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ADAPTIVE CONTROL IN THE PRESENCE OF SATURATION NON-LINEARITY

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SUMMARY

Saturation non-linearities are perhaps the most commonly present type of non-linearities in dynamic systems. It is therefore important for the controller present to function satisfactorily in the presence of input saturation. In this paper we present an adaptive controller which leads to satisfactory performance in the presence of magnitude saturation of the control input. For both continuous time and discrete time plants, the adaptive controller is shown to result in global stability if the plant is open loop stable and minimum phase, and local stability otherwise. Robustness properties of the resulting adaptive controller are established. The performance is verified through experimental results obtained from adaptive control of a precision machine tool axis.

KEY WORDS adaptive control; saturation non-linearity; stability properties; applications

1. INTRODUCTION

In a control problem, ensuring that the control input into the system does not exceed a certain magnitude is of paramount importance. From a practical point of view this facilitates safety and reliability of the automatic controller; from a theoretical point of view it permits the design of linear methods to be applicable by maintaining the deviations of the system variables from an operating point to be small.

The results for adaptive control of linear time-invariant plants, for continuous time and discrete time, are well known.[1–3] In most of these results the control input is assumed to be unrestricted. We provide in this paper a summary of adaptive control methods which ensure that the system will have bounded solutions and meet the control objectives in the presence of input saturation.

In References 4–15 the effect of saturation limits was considered. Discrete time plants are considered with a direct control approach in References 4–7 and an indirect approach in References 8–14. Continuous time plants are treated in References 15 and 16. However, in all cases the plants are restricted to stable, except in Reference 12 where this condition is somewhat relaxed to include plants with multiple poles at the origin. In References 17 and 18, this problem was addressed in detail and the class of plants which can be adaptively stabilized was delineated. In this paper we expand this class further to include robustness results and non-minimum phase plants and report the experimental results obtained by applying the controller to a precision machine positioning problem, thereby providing a complete and comprehensive set of results pertaining to adaptive control in the presence of saturation non-linearity.

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In Section 2 we discuss the behaviour of adaptive controllers in the context of continuous time plants. The problem statement and the controller structure are described in Section 2.1. The stability and robustness properties are described in Sections 2.2 and 2.3 respectively. Extensions to non-minimum phase plants are described in Section 2.2.1. Motivated by application problems in mechanical systems, a special class of low-order adaptive control in the presence of input saturation is described in Section 2.4. This is applied to control a precise machine tool axis in Section 2.5 and experimental results are reported illustrating the improvement in performance that can be obtained using the proposed controllers. In Section 3 adaptive controllers of discrete time plants in the presence of input saturation are considered and their stability and robustness properties are derived.

2. CONTINUOUS TIME SYSTEMS

2.1. Statement of the problem

An nth-order plant with an input–output pair $\{u(\cdot), y_p(\cdot)\}$ with a transfer function

$$W_p(s) = k_p \frac{Z_p(s)}{R_p(s)} \tag{1}$$

is to be adaptively controlled when the control input is subjected to a magnitude constraint

$$|u(t)| \leq u_0 \tag{2}$$

where u_0 is a known constant. It is assumed that the order, relative degree and sign of k_p are known. We further make the following assumption regarding the plant zeros:

$$Z_p(s) = Z_{pm}(s) Z_{pn}(s) \tag{3}$$

where $Z_{pm}(s)$ is Hurwitz and $Z_{pn}(s)$ has zeros in the right half-plane. Our objective is to let the plant output $y_p(t)$ follow a reference trajectory $y_m(t)$ as closely as possible, where the latter is described as

$$y_m(t) = W_m(s) r(t) = \left(k_m \frac{Z_m(s)}{R_m(s)} \right) r(t) \tag{4}$$

$W_m(s)$ is asymptotically stable and has relative degree n^*. The model zeros are such that

$$Z_m(s) = Z_{mm}(s) Z_{pn}(s)$$

where $Z_{mm}(s)$ is an arbitrary Hurwitz polynomial. The reference input $r(t)$ is assumed to be a uniformly bounded, piecewise continuous function of time, with $|r(t)| \leq r_0$.

A differentiator-free controller is chosen as in Reference 1, and can be described by the differential equations

$$\begin{aligned}
&\dot{\omega}_1(t) = \Lambda \omega_1(t) + l u(t), \qquad \dot{\omega}_2(t) = \Lambda \omega_2(t) + l y_p(t) \\
&\omega^T(t) = [r(t), \omega_1^T(t), y_p(t), \omega_2^T(t)], \qquad \theta^T(t) = [k(t), \theta_1^T(t), \theta_0(t), \theta_2^T(t)] \\
&v(t) = \theta^T(t)\omega(t), \qquad u = \begin{cases} v(t) & \text{if } |v(t)| \leq u_0 \\ u_0 \operatorname{sgn}[v(t)] & \text{if } |v(t)| > u_0 \end{cases}
\end{aligned} \tag{5}$$

where k, θ_0: $\mathbb{R}^+ \to \mathbb{R}$, θ_1, ω_1: $\mathbb{R}^+ \to \mathbb{R}^{n-1}$, θ_2, θ_2: $\mathbb{R}^+ \to \mathbb{R}^{n-1}$, Λ is an asymptotically stable matrix and $\det(sI - \Lambda) = \lambda(s)$. It is well-known that a parameter θ^* exists such that if

$\theta(t)=\theta^*$, the transfer function of the plant together with the controller matches that of the model. With the controller in (5) and the parameter errors

$$\begin{gathered}\psi(t)=k(t)-k^*, \qquad \phi_0(t)=\theta_0(t)-\theta_0^*, \qquad \phi_1(t)=\theta_1(t)-\theta_1^* \\ \phi_2(t)=\theta_2(t)-\theta_2^*, \qquad \phi(t)=[\psi(t),\ \phi_1^{\mathrm T}(t),\ \phi_2^{\mathrm T}(t)]^{\mathrm T}\end{gathered} \tag{6}$$

the overall system becomes

$$\dot{x}(t)=A_{\mathrm{mn}}x(t)+b_{\mathrm{mn}}[\phi^{\mathrm T}(t)\omega(t)+k^*r(t)+\Delta u(t)] \tag{7}$$

where $\Delta u(t)\triangleq u(t)-v(t)$ and

$$\begin{gathered}A_{\mathrm{mn}}=\begin{bmatrix}A_{\mathrm p}+b_{\mathrm p}\theta_0^*h_{\mathrm p}^{\mathrm T} & b_{\mathrm p}\theta_1^{*\mathrm T} & b_{\mathrm p}\theta_2^{*\mathrm T}\\ l\theta_0^*h_{\mathrm p}^{\mathrm T} & \Lambda+l\theta_1^{*\mathrm T} & l\theta_2^{*\mathrm T}\\ lh_{\mathrm p}^{\mathrm T} & 0 & \Lambda\end{bmatrix}, \qquad b_{\mathrm{mn}}=\begin{bmatrix}b_{\mathrm p}\\ l\\ 0\end{bmatrix}\\ h_{\mathrm{mn}}=[h_{\mathrm p}^{\mathrm T},0,0]^{\mathrm T}, \qquad x=[x_{\mathrm p}^{\mathrm T},\omega_1,\omega_2]^{\mathrm T}\end{gathered} \tag{8}$$

