Computing PCA

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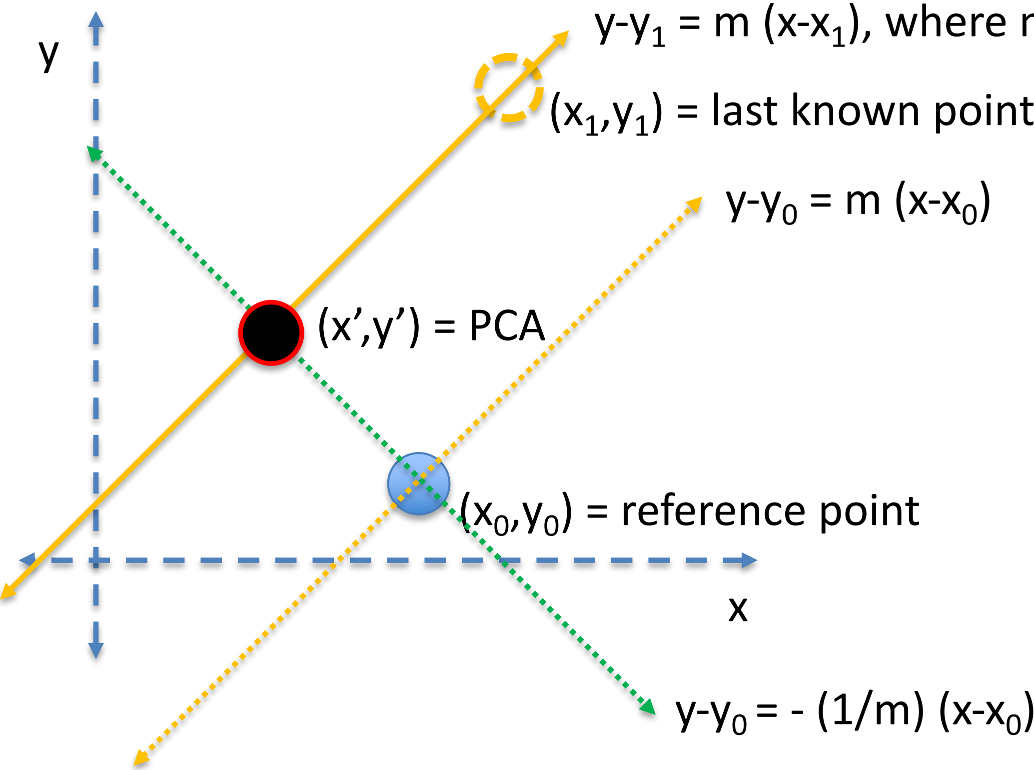
What PCA?

- Need to define reference point or region
  - Single point: origin, primary vertex, etc.
  - Region: beamspot in x-y, r-z, etc. (assume centered around origin?)
- Approximate track "helix" as a straight line or a circle in some plane
- Determine min (i.e. perpendicular) distance between track approx. and reference object
- Compute point from distance (i.e. PCA)
- Use computed PCA as input to our propagation methods (helixAtR or helixAtZ) to obtain propagated track parameters and uncertainties

PCA from line to point



- Know that the shortest distance between two parallel lines is a perpendicular line that intersects both
- Use reference point on the perpendicular line to define equation of a line
- Compute intersection point from original line and perpendicular line

$$x' = \frac{x_0 + m^2 x_1 + m(y_0 - y_1)}{1 + m^2}$$

$$y' = \frac{y_1 + m^2 y_0 + m(x_0 - x_1)}{1 + m^2}$$

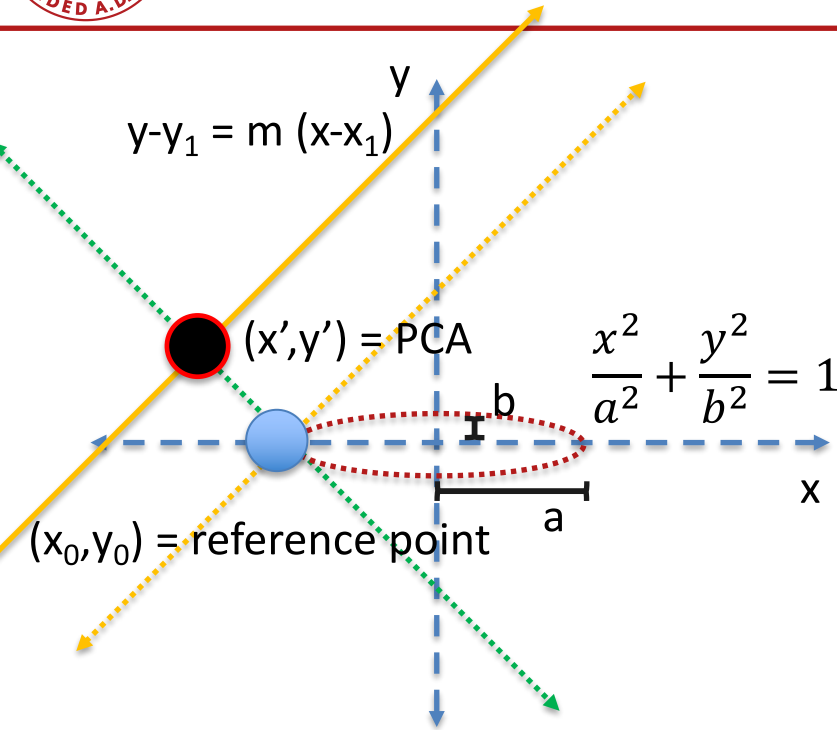
$$D_0 = \sqrt{(x_0 - x')^2 + (y_0 - y')^2}$$

$$D_0 = \frac{|m(x_1 - x_0) + y_0 - y_1|}{\sqrt{1 + m^2}}$$

PCA from line to point: Possible Interpretations

- Tracks are straight in r-z: project line back using last known (r,z) position and tanθ as slope, reference point is origin or PV
  - Solve for either r or z, and then use a single propagation method: helixAtR or helixAtZ
- Approximate helix as a straight line in x-y, solve for closest R

