Homework 2
Math 151A: Numerical Methods
Due: Wed, January 24

1 Pen and paper

1. Burden and Faires, Section 1.3, Problems 10,12.
2. Burden and Faires, Section 2.1, Problems 12,14.
3. Burden and Faires, Section 2.2, Problems 2,4.

What You Should Turn In: Submit your written solutions along with the work deriving them.

2 Programming

A common activity of a numerical analyst is to be responsible for learning about new computational techniques and implementing them for comparison purposes. Typically, this involves understanding the concept behind the new technique and then taking an existing program and modifying it to incorporate the new technique. The purpose of this assignment is to have you follow this process and implement an"improved" variant of the bisection method called Reguli-Falsi ("False Position"). You will be modifying a script that implements the bisection method given in bisect.m.

Reguli-Falsi Method for Root Finding: Consider the problem of finding a root x^* for a continuous function $f(x)$. The Reguli-Falsi method is a variant of the bisection method in which the approximate root at the k^{th} step, x_k, is taken to be the zero of the line through $(a_k, f(a_k))$ and $(b_k, f(b_k))$ rather than the midpoint of the interval $[a_k, b_k]$. Then, just as in the bisection method, one determines which interval $[a_k, x_k]$ or $[x_k, b_k]$ that contains the root and uses this new interval as the starting point for the next iteration. A bound for the error at the k^{th} step is $\frac{(b_k - a_k)}{2}$.

1. Create a copy of the m-file bisect.m and rename it falsep.m. Implement Reguli-Falsi by appropriately modifying the file falsep.m.
2. Compare the Reguli-Falsi method to the bisection method by using both to find the root of $f(x) = (x-1)(x-2)(x-3)$ with a starting interval of $[1.75, 2.95]$. Stop the iteration when the error bound for the root is 10^{-6}. How many iterations does each take? What is the value of the residual when the iteration terminates?
3. Repeat a comparison on the function $f(x) = x^6 - x - 1$ using an initial interval of both $[1.0, 1.2]$ and $[1.0, 2.0]$. Stop the iteration when the error bound for the root is 10^{-6}. How many iterations does each take? What is the value of the residual when the iteration terminates? Is Reguli-Falsi always better than the bisection method?

What You Should Turn In: A copy of your m-file falsep.m that implements the Reguli-Falsi method. In a plain text file, your answers to the questions concerning the comparisons done in computational problems; i.e. record and turn in the number of iterates, the size of the residual for each comparison, and your conclusion about the relative merits of the Reguli-Falsi method.