If $e_1=y_{\mathrm p}-y_{\mathrm m}$, the error model is given by

$$e_1(t)=\frac{k_{\mathrm p}}{k_{\mathrm m}}W_{\mathrm m}(s)[\phi^{\mathrm T}(t)\omega(t)]$$

in the absence of the limiter. With the limiter, if $\Delta u=u-v$, it follows in a straightforward manner that

$$e_1(t)=\frac{k_{\mathrm p}}{k_{\mathrm m}}W_{\mathrm m}(s)[\phi^{\mathrm T}(t)\omega(t)+\Delta u(t)] \tag{9}$$

2.2. *Stability*

Boundedness of parameter errors. The adaptive law for adjusting $\phi(t)$ is first determined. First, we compensate for the $\Delta u(t)$ term by subtracting from the output error the signal

$$e_\Delta(t)=k_\Delta(t)W_{\mathrm m}(s)\Delta u(t) \tag{10}$$

where $k_\Delta(t)$ is a time-varying gain, adapted according to an adaptive law that is to be determined. Define $e_{u1}=e_1-e_\Delta$, so that

$$e_{u1}(t)=\frac{1}{k^*}W_{\mathrm m}(s)[\phi^{\mathrm T}(t)\omega(t)]+\psi_\Delta(t)W_{\mathrm m}(s)\Delta u(t) \tag{11}$$

where $\psi_\Delta(t)=1/k^*-k_\Delta(t)$. Then we determine the auxiliary error and augmented error as

$$\begin{gathered}e_2(t)=[\theta^{\mathrm T}W_{\mathrm m}(s)I-W_{\mathrm m}(s)\theta^{\mathrm T}(t)]\omega(t)\\ \varepsilon_{1u}(t)=e_{u1}(t)+k_1(t)e_2(t)\end{gathered} \tag{12}$$

It follows that

$$\varepsilon_{1u}=\frac{1}{k^*}\phi^{\mathrm T}(t)\xi(t)+\psi_\Delta(t)\xi_\Delta(t)+\psi_1(t)e_2(t)+\delta(t) \tag{13}$$

where $\xi(t)=W_{\mathrm m}(s)I\omega(t)$, $\xi_\Delta(t)=W_{\mathrm m}(s)\Delta u(t)$, $\psi_1(t)=k_1(t)-1/k^*$, and $\delta(t)$ is an exponentially decaying term due to initial conditions. The adaptive laws can be found by inspection as

$$\begin{aligned}
\dot{\theta}(t) &= \dot{\phi}(t) = -\operatorname{sgn}(k_p)\frac{\varepsilon_{1u}(t)\xi(t)}{1+\xi^{\mathrm{T}}(t)\xi(t)} \\
\dot{k}_1(t) &= \dot{\psi}(t) = -\frac{\varepsilon_{1u}(t)e_2(t)}{1+\xi^{\mathrm{T}}(t)\xi(t)} \\
\dot{k}_\Delta(t) &= -\dot{\psi}_\Delta(t) = \frac{\varepsilon_{1u}(t)\xi_\Delta(t)}{1+\xi^{\mathrm{T}}(t)\xi(t)}
\end{aligned} \tag{14}$$

giving $\varepsilon_{1u}, \phi, \psi_1, \psi_\Delta \in \mathcal{L}^\infty$ with

$$V(\phi, \psi_1, \psi_\Delta) = \frac{1}{2}\left(\frac{1}{k^*}\phi^{\mathrm{T}}\phi + \frac{1}{\gamma_1}\psi_1^2 + \frac{1}{\gamma_\Delta}\psi_\Delta^2\right) \tag{15}$$

as a Lyapunov function. Hence $\| \phi(t) \| \leqslant \phi_{\max}, \forall t \geqslant t_0$.

Boundedness of the state. If the plant is open loop stable and minimum phase, the global boundedness of the remaining signals in the closed loop system immediately follows without any restriction on the initial conditions of the plant or the controller. We show below for unstable plants and non-minimum phase plants that boundedness follows when the initial conditions and the parameter error lie within a bounded set.

Define P as the symmetric positive definite solution to the Lyapunov equation and ρ as the ratio between the maximum and minimum eigenvalues of P so that

$$A_{\mathrm{mn}}^{\mathrm{T}}P + PA_{\mathrm{mn}} = -Q, \qquad \rho = \sqrt{\left(\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}\right)}$$

Let $q_0 = \lambda_{\min}(Q)$, $p_b = \| b_{\mathrm{mn}}^{\mathrm{T}}P \|$, $\bar{\omega} = [\omega_1^{\mathrm{T}}, y_{\mathrm{p}}, \omega_2^{\mathrm{T}}]^{\mathrm{T}}$ and $\bar{\phi} = [\phi_1^{\mathrm{T}}, \phi_0, \phi_2^{\mathrm{T}}]^{\mathrm{T}}$. If

$$C = \begin{bmatrix} 0 & I & 0 \\ h_{\mathrm{p}}^{\mathrm{T}} & 0 & 0 \\ 0 & 0 & I \end{bmatrix}$$

we have $\bar{\omega} = Cx$. Also, since $\phi = [\psi, \bar{\phi}^{\mathrm{T}}]^{\mathrm{T}}$, we have $\| \bar{\phi}(t) \| \leqslant \phi_{\max}$ and $| k(t) | \leqslant | k^* | + \phi_{\max}$ for all $t \geqslant t_0$. Theorem 1 establishes the conditions under which the overall adaptive system will have bounded solutions.

