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Integration of Mathematical Programming and Game Theory for Supply Chain Planning Optimization in Multi-objective competitive scenarios

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Abstract

This work develops a multi-objective MILP (Mixed Integer Linear Programming) model, devised to optimize the planning of supply chains using Game Theory optimization for decision making in cooperative and/or competitive scenarios. Three different optimization criteria are considered (total cost, tardiness and expenses of the buyers for the competitive problem). The multi objective problem has been solved using the Pareto frontier solutions, and both cooperative and non cooperative scenarios between supply chains are considered, so multiple optimization tools/techniques have been combined to analyze the different trade-offs associated to the resulting decision making: Game Theory, MILP based approach and Pareto frontiers. The resulting model is tested in a case study, based on the operation of two different supply chains in both competitive and cooperative situations.

Keywords: Supply Chain Management, Planning, Multi-objective Optimization, Game Theory.

1. Introduction

The problem of decision making associated to supply chain (SC) operational management (procurement of raw materials in different markets, allocation of products to different plants and distributing them to different customers), which is attracting the attention of the scientific community in the last years, becomes more complex when it is required to coordinately consider multiple criteria in the operation of the different sites of the resulting network (production sites and storage centers). The use of Multi-Objective Optimization (MOO) methods and tools becomes essential to improve the decision making in these problems, which usually exhibit tradeoffs between objectives but, as it is well known, these methods lead to many solutions, which provide greater degree of accuracy to the decision making, but they cannot offer a single final decision.

This complexity is additionally complicated by the need to consider external sources of uncertainty in the models used to predict/control the events that should be considered in the decision making problem, and one of this sources of uncertainty scarcely considered in the literature is the presence of other alternative SCs, able to compete or to cooperate. In a previous work (Zamarripa *et al.*, 2011) the use of the Game Theory (GT) was considered as a decision technique to determine the best SC operating strategy and the resulting production, inventory and distribution levels in a competitive planning scenario under uncertainty. Also, describes the competition behavior of the several coexisting SCs as an uncertainty source, and models the resulting problem to take into account the eventual decisions of the other SCs, since these decisions impact to the profit of the SC of interest (“own SC”), setting that the markets are embedded in this competitive scenario.

In order to deal with the complexity associated to the competition among markets, and also to keep looking at the different objectives simultaneously, this work proposes the development of a Multi-Objective Mixed Integer Linear Programming (MO-MILP) model to optimize the planning of SC in these competitive/cooperative environments, where decisions are the production, inventory and distribution profiles of the different echelons of the SC of interest over the time. When different objectives are to be considered, a final decision method is required – in this work, decisions are selected based on the normalized normal constraint method (Messac *et al.*, 2003). This will allow computing the payoff matrix and finally to locate the Nash equilibrium (John Nash, 1950), which represents the best solution for the several scenarios where the problem is defined.

2. Problem statement.

2.1 Supply Chain planning (cooperative and competitive scenarios)

The SC planning problem typically consists on determining the optimal production, storage and distribution variables associated to a SC network of production sites, distribution centers, costumers, etc. The main constraints of this problem result from mass balances (i.e.: linear), and others can be usually approximated through linear expressions, so the mathematical formulation for this problems typically leads to a mixed integer linear programming (MILP) model, introducing the binary variables due to the necessity of decide to produce or not, use or do not use the resources of the SC network, etc.

The model originally proposed by Zamarripa *et al.* (2011) has been adopted as a basis for the formulation presented in this paper. This formulation assumes the existence of several Supply Chains that may work in cooperative or competitive scenarios, in both cases, the mathematical constrains associated to the model will be the same, and the model will be used to minimize different Objective Functions according to the considered scenario.

In the cooperative scenario, the problem is formulated considering that the different SCs act as a single coordinated SC minimizing its overall cost, typically based on the economic terms (Eq. 1: production, inventory and distribution costs of each supply chain g) and eventually including other non-economic elements like the tardiness of products delivery to the consumers (Eq. 2).

