Homework 8
Math 151A: Numerical Methods
Due: Wed, March 7

1 Pen and paper

1. Give a derivation of Simpson's rule for integrating a function $f(x)$ over the interval $[p-h, p+h]$ by integrating the quadratic polynomial through the data $(p-h, f(p-h))$, $(p, f(p))$, $(p+h, f(p+h))$. A calculus fact you may find useful: $\int (t-a)(t-b)dt = \frac{1}{3}(t-a)^3 + \frac{1}{2}(a-b)(t-a)^2$.

2. Derive the composite rule for $\int_a^b f(x)dx$ based on the midpoint rule $\int_{-h}^{h} f(x)dx \approx 2hf(0)$.

3. Determine the value of the number of panels, N, so that the error bound for the composite Trapezoidal approximation of the integral of e^{-x^2} over $[0, 2]$ is less than 1.0×10^{-7}.

Submit In Class: Written solutions to these problems.

2 Programming

1. Download the scripts *IntegrateTest.m*, *trapInt.m*. Run the test script.

   (a) Implement Aitken extrapolation to estimate the rate of convergence. Modify the program so that the value of the integral and the rates of convergence are computed and printed to the screen or a file.

   (b) Test the program on the function $f(x) = \sin(x)$. Verify, by examining the rates of convergence, that the program is not working correctly. Fix the program and then verify that the corrected program is working properly. Save (and turn in) hardcopy of your results both before and after you have fixed the program.

2. Using the corrected program compute the integrals of the following functions over the interval $[0, 1]$.

   (a) e^{-x^2}

   (b) $x^{\frac{5}{2}}$

   (c) $\sqrt{x}$

   Use the number of panels $N = 2, 4, 8, 16, 32....256$. For each of these functions compute and save (to a file) the values of the integral and the rates of convergence estimated using Aitken extrapolation. Turn in copies of these values and explain the peculiar behavior of the computed approximation to the integral of $\sqrt{x}$.

Upload to CCLE: In a plain text file, your "before" and "after" estimated rates of convergence and errors for computational problem 1(b). In a plain text file, the values of the integrals and rates of convergence for 2(a)-(c) and your brief explanation of the results for $\sqrt{x}$.