Theorem 1

The system in equation (7) has bounded solutions if

$$\begin{aligned}
&\text{(i)} \quad x^{\mathrm{T}}(t_0)Px(t_0) < \lambda_{\min}(P)\left(\frac{2p_b u_0}{| 2p_b \| \theta^{*\mathrm{T}}C \| - q_0 |}\right)^2 \\
&\text{(ii)} \quad \sqrt{[V(t_0)]} < \frac{1}{\sqrt{[2|k^*|\lambda_{\max}(\Gamma)]}}\left(\frac{q_0 - |k^*|\left(\frac{r_0}{u_0}| 2p_b \| \theta^{*\mathrm{T}}C \| - q_0 |\right)}{\left(\rho\frac{r_0}{u_0}| 2p_b \| \theta^{*\mathrm{T}}C \| - q_0 |\right) + 2p_b \| C \|}\right)
\end{aligned} \tag{16}$$

Further,

$$x^{\mathrm{T}}(t)Px(t) \leqslant \lambda_{\min}(P)\left(\frac{2p_b u_0}{|2p_b\| \theta^{*\mathrm{T}}C\| - q_0}\right)^2, \qquad \forall t \geqslant t_0$$

and the output error e_1 is given by

$$|e_1(t)| = O(\sup_{\tau \leqslant t} \Delta u(\tau))$$

Proof. Define $W(x) = x^{\mathrm{T}}Px$ and

$$\mathscr{A} \triangleq \left\{ x \,\middle|\, \frac{2p_b(|k^*| + \phi_{\max})r_0}{q_0 - 2p_b\phi_{\max}\|C\|} < \|x\| < \frac{2p_b u_0}{|2p_b\|\theta^{*\mathrm{T}}C\| - q_0} \right\}$$

Boundedness can be established by considering the two cases (a) $|v| < u_0$ and (b) $|v| > u_0$ and showing in both cases that $\dot{W} < 0,\ \forall x \in \mathscr{A}$. It can also be shown that $\lim_{t\to\infty} \varepsilon_{1u}(t) = \lim_{t\to\infty} e_2(t) = 0$. Hence $|e_1(t)| = O[\sup_{\tau \leqslant t} \Delta u(\tau)]$. For very small initial conditions and small $r(t)$ it can also be shown that $e_1(t) \to 0$ as $t \to \infty$.[17] □

It should also be pointed out that the overall system becomes unstable if the open loop plant is unstable and the initial conditions are large.[17]

Remark 1

The local nature of the stability result when the plant has either unstable poles or zeros is to be expected, since the control input was restricted to lie within certain limits. In fact, even when the plant parameters are known, a bound on the control input implies that the initial condition of the state will have to be restricted to lie in a finite domain and, in addition, that the class of signals that the state can follow is correspondingly bounded. This condition can only be removed by restricting the open loop poles and zeros to lie in the left half plane. Theorem 1 extends this statement to the adaptive case, where the control parameter is also a state variable of the overall non-linear system and hence requires not only a bound on $|x(t_0)|$ but also the initial parameter error.

Remark 2

Condition (ii) implies that prior information regarding an upper bound on $|\theta^*|$ needs to be known so that q_0 and ρ can be chosen to make the right-hand side in condition (ii) positive. Since the feedback control input has a component due to the external input and a component due to the plant–model mismatch, it is not surprising that for a given parameter k^*, the scaling factor r_0/u_0 must be small. As the mismatch between the plant and the model is reduced, the factor r_0/u_0 can be increased.

2.2.1. Non-minimum phase plants. It is well-known that the presence of unstable zeros in a plant can lead to pole–zero cancellations in the right half-plane while adaptively adjusting the control parameter. In particular, in the controller given by (5), if at some time $t = t_1$ control parameter $\theta_1(t_1) = \bar{\theta}_1$ is such that

$$\bar{\theta}_1^{\mathrm{T}}(sI - \Lambda)^{-1}l = \frac{\bar{C}(s)}{\lambda(s)}, \quad \text{where } \det(sI - \Lambda) = \lambda(s)$$

and $\lambda(s) - \bar{C}(s)$ and $Z_{\mathrm{pm}}(s)$ have a common factor $\alpha(s)$, where $\alpha(s)$ has zeros in the right half-plane, then instability can occur if $|\dot{\theta}_1(t_1)|$ is arbitrarily small, since this implies that $\omega_1(t)$, $v(t)$, and therefore $\Delta u(t) \to \infty$ as $t \to \infty$. This can be established as follows. Suppose $\alpha(s) = (s - \alpha_0)\alpha_1(s)$, where $\alpha_0 > 0$. Since $\lambda(s) - \bar{C}(s)$ and $W_{\mathrm{m}}(s)$ have a common unstable factor, there exist initial conditions of the plant and the controller, at $t = t_1$, $\omega_1(t_1) = c_0 e^{\alpha_0 t_1}$, whereas $|e_{u1}(t_1)| < \varepsilon$, where ε is an arbitrary positive constant. It also follows that $e_2(t)$, $\varepsilon_{1u}(t)$ and $\xi(t)$ are arbitrarily small as well. As a result, $\dot{\phi}_1(t_1)$ is small, which leads to instability. Such instability occurs in the absence of input constraints as well. It therefore follows that in the parameter space there are points which correspond to unstable equilibrium points of the overall adaptive system, which can exist even if the plant is open loop stable. The restriction in the function $V(t_0)$ in Theorem 1 essentially avoids the inclusion of such equilibrium points, thereby leading to local stability.

We note therefore that unlike a minimum phase plant, even when it is open loop stable, the presence of unstable zeros leads to only local stability. Also, Theorem 1 establishes that for non-minimum phase plants, when the order, relative degree and sign of the high-frequency gain are known, the locations of the unstable zeros have to be known to determine a stable adaptive controller. This result can be extended to include small errors in the knowledge of unstable zeros.

The above argument also indicates that the adaptive controller in (5) leads to stability in the absence of input constraints only if the parameter error is constrained to lie in a compact set. This can be easily verified thus: The absence of input constraints essentially implies that $\Delta u(t)$ in (9) is zero. The same arguments as in Section 2.2 lead to boundedness of parameter errors. Proceeding along the same lines, the adaptive system equations are given by

$$\dot{x} = A_{\mathrm{mn}}x + b_{\mathrm{mn}}(\bar{\phi}^{\mathrm{T}}\omega + kr)$$

which implies

$$\dot{W}(x) \leq q_0 \|x\|^2 + 2P_b \|x\|^2 \phi_{\max} \|C\| + 2p_b \|x\|(|k^*| + \phi_{\max})r_0 < 0$$

if $\|\phi(t_0)\| < q_0/2p_b\|C\|$. It can further be shown that $e_1(t) \to 0$ as $t \to \infty$.

