Problem Set 1: Proof Techniques and Sets

1. Prove the following via direct proof:
   - sum of an even number and odd number is odd
   - product of two odd numbers is odd
   - the square of an even number is even

2. Prove by contradiction:
   - Show that $p^2 - q^2 = 1$ does not have positive integer solutions

3. Prove by Mathematical Induction that for any positive integer number n, $n^3 + 2n$ is divisible by 3.
   - Base Step
   - Inductive Hypothesis
   - Inductive Step

4. Show that the sum of the squares from 1 to n is (n(n+1)(2n+1))/6. Prove the claim via Mathematical Induction.

5. Determine if each of the following is true or false
   - $\varnothing \subseteq \varnothing$
   - $\varnothing \in \{\varnothing\}$
   - $\varnothing \subseteq \{\varnothing\}$
   - $\{a,b\} \in \{a,b,c, \{a,b\}\}$
   - $\{a,b\} \subseteq \{a,b, \{a,b\}\}$
   - $\{a,b\} \subset 2^{\{a,b,\ \{a,b\}\}}$
   - $\{a,b\} \in 2^{\{a,b,\ \{a,b\}\}}$
   - $\{a,b, \{a,b\}\} - \{a,b\} = \{a,b\}$

6. What are these sets?
   - $(\{1,3,5\} \cup \{3,1\}) \cap \{3,5,7\}$
   - $(\{1,2,5\} - \{5,7,9\}) \cup (\{5,7,9\} - \{1,2,5\})$
   - $2^{\{7,8,9\}} - 2^{\{7,9\}}$
   - $\cup\{\{3\},\{3,5\}, \cap\{\{5,7\},\{7,9\}\}\}$