Homework 3
Math 151A: Numerical Methods
Due: Wed, January 31

1 Pen and paper

1. Devise a Newton iteration for computing the cube root of any positive real number. In particular, determine the specific $g(x)$ so that the iteration $x_{k+1} = g(x_k)$ converges to the cube root of any positive real number (assuming a sufficiently close initial iterate).

2. (a) Give an example of a function for which the size of the residual at an approximate root, x_k, will over estimate the error in that root.

   (b) Give an example of a function for which the size of the residual at an approximate root, x_k, will under estimate the error in that root.

   (c) What general property should the function posses near the root so that the size of the residual is a good indicator of the error at an approximate root?

3. One modification of Newton's method that avoids the computation of the derivative is to use $f'(x_0)$ in the Newton iteration, rather than $f'(x_k)$ (where x_0 is the initial iterate). Show that the resulting method has an order of convergence 1.

What You Should Turn In: Submit your written solutions along with the work deriving them.

2 Programming

1. This problem requires you to work with the sequence of approximate roots generated by a root-finding method. You may use the script *newtonRoot.m* as a starting point for your work. You will be using the technique described in class to empirically determine the order of convergence for two different root finding methods. The two methods are Newton's method and a modified Newton's method where you use the Newton iteration, but with $f?(x_0)$ instead of $f?(x_k)$ for the derivative at the k^{th} step. (x_0 is the initial iterate).

   (a) For each method compute the positive root of $f(x) = x^2 - 3$. Use a starting guess of 5.0. Print out, and save in a text file, the error in the approximate root for each iteration. Stop your computation when the residual is less than $1e10^{-12}$.

   (b) Using the values of the error for each iterate, estimate the order of convergence for each of the methods, and save the results to a plain text file. Do your results agree with the theoretical predictions?

Tip: Use the computer to do the computation to estimate the order of convergence. For example, you could capture the root approximations in a vector and then use a loop to carry out the requisite computations.

What You Should Turn In: A listing of the codes you used, the errors for the approximations generated Programing Problem 1(a), and your convergence order estimates for each method Programing Problem 1(b). Don't forget to comment upon the agreement (or non-agreement) of your computational results with the theoretical predictions.