2.3. *Robustness*

We now consider the case where an additive uncertainty $\mu\Delta_{\mathrm{a}}(s)$, and a multiplicative uncertainty $\mu\Delta_{\mathrm{m}}(s)$ are present in the plant model, i.e.

$$y_{\mathrm{p}}(t) = W(s)u(t) = \{W_{\mathrm{p}}(s)[1 + \mu\Delta_{\mathrm{m}}(s)] + \mu\Delta_{\mathrm{a}}(s)\}u(t) \tag{17}$$

which satisfy the following assumptions.

(U1) $\Delta_{\mathrm{a}}(s)$ is an asymptotically stable, strictly proper transfer function.
(U2) $\Delta_{\mathrm{m}}(s)$ is an asymptotically stable transfer function.

With the controller and the same notation as in Section 2.2 the plant output is given by

$$y_{\mathrm{p}}(t) = W_{\mathrm{m}}(s)[r(t) + \phi^{\mathrm{T}}(t)\omega(t) + \Delta u(t)] + \mu v \tag{18}$$

where $\nu(t) = \Delta(s)u(t)$, $\Delta(s) = [1 - W_1(s)]W_m(s)\Delta_m(s) + [1 + W_2(s)W_m(s)]\Delta_a(s)$, and

$$W_1(s) = \theta_2^{*T}(sI - \Lambda)^{-1}l, \qquad W_2(s) = \theta_2^* + \theta_2^{*T}(sI - \Lambda)^{-1}l$$

The output error is therefore given by

$$e_1 = W_m(s)(\phi^T\omega + \Delta u) + \mu\nu \tag{19}$$

Since $|u| < u_0$ and $\Delta(s)$ is a proper and asymptotically stable transfer function, $\nu \in \mathcal{L}^\infty$ and therefore can be considered as a bounded disturbance with $|\nu(t)| < \nu_0$. The augmented error can be derived as in (13) as

$$\varepsilon_{1u} = \bar{\phi}^T\bar{\xi} + \mu\nu$$

where $\bar{\phi} = [\phi^T, \psi_\Delta, \psi_1]^T$ and $\bar{\xi} = [(\xi^T, e_2, \xi_\Delta]^T$.

Owing to the presence of an extra term $\mu\nu$ which is bounded, we modify the adaptive law in (14) as in the $|e_1|$-modification scheme as

$$\dot{\theta}(t) = -\frac{\varepsilon_{1u}\bar{\xi}}{1 + \bar{\xi}^T\bar{\xi}} - \gamma\frac{|\varepsilon_{1u}|\theta}{1 + \bar{\xi}^T\bar{\xi}}, \quad \gamma > 0 \tag{20}$$

This corresponds to adding a term of the form $-|\varepsilon_{1u}|\theta$ to the adaptive laws in (14). It can be shown that this ensures that the parameter error ϕ is bounded by following arguments similar to those in Reference 1. It can further be shown that the state variables of the adaptive system are bounded if $\mu\nu_0 < u_0$ and if the initial conditions of the plant, controller and control parameter lie within a certain set. This is summarized in Theorem 2.

Theorem 2

The system described by (1)–(5), (17) and (20) has bounded solutions if $\mu\nu_0 < u_0$, and

$$\text{(i)}\quad x^T(t_0)Px(t_0) < \lambda_{\min}(P)\left(\frac{2p_b|u_0 - \mu\nu_0|}{|2p_b\|\theta^{*T}C\| - q_0|}\right)^2$$

$$\text{(ii)}\quad \sqrt{[V(t_0)]} < \frac{1}{\sqrt{[2|k^*|\lambda_{\max}(\Gamma)]}}\left(\frac{q_0 - |k^*|\left(\rho\dfrac{|k^*|r_0 + \mu\nu_0}{u_0 - \mu\nu_0}|2p_b\|\theta^{*T}C\| - q_0|\right)}{\left(\rho\dfrac{r_0}{u_0 - \mu\nu_0}|2p_b\|\theta^{*T}C\| - q_0|\right) + 2p_b\|C\|}\right) \tag{21}$$

Boundedness is established as in Theorem 1 by considering the two cases (a) $|v| \leqslant u_0$, and (b) $|v| > u_0$, and showing that in both cases, if $W(x) = x^TPx$, $\dot{W} \leqslant 0$ for all x in an annulus $\mathcal{A}$. The annulus in this case is given by

$$\mathcal{A} \triangleq \left\{x \,\middle|\, \frac{2p_b(|k^*| + \phi_{\max})r_0 + \mu\nu_0}{q_0 - 2p_b\phi_{\max}\|C\|} < \|x\| < \frac{2p_b(u_0 - \mu\nu_0)}{|2p_b\|\theta^{*T}C\| - q_0|}\right\}$$

2.4. *A low-order controller with saturation*

The discussions above indicate that the order of the standard controller is directly proportional to that of the plant. Since the standard controller essentially functions like a pole placement controller and the goal of the adaptive controller is directed towards transfer function matching, it is not surprising that the assumptions involve the knowledge of the order as well as the relative degree and that the zeros lie in $\mathbb{C}^-$. In this subsection we develop a low-order adaptive controller for the case where the relative degree of the plant does not exceed two, when the control input is constrained to remain within certain limits.

The problem we shall consider is the adaptive control of a plant as given in (1) whose input is subjected to amplitude constraints as in (2). The plant output can be represented as

$$y_p = W_p(s)(v + \nu)$$

where $\nu = u - v$ can be considered as a disturbance introduced by saturation and v is the control input to be generated by the adaptive controller. The following assumptions are made regarding the plant transfer function $W_p(s)$.

(A1) The relative degree of $W_p(s)$ is less than or equal to two.
(A2) The sign of the high-frequency gain is known.
(A3) The plant zeros are in $\mathbb{C}^-$.

Without loss of generality we shall assume that $\mathrm{sgn}(k_p) > 0$. Assumptions (A1)–(A3) imply the existence of a feedback controller of the form

$$v = \theta_{0_c} \frac{s + z_c}{s + p_c} y_p, \qquad \theta_{0_c}, z_c, p_c > 0 \tag{22}$$

for which the closed loop transfer function $W_m(s)$ is asymptotically stable. Suppose that the control objective is for y_p to follow a reference trajectory y_{ref}. If $e_1 = y_p - y_{ref}$, we shall make the following assumption regarding a command signal $r(t)$ to ensure that $e_1(t) \to 0$ as $t \to \infty$.

(A4) There exists a bounded reference input $r(t) = \theta_m^{*T}\omega_m(t)$, where θ_m^* is an unknown parameter vector in $\mathbb{R}^m$ and $\omega_m(t)$ is a vector of measurable signals, such that $y_{ref}(t) = W_m(s)r(t)$.

