_{ad} + u_{s_n}. \quad (8)$$

The adaptive component u_{ad} is defined for $x \in \mathcal{D}$ to achieve accurate trajectory tracking in $\mathcal{D}$ as

$$u_{ad} = \frac{u_{a_n}}{g_n^o + \hat{g}_n + \beta_{g_n}} \quad (9)$$

$$u_{a_n} = -k_n\tilde{x}_n + \dot{x}_{nc} - f_n^o - \hat{f}_n - \beta_{f_n} - (g_{n-1}^o + \hat{g}_{n-1} + \beta_{g_{n-1}})\bar{x}_{n-1} \quad (10)$$

where the control gain k_n is a designer specified positive constant; functions β_{f_n} and β_{g_n} are similarly defined to handle the approximation errors as

$$\beta_{f_n} = \Psi_{f_n}^\top \Phi_{f_n}\omega\left(\frac{\bar{x}_n}{\epsilon}\right), \qquad \beta_{g_n} = \Psi_{g_n}^\top \Phi_{g_n}\omega\left(\frac{\bar{x}_n u_{an}}{g_l\epsilon}\right). \quad \diamond \quad (11)$$

For both cases, the sliding components u_{s_i} ($1 \le i \le n$) are designed to return state x to the approximation region $\mathcal{D}$ in finite time (i.e., to ensure that $\mathcal{D}$ is an attractive set) as is discussed in Appendix 7.3. For $x \in \mathcal{D}$, $u_{s_i} = 0$ ($1 \le i \le n$).

Note that, in the virtual control design of Eqs. (4)–(6) and the control design of Eqs. (8)–(10), large magnitude switching of u_{s_i} may happen for x outside of $\mathcal{D}$ because it is possible for $\bar{x}_i$ to switch signs without passing through $\mathcal{D}$. Within $\mathcal{D}$, all u_{s_i} terms are zero; the design of (4)–(6) and Eqs. (8)–(10) results in continuous signals α_i and u by their definitions. The β terms defined in Eqs. (7) and (11) may appear questionable; however, the $\omega(\cdot)$ function in Eqs. (7) and (11) is continuous.

The compensated tracking error signals $\bar{x}_i$ for $i = 1, \ldots, n$ are defined as

$$\bar{x}_i = \tilde{x}_i - \xi_i, \quad \text{for } i = 1, \ldots, n \quad (12)$$

where ξ_i is defined as (37) in Appendix 7.2. From Step (2) of the procedure described in Appendix 7.2, the variables ξ_i, $i = 1, \ldots, n-1$ are produced by filtering the unachieved portion of $x_{i+1,c}^0$. The variables $\bar{x}_i$ are referred as *compensated tracking errors* because they are obtained by removing the filtered unachieved portion of $x_{i+1,c}^0$ from the tracking error, as specified in Eq. (12). Due to space limitations, the dynamics of the compensated tracking errors are derived in Appendix 7.1. The command filtering technique for calculating $\dot{x}_{ic}$ is discussed in Appendix 7.2.

Given the control law defined above, Appendix 7.3 has shown that $\mathcal{D}$ is an attractive set. The remainder of the paper then considers performance within $\mathcal{D}$ using alternative parameter estimation methods.

4.2. Analysis for $x \in \mathcal{D}$

When $x \in \mathcal{D}$, the adaptive laws for $\theta_{f_i}, \theta_{g_i}, \Psi_{f_i}, \Psi_{g_i}$, $i = 1, \ldots, n$ are defined as

$$\dot{\theta}_{f_i} = \Gamma_{f_i}\left(\bar{x}_i\Phi_{f_i} + Q_{f_i}\right) \quad (13)$$

$$\dot{\theta}_{g_i} = \begin{cases} P_S\left\{\Gamma_{g_i}\left(\bar{x}_i x_{i+1}\Phi_{g_i} + Q_{g_i}\right)\right\}, & \text{if } i < n \\ P_S\left\{\Gamma_{g_n}\left(\bar{x}_n u_{ad}\Phi_{g_n} + Q_{g_n}\right)\right\}, & \text{if } i = n \end{cases} \quad (14)$$

and

$$\dot{\Psi}_{f_i} = \Gamma_{\Psi f_i}\left[\Phi_{f_i}\bar{x}_i\omega\left(\frac{\bar{x}_i}{\epsilon}\right) + Q_{\Psi f_i}\right] \quad (15)$$

