Homework 4
Math 151A: Numerical Methods
Due: Wed, February 7

1 Pen and paper

1. Use the analysis technique discussed in class to determine the expected order of convergence and asymptotic error constant for the following fixed point iterations. (These fixed point iterations arise from alternate fixed point iterations that could be used to find the root of $x^3 + 4x^2 - 10 = 0$ in the interval in $[1, 2]$).
   (a) $x_{k+1} = \frac{1}{2}(10 - x_k^3)^{\frac{1}{2}}$
   (b) $x_{k+1} = (\frac{10}{4+x_k})^{\frac{1}{2}}$
2. Burden and Faires Problem 14 in Section 2.5.
3. Burden and Faires Problem 5 in Section 2.6.

2 Programming

1. Given the sequence $\{p_n\}_{n=0}^{\infty}$, use Aitken's acceleration to compute the sequence $\{\hat{p}_n\}_{n=0}^{\infty}$. Plot the the two sequences from $n = 0 : 100$. Also plot the ratio $\frac{|\hat{p}_{n+1} - \hat{p}_n|}{|p_{n+1} - p_n|}$ for $n = 0 : 99$.
   (a) Use the sequence $\{p_n\}_{n=0}^{\infty}$ defined as $p_0 = .75$ and $p_n = (\frac{e^{p_n - 1}}{3})^{\frac{1}{2}}$.
   (b) Use the sequence $\{p_n\}_{n=0}^{\infty}$ defined as $p_n = \frac{(-1)^n}{n+1}$.
2. Use Steffesen's Method to accelerate the convergence of Newton to compute a root of $x^3 - x - 1 = 0$ in $[1, 2]$. Compare this to Newton's method for the same initial guess.