If the control input is of the form

$$v = \theta^T\omega - e_1\bar{\omega}^T\bar{\omega} \tag{23}$$

where $\theta = [\theta_m^T, \theta_0, p]^T$, $\omega = [\omega_m^T, y_p, \omega_1]^T$, $\dot{\omega}_1 = -z_c\omega_1 + v$, $\dot{\bar{\omega}} = -a\bar{\omega} + \omega$, $\dot{\theta} = -e_1\bar{\omega}$ and a, $z_c > 0$, the underlying error model can easily be shown to be

$$e_1 = \overline{W}_m(s)(\phi^T\bar{\omega} + \bar{\nu}), \qquad \bar{\nu} = \frac{1}{s + a}\nu \tag{24}$$

where $\overline{W}_m(s) = W_m(s)(s + a)$ is strictly positive real and ϕ is the parameter error. If certain prior information regarding the locations of the poles and zeros of the plant transfer function is available, then the transfer function $W_m(s)$ can be determined exactly. For instance, when $n^* = 2$, this information corresponds to an upper bound on $|\sum p_i - \sum z_j|$, where p_i and z_j correspond to the ith pole and jth zeros respectively. In such a case the following theorem establishes the stability property of the simple adaptive controller in the presence of saturation.

Theorem 3

If the parameter θ in (23) is adjusted as

$$\dot{\theta} = -e_u\bar{\omega} \tag{25}$$

where $e_u = e_1 - e_\nu$ and $e_\nu = \overline{W}_m(s)\bar{\nu}$, then (i) all signals are globally bounded if $W_p(s)$ is asymptotically stable, and (ii) all signals remain bounded for an arbitrary $W_p(s)$ if the initial conditions of the overall adaptive system lie within a compact set. In both cases $|e_1(t)| = O[\sup_{\tau \leqslant t} |\nu(\tau)|]$.

Proof. From the definition of e_u, it follows that

$$e_u = \overline{W}_m(s)\phi^T\bar{\omega}$$

Since $\overline{W}_m(s)$ is strictly positive real, the adaptive law in (25) implies that e_u and θ are uniformly bounded. If $W_p(s)$ is asymptotically stable, then it immediately follows that e_1 and $\bar{\omega}$ are bounded, leading to global stability. For an arbitrary $W_p(s)$ the local stability result follows from arguments similar to those in Section 2.2. The properties of the adaptive law ensure that e_1 is of the same order as ν. □

2.5. *Application to position control of a precision machine tool axis*

The testbed used to evaluate the adaptive control algorithms in the presence of input saturation was designed to be representative of a typical axis in a precision machine tool such as a small lathe. Figure 1 depicts the major components of the prototype machine tool axis. The testbed consists of a sliding mass supported by four recirculating ball linear motion bearing carriages and driven by a DC brushless motor through a preloaded precision ballscrew. An off-the-shelf motor driver board handles phase commutation and current signals for the motor and windings. A magnetic encoder mounted to the side of the slide provides position feedback with a resolution of 1·27 μm.

As with almost all mechanical systems, certain components of the prototype machine tool axis do not behave in a perfectly linear fashion. One reason is the non-linear damping of the bearing carriages. This introduces uncertainties in the plant model, necessitating adaptive control. Another non-linearity is input saturation, since power limits exist for all actuators. Voltage command signals are sent to the DC motor driver boards from a DSP card and the DSP's DAC (digital-to-analogue converter) limits the command signals to ±3 V. This implies that adaptive control has to be carried out in the presence of input saturation. The controller's task is to force the slide to follow a prescribed trajectory accurately.

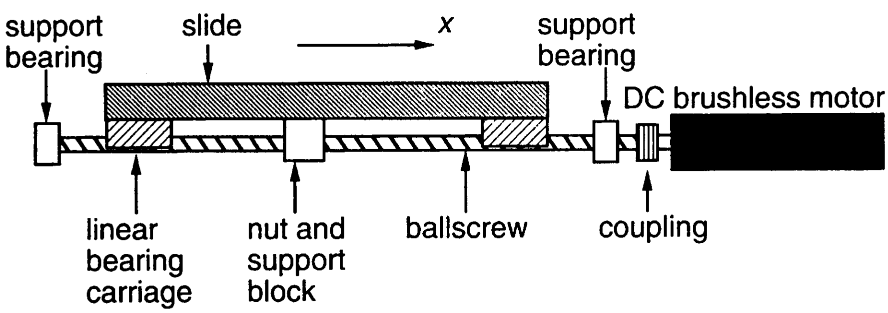

Figure 1. Principal components of precision machine tool axis

The prototype machine tool axis can be modelled as a linear plant for positioning accuracy greater than about 5–10 μm[19]. The equation of motion of the slide is given by

$$a_1\ddot{x}(t) + a_2\dot{x}(t) = u(t) \tag{26}$$

where the two states of the system are the slide position (x) and velocity ($\dot{x}$); u is the voltage command signal to the motor driver board, $\ddot{x}$ is the slide acceleration and a_1 and a_2 are parameters resulting from physical properties such as mass, inertia, damping coefficient, etc. The transfer function from the input u to the output x is of relative degree two, with no finite zeros, and has a positive high-frequency gain, with a magnitude restriction on the plant input due to the DAC voltage limit. Therefore the adaptive controller SW to developed in the previous subsection is directly applicable to this problem. Three different control algorithms were used to achieve position control with (a) a PID controller, (b) an adaptive controller without taking into account input saturation (i.e. e_ν was set to zero) and (c) the adaptive controller modified to accommodate the presence of saturation as discussed in Section 2.4. In case (a) the PID gains were carefully selected so as to yield the best possible performance for nominal plant conditions. The resulting performances are shown in Figures 2(a)–2(c), respectively, the case

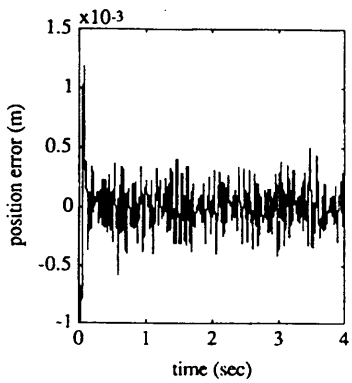

Figure 2(a). Position error for PID control with $m = 22$ kg and $x_d(t) = 0{\cdot}01\sin(2\pi t)$