$$\dot{\Psi}_{g_i} = \begin{cases} P_S\left\{\Gamma_{\Psi g_i}\left(\Phi_{g_i}\bar{x}_i x_{i+1}\omega\left(\frac{\bar{x}_i x_{i+1}}{\epsilon}\right) + Q_{\Psi g_i}\right)\right\}, & \text{if } i < n \\ P_S\left\{\Gamma_{\Psi g_n}\left(\Phi_{g_n}\bar{x}_n u_{ad}\omega\left(\frac{\bar{x}_n u_{an}}{g_l\epsilon}\right) + Q_{\Psi g_n}\right)\right\}, & \text{if } i = n \end{cases} \quad (16)$$

where Γ_{f_i}, Γ_{g_i}, $\Gamma_{\Psi f_i}$, and $\Gamma_{\Psi g_i}$, for $i = 1, \ldots, n$ are defined as positive definite matrices representing the learning rates. The functions Q_{f_i}, Q_{g_i}, $Q_{\Psi f_i}$, $Q_{\Psi g_i}$ for $i = 1, \ldots, n$ are auxiliary terms included to allow analysis of certain robust parameter estimation approaches in later sections. The function $P_S\{\cdot\}$ is a projection operation designed to ensure that $\hat{g}_i + \beta_{g_i}$ satisfies the controllability condition of Assumption 3. An example projection method is defined as (72) in Appendix 7.7.2. The projection operator is analyzed in Krstic et al. (1995).

We note that quantities of the form $m\omega(m/\epsilon)$ in Eqs. (15) and (16) are nonnegative; therefore, if the modification terms denoted by subscripted Q's were zero, then Ψ_{f_i} and Ψ_{g_i} would monotonically increase when the compensated tracking errors $\bar{x}_i$ were nonzero. The definition of the signals denoted by Q_Ψ's will ensure the boundedness of Ψ_{f_i} and Ψ_{g_i}.

For any $x \in \mathcal{D}$, the stability properties are analyzed by considering the following Lyapunov function

$$V = \sum_{i=1}^{n} V_i(\bar{x}_i, \tilde{\theta}_{f_i}, \tilde{\theta}_{g_i}, \tilde{\Psi}_{f_i}, \tilde{\Psi}_{g_i}) \quad (17)$$

where

$$V_i = \frac{1}{2}\left(\bar{x}_i^2 + \tilde{\theta}_{f_i}^\top\Gamma_{f_i}^{-1}\tilde{\theta}_{f_i} + \tilde{\theta}_{g_i}^\top\Gamma_{g_i}^{-1}\tilde{\theta}_{g_i} + \tilde{\Psi}_{f_i}^\top\Gamma_{\Psi f_i}^{-1}\tilde{\Psi}_{f_i} + \tilde{\Psi}_{g_i}^\top\Gamma_{\Psi g_i}^{-1}\tilde{\Psi}_{g_i}\right).$$

For $x \in \mathcal{D}$, when the projection is not active, the derivative of $V(t)$ is derived as

$$\dot{V} \le -\underline{k}\|\bar{x}\|^2 + \sum_{i=1}^{n}\Delta_i + \sum_{i=1}^{n}(\tilde{\theta}_{f_i}^\top Q_{f_i} + \tilde{\theta}_{g_i}^\top Q_{g_i}) \quad (18)$$

where $\underline{k} = \min_i(k_i)$ and $\bar{x} = [\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n]^\top \in \Re^n$ is the compensated tracking error vector. Each Δ_i represents the effect of β_{f_i} and β_{g_i} terms to handle the residual approximation errors δ_{f_i} and δ_{g_i} in the stability analysis. They are derived as, for $1 \le i < n$:

$$\Delta_i = \bar{x}_i\left(-\beta_{f_i} - \beta_{g_i}x_{i+1} + \delta_{f_i} + \delta_{g_i}x_{i+1}\right) + \tilde{\Psi}_{f_i}^\top\Gamma_{\Psi f_i}^{-1}\dot{\Psi}_{f_i} + \tilde{\Psi}_{g_i}^\top\Gamma_{\Psi g_i}^{-1}\dot{\Psi}_{g_i}, \quad (19)$$

and for $i = n$:

$$\Delta_n = \bar{x}_n\left(-\beta_{f_n} - \beta_{g_n}u_{ad} + \delta_{f_n} + \delta_{g_n}u_{ad}\right) + \tilde{\Psi}_{f_n}^\top\Gamma_{\Psi f_n}^{-1}\dot{\Psi}_{f_n} + \tilde{\Psi}_{g_n}^\top\Gamma_{\Psi g_n}^{-1}\dot{\Psi}_{g_n}. \quad (20)$$

Note that when the projection is active, inequality (18) is still preserved (Krstic et al., 1995).