Minimize the total cost:

$$Min\ z1 = \sum_{i \in I\_G(i,g)} \sum_n \sum_h a_{in} Q_{inh} (1+e_b)^h + \sum_{sc} \frac{1}{sc} \sum_{i \in I\_G(i,g)} \sum_n \sum_h c_{in} W_{scinh} (1+e_b)^h + \sum_{i \in I\_G(i,g)} \sum_n \sum_h d_{in} E_{scinh} (1+e_b)^h + \sum_{i \in I\_G(i,g)} \sum_n \sum_h \sum_j k_{inj} T_{scinhj} (1+e_b)^h \quad (1)$$

Minimize tardiness:

$$Min\ z2 = \sum_{i \in I\_G(i,g)} \sum_n \sum_h \sum_j [\frac{u_{inj}}{s_{inhj}}] T_{inhj} \quad (2)$$

In the competitive scenario, the GT is used to identify the different players (suppliers), who can consider two types of games: zero-sum and nonzero sum. In this work, the nonzero-sum game is proposed, since the SC of interest will not try to maintain the overall benefit of the system. This strategy is implemented through a payoff matrix, which is made up by the different potential strategies and shows the behavior for each action of the SC against the actions of its competitors.

To play this game (competition behavior), players should deal with the demand share (from the total demand) that customers really offer to each one, and this can be managed basically through their service policy: prices and delivery times. So, additionally to the already considered objectives, it is necessary to introduce a new objective representing the best deal for the customers (cost for the distribution centers). This has been done through the price rates ($Prate_g$), thus the prices associated at the source and the destiny of the products, Eq. (3) should be now considered as the objective function.

$$Min\ CST(g) = \sum_{i \in I\_G(i,g)} \sum_n \sum_h \sum_j Ps_{inj} T_{inhj}\ Prate_g + z1 \tag{3}$$

2.2. Multi Objective Optimization (MOO)

Multi objective optimization has been applied successfully in process systems engineering (PSE) problems, in order to improve the decision making in SC planning problems under uncertainty; this work develops a framework with more robustness in the decision making. The use of GT as a decision making tool has been proved in Zamarripa et al 2011, used to deal with competition behavior at the tactical level in SCM. Considering that several tradeoffs between the different objectives in SC planning problems must impact in the overall benefit of the SC network.

This work uses the Pareto solutions that are obtained by the normal constraint method, these solutions take place into the payoff matrix, and this matrix shows the different solutions for each scenario of the competitors and let us to choose the best solution for the problem (the Nash equilibrium point).

2.3. Game Theory and Equilibrium Point

The GT is based on the simulation of the results obtained by a set of players ($i = 1, ..., I$) following different strategies (S_n; $n = 1, ..., N$). These results are represented through a sort of payments ($P_{i,n}$; $i=1…I$; $n=1…N$) received by each player. In simultaneous games, the feasible strategy for one player is independent from the strategies chosen by each of the other players. Optimum strategies depend on the risk aversion of the players, so different strategies can be foreseen. Depending on the knowledge about the strategy of the other players, other solutions resulting from the concept of Nash equilibrium can be devised.

3. Case study

These concepts have been applied to a case study previously adapted by Zamarripa *et al.* (2011) from Wang and Liang, (2004, 2005) and Liang *et al.* (2008): The process involves de production and management of two products (P1 and P2), with an expected market demand for 3 months (time horizon), to be distributed from 4 distribution centers (Distr1 to Distr4). Two similar Supply Chains (SC1 and SC2) are able to deal with this demand in a collaborative or a competitive policy. The problem strategy is to maintain the work force level over the planning horizon and to supply as much product as possible. The complete data set, including initial storage, maximum and minimum production, distribution capacities, etc., can be found at http://cepima.upc.edu/papers/MOCompetitive_SCs.pdf.

