ISIS, Vanderbilt University

Compositional Control Design and Analysis

Passivity

$\boldsymbol{u(t)y(t) = \dot{S}(t) + \dot{D}(t)}$	**Power balance equation**
$\boldsymbol{< u, y >_t = S(t) + D(t) - S(0) - D(0)}$	**Energy balance equation**
$\boldsymbol{\dot{D}(t) \geq 0}$	**Dissipation condition**
$\boldsymbol{< u, y >_t \geq S(t) - S(0)}$	**Passivity condition**

$\boldsymbol{u(t)}$	**Input signal**
$\boldsymbol{y(t)}$	**Output signal**
$\boldsymbol{S(t)}$	**Stored energy**
$\boldsymbol{D(t)}$	**Dissipated energy**
$\boldsymbol{\|y_T\|_2^2}$	**Energy produced by the component so far (output) in interval T.**
$\boldsymbol{\|u_T\|_2^2}$	**Energy received by the component so far (input) in interval T.**
$\boldsymbol{< y, u >_T}$	**Correlation between input and output sample values in interval T. This measures dissipation.**
$\boldsymbol{a}$	**Real-valued sector lower bound.**
$\boldsymbol{b}$	**Real-valued sector upper bound.**

Passive systems exhibit robustness to time delays and quantization errors, preserving stability.

Parallel and feedback interconnections of passive systems are passive.

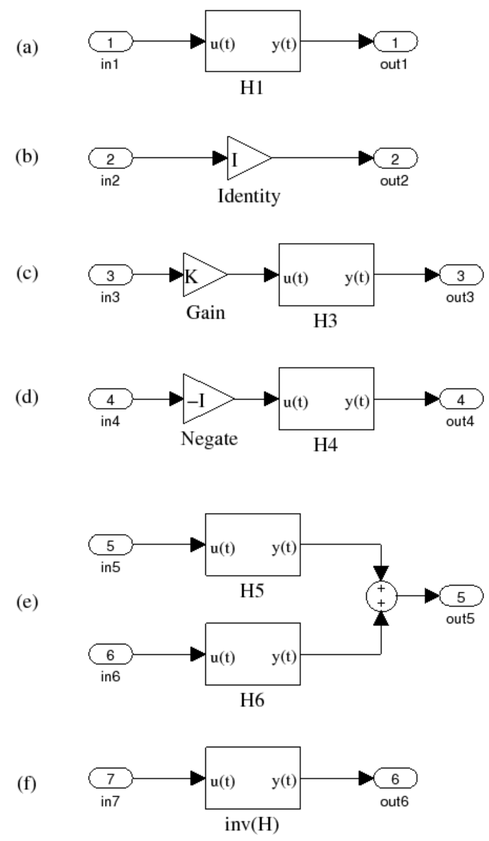
Sector Analysis

$$\|y_T\|_2^2 - (a+b) < y, u >_T + ab\|u_T\|_2^2 \leq 0$$

Conic systems are a superset of passive systems ([0,b]). Stability is easy to determine using gain conditions.

Sector analysis follows simple compositional rules.

1. I is in $[1, 1]$ (Fig. 1 (b))
2. kH is in $[ka, kb]$ (Fig. 1 (c))
3. $-H$ is in $[-b, -a]$ (Fig. 1 (d))
4. sum rule $H + H_1$ is in $[a + a_1, b + b_1]$ (Fig. 1 (e))
5. inverse rule(s) (Fig. 1 (f))
   (a) $a > 0 \rightarrow H^{-1}$ is in $[\frac{1}{b}, \frac{1}{a}]$.
   (b) $a < 0 \rightarrow H^{-1}$ is outside $[\frac{1}{a}, \frac{1}{b}]$.
   (c) $a = 0 \rightarrow (H^{-1} - (\frac{1}{b}I)$ is positive.



Online Stability Validation Using Sector Analysis

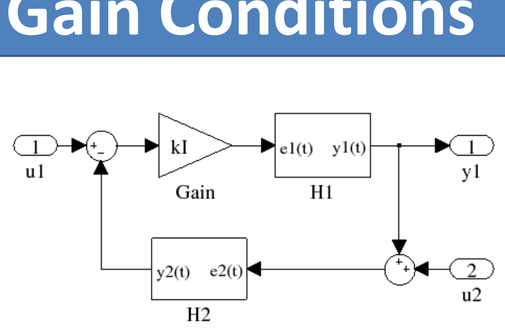
Joseph Porter, Graham Hemingway, Nicholas Kottenstette, Gabor Karsai, Janos Sztipanovits

Feedback Gain Conditions

Quadrotor helicopters – discrete-time controller is not quite passive[9]:

1. **Hold operation introduces small amounts of energy (quantified by sector bound *a*).**
2. **Fast dynamics aggravate the problem.**

