ADAPTIVE CONTROL FOR SYSTEMS WITH HARD SATURATION

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Abstract

The augmented error signal concept, recently introduced, can be useful in designing model reference adaptive systems for plants with inputs which are subject to hard saturation. Liapunov's Direct Method is used as the basis for these designs. Figure 1 shows the configuration for the types of systems considered. The adaptive control acts to null the augmented error, and, in the process, also assures that the true error between plant and model approaches zero.

Operation of the adaptive controller with the hard saturating element present relies on the auxiliary input, w, absorbing excess control signal, u_o; i.e. when u_o exceeds M, the difference u_o-M goes into generating the signal w. This condition cannot persist in the steady state, however, since a requirement for $e \to o$ is that $w \to o$. Therefore, the linear part of the plant, the level of M, and the amplitude of the reference input r must all be such that u_o will not exceed M in steady state.

The design proceeds using the same equations reported in [1]. These are repeated here for convenience.

A dynamic system (plant) is described by the nonlinear, nonautonomous differential equation

$$D_p(p)x(t) = D_u(p)u(t) + cf(x,t) \qquad (1)$$

where x(t) and u(t) are the plant output and input respectively, f(x,t) is a nonlinear time varying function of known form, p is the operator d/dt (the notation $\dot{x}$ is also used occasionally to indicate dx/dt) $D_p(p) = p^n+a_1p^{n-1}+a_2p^{n-2}+\ldots a_n$, $D_u(p) = b_0p^m+b_1p^{m-1}+\ldots b_n$, and the coefficients a_i, b_j and c are constant (or slowly varying) and unknown. The term cf(x,t) may in fact be a sum of terms, e.g., $\sum_{i=1}^{q} c_i f_i(x,t)$, where the c_i's are unknown and f_i's are known. Each term in the design is handled exactly the same. There is no loss in generality, therefore, in carrying only the one term.

In (1) it is assumed that (a) the function f satisfies the continuity conditions necessary for solutions of (1) to exist and be unique, (b) all roots of $D_u(s)$ are in the open left half plane, (c) $b_o \neq 0$, and (d) $m \leq n-1$.

The design objective is to have the plant output x follow a model output x_m, where the model is defined by the equation

$$D_m(p)x_m(t) = K_0r(t) + g(\underline{x}_m, r, t) \qquad (2)$$

where $D_m(p) = p^n + a_{d1}p^{n-1} + a_{d2}p^{n-2} + \ldots a_{dn}$, r is the model reference input, g is a nonlinear time varying function with the smoothness properties required to ensure existence and uniqueness of solutions to (2) and $\underline{x}_m$ is the model state vector. As was mentioned in connection with the nonlinear term f in (1), g may also be replaced by a linear combination of nonlinear terms. In (2), r may be $r = D_r(p)\, r^1$ where $D_r(p) = p^m + R_1p^{m-1} + \ldots R_m$.

The augmented error signal, augmented by the addition of the signal y, is defined as

$$\eta(t) = e(t) + y(t) \qquad (3)$$

where y is the output of the error augmenting filter defined by

$$D_m(p)y = D_w(p)\, w(t) \qquad (4)$$

where $D_w(p) = p^{n-1} + c_1p^{n-2} + c_3p^{n-3} + \ldots c_n$, w(t) is an auxiliary system input which is to be determined, and coefficients c_i are chosen such that the transfer function $D_w(s)/D_m(s)$ is a positive real function of s.

A differential equation for e is formed by subtracting (1) from (2) to obtain

$$D_m(p)e = K_0r + g - D_u(p)u - cf + D_\Delta(p)\, x \qquad (5)$$

where $D_\Delta(p) = D_p(p) - D_m(p) = \Delta a_1p^{n-1} + \Delta a_2p^{n-2} + \ldots \Delta a_n$ and $\Delta a_i = a_i - a_{di}$ for i = 1 to n.