PCA from line to ellipse



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

1. Know that shortest (and longest) distance to ellipse from a line is when line tangent to ellipse is parallel to approach
2. Implicit differentiation of ellipse yields slope, set equal to slope of line
3. Plug result into equation of ellipse to obtain two points on ellipse
4. Compute distance from each point to line, and take the shorter one
5. Plug in this point as the "reference point" from first example

$$\frac{\partial y}{\partial x} = -\frac{x}{y}\frac{b^2}{a^2} = m$$

$$(x_0, y_0) = \frac{1}{\sqrt{b^2 + a^2 m}}(\pm a^2 m, \mp b^2)$$

$$D_0 = \frac{\left| \mp \frac{a^2 m}{\sqrt{b^2 + a^2 m}} \pm \frac{b^2}{\sqrt{b^2 + a^2 m}} + m x_1 - y_1 \right|}{\sqrt{1 + m^2}}$$

$$x' = \frac{x_0 + m^2 x_1 + m(y_0 - y_1)}{1 + m^2}$$

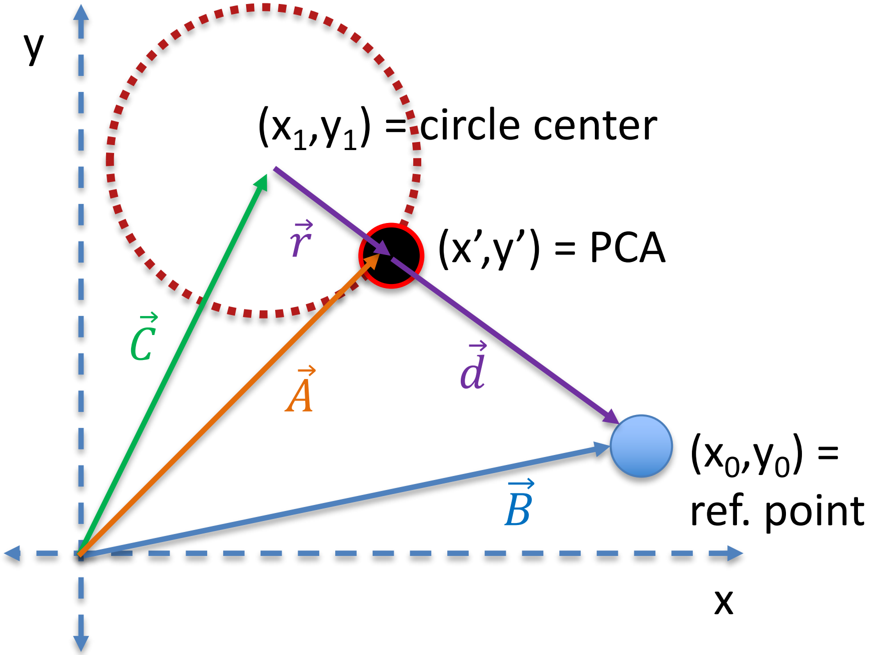
$$y' = \frac{y_1 + m^2 y_0 + m(x_0 - x_1)}{1 + m^2}$$

PCA from line to ellipse: Possible Interpretations

- Tracks are straight in r-z: project line back using last known (r,z) position and tanθ as slope, ellipse is beamspot
  - Solve for either r or z, and then use a single propagation method: helixAtR or helixAtZ

- Approximate helix as a straight line in x-y, use ellipse for beamspot in x-y, solve for closest R
  - However, beamspot in x-y is a circle… do we really gain anything here?

PCA from circle to point



- Know that shortest distance to point is along a radial vector
- Use vector algebra to construct system of vectors to easily compute PCA

$$\vec{C} + \vec{r} = \vec{A}$$

$$\hat{r} = \frac{(\vec{B} - \vec{C})}{\|\vec{B} - \vec{C}\|}$$

$$\vec{C} + r\frac{(\vec{B} - \vec{C})}{\|\vec{B} - \vec{C}\|} = \vec{A}$$

$$x' = \frac{x_0 - x_1}{\sqrt{(x_0 - x_1)^2 + (y_0 - y_1)^2}} r + x_1$$

$$y' = \frac{y_0 - y_1}{\sqrt{(x_0 - x_1)^2 + (y_0 - y_1)^2}} r + y_1$$

PCA from circle to point: Possible Interpretations

- Project helix into x-y plane as a circle
  - Radius inversely proportional to R
  - Use phi to determine center of circle

- Compute PCA to origin, PV

Other ideas

- Could generalize PCA from line to ellipse to a shifted, tilted ellipse… a bit more algebra

- Could also generalize PCA from circle to point to PCA from circle to ellipse (although this is already known to be quite difficult: https://en.wikipedia.org/wiki/Distance_of_closest_approach_of_ellipses_and_ellipsoids)

- Generic: helix to ellipsoid for full 3D PCA… sounds like a lot of algebra and nasty