4. Case study Results.

To better compare the different alternatives considered, the best standalone results for each SC are displayed in the Table 1, as well as the results obtained using the original information (Liang 2008).

Table 1: Comparative results between SC (original data and standalone cases)

	SC1 Original data	SC1 Standalone	SC2 Standalone
Obj. Funct.	min z1+z2	min z1+z2	min z1+z2
z1($)	719 990	838 652	840 904
z2(hours)	1 887	1 700	1 747
Benefit ($)	3 784 060	3 665 347	3 663 095
CST ($)	5 223 939	5 342 652	5 344 904

The optimal solution for SC1 (standalone) is driven by the geographical conditions (nearest delivery), although different solutions are obtained depending on the specific objectives considered. Differences between SC1 and SC2 standalone solutions are associated to the different distances from the production sites of SC2 to the markets. More detailed results can be found at http://cepima.upc.edu/ papers/MOCompetitive_SCs.pdf

The solutions obtained for the cooperative case (when SC1 and SC2 working together to meet the overall market demand) are shown in the Figure 1, which includes the Pareto front ("+") for the multi objective problem (tardiness vs. total cost), the Anchor Points (that represent the best optimal solutions for each objective), the Utopia Point ("*", unfeasible point resulting from the combination of the best individual values of each objective). As previously advanced, this set will be considered represented by the solution closer to the utopian point, especially for comparison purposes (Table 3)

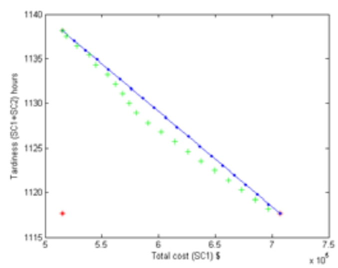

Figure 1. Pareto solutions for the cooperative case.

For the competitive case, the model takes into account the consumers preferences, just based on service and customers cost. A nominal selling price has been introduced to maintain the data integrity. The payoff matrix (Table 2, http://cepima.upc.edu/ papers/MOCompetitive_SCs.pdf) is built with the solutions obtained from the Pareto frontier for each scenario of the problem. The Nash equilibrium point of the payoff matrix represents the best solution of the non-cooperative problem (Table 3).

Also, the Table 3 compares the solutions from the Pareto frontiers obtained for the cooperative case. These results and the analysis from the previous work (Zamarripa et al 2011), shows how the multiple objective optimization, acts as a bargaining tool; since even when there is a competition and each SC modifies their behavior looking to obtain more benefit, the MOO looks for the best tradeoffs between objectives and this supposes a cooperation in the decision making.

Table 3. Nash Equilibrium of the payoff matrix

	Cooperative Solutions		Competitive Solutions	
	SC1	SC2	SC1	SC2
Discount	-	-	0.1	0.0
Obj. Funct.	Min Total cost and Tardiness		min CST and Tardiness	
z1($)	580 422	222 259	621 642	181 203
Total cost	802 681		802 845	
z2(hours)	1 124		1 124	
Benefit ($)	2 714 078	987 240	2 866 066	831 596
CST ($)	3 874 922	1 431 759	4 109 351	1 194 003

5. Conclusions

This work proposes and describes the integrated use of the Game Theory in an optimization based decision support system to determine the production, inventory and distribution profiles to face in the SC planning problem. The cooperative and non cooperative multi objective problems have been modeled and solved using mathematical programming techniques (MILP models) and GT optimization strategies as the payoff matrix and the Nash equilibrium point.

The proposed approach introduces the use of a robustness metric, emphasizing the role of competitors as sources of uncertainty in typical SC planning problems. Also, the integrated use of the different considers the use of different methodologies to improve the decision making associated to the new challenges of the present and future market scenarios problems, like inventory reduction, market/competitors reactive policies, increased competition pressure, production capacity changes/flexibility, etc.

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[1] Complementary material can be found at: http://cepima.upc.edu/papers/MOCompetitive_SCs.pdf

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