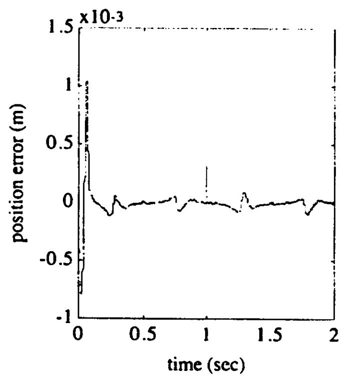

Figure 2(b). Position error for adaptive control (without saturation) with $m = 22$ kg and $x_d(t) = 0{\cdot}01\sin(2\pi t)$

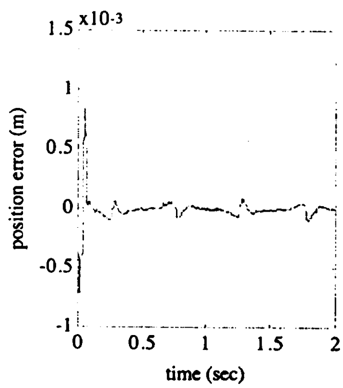

Figure 2(c). Position error for adaptive control (with saturation) with $m = 22$ kg and $x_d(t) = 0{\cdot}01\sin(2\pi t)$

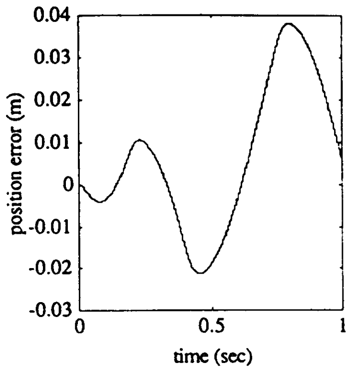

Figure 3(a). Position error for PID Control with $m = 162$ kg and $x_d(t) = 0{\cdot}01\sin(2\pi t)$

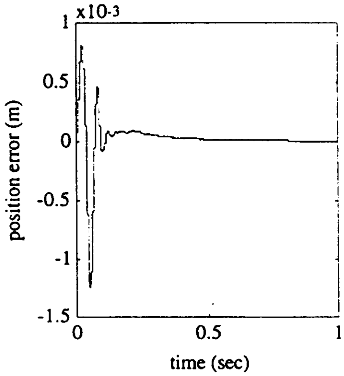

Figure 3(b). Position error for adaptive control (without saturation) with $m = 162$ kg and $x_d(t) = 0{\cdot}01\sin(2\pi t)$

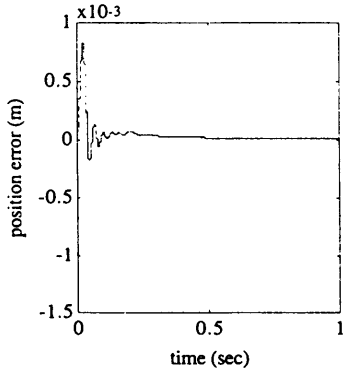

Figure 3(c). Position error for adaptive control (with saturation) with $m = 162$ kg and $x_d(t) = 0{\cdot}01\ \sin(2\pi t)$

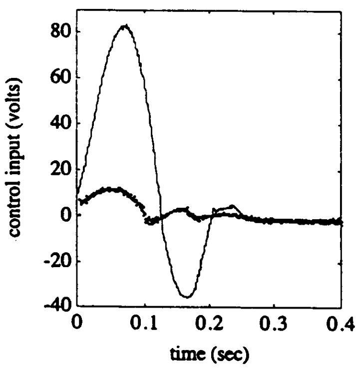

Figure 4. Calculated control inputs for adaptive control (with (*) and without saturation) with $m = 162$ kg and $x_d(t) = 0{\cdot}01\sin(2\pi t)$

when the prescribed trajectory x_d that x was required to follow was a sinusoid. The figures clearly show that the output error is much smaller with adaptive controllers and is improved further when the saturation is taken into account explicitly in the adaptive algorithm. The robustness of these algorithms was evaluated by adding a load of 140 kg to the slide (initial mass of the slide, 22 kg) (see Figures 3(a)–3(c)), indicating the superior performance provided by the adaptive controllers. The control inputs obtained with the additional mass and these controllers, without and with the saturation included in the design, over a period of 0·2 s are shown in Figure 4. The transients indicate the improvement that can be obtained with our proposed adaptive controller, which explicitly took into account the presence of saturation in the design of the adaptive law in (25). The reader is referred to Reference 20 for further details.

3. DISCRETE TIME SYSTEMS

In this section we include a brief discussion to indicate that adaptive controllers of discrete time systems behave satisfactorily in the presence of input saturation. The main results are summarized in two theorems without proofs and the reader is referred to Reference 18 for further details.

3.1. *Statement of the problem*

The plant to be controlled is described by

$$A(q^{-1})y(t) = q^{-d}B(q^{-1})u(t) \tag{27}$$

where q^{-1} is the backward shift operator, d stands for pure time delay, $A(q^{-1})$ and $B(q^{-1})$ are coprime polynomials and the coefficients of $A(\cdot)$ and $B(\cdot)$ are assumed to be unknown. The control input is such that $|u(t)| \leqslant u_0$ for all $t \geqslant t_0$ and the objective is for the plant output $y(t)$ follow some desired trajectory $y^*(t)$ as closely as possible. Since $A(\cdot)$ and $B(\cdot)$ are coprime, Equation (27) can be rewritten as[2]

$$u(t) = \bar{\phi}^{\mathrm{T}}(t)\theta_0 \tag{28}$$

where

$$\theta_0 = [\theta_{0y}^{\mathrm{T}}, \theta_{0u}^{\mathrm{T}}, \theta_{0r}^{\mathrm{T}}]^{\mathrm{T}}, \qquad \theta_{0y} = [\alpha_0', \ldots, \alpha_{n-1}']^{\mathrm{T}}, \qquad \theta_{0u} = [\beta_1', \ldots, \beta_{m+d-1}']^{\mathrm{T}}, \qquad \theta_{0r} = \left[\frac{1}{\beta_0}\right]$$

$$\bar{\phi}(t) = [-y(t), \ldots, -y(t-n+1), -u(t-1), \ldots, -u(t-m-d+1), y(t+d)]^{\mathrm{T}}$$

$$\alpha_i' = \frac{\alpha_i}{\beta_0}, \quad i = 1, \ldots, n-1, \qquad \beta_j' = \frac{\beta_j}{\beta_0}, \quad j = 1, \ldots, m+d-1$$

