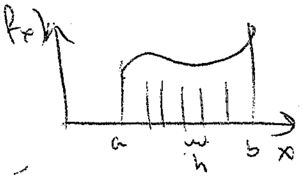
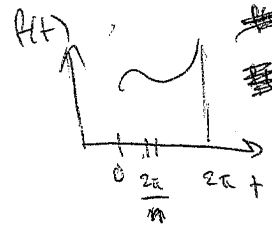
Olver 2012 Notes





$x_0 = t_0 = 0$

$x_{n-1} = \frac{2(n-1)\pi}{n} = t_{n-1}$

$\vec{f} = (f_0 \dots f_{n-1})^T = (f(x_0) \dots f(x_{n-1}))^T$

↳ sample vector

"Sampling at n equally spaced sample points cannot detect periodic signals of frequency n" ?

→ corollary: $e^{i(k+n)x}$ + e^{ikx} are indistinguishable?

"We need use only the first n periodic complex exponentials to represent any 2π periodic sampled signal:

$f_0(x) = 1, \quad f_1(x) = e^{ix}, \quad f_{n-1}(x) = e^{(n-1)ix}$ "

C: When sampled with n points over 2π domain, the signals e^{-ikx} and $e^{i(n-k)x}$ are indistinguishable. → aliasing

"Discrete fourier transformation decomposes sampled $f(x)$ into linear combination of n exponentials.

→ $f(x) \sim p(x) = c_0 + c_1 e^{ix} + c_2 e^{2ix} + \dots + c_{n-1} e^{(n-1)ix} = \sum_{k=0}^{n-1} c_k e^{ikx}$ (i)

$f(x_j) = p(x_j)$ $\quad j = 0, \dots, n-1$, C: $x_j = \frac{2\pi}{n} j$

C: Now, sample each exponential e^{ikx} with the domain values $\{x_0 \dots x_{n-1}\}$:

$\vec{w}_k = \{e^{ikx_0}, e^{ikx_1} \dots e^{ikx_{n-1}}\}^T$, or using $x_j = \frac{2\pi}{n} j$ for $j = 0, n-1$,

$= (1, e^{\frac{2\pi}{n} ik} \dots e^{\frac{2\pi(n-1)}{n} ik})^T$

So, in terms of the samples $\vec{x} = (x_0 \dots x_{n-1})^T$, we can write:

$\vec{f} = c_0 \vec{\omega}_0 + c_1 \vec{\omega}_1 + \dots c_{n-1} \vec{\omega}_{n-1}$

Note for $n \le i \le n-1$, $\omega_0 \dots \omega_{n-1}$ form orthonormal basis of $\mathbb{C}^n$.

Using orthonormality, we can take dot products to compute coefficients $c_0 \dots c_{n-1}$:

$c_k = \langle \vec{f}, \vec{\omega}_k \rangle$

Problem with (i): Higher order frequencies in second half. Assuming $n = 2m$, use identity for $c_k^1 e^{-ikx}$, $c_k^2 e^{i(n-k)x}$, $c_k^1 = c_k^2$. ⇒ e^{-imx} ... e.g. for $m=4$ $c_0\ c_1\ c_2\ c_3 \Rightarrow c_{-2}\ c_{-1}\ c_0\ c_1$

$c_{-2} = c_2 \quad c_{-1} = c_3$.

So: $\hat{p}(x) = \sum_{k=-m}^{m-1} c_k e^{ikx} = \sum_{k=-n/2}^{n/2-1} c_k e^{ikx}$ → low frequency interpolant