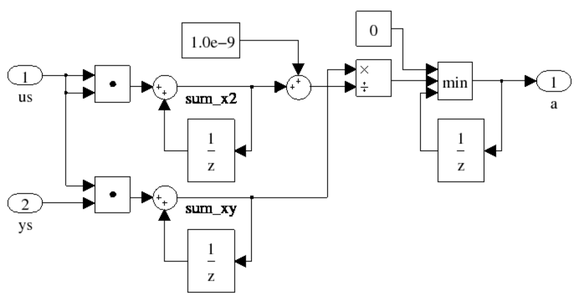
Sufficient gain condition for L2 stability:



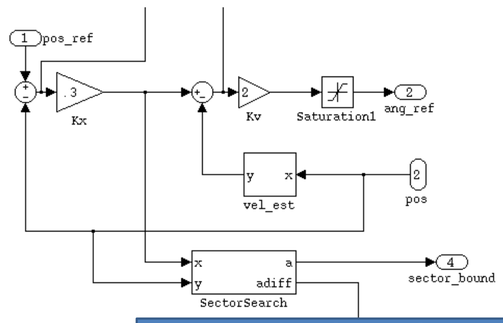
$$-\frac{1}{\max\{|a|, b\}} < k < -\frac{1}{a_1}, \text{ if } a_1 < 0$$

$$-\frac{1}{b} < k < \infty, \text{ otherwise.}$$

Online Analysis



Analyzer

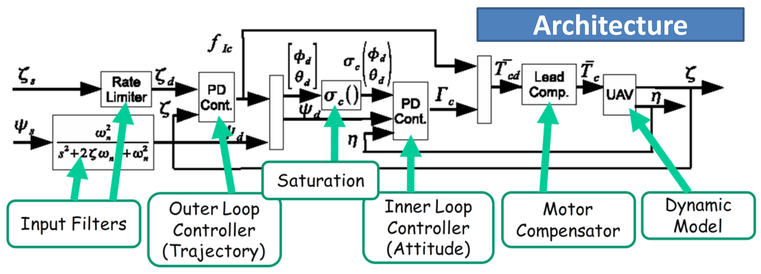


Connecting the analyzer

Limitations
1. Sector condition is only sufficient.
2. Sector analyzer approximates.
3. Analysis requires high precision and worst-case inputs.
4. Analysis must starting at the beginning of time with reasonable initial conditions.
5. Evaluation is criteria are design-specific – use carefully.