Since θ_0 is unknown and the control is subjected to saturation, we use the controller

$$\begin{aligned} v(t) &= \phi'^{\mathrm{T}}(t)\theta(t) \\ u(t) &= \begin{cases} v(t) & \text{if } |v(t)| \leqslant u_0 \\ u_0 \operatorname{sgn}[v(t)] & \text{if } |v(t)| > u_0 \end{cases} \\ \phi'^{\mathrm{T}}(t) &= [-y(t), \ldots, -y(t-n+1), -u(t-1), \ldots, -u(t-m-d+1), y^*(t+d)] \end{aligned} \tag{29}$$

where $\theta(t)$ is a parameter estimate of θ_0 adjusted according to the adaptive law

$$\theta(t) = \theta(t-1) + \frac{a\bar{\phi}(t-d)}{c + \bar{\phi}^{\mathrm{T}}(t-d)\bar{\phi}(t-d)}[u(t-d) - \bar{\phi}^{\mathrm{T}}(t-d)\theta(t-1)], \quad 0 < a < 2, \quad c > 0 \tag{30}$$

and the estimate of θ_{0r} is kept away from zero by using standard projection methods. Standard results from Reference 2 can be used to show that $\|\theta(t)\|$ is bounded.

3.2. Stability

The stability of the closed loop system is provided by considering two cases separately.

Case (a). All poles of the plant are inside the unit circle.

Global boundedness follows immediately if the plant is asymptotically stable, since the boundedness of the plant input u leads directly to boundedness of $\bar{\phi}$.

Case (b). At least one pole of the plant is on or outside the unit circle.

Let

$$\begin{gathered}
\text{Let } \theta(t) = [\theta_y^{\mathrm{T}}(t),\ \theta_u^{\mathrm{T}}(t),\ \theta_r(t)]^{\mathrm{T}} \\
\tilde{\theta}_y(t) = \theta_y(t) - \theta_{0y}, \qquad \tilde{\theta}_u(t) = \theta_u(t) - \theta_{0u}, \qquad \tilde{\theta}_r(t) = \theta_r(t) - \theta_{0r} \\
\tilde{\theta}^{\mathrm{T}}(t) = [\tilde{\theta}_y(t),\ \tilde{\theta}_u^{\mathrm{T}}(t),\ \tilde{\theta}_r(t)]^{\mathrm{T}} \\
\psi_y(t) = [-y(t+d-1),\ y(t+d-2), \ldots, -y(t-n+1)]^{\mathrm{T}} \\
\psi_u(t) = [-u(t-1),\ -u(t-2), \ldots, -u(t-m-d+1)]^{\mathrm{T}} \\
n_d = n+d-1, \qquad m_d = m+d-1, \qquad |\, y^*(t)\,| \leqslant y^*_{\max} \\
P \triangleq \mathrm{diag}(n_d,\ n_d-1, \ldots, 1)
\end{gathered} \tag{31}$$

From the definition of Case (b) it can be shown that

$$|\beta_0|\,\|\theta_{0y}\| - \frac{1}{\sqrt{n_d}} > 0 \tag{32}$$

The main result is stated in the following theorem.

Theorem 4

In Case (b), The adaptive system defined by (28)–(30) has bounded solutions if

$$\text{(i)} \quad \|\tilde{\theta}(t_0)\| \leqslant \frac{(1 - \sqrt{m_d}\,\|\theta_{0u}\|) - n_d \dfrac{y^*_{\max}}{|\beta_0|\,u_0}\left(|\beta_0|\,\|\theta_{0y}\| - \dfrac{1}{\sqrt{n_d}}\right)}{\sqrt{n_d}\,|\beta_0|(1 - \sqrt{m_d}\,\|\theta_{0u}\|) + n_d\left(\sqrt{m_d} + \dfrac{y^*_{\max}}{u_0}\right)\left(|\beta_0|\,\|\theta_{0y}\| - \dfrac{1}{\sqrt{n_d}}\right)}$$

$$\text{(ii)}\quad \psi_y^{\mathrm{T}}(t_0 - d + 1)P\psi_y(t_0 - d + 1) \leqslant \left(\frac{u_0(1 - \sqrt{m_d}\,\|\theta_{0u}\|}{\|\theta_{0y}\| - \dfrac{1}{\sqrt{n_d}\,|\beta_0|}}\right)^2$$

We note that since equations (31) hold in Case (b), it immediately follows that the upper bounds in (i) and (ii) are finite. For them to be positive, in addition to $y^*_{\max}/u_0$ being small, we also require that

$$\sqrt{m_d}\,\|\theta_{0_u}\| < 1 \tag{33}$$

Inequality (32) essentially implies that if a transfer function

$$W(z) = \frac{\beta_1' z^{m_d - 1} + \beta_2' z^{m_d - 2} + \cdots + \beta_{m_d - 1}}{z^{m_d}}$$

then for all z

$$|W(z)| \leqslant \sqrt{m_d}\,\|\theta_{0u}\| < 1 \tag{34}$$

In the case where $d = 1$, since $\beta(q-1) = B(q^{-1})$, inequality (33) implies that the plant zeros must lie within the unit circle but places no restrictions on the plant poles. When $d > 1$, since $\beta = FB$, inequality (33) requires that both $F(q^{-1})$ and $B(q^{-1})$ be stable polynomials, which restricts the location of the plant poles and zeros. It should be noted that this restriction arises because we are attempting to accomplish d-step-ahead predictive control for arbitrary plants with a bounded control input. It is therefore not surprising that as the time delay is increased, the stability result places further demands on the locations of the plant poles and zeros. Less restrictive conditions on the plant parameters can be derived by using a model reference controller as in Section 2. Also, an $\mathscr{L}^1$-norm instead of the Euclidean norm used in the derivations here will lead to tighter bounds.

Steady state behaviour: Once the boundedness of all signals has been established, bounds on the tracking error can be derived. We can also show that $\Delta\tilde{\theta}(t)$ tends to zero as $t \to \infty$. Since $\bar{\phi}(t)$ is bounded,

$$\bar{\phi}^{\mathrm{T}}(t-d)\tilde{\theta}(t-1) \to 0 \quad \text{as } t \to \infty$$

Since $v(t) = \phi'^{\mathrm{T}}(t)\theta(t)$, this in turn implies that

$$|[\bar{\phi}^{\mathrm{T}}(t-d)]\theta(t-1) + \Delta u(t-d)| \to 0 \quad \text{as } t \to \infty$$

If the tracking error is defined as $e(t) = y(t) - y^*(t)$, from the definition of ϕ' and $\bar{\phi}$ and from the fact that $\theta_r(t)$ is bounded away from zero, it follows that

$$|e(t)| = O(\sup_{\tau \leqslant t} |\Delta u(\tau)|) \tag{35}$$

It can be shown that if $\|\tilde{\theta}(t_0)\| \leqslant \varepsilon_1$, $\|\psi_u(t_0)\| \leqslant \varepsilon_2$, $\|\psi_y(t_0)\| \leqslant \varepsilon_3$ and $|r(t)| < \varepsilon_4 u_0$, then for sufficiently small ε_i, $i = 1, \ldots, 4$, $|v(t)| \leqslant u_0$, $\forall t \geqslant t_0$. Inequality (34) in turn implies that $\lim_{t\to\infty} e(t) = 0$.