Eq. (18) will be used at various locations in the following sections. First, Section 4.3 will briefly consider the ideal case in which $\Delta_i = 0$, $i = 1, \ldots, n$.

4.3. Ideal special case

In the ideal case where perfect approximation is known to be possible, $\delta_{f_i} = \delta_{g_i} = 0$ for $i = 1, \ldots, n$. Therefore, the designer could define $\beta_{f_i} = \beta_{g_i} = 0$ and $\Psi_{f_i} = \Psi_{g_i} = 0$, so that $\Delta_i = 0$.

We start from Eq. (18) with $Q_{f_i} = Q_{g_i} = 0, i = 1, \ldots, n$ (i.e., without any robust parameter estimation approach in the adaptation laws defined by Eqs. (13) and (14)). Then the derivative of V reduces to

$$\frac{\mathrm{d}V}{\mathrm{d}t} \leq -\underline{k}\|\bar{x}\|^2 \tag{21}$$

which is negative semi-definite.

Theorem 1 (*Ideal Case*). *For the higher order system described by Eqs. (1) and (2) with the adaptive feedback control law of Eqs. (8)–(10), and the parameter adaptation laws of Eqs. (13) and (14) with $Q_{f_i} = Q_{g_i} = 0, i = 1, \ldots, n$, we have the following stability properties:*

(1) *For $x(0) \notin \mathcal{D}$, $x(t)$ for $t > 0$ enters $\mathcal{D}$ in finite time.*
(2) *When $x \in \mathcal{D}$ and $\delta_{f_i} = \delta_{g_i} = 0$:*
  (a) *$\bar{x}_i \in \mathcal{L}_2$;*
  (b) *$\bar{x}_i \to 0$ as $t \to \infty$;*
  (c) *$\bar{x}_i, \theta_{f_i}, \theta_{g_i} \in \mathcal{L}_\infty$.*

In more realistic situations, the Δ_i terms are expected to be small but nonzero on $\mathcal{D}$. In such cases, some form of robust parameter adaptation is required to ensure that the parameter estimates do not diverge toward infinity and to ensure that the tracking performance is maintained. In the presence of the nonzero Δ_i terms, it is not possible to prove convergence of the tracking errors to zero; instead, we will show boundedness of the tracking errors. To be useful to a designer, the bounds should be known functions of the design parameters so that the designer is able to select those parameters to make the bounds suitably small.

5. Localized adaptive bounding methods

Section 4.3 considered the stability results applicable in the ideal case where perfect approximation was possible. In most applications, perfect approximation is not possible; therefore, we are interested in developing bounds on the approximation error and using those bounds in the control law to achieve robustness to the approximation error. In addition, we are interested in analysis of the achievable tracking performance. In this section, we consider the case where $x \in \mathcal{D}$ and there are residual approximation errors, i.e., $\delta_{f_i} \neq 0$ or $\delta_{g_i} \neq 0$, $i = 1, \ldots, n$. The case of $x \notin \mathcal{D}$ is discussed in Appendix 7.3.

With the definitions for $\beta_{f_i}, \beta_{g_i}, i = 1, \ldots, n$ in Eqs. (7) and (11), starting from (19) and (20) and substituting the adaptive laws of $\dot{\Psi}_{f_i}, \dot{\Psi}_{g_i}, i = 1, \ldots, n$ given in Eqs. (15) and (16), we can reduce the expression for $\sum_{i=1}^n \Delta_i$ to

$$\sum_{i=1}^n \Delta_i = \eta\epsilon \sum_{i=1}^n \left((\Psi_{f_i}^M)^\top \Phi_{f_i} + (\Psi_{g_i}^M)^\top \Phi_{g_i}\right) + \sum_{i=1}^n \left(\tilde{\Psi}_{f_i}^\top Q_{\Psi f_i} + \tilde{\Psi}_{g_i}^\top Q_{\Psi g_i}\right),$$