By adding (4) and (5), one obtains the differential equation for the augmented error signal

η. It is

$$D_m(p)\eta = K_0 r + g - D_u(p)u - cf + D_\Delta(p)x + D_w(p)w \qquad (6)$$

Equation (6) is the starting equation for the synthesis procedure. The design problem is to choose u and w (both independent of derivatives of x) in a way which guarantees that $e \to 0$ and $w \to 0$ (hence $y \to 0$) as $t \to \infty$.

Consider one of the special cases investigated in [1] where $m = 0$, $D_\Delta(p) = 0$, $D_u(p) = b_o$, $c = 0$, and $g = 0$. Then (6) becomes

$$\dot{\underline{\eta}} = A\underline{\eta} + \underline{d}\,(-b_0 z_0 + K_0 z_1 + w) \qquad (7)$$

where z_o and z_1 are defined by

$$D_w(p)z_0 = u \quad ; \quad D_w(p)z_1 = r \qquad (8)$$

In (7), let $w = (1 + k_1(t))w_1$ and take the linear combination $z_o + w_1$ to be

$$z_0 + w_1 = k_1(t)z_1 \qquad (9)$$

If (9) is substituted into (7), then (7) may be put in matrix vector form

$$\dot{\underline{\eta}} = A\underline{\eta} + \underline{d}\,(\delta_1(t)z_1 + \delta_2(t)w_1) \qquad (10)$$

where $\delta_1 = (-b_0 k_1(t) + K_0)$ and $\delta_2(t) = (k_2(t) + b_0 + 1)$.
In (10) $\eta_1 = \eta$, A is in controllable canonical form, and the elements of d are functions of the a_{di}'s and c_i's.

As shown in [1], the adaptive laws

$$\dot{k}_1 = B_1\, \eta\, z_1 \text{ and } \dot{k}_2 = -B_2\, \eta\, w_1 \qquad (11)$$

cause $\underline{\eta} \to \underline{0}$. In (11) B_1 and B_2 are arbitrary positive constants.

Signals u and w_1 are found by applying the operator $D_w(p)$ to both sides of (9) to get

$$D_w(p)(z_0 + w_1) = u + D_w(p)w_1 = k_1 r + \sum_{i=0}^{n-2} D_{wi}(p)((pk_1)(P^{n-2-i}z_1)) \qquad (12)$$

In plants without saturation, the signals u_0, u and w_1 are chosen to satisfy

$$u_0 = u = k_1 r \quad ;$$

$$D_w(p)w_1 = \sum_{i=0}^{n-2} D_{wi}(p)((\dot{k}_1)(p^{n-2-i}z_1)) \qquad (13)$$

This choice assures that $e \to 0$. In (13)

$$D_{w0}(p) = 1,\ D_{w1}(p) = (p+c_1),\ D_{w2}(p) = p^2 + c_1 p + c_2,\ \ldots,\ D_{w(n-2)}(p) = p^{n-2} + c_1 p^{n-3} + c_2 p^{n-4} + \ldots c_{n-2} \qquad (14)$$

In plants with saturation, a modification is required in the design equations (13), since $|u| \le M$. The following is one modification which can be made:

$$u_o = k_o r$$

$$u = \begin{cases} +M & \text{if} \quad u_o > +M \\ u_o & \text{if} \quad -M \le u_o \le +M \\ -M & \text{if} \quad u_o < -M \end{cases} \qquad (15)$$

$$D_w(p)w_1 = \sum_{i=0}^{n-2} D_{wi}(p)(\dot{k}_1)(p^{n-2-i}z_1) + u_o - u$$

This choice for u and w_1 satisfies (12) and guarantees that the overall adaptive design is stable, i.e. $\eta \to 0$. Simulation results show that, in many cases, $e \to 0$ in steady state provided system parameters are such that this can be achieved, i.e. $|u| > M$ is not required to have $x = x_m$. Theoretical results which can predict conditions under which $e \to o$ can be guaranteed are not available. Simulation results for stable and unstable plants operating under a variety of conditions are discussed in the paper.

