1.

The sum of an even number and odd number is odd.

$a = 2k$

$b = 2j + 1$

$Where\ k\ and\ j\ are\ \in Z$

$a + b = 2k + 2j + 1$

$a + b = 2(k + j) + 1$

$let\ m = k + j$

$m \in Z\ due\ to\ the\ closure\ property\ of\ addition$

$a + b = 2m + 1$

$h.n.$

The product of two odd numbers is odd.

$a = 2k + 1$

$b = 2j + 1$

$Where\ k\ and\ j\ are\ \in Z$

$a \times b = (2k + 1) \times (2j + 1)$

$a \times b = 4kj + 2k + 2j + 1$

$a \times b = 2(2kj + k + j) + 1$

$let\ m = 2kj + k + j$

$m \in Z\ due\ to\ the\ closure\ property\ of\ multiplication\ and\ addition$

$a \times b = 2m + 1$

$h.n.$

The square of an even number is even.

$$a = 2k$$

$$Where\ k\ is \in Z$$

$$a^2 = 2k \times 2k$$

$$a^2 = 2(k \times 2k)$$

$$Let\ m = k \times 2k$$

$$m \in Z\ due\ to\ the\ closure\ property\ of\ multiplication$$

$$a^2 = 2m$$

$$h.n.$$

2.

Show that $p^2 - q^2 = 1$ does not have positive integer solutions.

$Assume\ the\ opposite\ is\ true$

$p^2 - q^2 = 1\ have\ positive\ integer\ solutions.$

$(p - q)(p + q) = 1$

$(p - q) = 1\ and\ (p + q) = 1$

$(p - q) + (p + q) = 2$

$2p + 0q = 2$

$p = 1, q = 0$

or

$(p - q) = -1\ and\ (p + q) = -1$

$(p - q) + (p + q) = -2$

$2p + 0q = -2$

$p = -1, q = 0$

3.

Prove by Mathematical Induction that for any positive integer number n, $n^3 + 2n$ is divisible by 3.

Step 1. Show that $n^3 + 2n$ is divisible by 3 when $n = 1$

$1^3 + 2(1) = 3$, 3 is divisible by3.

Step 2. Assume $n^3 + 2n$ = 3m is true for $n = k \; and \; for \; some \; integer \; m$

Step 3. Prove $n^3 + 2n = 3m$ is true for $n = k + 1 \; and \; for \; some \; integer \; m$

$$
\begin{aligned}
n^3 + 2n &= (k+1)^3 + 2(k+1) \\
&= (k^2 + k + k + k + 1)(k+1) + 2k + 2 \\
&= k^3 + 3k^2 + 3k + 1 + 2k + 2 \\
&= k^3 + 2k + 3k^2 + 3k + 3 \\
&= k^3 + 2k + 3k^2 + 3k + 3 \\
&= 3m + 3k^2 + 3k + 3 \\
&= 3(m + k^2 + k + 1)
\end{aligned}
$$

$n^3 + 2n = 3(m + k^2 + k + 1)$ for $n = k + 1 \; and \; for \; some \; integer \; m$

$Let \; c = m + k^2 + k + 1$

$c \in Z \; due \; to \; the \; closure \; property \; of \; addition$

$n^3 + 2n = 3c$

h. n.

4.

Step 1. Show that $(n(n+1)(2n+1))/6$ is the sum of the squares from 1 to n when n = 1.

$$\sum_{i=1}^{n} i^2 = (1(1+1)(2(1)+1))/6$$

$$= (2 \times 3)/6$$

$$= 1$$

Step 2. Assume $n = k$ is true for an integer k.

$$1^2 + 2^2 + 3^2 + \cdots + k^2 = \frac{k(k+1)(2k+1)}{6}$$

Step 3. Prove it's also true when $n = k + 1$

$$1^2 + 2^2 + 3^2 + \cdots + k^2 + (k+1)^2 = \frac{(k+1)(k+2)(2(k+1)+1)}{6}$$

$$\frac{k(k+1)(2k+1)}{6} + (k+1)^2 = \frac{(k+1)(k+2)(2(k+1)+1)}{6}$$

$$\frac{(k^2+k)(2k+1)+6(k^2+2k+1)}{6} = \frac{(k^2+3k+2)(2k+3)}{6}$$

$$\frac{(2k^3+3k^2+k)+(6k^2+12k+6)}{6} = \frac{2k^3+3k^2+6k^2+9k+4k+6}{6}$$

$$\frac{2k^3+9k^2+13k+6}{6} = \frac{2k^3+9k^2+13k+6}{6}$$

$h.n.$

5.

False

True

True

True

True

True

False

6.

{3, 5}

{7, 9}

{{8}, {7, 8}, {8, 9}, {7, 8, 9}}

{{3}, {3, 5}, {7}}