and the derivative of $V(t)$ in (18) for $x \in \mathcal{D}$ and $\delta_{f_i} \neq 0$ or $\delta_{g_i} \neq 0$ becomes

$$\dot{V} \leq -\underline{k}\|\bar{x}\|^2 + \bar{d} + \sum_{i=1}^n \left(\tilde{\theta}_{f_i}^\top Q_{f_i} + \tilde{\theta}_{g_i}^\top Q_{g_i} + \tilde{\Psi}_{f_i}^\top Q_{\Psi f_i} + \tilde{\Psi}_{g_i}^\top Q_{\Psi g_i}\right) \tag{22}$$

where

$$\bar{d} = \eta\epsilon \sum_{i=1}^n \left((\Psi_{f_i}^M)^\top \Phi_{f_i} + (\Psi_{g_i}^M)^\top \Phi_{g_i}\right) \tag{23}$$

is a positive bounded scalar variable.

The following lemma will be used frequently in the subsequent analysis.

Lemma 1. *Let $M(\mathbf{z}, \Theta_1, \ldots, \Theta_p, t) \in \Re^1$ be a positive definite function with $\mathbf{z} = [z_1, \ldots, z_p]^\top \in \Re^p$, $\Theta_i \in \Re^q (1 \leq i \leq p)$, $t \in [0, t_f]$, and*

$$\dot{\Theta}_i = \Gamma[\Phi(\mathbf{z})z_i - c_0 R(\mathbf{z})(\Theta_i - \Theta_i^0)]$$

where $\Phi(\mathbf{z}) : \Re^p \to \Re^q$ is a vector of nonnegative, bounded functions; and $R(\mathbf{z}) : \Re^p \to \Re^{q\times q}$ is a square diagonal matrix with nonnegative diagonal components; $\Theta_i^0 \in \Re^q$ is a constant vector; $\Gamma \in \Re^{q\times q}$ is a positive definite matrix; and $c_0 > 0$ is a scalar constant. If there exist $\varphi_1, \varphi_2 \in \mathcal{KR}$ such that

(1) $\varphi_1(\cdot) \leq M(\mathbf{z}, \Theta_1, \ldots, \Theta_p, t) \leq \varphi_2(\cdot)$,
(2) $\frac{\mathrm{d}}{\mathrm{d}t} M(\mathbf{z}, \Theta_1, \ldots, \Theta_p, t) \leq -c_1\|\mathbf{z}\|^2 + c_2$,

for $t \in [0, t_f]$, where the dependence on $\|\mathbf{z}\|, \|\Theta_1\|, \ldots, \|\Theta_p\|$ is dropped from the functions φ_1 and φ_2 for simplicity, and c_1, c_2 are positive scalar constants; then, $|z_i(t)|, \|\Theta_i(t)\|, i = 1, \ldots, p$, and M are bounded on $[0, t_f]$.

The proof of Lemma 1 can be found in Appendix 7.7.3.

Although three alternative localized robust adaptation approaches have been considered for this paper, the following subsections will only present the σ modification method due to the space limitation. The other two approaches (i.e., $\bar{x}$ modification and deadzone) will be discussed in Appendix 7.4 and 7.5, respectively. The standard modification approaches are described in Ioannou and Sun (1996), Jiang and Praly (1998) and Polycarpou (1996). We will also discuss global forgetting features of standard modification approaches and show the advantage of the localized methods.

5.1. σ modification

To implement a localized σ-modification approach, for $i = 1, \ldots, n$, the designer selects parameter vectors $\theta_{f_i}^0, \theta_{g_i}^0, \Psi_{f_i}^0$ and $\Psi_{g_i}^0$ and the Q terms in the parameter adaptation laws are defined as Zhao et al. (2004)

$$Q_{f_i} = -\sigma_{f_i} R_{f_i}(\theta_{f_i} - \theta_{f_i}^0) \tag{24}$$

$$Q_{g_i} = -\sigma_{g_i} R_{g_i}(\theta_{g_i} - \theta_{g_i}^0) \tag{25}$$

$$Q_{\Psi f_i} = -\sigma_{\Psi f_i} R_{f_i}(\Psi_{f_i} - \Psi_{f_i}^0) \tag{26}$$

$$Q_{\Psi g_2} = -\sigma_{\Psi g_i} R_{g_i}(\Psi_{g_i} - \Psi_{g_i}^0). \tag{27}$$

These terms ensure that the adapted parameter estimates do not drift too far from the design parameter vectors. The matrices $R_{f_i} = \mathrm{diag}(\Phi_{f_i}) \in \Re^{N\times N}$ and $R_{g_i} = \mathrm{diag}(\Phi_{g_i}) \in \Re^{N\times N}$, where $\mathrm{diag}(\upsilon)$ is the square diagonal matrix with diagonal components equal to the vector υ. The standard σ-modification approach does not include the R matrices. The inclusion of the R matrices localizes the effect of the σ-modification for any specific parameter to the region of the operating envelope where that parameter actually affects the approximator.