REFERENCE

[1] Monopoli, R.V., "Model Reference Adaptive Control with an Augmented Error Signal," IEEE TAC Vol. AC-19, No. 5, Oct. 1974, pp. 474-484.

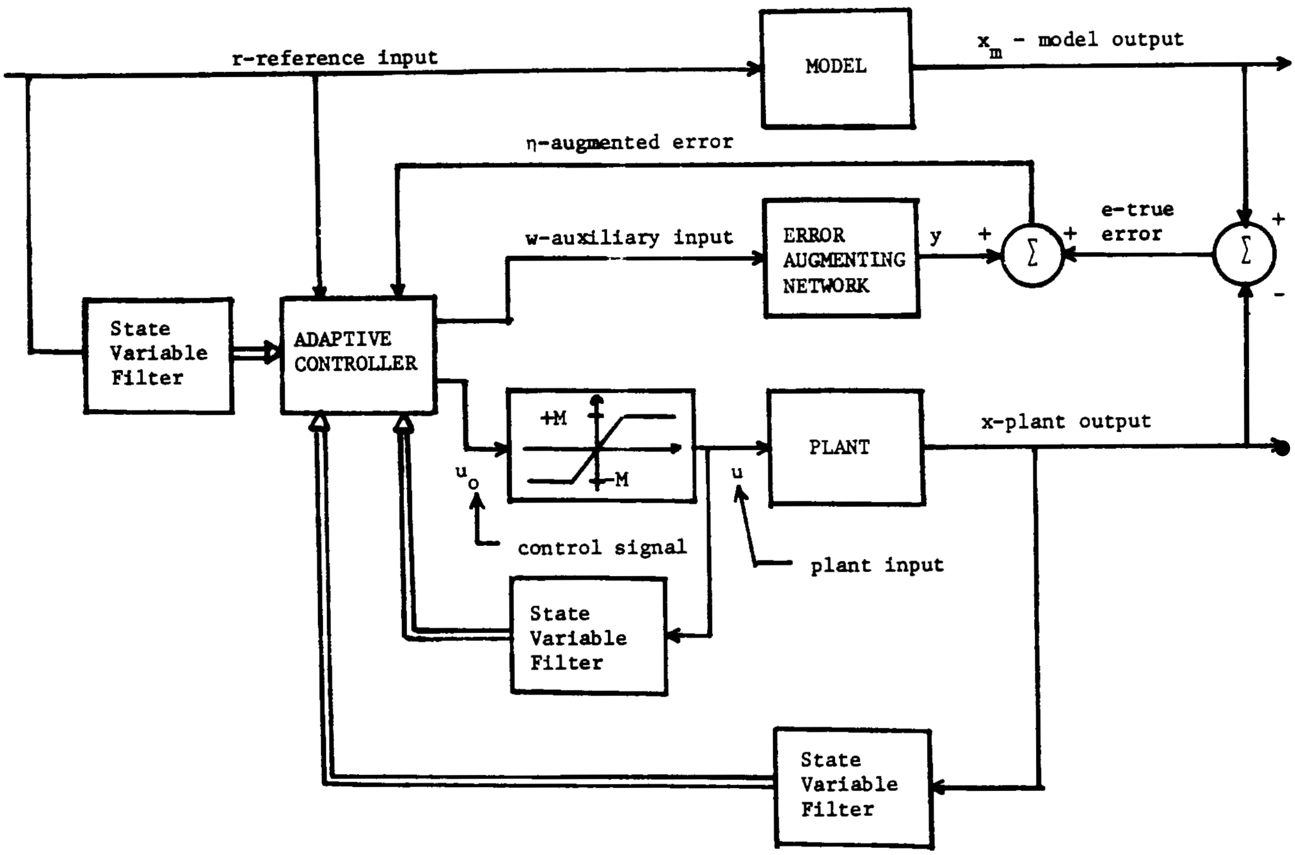

Figure 1 - System Configuration