The results of this subsection indicate that local stability of a class of discrete time plants can be adaptively controlled even in the presence of input constraints. The class of plants is somewhat smaller than that in the continuous time case. We did not modify the adaptive law as in Section 2.2 but used the input error formulation to ensure bounded parameters.

3.3. Robustness

We now consider the adaptive control problem when the plant has modelling errors, so that equation (27) is a reduced-order design model. As in Reference 21, we assume that the actual plant output can be described as

$$A(q^{-1})y(t) = q^{-d}B(q^{-1})u(t) + \eta(t)$$

where $\eta(t)$ is the truncation error due to low-order modelling and bounded disturbance, and is assumed to be of the form

$$\eta(t) = A(q^{-1})[\Delta(q^{-1})u(t-1) + \eta_b(t)]$$

where η_b is a bounded disturbance with $|\eta_b(t)| \leq \eta_0$ and $\Delta(q^{-1})$ is an additive mismatch. We assume that the operator $\Delta(\cdot)$ is such that

$$|\Delta(q^{-1})x(t)| \leq d_o |x(t)|, \quad \forall x(\cdot) \in \mathbb{R} \tag{36}$$

and d_o is a finite constant.

The problem is to adaptively control such a plant in the presence of-parametric uncertainties and input saturation so that all the signals in the closed loop system remain bounded and the output y tracks a desired trajectory y^* as closely as possible. The plant equations can be rewritten as

$$y(t+d) = \alpha(q^{-1})y(t) + \beta(q^{-1})u(t) + \nu(t) \tag{37}$$

where the polynomials $\alpha(\cdot)$ and $\beta(\cdot)$ are defined as in Section 3.1, and $\nu(t)$ satisfies the relation

$$\nu(t) = [\alpha(q^{-1}) - q^d][\Delta(q^{-1})u(t-1) + \eta_b(t)] \tag{38}$$

Since $\Delta(q^{-1})$ satisfies equation (35) and $\alpha(q^{-1})$ has bounded coefficients, it follows that $|\nu(t)| \leq \nu_0$, a finite positive constant. Hence relation (36) can be expressed as

$$u(t) = \bar{\phi}^{\mathrm{T}}(t)\theta_0 - \bar{\nu} \tag{39}$$

where $\bar{\phi}$ is defined as in Section 3.2 and $|\bar{\nu}(t)| = |\nu/\beta_0| \leq \nu_0/|\beta_0|$. The problem reduces to establishing robustness in the presence of the bounded disturbance term $\bar{\nu}$.

The robust adaptive control problem in the presence of bounded disturbances has been investigated in depth in both the continuous time and discrete time cases. A constrained parameter estimation scheme as in Reference 21 can be used in the presence of input constraints as well. Such a scheme is described below:

$$\begin{aligned}
\bar{\theta}(t) &= \bar{\theta}(t-1) + \frac{a\bar{\phi}(t-d)[u(t-d) - \bar{\phi}^{\mathrm{T}}(t-d)\bar{\theta}(t-1)]}{c + \bar{\phi}^{\mathrm{T}}(t-d)\bar{\phi}(t-d)} \\
\theta(t) &= \begin{cases} \bar{\theta}(t) & \text{if } \|\bar{\theta}\| \leq \theta^*_{\max} \\ \dfrac{\bar{\theta}}{\|\bar{\theta}\|}\theta^*_{\max} & \text{if } \|\bar{\theta}\| \geq \theta^*_{\max} \end{cases}
\end{aligned} \tag{40}$$

where $\theta^*_{\max}$ is a known upper bound on θ_0. The adaptive law in (39) trivially implies that the parameter error $\|\tilde{\theta}(t)\| \leq \tilde{\theta}_{\max} = 2\theta^*_{\max}$.

As in the disturbance-free case, the controller is chosen as in (29). The robustness result is summarized in Theorem 5.

Theorem 5

The adaptive system given by (36), (39) and (29) has bounded solutions if

$$\text{(i)}\quad \tilde{\theta}_{\max} \leqslant \frac{1-\sqrt{m_d}\,\|\theta_{0u}\| - \dfrac{\bar{v}_0}{u_0} - n_d\left(\dfrac{\bar{v}_0}{u_0} + \dfrac{y^*_{\max}}{|\beta_0|\,u_0}\right)\left(|\beta_0|\,\|\theta_{0y}\| - \dfrac{1}{\sqrt{n_d}}\right)}{\sqrt{n_d}\,|\beta_0|\left(1-\sqrt{m_d}\,\|\theta_{0u}\| - \dfrac{\bar{v}_0}{u_0}\right) + n_d\left(\sqrt{m_d} + \dfrac{y^*_{\max}}{u_0}\right)\left(|\beta_0|\,\|\theta_{0y}\| - \dfrac{1}{\sqrt{n_d}}\right)}$$

$$\text{(ii)}\quad \psi_y^{\mathrm{T}}(t_0-d+1)P\psi_y(t_0-d+1) \leqslant \left(\frac{u_0(1-\sqrt{m_d}\,\|\theta_{0u}\|)}{\|\theta_{0y}\| - \dfrac{1}{\sqrt{n_d}\,|\beta_0|}}\right)^2$$

where $\psi_y(t)$, θ_{0u}, θ_{0y}, n_d, m_d, $y^*_{\max}$ and P are defined as in (31).

CONCLUSIONS

In this paper we have provided complete and comprehensive results on adaptive control in the presence of input saturation. Both continuous time and discrete time plants were considered. We showed that adaptive control results in global stability for plants with stable zeros and poles, and stability provided that the system initial conditions lie within a bounded set, by including the presence of input saturation in the derivation of the adaptive law. Robustness properties of the resulting adaptive controller in the presence of unmodelled dynamics were derived. A stable low-order controller is also developed for a class of plants in the presence of input saturation. Experimental results derived while applying this controller to a precision machine tool axis are included, indicating the improvement in performance that can be obtained with the adaptive controllers proposed in this paper.

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