Substituting the definitions of the Q terms as (24)–(27) into (22), we have the derivative of $V(t)$ as

$$\dot{V} \leq -\underline{k}\|\bar{x}\|^2 + \bar{d} + \rho_1 \tag{28}$$

where ρ_1 is a bounded, positive scalar variable defined as

$$\rho_1 = \frac{1}{2}\sum_{i=1}^{n}\Big(\sigma_{f_i}(\theta_{f_i}^* - \theta_{f_i}^0)^\top R_{f_i}(\theta_{f_i}^* - \theta_{f_i}^0) + \sigma_{g_i}(\theta_{g_i}^* - \theta_{g_i}^0)^\top R_{g_i}(\theta_{g_i}^* - \theta_{g_i}^0) + \sigma_{\Psi fi}(\Psi_{f_i}^M - \Psi_{f_i}^0)^\top R_{f_i}(\Psi_{f_i}^M - \Psi_{f_i}^0) + \sigma_{\Psi gi}(\Psi_{g_i}^M - \Psi_{g_i}^0)^\top R_{g_i}(\Psi_{g_i}^M - \Psi_{g_i}^0)\Big). \quad (29)$$

Therefore, we can summarize these results in the following theorem.

Theorem 2 (σ-*modification*). *For the higher order system described by (1) and (2) with the adaptive feedback control law of Eqs. (8)–(10), and the parameter adaptation laws of Eqs. (13)–(16) with modification terms defined in (24)–(27), we have the following stability properties, for $i = 1, \dots, n$,*

(1) $\bar{x}_i, \tilde{\theta}_{f_i}, \tilde{\theta}_{g_i}, \tilde{\Psi}_{f_i}, \tilde{\Psi}_{g_i} \in \mathcal{L}_\infty$;
(2) $x_i, \theta_{f_i}, \theta_{g_i}, \Psi_{f_i}, \Psi_{g_i} \in \mathcal{L}_\infty$;
(3) $\dot{\bar{x}}_i, \dot{\theta}_{f_i}, \dot{\theta}_{g_i}, \dot{\Psi}_{f_i}, \dot{\Psi}_{g_i} \in \mathcal{L}_\infty$;
(4) *$\bar{x}$ is small in the mean square sense, satisfying*

$$\int_t^{t+T} \|\bar{x}(\tau)\|_2^2 \mathrm{d}\tau \le \frac{1}{\underline{k}}V(t) + \frac{1}{\underline{k}}(\bar{d} + \rho_1)T. \quad (30)$$

The previously existing approach in the literature (Polycarpou, 1996) used parameter updates with the standard leakage modification, which did not include the R_{f_i} and R_{g_i} terms as in Eqs. (24)–(27). Consider the adaptive laws with standard σ-modification for θ_{f_i} as an example:

$$\dot{\theta}_{f_i} = \Gamma_{f_i}\left(\bar{x}_i\Phi_{f_i} - \sigma_{f_i}(\theta_{f_i} - \theta_{f_i}^0)\right), \quad i = 1, \dots, n. \quad (31)$$

