Homework 9
Math 151A: Numerical Methods
Due: Wed, March 14

1 Pen and paper

1. The following table lists the results obtained with the composite Trapezoidal method to compute and approximation $\int_0^1 e^{-x^2} dx$ with node spacing corresponding to $n = 4$, 8 and 16 panels.

Node spacing	Approximate Integral Value
$h = .25$	0.742984097800
$h = .125$	0.745865614846
$h = .0625$	0.746584596788

   (a) Give the approximate value for the integral that results after 2 steps of Richardson extrapolation.

   (b) What is the order of accuracy of values computed using 2 steps of Richardson extrapolation?

2. Assume that you have an integration formula for functions defined over the interval $[-1, 1]$ with nodes x_i and weights w_i of the form

$$\int_{-1}^{1} f(x)dx \approx \sum_{i=1}^{n} f(x_i)w_i \tag{1}$$

What is the corresponding formula for functions defined over an interval $[a, b]$? Specifically, give expressions for the nodes x_i' and weights w_i' of the integration formula in terms of the nodes and weights of the integration formula (1)

$$\int_{a}^{b} f(x)dx \approx \sum_{i=1}^{n} f(x_i')w_i'. \tag{2}$$

3. Give the value of an approximation to $\int_0^1 e^{-x^2} dx$ that is obtained using 5-point Gauss-quadrature. The nodes and weights corresponding to a 5 point Gauss-Quadrature approximation to $\int_{-1}^{1} f(x)dx$ are

i	x_i	w_i
1	-0.906179845938664	0.236926885056182
2	-0.538469310105683	0.478628670499366
3	0.000000000000000	0.568888888888889
4	0.538469310105683	0.478628670499366
5	0.906179845938664	0.236926885056182

4. For the following two systems, use Gaussian elimination and turn in (i) the upper triangular part of the augmented matrix used for the back-substitution step and (ii) the solution $\mathbf{x}$.

(a)
$$\begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

(b)
$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 1 \\ 1 & 2 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

Submit In Class: Written solutions to these problems.