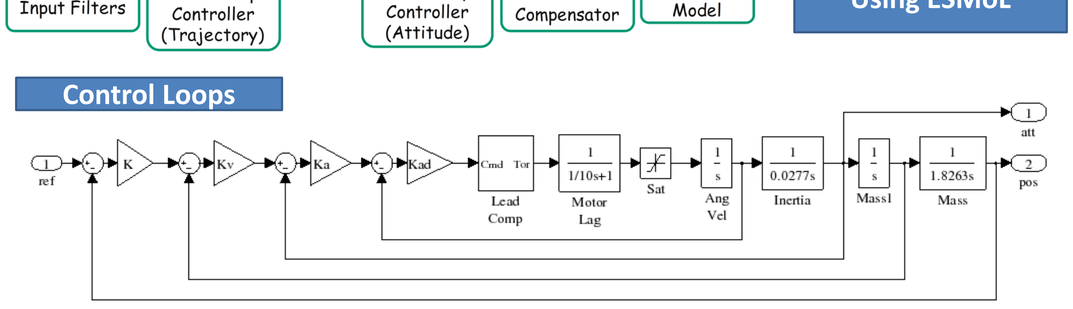
Simplified Quadrotor Example

Architecture

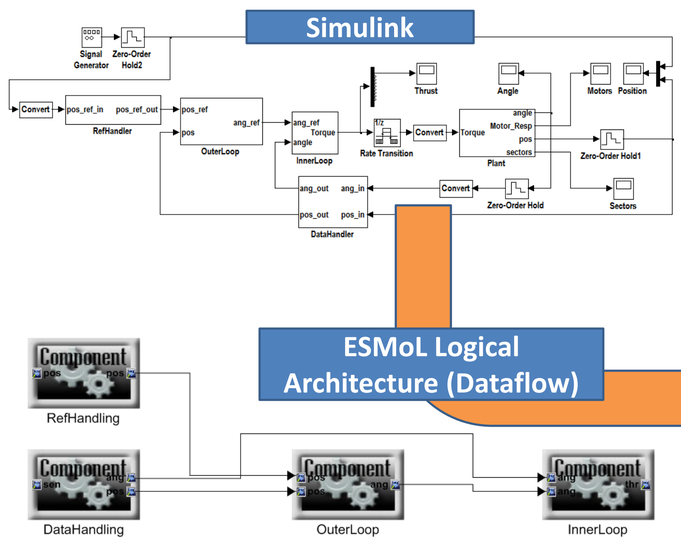


Model-Based Distributed Control Software Implementation Using ESMoL

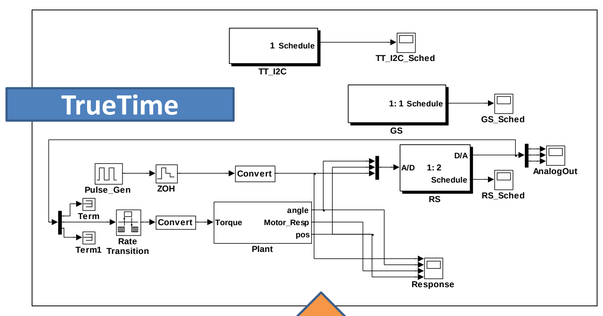
Control Loops



Simulink



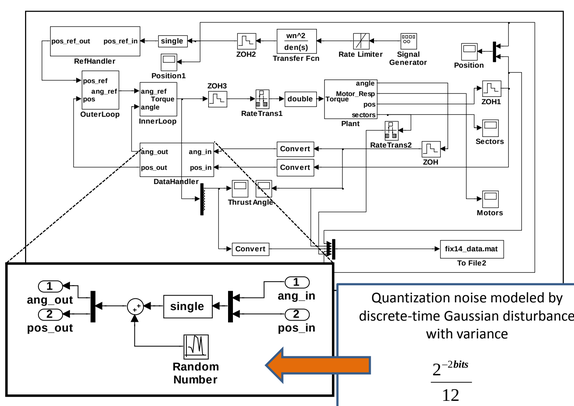
TrueTime



ESMoL Logical Architecture (Dataflow)

ESMoL Deployment Spec

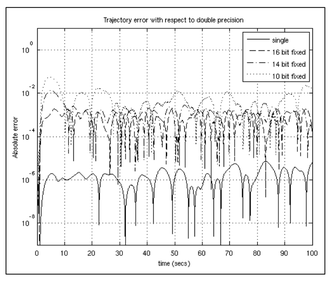
Evaluation of Analyzer



Quantization noise modeled by discrete-time Gaussian disturbance with variance

$$\frac{2^{-2bits}}{12}$$

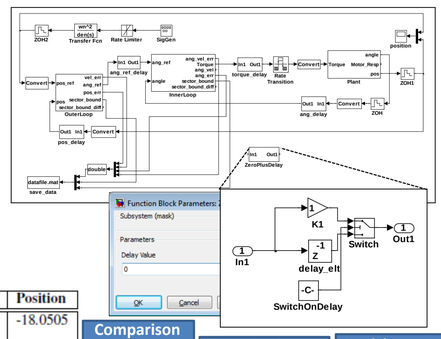
Quantization effects via sensor noise injection



Quant Level	Ang Velocity	Angle	Velocity	Position
Gain bound $a > -\frac{1}{K}$	-4.0000	-1.0951	-1.0000	-18.0505
double	-3.0116	-0.9237	-0.7252	-1.0625
single	-3.0127	-0.9237	-0.7267	-1.0625
fix16	-3.0127	-2.1673	-0.7266	-12.0967
fix14	-3.0126	-2.1834	-0.7258	-26.7733
fix10	-7.5519	-2.1845	-0.5887	-29.1194
Gain bound	-4.0000	-1.0951	-1.0000	-18.0505

Comparison of measured bound with gain bound for different quantization levels.

Delay Effects via Brute Force

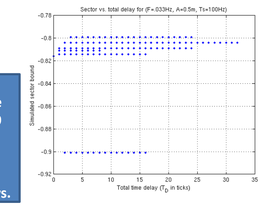


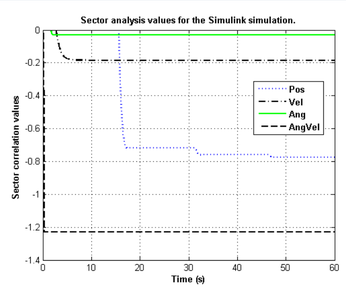
Quantization sensitivity looks good, but delay sensitivity is inconclusive.

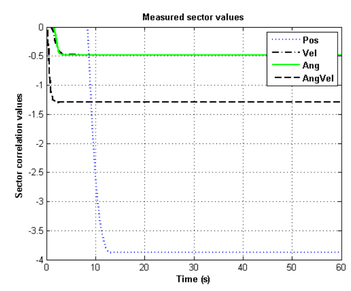
For delay sensitivity, the outliers at -0.9 roughly correspond to sensor and actuator delays.

HIL Evaluation of Platform Effects

Quantity	Simulation Values
Delay from Plant to Data Handler D_{PH}	0–8 (ticks)
Delay from Data Handler to Outer Loop D_{HO}	0–8 (ticks)
Delay from Outer Loop to Inner Loop D_{OI}	0–8 (ticks)
Delay from Inner Loop to Plant D_{IP}	0–8 (ticks)
Reference input frequency	.20, .10, .033 (Hz)
Sample rate	25, 35, 50, 75, 90, 100 (Hz)
Reference input amplitude	0.5, 1, 2, 4 (m)







Signal	Original Bound	Simulated Sector	Measured Sector	Delta	New Bound	New Sector
Angular Velocity	-1.333	-1.2292	-1.2963	-0.0671	-2.667	-1.4568
Angle	-0.5	-0.0295	-0.4831	-0.4536	-1.0	-0.0068
Velocity	-0.5	-0.1856	-0.4830	-0.2974	-1.0	-0.9324
Position	-3.333	-.7757	-3.8811	-3.1054	-6.667	-1.6081