The disadvantage of (31) is that when either $\bar{x}_i$ or $\Phi_{f_i,k}$ is zero, $\theta_{f_i,k}$ (i.e., the kth element of the θ_{f_i} vector) will converge toward $\theta_{f_i,k}^0$ (i.e., the kth element of the $\theta_{f_i}^0$ vector). Remember from Section 3 and Fig. 1 that $\Phi_{f_i,k}$ is only nonzero on $S_{f_i,k}$ which is small relative to $\mathcal{D}$. Therefore, $\theta_{f_i,k}$ only affects the f_i approximation on $S_{f_i,k}$. When $x \in S_{f_i,k}$, then $\theta_{f_i,k}$ will converge toward the set of values capable of yielding $\underline{k}\|\bar{x}\|^2 < \bar{d} + \rho_1$ on $S_{f_i,k}$, but when $x \in \mathcal{D} - S_{f_i,k}$ it is the case that $\theta_{f_i,k} \to \theta_{f_i,k}^0$. This cause the approximated functions and bounds to lose their local accuracy on $S_{f_i,k}$ when x is outside $S_{f_i,k}$. When the state later returns to $S_{f_i,k}$, $\theta_{f_i,k}$ will need to be estimated again to satisfy the tracking bound. This is due to the forgetting caused by σ_{f_i} having a global effect. Similarly, for the other parameter estimates (i.e., $\theta_{g_i}, \Psi_{f_i}, \Psi_{g_i}$), standard leakage terms will result in the problem of global forgetting.

Using the localized adaptive laws proposed in Eqs. (13) and (14) and (15) and (16) with the modification terms as defined in Eqs. (24)–(27), it is possible to eliminate the problem with global forgetting by localizing the effects of the leakage terms to the vicinity of the present operating point. The localized adaptive approach also decreases the required amount of online computation, since all parameters associated with zero elements of basis vectors are left unchanged. In addition, due to the inclusion in ρ_1 of R_{f_i} and R_{g_i}, which are local functions of the operating point, the m.s.s. bound can be shown to be significantly smaller than the bound derived from the previously existing approaches.

The σ-modification is an effective method to prevent the estimated parameters from increasing without bound; however, it does allow the estimated parameters to drift (toward $\theta_{f_i}^0$) in a bounded fashion when the $\bar{x}_i$ is small. Also, the guaranteed $\mathcal{L}_\infty$ property of $\bar{x}_i$ does not provide a useful bound on $\|\bar{x}\|$ that the designer can directly influence. Note in particular, that the theorem does not imply that (ultimately) $\underline{k}\|\bar{x}\|^2 < \bar{d} + \rho_1$.

5.2. Discussion

We have considered robust adaptation laws with localized forgetting using the σ-modification. Two other approaches (i.e., $\bar{x}$ modification and deadzone) will be discussed in Appendix 7.4 and 7.5, respectively. In Polycarpou (1996) and Jiang and Praly (1998), the adaptive bounding approach was introduced using the standard leakage approach that has global features. To remove the issue of global forgetting, localized leakage-based adaptation was proposed in Zhao et al. (2004) for simple scalar systems. In Zhao and Farrell (2006) those results are extended in various directions: second order systems, nonconstant g_1^o, unknown g_2, and alternative robust estimation approaches. Article (Zhao & Farrell, 2005) extended the localized adaptive bounding technique to a wider class of single input systems: nth order, and unknown g_i, $i = 1, \dots, n$; however, only deadzone with parameter projection modification was presented for robust adaptation. This paper studies nth order single input systems and compares alternative robust estimation approaches.

Note that the bounds of Eqs. (30), (54), and (60) although they look similar are subtly different. The $V(t)$ term of the bound of (60) is nonincreasing, since the Lyapunov function in the case of the deadzone can never increase. The $V(t)$ term in the right hand side of either (30) or (54) can increase (in a bounded fashion), since the Lyapunov functions in the leakage approaches can increase while the tracking error is suitably small.

6. Conclusions

We have considered robust, adaptive, approximation-based control design for the class of nth order uncertain strict-feedback nonlinear systems. A novel robust adaptive backstepping design procedure is proposed by incorporating locally learned adaptive bounding functions on the *inherent approximation errors*. This adds robustness to previous backstepping-based adaptive nonlinear controllers (see Krstic et al. (1995) and Polycarpou (1996)), where function approximation error was assumed to be zero. Furthermore, the complexity of calculating the time derivatives of the intermediate state commands for the backstepping approach (Khalil, 2002; Krstic et al., 1995) is addressed by the filtering techniques proposed in Farrell et al. (2004) and Farrell and Polycarpou (2006). By Lyapunov stability analysis, we have proved that the overall adaptive scheme guarantees the boundedness of both actual tracking errors and compensated tracking errors. Numerical simulations are provided in Appendix 7.4 to demonstrate the effectiveness of the proposed method.

Appendix. Supplementary data

Supplementary data associated with this article can be found, in the online version, at doi:10.1016/j.automatica.2008.02.013.

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